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THE STORAGE OF BITUMINOUS COAL

BY

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CONTENTS

	PAGE
I. INTRODUCTION	7
1. Preliminary Statement	7
2. Acknowledgments	9
3. Summary of Conclusions	9
II. REASONS FOR STORING COAL	14
4. Present Mining Conditions	14
5. Benefits of Storage	15
III. PLACES OF STORAGE FOR DIFFERENT PURPOSES	18
6. Storage near the Point of Consumption	18
7. Storage for Domestic Use and for Small Power Plants	18
8. Storage by Railroads	22
9. Storage at the Mine	23
IV. COAL STORAGE PRACTICE	26
10. Kinds and Sizes of Coal which may be Safely Stored	26
11. When Coal should be Stored	28
12. Storage Piles	29
13. Ventilation of Coal Piles	31
14. Testing for Fires	34
15. Handling Fires	35
16. City Ordinances Affecting Storage	36
V. STORAGE SYSTEMS	39
17. Choice of a Storage System	39
18. Hand Operated Storage Systems	40
19. Storage by Motor Truck	40
20. Pile Storage from Cars without a Trestle	47
21. Trestle Storage	48
22. Storage with Side Dump Cars	51

CONTENTS (CONTINUED)

	PAGE
23. Side Hill Storage	56
24. Self-filling Buckets	56
25. Use of Mast and Gaff Arrangement in Storage	56
26. Locomotive Crane Storage	59
27. Parallel Track Storage	63
28. The Trestle and Crane System	63
29. Circular Storage	67
30. Steeple Towers	75
31. The Hunt System	75
32. Bridge Storage	76
33. Deep Reinforced Concrete Storage Bins	86
34. Under-water Storage	86
 VI. EFFECTS OF STORAGE UPON THE PROPERTIES OF COAL	 107
35. Appearance	107
36. Loss of Heating Value	107
37. Firing Qualities	112
38. Spontaneous Combustion	114
39. Coking and Gas Making Properties	118
40. Degradation or Breakage	120
41. Loss in Weight	127
 VII. EXPENSE OF STORING COAL	 127
 APPENDIX I. QUESTIONNAIRE ON COAL STORAGE DATA	 130
 APPENDIX II. SUMMARY OF CONCLUSIONS AND SUGGESTIONS REGARDING COAL STORAGE	 132
 APPENDIX III. EXPERIENCES OF FIRMS AND INDIVIDUALS STOR- ING COAL	 159
42. Storage for Domestic Use and by Coal Dealers	159
43. Light, Heat and Power Companies	165
44. Storage at Metallurgical Plants	171
45. Storage for Coke Ovens and Blast Furnace Plants	172
46. Mine Storage	175
47. Storage by Railroads	178

LIST OF FIGURES

NO.	PAGE
1. Method of Ventilating Coal Piles Employed by the Canadian Pacific Railway	32
2. Coal Storage Plat and Storage Pile at the University of Illinois . .	43
3. Placing the First Layer of Coal on Storage Plat at the University of Illinois	43
4. Placing the Second Layer of Coal on Storage Pile at the University of Illinois	44
5. Electrically Operated Wagon and Truck Loader Used at the University of Illinois	44
6. Railroad Storage Pile Showing Tracks	45
7. Railroad Storage Pile Showing Crib for Supporting Track	45
8. Railroad Storage Pile Showing Men at Work Raising Track	46
9. Trestle for Coal Storage as Suggested by Fuel Station Committee of the International Railway Fuel Association	49
10. Trestle and Tunnel Systems, Showing an Above-ground and a Below-ground Tunnel	51
11. Bin and Tunnel Type of Storage System with Bulkheads	52
12. Parallel Trestle System of Storing Coal	52
13. Storage Track and Side Dump Cars at the Orient Mine of the Chicago, Wilmington and Franklin Coal Company	53
14. Loading Dump Cars at the Tipple, Orient Mine of the Chicago, Wilmington and Franklin Coal Company	53
15. Side Hill System of Storage	54
16. Clam-shell Bucket and Orange-peel Bucket	55
17. Automatic Grab on Mast and Gaff with Shuttle Cable Railway	57
18. Traveling Cableway	59
19. Locomotive Crane with Clam-shell Bucket Unloading Coal	57
20. Arrangement of Tracks and Storage Piles Employed by the Commonwealth Edison Company in Open Storage	62
21. Parallel Track Storage System	63
22. Railway Trestle and Crane System of Storage	63
23. Locomotive Crane Placed on an Elevated Traveling Platform	65
24. Dodge Type of Circular Storage System	68
25. Twenty-five-Thousand-Ton Storage Plant	69
26. Arrangement of the Storage Plant of the Old Ben Coal Corporation at West Frankfort, Illinois	71

LIST OF FIGURES (CONTINUED)

NO.		PAGE
27.	Semicircular Storage Adapted to Locations near Power House or Coaling Stations	72
28.	Locomotive Crane Reclaiming Coal and Dumping into Bins above Railroad Tracks	73
29.	L. and N. R. R. Type of Mechanical Coaling Station	75
30.	Towers Employed in Handling Coal, Western Coal and Dock Company, Waukegan, Illinois	77
31.	Traveling Tower or Direct Unloader	77
32.	Hunt System of Handling Coal	78
33.	Bridge Gantry Crane with Electrically Operated Man Trolley	78
34.	Coal Handling Plant of the Reiss Coal Company at Superior, Wisconsin.	79
35.	Man Trolley Coal Storage Bridges of the Indiana Steel Company . . .	79
36.	Belt Conveyor Type of Bridge of the Inland Steel Company	80
37.	Movable Bridge with Side Dump Cars	80
38.	Traveling Bridges of the Clarkson Coal and Dock Company, Duluth, Minn.	83
39.	Parallel Track Storage Yard for a Locomotive Coaling Station	87
40.	Layout of Storage Yard for Locomotive Coal Station	87
41.	Movable Screening Plant of Berwind Coal Company	83
42.	Swivel Bridge Designed for Semicircular Pile	89
43.	Pivoted Bridge of the Milwaukee Coke and Gas Company	91
44.	View looking toward Pivoted End Bridge of the Milwaukee Coke and Gas Company	92
45.	Bulkhead at the Storage Plant of the Milwaukee Coke and Gas Company	93
46.	View showing Construction of the Bulkhead at the Plant of the Milwaukee Coke and Gas Company	93
47.	Hulett Unloaders of the Canadian Pacific Railway at Fort William, Ontario	94
48.	Reinforced Concrete Storage Bins of F. W. Stock and Sons, Hillsdale, Michigan, designed and built by Macdonald Engineering Company, Chicago	95
49.	View of the Under-water Storage Pit of the Western Electric Company at Hawthorne, Illinois	96
50.	View of the Under-water Storage Pit of the Western Electric Company at Hawthorne, Illinois	96
51.	Coal Storage Pit of the Illinois Traction Company at Mackinaw, Illinois	102
52.	Storage Pit of the New Kentucky Coal Company, Kankakee, Illinois . .	97
53.	Concrete Storage Pit of the Indianapolis Light and Heat Company . .	97
54.	Storage Pit of the Metropolitan Water District of Omaha	98

LIST OF FIGURES (CONTINUED)

NO.		PAGE
55.	Storage Pit and Bridge of the Western Clock Works	99
56.	Apparatus Used to Determine the Extent of Breakage in Coal Caused by Dropping in Tests at the University of Illinois	125
57.	Chart showing Extent of Breakage in a 50-pound Sample of Screened Lump Coal (over 2 inches) Dropped from a Height of Six Feet in Per Cent of Original Sample	123
58.	Typical Storage Yard of the Consumers' Company, Chicago	164
59.	Twenty-five-Thousand-Ton Pile of Run of Mine Coal stored by the Union Light and Power Company, St. Louis, Missouri	169
60.	Suggested System of Storage for a Mine in a Hilly Country	177
61.	Suggested System of Storage for Screenings at the Tipple	177
62.	Coal Pile of the C. C. C. and St. L. R. R. near Hillary, Illinois	185
63.	Another View of the Burning Coal Pile of the C. C. C. and St. L. R. R. near Hillary, Illinois, in the Summer of 1917	186

LIST OF TABLES

NO.		PAGE
1.	Quantity of Coal and Coke Used in Chicago during 1912	20
2.	Days of Active Operation of Mines for Seventeen Years	25
3.	Horizontal Pressure Exerted by Bituminous Coal against Vertical Retain- ing Walls per Foot of Length	50
4.	Capacity of Circular Crane Storage (Link-Belt Company)	70
5.	Decrease in Heating Value (B. t. u.) of Illinois Coals	108
6.	Increase in Fine Material after One and One-Half and Six Years of Storage	121
7.	Temperature of Coal Stored at Hillary, Illinois, by the C. C. C. & St. L. R. R.	187

THE STORAGE OF BITUMINOUS COAL

I. INTRODUCTION

1. *Preliminary Statement.*—The coal shortage during the winter of 1917-18, and the resulting inconvenience, suffering, and industrial loss have emphasized both the importance and the complexity of the problem of guarding against a recurrence of the situation. The problem is one which concerns not only the Federal Fuel Administration and the State Councils of Defense, but the individual industry and even the individual household. It is generally recognized that perhaps the most important factor contributing to the shortage of fuel has been the inability of the transportation system to handle the heavy winter fuel tonnage in addition to the abnormally large tonnage from other sources. It is evident, therefore, that if large quantities of coal can be stored at or near the point of use during those seasons when consumption is lightest and when the railroads are best able to handle the traffic, an important step will have been taken in the solution of the problem.

It is the purpose of this circular to present a review of modern practice governing the storage of coal and a statement of the facts which have developed in the experience of those who have successfully or otherwise undertaken to store coal. The discussion is confined largely to bituminous coal which has given so much trouble owing to its tendency toward spontaneous combustion while stored, and to storage systems and mechanical devices.

The storage of coal should be regarded not merely as a war measure, but as part of the solution of the general coal problem. Storage of coal will do much to help in stabilizing an industry of fundamental importance by permitting mining operations to proceed throughout the year at a fairly uniform rate, and it will serve in a large measure as insurance against the losses accompanying a shortage to the fuel consuming industries.

At the present time large quantities of coal are stored by distributing companies on the Great Lakes and along the seaboard; fairly large quantities by coke, steel, gas, and power plants, by a

few wholesalers situated at a distance from the coal fields, and by many railroads; smaller quantities by small power plants and a few mining companies; and still smaller quantities by the ordinary householder.

Interest in the subject of coal storage is rapidly increasing, a fact which is shown by the large number of letters received by the Department of Mining Engineering and the Engineering Experiment Station of the University of Illinois, and the amount of coal stored will no doubt increase as the feasibility and advantages of storage become more fully understood.

The data presented have been compiled from the record of correspondence and interviews with a large number of persons in charge of storage yards, with the manufacturers of storage machinery, and with the officials of various coal associations. Visits have been made to a number of plants in Illinois and the adjoining states, and the literature of the subject has been freely consulted. It has thus been possible to compile within a comparatively short period the results of the experience of those who have been storing coal. In order to reach as many different sources of information as possible a questionnaire was prepared and sent to about 175 individuals and corporations.* Groups of industries were selected in order that as many different points of view as possible might be presented. These groups included the zinc smelting companies in Illinois, the steel companies and the by-product coke companies of Illinois, Indiana, Wisconsin, and Missouri, large public service corporations, many of the steam and electric railroads, representative wholesale and retail dealers suggested by I. L. Runyan, Secretary of the Illinois and Wisconsin Coal Dealers Association, the secretaries of all the coal operators' associations in the United States, the dock owners about the Great Lakes, large manufacturers, and other users of coal. The fire departments of a large number of cities were also consulted. The careful and prompt attention given to the requests for information indicated a wide-spread interest in the subject, and the responses were most gratifying.

The theoretical considerations connected with the spontaneous combustion of coal and the other phenomena incident to coal storage have been investigated by S. W. Parr, H. C. Porter, F. K. Ovitz, and J. B. Porter, and the results of much experimental work by these and

* The form of this questionnaire is shown on p. 130.

other investigators are contained in the publications of the Engineering Experiment Station of the University of Illinois, the U. S. Bureau of Mines, and the Canadian Department of Mines. There is presented here only such a summary of these results as is necessary for a consideration of the engineering features of coal storage.

The International Railway Fuel Association as the result of a paper on coal storage by its former secretary, C. G. Hall,* has carried on a most valuable work through its Committees on Coal Storage and Fuel Stations. The Committee on Prime Movers of the National Electric Light Association also presented a useful report,† in 1917, on the subject of coal storage.

This circular represents merely a report of progress in an investigation which it is hoped will be continued. Its purpose is to bring to the attention of those who are considering the advisability and possibility of insuring themselves a regular fuel supply as much in the way of useful and helpful suggestions as possible. Additional data or suggestions from those into whose hands the circular may fall will be welcomed by the author.

2. *Acknowledgments.* — Grateful acknowledgment is made of help accorded by PROFESSOR S. W. PARR, Professor of Applied Chemistry, University of Illinois, and PROFESSOR E. A. HOLBROOK, Supervising Mining Engineer, Metallurgist and Superintendent of U. S. Bureau of Mines, Urbana, Illinois, and formerly Professor of Mineral Preparation and Utilization at the University of Illinois, who have coöperated most heartily and have contributed data gathered in connection with their own investigations of the subject.

Wherever it has been possible, full credit has been given for definite data used and for direct quotations from other publications, but it has been impossible to do so in all cases, and many of the opinions expressed and the suggestions made represent composite ideas based upon a large amount of correspondence and much reading.

3. *Summary of Conclusions.* — Based upon the facts and information hereinafter presented certain definite statements are justified regarding the reasons for storing coal, the kinds and sizes of coal to be stored, the methods of piling, and other factors entering into

* "Storage of Coal: Its Feasibility and Advantages to Producer, Carrier and Consumer," Proceedings, International Railway Fuel Association, Vol. VI, pp. 109-130, 1914.

† Published by the National Electric Light Association, New York City.

the problem of successful coal storage. These represent very largely a digest of opinions expressed by many persons who have had experience in storing coal and, it is believed, constitute a comprehensive statement of the present knowledge of safe coal storage practice. These conclusions are presented as follows:

(1) It is practical and advantageous to store coal not only during war times but also under normal conditions, at the mine, near the point at which it is to be used, or at some intermediate point. If possible, it should be stored near the point of consumption to avoid rehandling and the resultant breakage.

The reasons for storing coal are:

- (a) To insure the fuel consumer a supply of coal at all times.
- (b) To take advantage of low freight rates, or of low prices for coal at certain seasons of the year.
- (c) To permit the railroads to utilize their cars and equipment to the best advantage.
- (d) To maintain a uniform rate of production at the mines.

(2) Kinds and sizes of coal which may be safely stored:

(a) Although it is undoubtedly true that some coals may be stored with greater safety than others, the danger from spontaneous combustion is due more to improper preparation and piling of coal than to the kind of coal stored.

(b) Most varieties of bituminous coal may be safely stored if of proper size and if free from fine coal and dust. The coal must be handled in such manner as to prevent excessive breakage and the consequent production of much fine coal and dust when being placed in storage, because spontaneous combustion is due mainly to the oxidation of the coal surface. The danger of spontaneous combustion in storing coal is very greatly reduced, if not entirely eliminated, by storing only lump coal from which the dust and fine coal have been removed. Of two coals, the one which is least friable should be chosen for storage purposes because less dust and fine coal will be produced in its handling. All varieties of bituminous coal can be stored under water, since water excludes the air and prevents spontaneous combustion.

(c) Fine coal or slack has sometimes been safely stored in cases in which air has been excluded from the interior of the

pile. Exclusion of air from the interior may be accomplished (1) by a closely sealed wall built around the pile or (2) by close packing of the fine coal. Any pile of slack requires careful watching to detect evidences of heating and means should be provided for moving the coal promptly if heat develops. The only absolutely safe way to store slack or fine coal is under water.

(d) Many varieties of mine run coal cannot be stored safely because of the presence of fine coal and dust mixed with the lumps.

(e) Coal exposed to the air for some time may become "seasoned" and thus may be less liable to spontaneous combustion because of the oxidation of the surfaces of the lumps of coal, but opinions are by no means unanimous on this point.

(f) It is believed by many that damp coal or coal stored on a damp base is peculiarly liable to spontaneous combustion, but the evidence on this point is by no means conclusive. It is safer not to dampen the coal when or after it is placed in storage.

(3) The effect of sulphur on spontaneous combustion :

It has been shown by experimentation that the sulphur contained in coal in the form of pyrites is not the chief source of spontaneous combustion, as was formerly supposed, but the oxidation of the sulphur in the coal may assist in breaking up the lumps of coal and thus may increase the amount of fine coal which is particularly liable to rapid oxidation. Even this opinion is not unanimously endorsed. In spite of experimental data showing that sulphur is not the determining element in spontaneous combustion, the opinion is wide-spread that, if possible, it is well for storage purposes to choose a coal with a low sulphur content.

(4) Method of piling coal :

(a) To prevent spontaneous combustion, coal should be so piled that air may circulate freely through it and thus may carry off the heat due to oxidation of the carbon, or it should be so closely packed that air cannot enter the pile and oxidize the fine coal.

(b) Stratification or segregation of fine and lump coal should be avoided, since an open stratum of coarse lumps of coal provides a passage through which air may enter and come in

contact with the fine coal, thus oxidizing it and starting combustion.

(c) If space permits, low piles are preferable since in low piles the coal is more fully exposed to the air and is better cooled than in high piles and in case of heating it can be more readily and quickly moved. A disadvantage of high piles lies in the greater difficulty of moving the coal quickly, if necessary. The idea that a high pile causes heating at the bottom is erroneous, since as many fires take place near the top as near the bottom, and as many near the outside as near the interior of the pile. If possible, the coal should be divided by alleyways so as to facilitate rapid loading out of the coal in case of necessity, and so that an entire coal pile may not be endangered by a local fire.

(d) Much of the attempted ventilation of coal piles has been inadequately done through the use of only an occasional ventilation pipe, which has been not much more than a place in which to insert a thermometer. The practice of placing ventilating pipes closely together has been used in Canada and is reported to be effective.

(e) Water is an effective agent in quenching fire in a coal pile only if it can be applied in sufficient quantities to extinguish the fire and to cool the mass. A small quantity of water is not effective. Unless there is an ample supply of water thoroughly to quench the fire and to cool the pile, it is very dangerous to add any water to a coal pile.

(f) Coals of different varieties should if possible not be mixed in storage, for the coal possessing the greatest tendency toward spontaneous combustion may jeopardize the safety of other varieties not so liable to spontaneous combustion.

(5) The effects of storage upon the properties of coal:

(a) The heating value of coal as expressed in B. t. u. is decreased very little by storage, but the opinion is wide-spread that storage coal burns less freely than fresh coal. Experiments indicate that much of this deficiency may be overcome by keeping a thinner bed on the grate and by regulating the draft.

(b) The coking properties of most coals seem to decrease as a result of storage, but coals vary greatly in this respect.

(c) The deterioration of coal stored under water is negligible, and such coal absorbs very little extra moisture. If only part of a coal pile is submerged, the part exposed to the air is still liable to spontaneous combustion.

(6) Additional precautions:

(a) The best preventive of loss in coal storage is regularly to inspect the pile. If the temperature reaches 150 degrees F., the pile should be carefully watched and if the temperature rises to 175 or 180 degrees F. the coal should be removed as promptly as possible. The coal should be thoroughly cooled before being replaced in storage.

(b) Storage appliances and arrangements should be so designed as to make it possible to remove the coal quickly if necessary, and coal should not be stored in large piles unless provision is made for loading it out quickly.

(c) Pieces of wood, greasy waste, or other easily combustible material mixed in a coal pile may form a starting point for a fire, and every effort should be made to keep such material from the coal as it is being placed in storage.

(d) It is important that coal in storage should not be subjected to such external sources of heat as steam pipes, because the susceptibility of coal to spontaneous combustion increases rapidly as the temperature rises.

Warning: Special emphasis is laid upon the fact that safety in the storage of coal depends upon a very careful and thorough consideration of and attention to the details referred to in the foregoing. Lack of attention to these details and lack of care in handling will in many cases result in losses due to dangerous fires. Do not undertake to store coal until you are sure you know how to do it properly and safely.

II. REASONS FOR STORING COAL

4. *Present Mining Conditions.*—An ideal condition with regard to the production, distribution, and utilization of coal would permit the steady operation of mines without requiring the storage of large quantities of coal. There would be just enough mines to produce the coal required with only a suitable excess as a safety factor; these mines would be operated steadily for from 300 to 310 days per year and the coal would be taken away as rapidly as produced and delivered to the consumers as needed. This ideal condition is, however, far from being fulfilled. The actual condition is that an excessive number of mines is being operated, variously estimated as capable of producing from 50 to 150 per cent above the normal demands of the market, that these mines are irregularly worked for a total of only about 200 days per year producing much less during the spring and summer than during the fall and winter, and that the railroads have not adequate equipment to handle all the coal if all the mines were operated full time.

The storage of coal will not increase the amount used, and it may result in closing down unprofitable mines and in discouraging the opening of new mines until the readjusted demand may justify the operation of such mines on a profitable basis. Storage should make possible fairly uniform operation throughout the year and should help make the coal industry stable instead of intermittent and uncertain as at present. More nearly uniform operation of the mines and a greater number of days of work per year are necessary if coal is to be produced at a minimum cost and if the miners are to be kept satisfied, for no body of workmen can be satisfied with the irregular working conditions which have prevailed for many years in the coal mines of the United States. Regular operation also means the more systematic removal of the coal, greater extraction, and greater conservation of a natural resource.

The variation in the price of screenings has frequently been very great. In the summer, when the demand for coal has been small and the demand for fine sizes relatively large, screenings have commanded a price of one dollar or more per ton, while in the fall and early winter, when the demand for coal is greatest, the production of screenings has sometimes been so much in excess of the immediate

demand as to reduce the price at the mine to as little as ten cents per ton.

The variations in prices for different sizes of coal, as given by Prof. E. A. Holbrook,* show the advisability, from the standpoint of the buyer, of stocking coal to secure the advantage of lower prices during certain seasons. The general or extensive storage of coal should equalize these widely varying prices.

5. *Benefits of Storage.*—The reports of the Coal Storage Committee of the International Railway Fuel Association have shown that transportation lines should store coal, first, to assure a continuous supply of fuel for railroad operation at times when the supply from the mines may be cut off by floods, storms, labor disturbances, or other causes, and secondly, to avoid the necessity of providing the excessive equipment of locomotives and coal cars which are needed if all the fuel demands of the winter season must be met during that season. This excessive equipment is a non-productive investment during a part of the year.

Hall† says:

“The storage of coal in the summer months will enable the carriers to move a percentage of the tonnage during that season, which they are now called upon to move during sixty to eighty days of each fall and early winter, and during which period no railroad in this country now has, or ever can afford to have sufficient power and cars to serve the mines and move the coal to accord with the demand. To do this, we will say five of the large coal moving roads of Illinois and Indiana would each have to buy fifty new locomotives and six thousand new coal cars:

A total of 250 locomotives at \$25,000.00	\$6,250,000
A total of 30,000 cars at 800.00	24,000,000

Total Investment of	\$30,250,000
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the annual interest on which, at five per cent, would amount to \$1,512,500, and a greater percentage of such cars and power would be idle eight months of the year.

“This is only a small portion of the total investment that even this small group of railroads would have to make in order to handle the coal as the consumers now demand, as the expense of additional double and triple tracks and yard facilities would far exceed the cost of additional equipment and power. It is therefore very apparent that the carriers would have to spend so much money to provide 100 per cent service during the short abnormal offerings of coal that

* “Dry Preparation of Bituminous Coal at Illinois Mines.” Univ. of Ill. Eng. Exp. Sta., Bul. No. 88, 1916.

† Op. Cit., pp. 110-111.

a substantial increase in freight rates would be the inevitable result, which of course would mean an increase in the ultimate delivered cost of the coal. Thus the consumers would not only be called upon to pay the interest on an excessive amount of capital invested in mines but would also have to carry the burden of an excessive investment in equipment and facilities on the part of the carriers."

Between May 15 and July 15, 1914, the railroads had a surplus of 95,564 coal cars, representing an investment of \$105,120,400. This amount of money would provide extensive storage facilities which would probably have a much longer life, and which would cost less for maintenance and depreciation than the surplus of coal cars. In the case of the railroads, one of the greatest advantages of storage lies in the opportunity it affords for moving railroad coal, which is not a revenue producing element, at a time when revenue producing freight shipments are least. Railroads need storage to control traffic, and the coal operators need it to regulate production.

Storage assures the wholesale and retail distributors of coal a supply in times of failure of railroads and of mines to furnish the regular daily supply owing to strikes and transportation disturbances. It permits distributors to take advantage of low freight rates, particularly through the use of water transportation in the summer time, and possibly also of lower prices of coal at the mines during slack mining time.

Hall* reached the following conclusions regarding coal storage:

"The storage of bituminous coal can only be made feasible by the producer, carrier and consumer coöperating closely to carry on the work in a systematic and economical manner.

"One million dollars spent by producers for storage facilities will afford more relief to the coal industry than five millions spent in the development of new mines.

"One million dollars invested in storage facilities by railroads would go further toward relieving operating tangles and preventing car shortage than three million invested in cars and locomotives.

"The purchase of coal by the industrial and domestic consumers, during the slack periods, will result in a saving of a very high per cent on the investment."

The reasons for storing may be summarized as follows:

(1) To assure the consumer an adequate supply which protects him against strikes, other labor disturbances, and uncertain railroad deliveries.

* Op. Cit., p. 130.

(2) To take advantage of water transportation and low freight rates.

(3) To secure the advantages of low prices.

(4) To equalize the prices on the different sizes of coal.

(5) To avoid the maintenance by the railroads of equipment which is used for only part of the year.

(6) To maintain a uniform rate of production at the mines

III. PLACES OF STORAGE FOR DIFFERENT PURPOSES

6. *Storage Near the Point of Consumption.*—The place of storage should be near the point at which the coal is to be used, not only to assure a constant supply to the user, but to avoid the extra cost and extra breakage incident to each rehandling and to utilize transportation equipment to the best advantage.

Hall* says:

“While unquestionably the best and most economical results can be obtained by moving the coal to final destination before storing, however, it may be desirable and necessary to establish storage yards at principal distributing centers such as Chicago, Peoria, St. Paul, and Omaha, and in doing this the coöperation of producers and carriers will be essential. In fact, it would probably be feasible and desirable for the producers to control the storage companies.

“These central storage yards should have a capacity of 100,000 to 500,000 tons, a screening plant to be installed in conjunction with the yard.”

7. *Storage for Domestic Use and for Small Power Plants.*—The domestic consumer does not always realize that if his winter supply of coal is delivered in October or November, when he begins to use it, or in small amounts throughout the winter, he is contributing to the car shortage and is helping to make necessary the maintenance of more mines than the actual yearly consumption will justify. The householder should realize that there is no other item in his living expenses which will pay as large a return on the investment as to lay in a supply of coal in May or June, when it may usually be procured from fifty cents to one dollar per ton cheaper than in October or November.

Although an increase in domestic storage during the summer is one of the simplest ways of helping to solve the storage problem, many householders cannot afford to pay their annual coal bill in a lump sum. Knowing this, retailers often ask on the basis of deferred or monthly payments as much as twenty-five cents a ton more than the cash price. The plan of making a definite reduction in price in the spring might be more generally tried. In the anthracite region of Pennsylvania, for instance, a reduction of fifty cents per ton in the retail price in usually made April 1 and an increase of ten cents per month is added until by September 1 this reduction has been

* Op. Cit., p. 113.

made up. Some such stimulus is justifiable to help keep the mines operating more steadily during the summer months and to allow the railroads to transport the coal when the car supply is available. Systematic advertising campaigns calling attention to the advantages of buying during the summer have been carried on by a number of coal dealers, notably in Springfield, Illinois, where full page advertisements have appeared in the daily and Sunday papers.

In small cities like Urbana and Champaign where the population is from 25,000 to 30,000 the amount of coal used per year is from 75,000 to 100,000 tons. The dealers estimate that an ordinary eight room house requires from fifteen to twenty tons per year, and that seventy per cent of this amount are stored in basements during the summer months, when the price is about fifty cents per ton less than in winter. In towns which have a large industrial population the percentage stored during the summer is less and the sales are distributed throughout the year. In towns where the local supply comes from neighboring mines, the incentive for storage is not so great since there is not often any variation in price and no likelihood of a scarcity.

Hall* says:

“If consumers are asked to coöperate with producers and carriers to the extent of storing some coal during the spring or summer they will expect to be shown wherein they will be benefited, and this also holds true in the case of dealers or jobbers who may plan to go into the storage proposition on a large scale.

“The producer has in the past, and will in the future unquestionably make sufficient price concessions during the spring and summer months to cause that to be an attractive feature of storage.

“The carriers should also offer inducements in the nature of freight rates with ‘storage in transit’ privileges and should make lower rates during the months that is to the advantages of all interests to store coal.

“The total operating expenses of the railroads moving the Illinois and Indiana coal to Chicago and the adjacent territory north and west show an increase of 31 per cent in proportion to gross earnings in a severe winter month as compared with a spring or summer month. It is therefore entirely fair, and in fact a sound business proposition for these carriers to make a corresponding variation in freight rates. For example the present rate to a given point is \$1.00 per ton—why should not a rate of 85 cents be made for the months of April, May, and June; 95 cents for July, August and September, and \$1.10 for the remaining months of the year? ”

In this connection Coal Tariff 2338 of the Illinois Central Railroad, issued December 6, 1915, is of interest:

* Op. Cit., pp. 112-113.

“Storage in transit at Kankakee, Ill., of Bituminous Coal, C. L.

“Bituminous Coal, earloads, originating at Mines and Stations on Illinois Central Railroad and connections in Illinois, and destined to Chicago, Illinois, may be unloaded and stored in transit at Kankakee, Illinois.

“Shipments will be way-billed to Chicago, Ill., with notation on way-bill ‘To be unloaded and stored at Kankakee, Ill.’

“The through rate, lawfully on file with the Illinois Public Utilities Commission, in effect on the date of forwarding from original point of shipment to Chicago, Ill., plus \$2.00 per car for reconsigning and extra service, will be protected when shipments are reloaded and forwarded to Chicago, Ill.

“On reshipments of storage coal the deduction for moisture, on shipments of screenings, from the actual net weight as ascertained on track scales at Kankakee, Ill., will be five (5) per cent.

“Agent at Kankakee, Ill., to show gross, tare and net weight, and deduction for moisture on face of way-bill.

“No deduction will be allowed if weighed en route or at destination.

“If the shipments are not forwarded from Kankakee, Ill., within six months from the date they were forwarded from original point of shipments, through rate, lawfully on file with the Illinois Public Utilities Commission, in effect from the original point of shipment to Chicago, Ill., will be collected at that time.

“Owners will be required to unload and reload the coal.

“Shipments will be subject to Car Demurrage Rules lawfully on file with the Illinois Public Utilities Commission.

“Note:—Applicable only on traffic having point of origin, destination and entire transportation within the State of Illinois.”

In this connection, it is interesting to note the distribution among different classes of service of coal used in the city of Chicago.* A

TABLE 1
QUANTITY OF COAL AND COKE USED IN CHICAGO DURING 1912

SERVICE	TONS	PER CENT
(1) Steam Locomotives	2,815,400	13.27
(2) Steam Vessels	92,368	0.44
(3) High Pressure Steam Stationary Power and Heating Plants: Including public service corporations, municipal, steam railroads, office buildings, hotels, schools, power plants, or boiler plants . . .	9,147,344	43.13
(4) Low Pressure Steam and Other Stationary Heating Plants: Including large and small buildings, large and small apartments and residences	4,646,910	21.91
(5) Gas and Coke Plants: Excluding boiler power plants	253,867	1.20
(6) Furnaces for Metallurgical, Manufacturing and Other Processes: Including steel plants, foundries, forges and allied processes; brick, pottery and allied processes, and miscellaneous manu- facturing, rendering and other processes, excluding boiler power plants	4,253,007	20.05
Totals	21,208,886	100.00

* Report of the Chicago Association of Commerce Committee of Investigation on Smoke Abatement and Electrification of Railway Terminals, p. 110, 1915.

study of this table will indicate the points of attack in a campaign to increase the amounts of coal to be kept in storage.

The average retailer is in much the same condition as the householder, to the extent that he does business on limited capital and cannot afford to lay in a heavy stock, even if he has the required space and facilities. According to *The Black Diamond*,* there is one retail dealer in the United States for every two thousand inhabitants, which means that the average retailer handles only about 7500 tons per year. Since there is usually in every town some corporation which handles a large proportion of the retail coal business the average dealer's business will probably amount to much less than 7500 tons. There are at least twenty coal dealers in Champaign and Urbana for a population of about 25,000. Since the consumption in these towns is probably not over 100,000 tons per year, the average sales per dealer are not more than 5,000 tons. The ordinary retail dealer cannot afford to put much money into costly storage appliances which add greatly to his overhead charges, nor to buy the land necessary to store great quantities. The comparatively small amounts stored are kept usually in small bins under cover and are stored for only a short time.

One way out of the difficulty would be to establish coöperative storage plants which could serve as common storage facilities for a number of dealers. Such coöperative schemes have not, however, been generally successful. In Rock Island, Illinois, a company has built a number of storage bins which are rented. In a large city, storage facilities for any considerable quantity of coal would usually have to be located in the outlying districts to keep the rental cost for the land within reasonable limits. A ton of coal occupies about forty cubic feet; hence to store one thousand tons in a pile ten feet high would require a space sixty-three feet square.

Large office buildings, hotels, and stores are the largest users of coal per foot of floor area. In Chicago the amount used per building, according to *The Black Diamond*, varies from seven to sixty tons per day.

For large buildings two kinds of storage must be provided, namely, the current daily working storage and the reserve or emergency storage to tide over strikes, bad weather conditions, transportation difficulties, and other contingencies. Provision for the

* February 3, 1917.

storing of coal for the current supply is a necessity and need scarcely be considered in connection with the broader phases of the question, since it is a matter of architectural design and depends largely upon the space available in a given building. If a building requires 40 tons per day, a storage space of 1600 cubic feet is necessary for a day's supply and a room approximately 30 by 30 by 10 feet for a week's supply. According to *The Black Diamond*,* this storage space in the loop district of Chicago is worth a rental of three dollars per square foot per annum for power purposes. The Continental and Commercial National Bank uses this space for its vaults, and their rental value is about eight dollars per square foot. That makes its value two and two-thirds times the value of the same space for power generating purposes. The Rector building in Chicago rents its basement to a restaurant for six dollars per square foot. This fact shows that the basement is worth almost as much as the ground floor space, and more than twice as much as the office space of the upper stories. Such buildings must have a regular daily supply, and they must use their own space at a high rental loss, or some provision for storage must be made by the wholesalers, or by the railroads. Local storage of large quantities of coal for such a district as the loop in Chicago is, therefore, impracticable without an unreasonable increase in the fuel bill, and some other solution of this part of the problem must be worked out.

Public utilities companies, owing to the constantly increasing governmental supervision which requires uninterrupted service, and the by-product coke companies, the steel companies, and other industries which must run continuously must necessarily regard storage as a matter of necessity, unless they are situated near the mines. Many of the zinc smelters in Illinois, although in mining towns, have ample storage facilities.

8. *Storage by Railroads.*—The railroads are interested in storage both because they are the largest users of coal and because they are interested in utilizing their coal carrying equipment to the best advantage.

According to the United States Geological Survey, the railroads of the United States used, in 1916, 136,000,000 tons of bituminous coal (27 per cent of the total production), 6,735,000 tons of anthra-

* Op. Cit

cite (7.1 per cent of the production), and 22,950 tons of coke. The railroad consumption of bituminous coal in 1916 showed an increase of 14,000,000 tons, or 11.5 per cent over 1915. The quantity of bituminous coal used by the railroads in the eastern district increased from 56,500,000 to 62,700,000, or 11 per cent. The increase in the southern district was from 22,000,000 to 23,300,000 or 5.1 per cent, and in the western district, from 43,500,000 to 50,000,000, or 15 per cent.

There is a difference of opinion concerning the place at which a railroad should store coal. Some recommend storage at the using point, i. e., at each coaling station, while others suggest some central point from which a number of coaling stations may be supplied. Even though the cost of providing storage facilities at each coaling station may be prohibitive at present, many advise that in locating coaling stations consideration be given to the possibility of subsequently erecting storage facilities at such points. Some recommend a central storage plant for each division, so that each superintendent may be responsible for his own fuel supply. Others recommend only general terminal supply stations. The cost of installation and operation of one large plant must be compared with that of a number of smaller ones, and the item of additional breakage must also be considered.

9. *Storage at the Mine.*—There is a wide diversity of opinion concerning the advisability of storing at the mine, many claiming that it is not desirable, since it neither increases the output nor helps the transportation problem. It is as logical economically for the producer of coal to store coal at the mine as it is for the manufacturer of cement to store cement at his plant.

Storage near the point of consumption is, of course, the best method, but storage at the mine possesses such advantages as to warrant its consideration. Even a small storage pile at the mine may permit the mine to begin running in the morning when railroad cars have not been delivered and when ordinarily the mine would not start owing to the uncertainty of a car supply later in the day. It may also often tide over periods of the day when additional cars are needed. During the early part of the week the car supply is usually good, but it falls off during the latter part, and storage should help to increase the regularity of running.

Carl Scholz* says:

"Lump and egg coal should be stored at destination by railroads, industrial plants, and domestic users during the period from April to August. Screenings should be stored at mines by coal operators during the period from September to January."

By storing at the mine, the operators are fortified against fluctuations in demand. Occasionally an operator cannot increase his output rapidly owing to insufficient mine development; whereas if he has a stored supply upon which to draw, he may retain the business and at the same time proceed with the development until his capacity is increased to meet the extra demand. Suitable storage facilities at the mine will make possible more rapid working and better extraction. Storage will also help to equalize the difference in demand for different sizes, and will permit shipments to continue when the mine is shut down. Operating expenses at a mine, as with the railroads, are usually greatest in the winter and mining would, therefore, be more profitable if the mines could be worked steadily during the summer and spring months.

E. A. McAuliffe† has stated the advantages of storing as affecting labor conditions at the mine as follows:

"A shop man ordinarily receives some limited notice relative to short hours or reduction of force. Engine and trainmen have learned by experience that business falls off at certain seasons, but they know at the same time that they will be permitted to make a mileage at least reasonably commensurate with their domestic and their financial requirements; not so with the miner, with the fellow that digs the coal; he works to the extreme limit of his ability, or at least he has the opportunity to work to the limit of his ability, one week, when suddenly, without notice, due possibly to an unforeseen weather condition, business slumps and he gets one day or two days a week. In the meantime his operating expenses are running on just the same. I think that condition is very largely responsible for the unrest, for the dissatisfaction, that the mine worker labors under. That he does not take a greater interest in the affairs of his employer, in the permanency of his vocation, I think is very largely due to the fact that he does not know what he is going to earn next week. When night comes he can sum up what he earned today, but he does not know what tomorrow holds in store for him. There is no business on earth subject to the tremendous and violent fluctuations that the production of coal is subject to. Today we demand the maximum, or if possible, more than the maximum output of all the mines; tomorrow we are quite indifferent to whether the operator sells his product or otherwise. The storage of coal, and the movement from the mines to the stock piles during these temporary depressed periods is the only same solution of that problem."

* Proceedings, International Railway Fuel Association, Vol. VII, p. 267, 1915.

† Proceedings, International Railway Fuel Association, Vol. VI, p. 143, 1914.

The small number of working days at Illinois mines is shown by the following table, taken from the Illinois Coal Report.

TABLE 2
DAYS OF ACTIVE OPERATION OF MINES FOR SEVENTEEN YEARS

Year	All Mines	Shipping Mines
	Days	Days
1900	183	214
1901	174	204
1902	179	210
1903	192	222
1904	198	213
1905	174	198
1906	172	189
1907	184	209
1908	171	191
1909	168	189
1910	171	179
1911	165	169
1912	160	172
1913	170	179
1914	162	174
1915	158	172
1916	163	185

IV. COAL STORAGE PRACTICE

10. *Kinds and Sizes of Coal which may be Safely Stored.*—The opinion is current that the locality from which a coal comes determines its suitability for storage. One frequently hears such a remark as, “Eastern coals (meaning those from Pennsylvania and West Virginia) can be easily stored but Western coals (meaning those from Illinois and Indiana) cannot be since they are much more liable to spontaneous combustion.”

This statement is too general to fit the facts, because scientific research and the experience of those storing coal have shown that while there are undoubtedly inherent differences in coals which affect their liability to spontaneous combustion and to degradation, these differences are of less importance than the size of the coal and the way in which it is stored. The answers to the questionnaire (Appendix II) indicate that almost any coal may be stored if it is properly piled, and that almost any coal improperly stored will heat and may fire.

The C. W. Hunt Company, of Staten Island, makes the following statement:

“Any coal of fair quality, both anthracite and bituminous, can be successfully stored for an indefinite length of time. There is a slight deterioration both chemical and physical, but this is not serious, provided the coal does not heat. Low grade bituminous coals are subject to heating and should not be stored above twenty feet in depth or for very long periods. Any storage of bituminous coal should be watched and if found to heat, should be moved.”

The Coal Storage Committee of the International Railway Fuel Association says with reference to storage by roads using Ohio, Pennsylvania, and West Virginia coals:

“The consensus of opinion in regard to the coals which may be screened without excessive breakage seems to be that the coal best adapted for storage is the three-fourths inch to the one and one-fourth inch lump. In the case of very friable coals such a separation of sizes is not practicable, for which reason they should be stored as run of mine and the storage piles so made as to minimize the danger attendant upon storing the coal of mixed sizes.”*

With reference to storage by the railways in the southeastern part of the United States the Committee says:

* Proceedings, International Railway Fuel Association, Vol. VII, p. 260, 1915.

“It is agreed that lump coal of a firm quality, in order to resist powdering, is the best. The Central of Georgia Railway has been very successful in storing slack with a large percentage of nut. From the operators’ viewpoint, run of mine should be stored, as it leaves nothing on their hands.” *

With reference to southwestern railways the Committee says:

“It is generally agreed that lump and nut coal with at least twenty-five per cent of slack removed is the best kind to store, that is, with the coals which will store without great danger of spontaneous combustion.

“All who have tried to store Texas and part of the Arkansas coal have found that it is a failure on account of the liability to spontaneous combustion.

“All other coals in the second territory—Oklahoma, Kansas, Missouri, and Northern Arkansas—can be stored and the loss in heating value and weight will be from two per cent to eight per cent.” †

With reference to the railroads using Indiana, Illinois and Iowa coals the Committee says:

“Lump and egg coal should be stored at destination by railroads, industrial plants and domestic users during months April to August. Screenings should be stored at mines by coal operators during period September to January.” ‡

W. L. Abbott, Chief Engineer of the Commonwealth Edison Company, Chicago, says:

“The experience of the Commonwealth Edison Company after storing large amounts of all varieties of coal and particularly Illinois coals for a number of years may be summarized as follows:

“Nearly any coal which has gone over a 1½-inch screen can be stored. Any size of coal with duff left in will heat.

“Pea coal over one-half inch through three-fourths inches has been in storage for more than a year without heating. Coal with screenings removed has been kept in storage eight years without firing.

“Heating usually occurs within three months after the coal has been stocked, and the tendency to heat decreases rapidly after that period.

“Coal in storage piles shows no measurable loss of heating power, although weathering reduces the lumps on the outside of the piles to slack.

“As insurance, cost of handling, etc., are the same for all grades of coal, regardless of heat value, it is more economical to store the better grades.”

These statements are endorsed by others in Chicago who store large quantities of coal.

* Ibid., p. 262.

† Ibid., p. 264.

‡ Ibid., p. 267.

Hall * says:

“On account of the cheaper prices which have prevailed on slack coal, the industrial concerns of the country have gradually been installing steam plants so equipped as to burn successfully that grade, resulting in a proportionately stronger demand for screenings, from April to August of each year, than any other size. This creates an ideal condition for storage as it enables the operator to supply the railroads, and other concerns that require it, sized coal such as nut and egg for current use and the large lumps for storage. There has been a surplus of egg as well as lump coal during the spring and summer and this grade can also be stored to good advantage. The six-inch lump coal when stored and reloaded should be rescreened if any part of it is to be offered to the householder. Lump or egg when stored by the ultimate consumer and at the point of consumption can be recovered without appreciable degradation.

“During the fall and early winter, conditions are reversed and screenings become the unsalable size. This is the period that screenings should be stored at the mines.”

A. Bement† says: “It appeared that egg coal was the best size to put in storage and there has been an enormous tonnage of it stored in the open with complete success.”

11. *When Coal should be Stored.*—To keep the mines running regularly and to relieve the congestion of the railroads, coal should be stored during the spring and summer. Coal which is moved to the storage point by water must be stored during the summer and early fall before the close of water navigation.

From the standpoint of heating of the coal it is preferable to store during the cool days of the fall or winter, but many prefer to store during July and August, because then the coal is drier. Others prefer to store in May and June, because then labor is more easily obtained and more can be accomplished than during the hot summer.

W. D. Langtry‡ suggests the advisability of cooling steel coal cars before they are loaded and says:

“One item which might prevent trouble in storing is the condition of railroad cars in the summer when coal is loaded therein at the mines. We found that steel cars, for instance, would absorb the heat, to a great extent, from the sun, and if the coal was loaded into these cars this heat would be transmitted to the coal, thus giving it a good start for spontaneous combustion to continue. Some coals are more or less damp when freshly mined, and this and the heat are very good factors in giving the coal a good start. If the cars could be cooled in some way, by sprinkling them with a hose, when it is known that the coal is going in storage it might help somewhat.”

* *Proceedings, International Railway Fuel Association*, Vol. VI, p. 125, 1914.

† *The Black Diamond*, April 7, 1917.

‡ *Personal Communication*.

The following statement by F. W. Gray* is of interest with relation to the effects of the temperature at which coal is placed in storage:

“The most extensive storage yet undertaken was in the winter months of 1913-1914, when 650,000 tons of coal were ‘banked’ by the Dominion Coal Company at their Glace Bay mines. The coal is lifted in the summer by steam shovels, rescreened and shipped. There has never yet been an actual fire in the round coal banks, although the first coal banked out must remain in the center of the pile for over six months before it is lifted. Heating sometimes takes place, but with proper methods this can be speedily checked and dissipated. The temperature of the air at the actual time of banking is an important consideration, as generally speaking the banked coal seems to remain at about the temperature which it had when placed in the bank. The bulk of the coal placed on the ground is, of course, put there in cold or freezing weather. If a thermometer is lowered down a pipe into the interior of the bank it will usually register a temperature near to the freezing point, a fact that it is interesting to observe on a hot summer day, when the surface of the banked coal is quite warm to the hand. The coal is banked up to a height of from 40 to 46 feet, and over 300,000 tons have been stored in a continuous pile.”

There is the greatest danger when coal is stored during the hot months, July and August, and the liability is greatly reduced if it is stored in May, June, September, or October.

12. *Storage Piles.*—Storage piles are usually in the form of truncated cones or pyramids, the size depending mainly upon the appliances used for storing and reclaiming and upon the space available.

The different sizes of coal are sometimes, if space permits, kept separate in storage, but often this plan is impracticable and the coal must be screened, if at all, as it is taken out of storage.

If piled on the ground, the space should be clean, level and free from water or moisture. Many large piles are placed upon a board or concrete foundation.

There is a wide diversity of opinion concerning the height of piles, and many think that piles should be not more than ten feet high. The opinion often expressed that the height of the pile is an important factor affecting the extent to which coal at the bottom of the pile may be crushed does not seem to be substantiated by the facts. Tests made in the Materials Testing Laboratory of the University of Illinois on Illinois coal showed a maximum crushing strength of 2090 pounds per

* “The Coal Fields and Coal Industry of Eastern Canada.” Canada Department of Mines, Bul. 14, pp. 11-12, 1917.

square inch and a minimum of 1280 pounds. Similar tests on Connellsville coal showed a maximum strength of 3430 pounds per square inch and a minimum of 1310 pounds. Tests in England required 1.27 tons per square inch to start cracking and 1.52 tons to crush the coal.

Coal is seldom piled at present more than fifty feet high and at this height the pressure at the bottom of the pile would be only about fourteen pounds per square inch which is so small compared with the crushing strength as to be negligible. Any increase in fine material in high piles is probably due mainly to the handling of the coal, as, for example, dropping it from a considerable height or allowing it to roll down a long slope and thus produce breakage and abrasion. None of those who pile coal to a considerable height reports any crushing due to the height of the pile.

Coal has been successfully stored in piles varying in height from six to sixty feet and it has also frequently fired in very low piles. The depth is not so important as the manner in which the coal is placed in storage and the facilities available for quickly removing it in case of firing.

An examination of piles which have fired shows that the fires have started frequently near the top and sides and not at any great depth in the pile. A current of air in the pile too sluggish to carry off the heat, a piece of wood, or oily waste may furnish a starting place for fire. The theory has been advanced that the temperature of a current of air rising through a pile of coal in which oxidation is taking place receives sufficient heat from the oxidizing coal to increase the tendency to spontaneous combustion near the top.

One objection to high piles lies in the difficulty of testing for heating. Low piles are advantageous because they can be easily watched and tested for a rise in temperature, and if necessary the coal can be removed quickly.

In connection with experiments made on coal stored by the Canadian Pacific Railway near Montreal, Canada, Prof. J. B. Porter* says:

"It is interesting to note that the series of observations prove very clearly that the maximum heating was comparatively near the surface of the pile. As a matter of fact the hottest points were apparently not more than five or six feet from the surface, although the weather at the time was extremely severe. The common opinion of practical men in charge of coal storage is that fires usually

* "Weathering of Coal." An Investigation of the Coals of Canada, Extra Vol., p. 164, 1915.

occur close to the bottom, but in this case it is almost certain that had the pile been left unventilated it would have ignited within a week or two at a depth of not over six feet, and in certain other cases which the author has observed, fires in large piles have actually originated at this depth. The probable reason is that in a pile of great extent and depth, the coal in the lower portions is so heavily compressed and so isolated from supplies of fresh air that it does not receive sufficient oxygen. It must be confessed, however, that the persons responsible for coal storage are not willing to act upon this theory, which, if followed to a logical conclusion, would lead to the storage of coal in very deep piles on ground impermeable to air."

A pile should be divided by alleyways into small units so that all parts may be easily accessible, and thus the danger of a fire spreading may be minimized.

13. *Ventilation of Coal Piles.*—It is a generally accepted theory that if the air supply is entirely shut off from the coal, as with underwater storage, spontaneous combustion cannot occur. It is also agreed that if ample ventilation can be furnished to carry off the heat and keep down the temperature in a coal pile, spontaneous combustion will not occur. It is the intermediate condition which is dangerous, that is, a condition in which enough air is admitted to permit the coal to oxidize and heat and not enough to carry off the heat as rapidly as it is generated. For this reason lump coal may be safely stored, because there is good circulation through the pile. On the other hand, run of mine often cannot be safely stored, not only because of the presence of an excessive amount of fine coal which oxidizes readily, but because the openings between the lumps are filled to a considerable extent by the fine coal and the free circulation of air is prevented. Alternate stratification of coarse and fine coal, therefore, is undesirable, and air passages formed by the large lumps rolling to the bottom of a pile should be avoided. Such passages form a duct or flue for a sufficient amount of air to reach the fine material inside the pile to start oxidation.

The practicability of properly ventilating a coal pile has been disputed and while the consensus of opinion in the United States is against ventilation by pipes, it is probable that many of the opinions expressed are based upon unfavorable results secured through improperly installed and inadequate ventilation systems. Many so-called pipe ventilation systems have consisted merely of an occasional pipe into which a thermometer may be inserted for reading tempera-

tures. There are of record few adequate ventilation systems being installed in the United States, because such systems are expensive and interfere to some extent with the rapid handling of the coal; such systems are also considered dangerous.

It is also stated by many that closely packed coal is so poor a conductor of heat that fire may start close to a ventilating pipe, and that it is impossible adequately to ventilate a coal pile.

J. H. Hibben* cites an instance of apparently successful ventilation† which he describes as follows: "In southern Texas several years ago we stored a large amount of Oklahoma mine run coal at Smith-

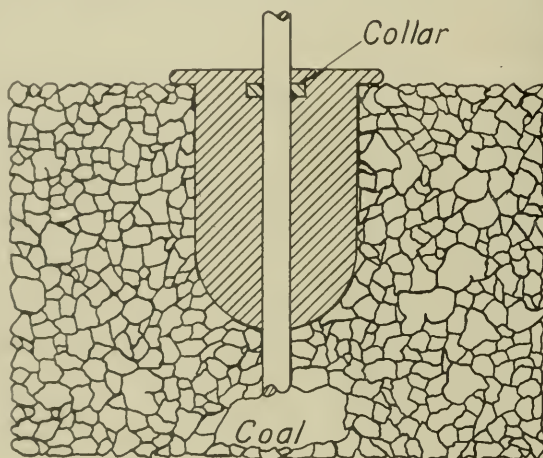


FIG. 1. METHOD OF VENTILATING COAL PILES EMPLOYED BY THE CANADIAN PACIFIC RAILWAY

ville, a coaling station near San Antonio. I experimented with one pile of coal by placing one six-inch glazed tile on the bottom for the entire length of the pile. This was the ordinary bell-shaped tile and it could be put together without cement, the ends butting against each other. In another adjoining pile an equal quantity of coal was stored without the use of tile and the result was that in the pile where the tiling had been placed, there was no fire, but in the other pile not tiled, there was a great deal of trouble with spontaneous combustion." Hibben says further: "I am of the opinion that pipes or flues do not always produce the desired results."

A comprehensive system of ventilation installed in Canada by the Canadian Pacific Railway is described by Porter‡ as follows:

* Op. Cit., pp. 152-153.

† Another instance of possible successful ventilation is referred to on page 150.

‡ Op. Cit., pp. 152-153.

“The method in general is: first to level off a triangular piece of ground so that temporary tracks may be laid upon it and a steam shovel used if desired. A track is then laid in and hopper-bottom coal cars unload on this track, which is then raised on the coal and the operation repeated until an embankment of considerable height is made, with an inclined track from the apex of the triangle to the main line. When a sufficient height is reached, the coal is side dumped or side shovelled and the track is gradually shifted to one side and the width of the pile thus increased to any desired extent. The height of the pile on the approach increases from nothing to about sixteen feet, and this latter height is maintained for the major portion of the pile.

“As soon as possible after each portion of the pile reaches its full height it is ventilated by driving pointed iron rods, $1\frac{1}{2}$ inches or 2 inches in diameter, vertically down through it. After the rod reaches the bottom a bell-shaped collar is slipped over it and forced down about six inches into the coal, as shown in Fig. 1. The rod is then tapped to compact the coal around it and to loosen the rod itself and both collar and rod are withdrawn. A rough funnel of tar paper is then put in the bell-shaped top of the hole to prevent pieces of coal falling in or being washed in by the rain. The walls of the lower part of the hole maintain themselves without protection, and holes driven in this way ordinarily last for many months. The distance between the holes varies somewhat with the circumstances; but generally it is about sixteen inches from center to center, and the cost, which is almost wholly labor, amounts to approximately five cents per ton.

“Another method, which is frequently effective, and in certain cases economically possible, is to pile the coal in layers of about two feet, allowing each layer to be exposed to the air for at least two days, and if possible much longer, before it is covered by the next layer. This method is usually effective in preventing fire, if the piling is done in cool weather, but in midsummer, particularly in hot and sunny weather, it is of doubtful value to say the least.”

This description is supplemented by Dr. Porter in a private communication, as follows:

“I have had personal experience of the storage in Montreal of very large quantities of coal from Nova Scotia and elsewhere, and of one case in England of the Storage of Welsh coal, where it has been found possible to ventilate the piles cheaply and effectively by air passage ways from the surface. To my personal knowledge these methods are still in successful use on a very large scale.

“In the majority of cases above referred to the ventilation was arranged at the time that the coal was stored, but in one or two cases under my own instruction ventilation was successfully applied to storage piles which had begun to heat, and in no single case under my observation was there any difficulty in cooling the coal; although in one instance at least the temperature had risen to 133 degrees F. and was going up with a constantly increasing rapidity.

“I am confident that the method can be successfully applied to the great majority of cases of storage. If the coal is known or suspected to be liable to heating, the ventilation holes should be driven at the time of piling and should

be inspected from time to time to see that they are kept open. If the coal is of a less dangerous character, or if for any reason ventilation cannot be provided at the start, it will suffice to make frequent observations of its temperature, and ventilate if serious heating begins.

"I may add that the method referred to is not of my own invention or discovery, but that circumstances made it possible for me to examine and test it very thoroughly on a very large scale with the result that I am firmly convinced that it is a perfectly practicable and reasonably inexpensive method for commercial storage on a very large scale.

"In spite of the numerous fires in coal piles in Canada during 1917, I am informed that not one single ventilated coal pile belonging to the Canadian Pacific Railway has caught fire either this year or for several years. The method of ventilation used by the C. P. R. and copied by various other concerns is set forth in detail in 'The Weathering of Coal,' pages 152-166; the only difference is that their present specifications are somewhat simpler as they find that one-inch holes suffice driven from twenty to twenty-four inches center to center in parallel rows. When I described the method the company ventilated its own piles, but now it does the work by contract in the majority of cases at a cost of about five cents per ton. The Canadian Pacific Railway Company stores from a million and a half to two million tons of coal each year; the main part of this coal being piled in the summer and used during the winter and spring; and the statement that they have no fires where they ventilate is therefore of very great weight. It is interesting to note that at a few C. P. R. storage points it was not found practicable to ventilate the piles this year, and Mr. Britt, the Fuel Agent of the Company, informs me that at these points heating has occurred and that at the most important of them (Fort William) they have had a very bad fire.

"Under ordinary conditions the main part of the Montreal coal supply comes from Nova Scotia in ships and is quite cold when it reaches here; in fact I have seen ice in the coal when it was being loaded in these ships at Sydney in mid-summer. This cool coal is ordinarily handled very quickly to the storage piles, and the piles themselves so laid out that as little as possible sun-heated coal is buried."

14. *Testing for Fires.*—The common methods of testing for the heating of a coal pile are:

(1) By watching to detect evidences of steaming in the pile.

(2) By noting the odor given off. The bituminous odor of burning coal or the odor of burning sulphur are evidences of heating.

(3) By inserting an iron rod into the pile and, when drawn out, noting its temperature by touching with the hand.

(4) By noting places where snow on a pile has melted.

(5) By means of maximum temperature thermometers inserted into pipes driven into the pile at intervals. These pipes

should have a conical plug in the bottom to facilitate driving them into the pile. After driving to the desired depth the plug may be withdrawn and the pipe raised a short distance from the bottom of the hole so that the actual temperature of the coal may be taken. The top of the pipe should be kept closed to reduce the effect of outside temperatures. Instead of leaving a pipe in the coal pile it is sometimes necessary only to drive it and then withdraw it, the hole remaining open sufficiently to permit the insertion of the thermometer. This method obviates one of the greatest objections to pipes placed in the pile; i. e., interference with the apparatus used for removing the coal. Self-registering thermometers protected by a metal casing may be bought from any dealer in scientific instruments. They may be had graduated according to the Fahrenheit or the Centigrade scale and adjusted to various ranges of temperature, the best range for this work being from 0 to 220 deg. F. The cost per instrument is about \$2.25 without armor and \$4.50 with armor.

In the experimental work carried on by Prof. J. B. Porter, of McGill University, a Richard thermograph was also used for recording temperatures.*

15. *Handling Fires.*—Opinions differ widely concerning the critical or dangerous temperature in a coal pile. Parr† says, "Bituminous coal can be stocked without appreciable loss of heat value provided the temperature is not allowed to rise above 180 degrees F." How near to this temperature a pile should be allowed to heat is largely a matter of judgment. If the rate of rise in temperature is decreasing rapidly, it may be safe to allow the temperature to approach 180 degrees, but if the rise is steady and regular it is wise to load out the pile before this danger point is reached. The extent of rise allowable also depends upon the means available for loading out. At a plant equipped with large grab buckets or other means for rapidly handling the coal a higher temperature can be permitted than in cases in which a considerable period is necessary to load out the coal. A person in charge of a certain kind of coal under certain climatic conditions will with a little experience be able to determine

* Porter, J. B., "Weathering of Coal," An Investigation of the Coals of Canada, Extra Vol., p. 159, 1915.

† "Effects of Storage Upon the Properties of Coal." Univ. of Ill. Eng. Exp. Sta., Bul. 97, pp. 7, 38, 1917.

the danger point. It is impossible to set any critical temperature which will apply to all coals under varying storage conditions. One very safe rule is to be ready to remove the coal if the temperature reaches 150 degrees F. and to load it out if the temperature rises to 175 degrees.

Water has often not proved effective in putting out fires, doubtlessly because of the fact that it was not applied in sufficient quantities to cool the entire mass thoroughly. An insufficient amount of water will aggravate rather than stop an incipient fire. One large pile in Chicago was soaked as completely as possible with streams from river fire tugs and while the fire was at the time apparently extinguished, it began burning again within two or three days. If the coal can be spread out and thoroughly saturated with water, the fire can be extinguished, but often there is not sufficient ground available to permit proper spreading.

In a private communication Dr. J. B. Porter says:

"I fully appreciate the fact that nearly everybody experienced in the storage of coal objects to the use of water for quenching fires in storage piles. I express scepticism as to the harmfulness of water quenching. Recent information strengthens this scepticism, and I have come across several cases of successful fire fighting by the intelligent use of water. The fuel agent of the Canadian Pacific Railway states that he always recommends the use of water if the fire is a small one, and particularly if it is detected in an incipient stage. His practice is to locate the hot spot by driving test rods into the pile and then to dig a pit one or two feet deep right over the center of trouble; to drive and pull pointed rods or open pipes from it down into the heating mass and then to fill the pit with water, thus quenching the fire at its very center. At the same time if the fire is a large one he surrounds the whole heated part with a water curtain made by digging a ring ditch one or two feet deep and perforating its bottom with a row of holes as in ventilation. This ditch like the central hole is kept full of water from the hose, and if there is any tendency for the fire to be driven outward from the center, it is quenched by the water curtain.

"This method of putting out a fire is of course costly, but it is enormously quicker and less costly than that of digging out and results in far less loss of material. Personally, I am confident that it will prove successful in any ordinary case."

Inert gases, such as carbon dioxide, have been tried as fire extinguishing agents.

16. *City Ordinances Affecting Storage.*—In order to determine if the heating of coal piles and the occasional fires resulting from them increase fire risks, letters were sent to the fire departments of

a number of cities. The following cities report that they have no city ordinances covering the storage of coal: Bloomington, Danville, Galesburg, Rockford, Decatur, Peoria, Illinois; Detroit, Michigan; Terre Haute, Indiana; Buffalo, New York; Superior and Milwaukee, Wisconsin; St. Louis, Missouri.

The Chief of the Fire Department of Toledo, Ohio, says: "We have no city ordinance governing the storage of coal, but have had several stubborn, but no disastrous fires in large coal piles due to spontaneous combustion."

J. C. McDonnell, Chief of the Bureau of Fire Prevention and Public Safety of Chicago, says:

"Every fall we have fires due to spontaneous combustion of coal in piles but this year they started earlier than usual. Since July 1, 1917 to date (November 14, 1917) there have been sixty-three fires in coal piles stored outside of buildings. The quantities involved varied from 20 to 15,000 tons. In fifty cases the pile laid in the open and in thirteen cases only a shelter roof was provided. There were for the same period thirty-nine fires in coal piles inside of buildings. Thirty of these interior fires were in apartment buildings and the amounts involved varied from 5 to 1,000 tons; stored in all cases on a concrete floor. In five of the inside fires provision was made for ventilating the pile by means of pipes. Some of the outside piles have been burning for three months and are still on fire. One large pile at the stockyards has completely changed itself into coke. Water has no effect on these fires. All the coal was a poor grade of soft coal."

The City of Chicago has the following ordinance:

"Coal—Storage in Buildings: Soft coal shall be stored away from the brickwork of boilers and furnaces and shall be kept only in incombustible rooms."

The following regulations covering fire insurance on coal docks show the points to be considered in connection with the construction and operation of such large storage plants:

"Standard Dock: Sand or earth filled, cement or concrete floor, with all iron or fireproof superstructure, and (or) storage sheds.

"Hoisting Apparatus: To be equipped with the 'Brown' or similar hoisting apparatus with automatic shovels or buckets known as the 'Clam' or 'Pick-up' type, for removal of coal in case of fire.

"Boilers: To be in fireproof house.

"Steam Pipes: To be free from wood or combustible material.

"Lubricating Oils: Must be limited to two barrels if in house exposing dock, sheds, or superstructure.

"Watchmen: To report through A. D. T. System of signal boxes to central station.

“Lighting or Power: If electricity, to be in accordance with rules of National Board of Fire Underwriters.

“Fire Protection: To be protected by city water and city fire department or private fire pump of at least five hundred gallons capacity per minute, supplying six-inch main extending entire length of dock, laid on ‘water’ dock so as to be properly drained, with sufficient number of hydrants and amount of 2½-inch hose attached or on cart to cover and reach all parts of the dock, also by standard fire boat service.

“Telegraph Fire Alarm: There shall be a telegraph fire alarm station or box within two hundred feet, with key at the dock.

“Fire Casks and Pails or Chemical Fire Extinguishers: Should be protected by an adequate number of fire casks and pails or approved chemical fire extinguishers (at least one at every Engine or Motor room on Dock or superstructure).

“Coal: A space of at least two feet at the base shall always be maintained between open piles of Bituminous Slack, Bituminous Coal, and Anthracite Coal.

“Anthracite coal piled or stored in frame covered or enclosed shed with bituminous coal shall take bituminous coal rate.

“Anthracite coal inside non-fireproof structure must be separated from bituminous coal or bituminous slack outside of said structure by at least two feet space at base of piles, and fifty feet space from non-fireproof structure containing bituminous coal or bituminous slack.

“Bituminous Slack Coal: Considering the disastrous experience on bituminous slack coal and the many fires resulting therefrom, it is not considered within the province of this schedule to name a rate thereon in any situation; and all policies covering bituminous coal should contain stipulation that the same is not intended to cover bituminous slack coal or screenings.

“Bituminous Coal: If placed on, in or within two feet, if dock is filled,—or within ten feet if dock is not filled,—of any frame building, shed, covered or enclosed superstructure (excepting loading pockets or loading bins), or of any anthracite coal or bituminous slack, add fifty cents to the base rate.

“Bituminous Slack: If placed on, in or within two feet, if dock is filled,—or within ten feet if dock is not filled,—of any frame building, shed, covered or enclosed superstructure (excepting loading pockets or loading bins), or any bituminous or anthracite coal, add one dollar to the base rate.”

V. STORAGE SYSTEMS

17. *Choice of a Storage System.*—In the choice of a storage system the following points should be considered:

(1) The location, size, and topography of the available storage ground.

(2) The capacity of the desired installation, that is, the amount of coal which it is desired to load and unload in a given time.

(3) The cost of the plant.

(4) The cost of maintenance.

(5) The cost of operation.

(6) The amount of breakage to be permitted in handling the coal.

(7) The way in which the coal is received, in open or box cars, or in boats.

(8) The length of time the coal must be kept in storage.

(9) Climate; in very cold countries under-water storage is impracticable for a part of the year.

The requirements of an ideal plant are:

(1) Adequate ground area, so that several sizes and varieties of coal may be stored separately. Separation into sizes has not been considered so important for bituminous as for anthracite coal, but it is becoming more important because of the increasing attention being given to preparation of coal for domestic use, and because of the fact that danger of spontaneous combustion is decreased by keeping different sizes separate in storage.

(2) Adequate facilities for rapidly and economically transferring coal from cars or from boats into storage.

(3) Adequate facilities for rapidly and economically reclaiming the coal and for rapidly moving any part of the pile which shows evidences of taking fire.

(4) Adequate track facilities, with gravity facilities, if possible, for handling cars.

(5) Means for preventing undue breakage in handling.

(6) Facilities for rescreening the stored coal, which, of course, increase the cost.

(7) Adequate available water supply.

(8) Low cost of installation, maintenance, and operation per ton of capacity. A storage plant is in operation very irregularly and costs are likely to be correspondingly higher because of the heavy fixed charges, especially interest and depreciation.

Few, if any, storage plants possess or require all these ideal conditions. In a coke plant, for instance, breakage need not be considered, except in connection with spontaneous combustion, since the coal is ground fine before being charged into the ovens. Storage facilities must of course be adapted to the various requirements and limitations in coal yards, power plants, railroad yards, boat docks, steel plants, and other establishments.

18. *Hand Operated Storage Systems.*—The simplest form of storage consists of dumping or shoveling the coal from a car or cart upon a pile or into a bin or bunker, or merely of dumping it on the ground. From this pile it may be shoveled directly into the furnace, or conveyed by wheelbarrow, scraper, or bucket line to the place of consumption. Under this classification may be included most of the systems of storage used by domestic consumers, retail coal yards, and small power plants. The equipment required for such a system is simple, and although it is sometimes the only system applicable to a given situation, it is not necessarily the cheapest form of storage.

The quantities stored in hand operated plants are relatively small, and the cost of storage is not usually separable from the other operating costs of the furnace or power plant. According to C. K. Baldwin,* “In transporting by wheelbarrows, gangs should be arranged to give room for the wheeler to load his own barrow. Should two men load with the wheeler idle, add thirty-five per cent to the time and cost of loading; when one loads with the wheeler, add twenty-five per cent. In carrying loads up and down a slope, add five per cent for each 4 degrees of slope. Hauling by wheelbarrows is more economical than by carts up to a distance of about 250 feet.”

19. *Storage by Motor Truck.*—An interesting experiment is being carried on at the University of Illinois in the stocking of Illinois coal from Seam Number 6 at Georgetown under the general

* Marks, Lionel S., “Mechanical Engineers’ Handbook,” p. 1140, 1916.

supervision of J. M. White, Supervising Architect, and J. A. Morrow, Superintendent of Buildings. The annual consumption of the University is about 30,000 tons, the daily minimum being 50 tons and the maximum 150 tons. For several years it has been customary to stock from 4,000 to 5,000 tons on the ground in piles about twelve feet high. The coal is thrown by hand from railroad cars upon the pile, distributed by scrapers, and then hauled by wagons to the power house. At times fires have occurred in these piles.

At the present time an area 114 by 196 feet is being used, and since this space was formerly used for tennis courts, the base is of firm, smooth clay. The storage space has on three sides a plank fence seven feet high, the posts being tied by wire rope to pegs within the enclosure. When the enclosure is filled to a depth of fifteen feet it contains about 10,000 tons. This storage plat is about one thousand feet from the power house where coal is received and in which is located the machinery for the necessary crushing and screening of the coal prior to storage. The coal is dumped from coal cars into a track hopper from which it is elevated. When intended for storage, the coal is diverted from the bunkers which feed the boilers. If the coal consists of screenings or the size of lump desired for storing, the railroad car dumps it into a pit from which it is elevated to a bin and then discharged into an end dumping motor truck. If lump coal or a size not desired for storing is received, it is crushed, if necessary, and screened; then by means of the storage truck which holds $31\frac{1}{2}$ tons it is taken to the storage ground. An illustration of the storage ground is shown as Fig. 2.

At the storage plat a truck running on the ground first builds up a bed of coal from two to five feet thick from the fence toward the center as shown in Fig. 3. When this center becomes too small to provide space for handling the truck, the operation continues on top of the first layer already in place as shown by Fig. 4, and a second layer is similarly deposited, the bed thus being built layer upon layer. To permit the truck to work on top of the pile of coal a track is built of pieces of scrap plank, two by three feet, woven together with galvanized wire or cable, as shown in Fig. 2. The cable is more flexible but more costly than the wire. This track is in sections, of from five to eight feet, and two lines are laid on which the truck runs. A track of wire fencing has been tried, but it is not stiff enough to provide a satisfactory running base for the truck.

An effort is made in storing the coal to have it thoroughly packed and to exclude the air as much as possible. Regular temperature observations are taken with an iron rod. This method differs from the ordinary pile storage in the tamping of the fine coal to exclude the air, and the experiment is being watched with a great deal of interest.

A plank track for the truck is laid between the storage pile and the power house, and by rapid loading and fast driving a round trip is made in from seven to eight minutes; thus nineteen tons are stored per hour. Three men are used on the pile for cleaning out the truck and spreading the coal. The coal is separated into two parts, screenings being placed on one side of the storage space and lump coal on another, but the two piles come together. The coal is reclaimed with an electrically operated Jeffrey wagon and truck loader (Fig. 5).

The expenses of handling are given in the following statement:

EXPENSE OF HANDLING COAL AT UNIVERSITY OF ILLINOIS POWER PLANT

Expense of labor for unloading coal per ton by hand from flat bottom cars at hopper with labor at 25 cents per hour.

Lump Coal	\$0.10
Screenings	0.08

(With Motor Truck Based on 15 Tons per hour and on Two Blocks Haul.)

Expense of labor and teams for loading screenings by hand and hauling in wagons from storage pile to track hopper 300 to 500 feet away, per ton.

Labor	\$0.13
Team	0.07
Total	\$0.20

Expense of labor and team for unloading screenings from bottom dump cars to hopper and for loading wagons from overhead bunker and hauling to storage pile, 300 to 500 feet away, per ton.

Labor	\$0.08
Team	0.03
Total	\$0.11

One team will haul 20 tons per hour to storage if loaded from overhead bunker and will return 7½ tons per hour from storage pile to hopper if loaded by hand.



FIG. 2. COAL STORAGE PLAT AND STORAGE PILE AT THE UNIVERSITY OF ILLINOIS



FIG. 3. PLACING THE FIRST LAYER OF COAL ON STORAGE PLAT AT THE
UNIVERSITY OF ILLINOIS



FIG. 4. PLACING THE SECOND LAYER OF COAL ON STORAGE PILE AT THE
UNIVERSITY OF ILLINOIS



FIG. 5. ELECTRICALLY OPERATED WAGON AND TRUCK LOADER USED AT THE
UNIVERSITY OF ILLINOIS



FIG. 6. RAILROAD STORAGE PILE SHOWING TRACK



FIG. 7. RAILROAD STORAGE PILE SHOWING CRIB FOR SUPPORTING TRACK



FIG. 8. RAILROAD STORAGE PILE SHOWING MEN AT WORK RAISING TRACK
THIS PILE CONTAINS 25000 TONS OF COAL

Expense of Storing Coal with Truck
(Truck will handle nineteen tons per hour)

Expense of hauling coal with truck	\$0.08
Expense of trimming pile, building roads, etc.,	0.06
Expense of unloading lump coal by hand from flat bottom cars, crushing, elevating, and loading trucks	0.20
Expense of unloading screenings by hand from flat bottom cars, elevating, and loading trucks	0.13
Expense of unloading screenings from bottom dump cars, elevat- ing, and loading trucks	0.07
Total	\$0.21—0.34

Expense of Removing Coal from Storage

1 Man running loader at 30 cents per hour	\$0.30
1 Man running truck at 30 cents per hour	0.30
1 Man leveling load at 30 cents per hour	0.30
1 Man at top of pile at 30 cents per hour	0.30
2 Men at plant hopper at 30 cents per hour	0.60
2 Men at feeding loader at 30 cents per hour	0.60
Truck operation and maintenance	0.60
Total expenditure per hour	\$3.00
Expenditure per ton	0.20

20. *Pile Storage from Cars without a Trestle.*—A method of storage commonly used by railroads consists of starting a pile on the ground, and raising the track gradually on top of the coal pile until a height of from ten to twenty feet is reached, the end of the track being supported on a crib as shown in Figs. 6 and 7. Fig. 8 shows such a coal pile with men raising the track. The coal is reclaimed with a locomotive crane, with a steam shovel, or by hand shoveling into cars. This is a simple form of storage, but the conditions are favorable to spontaneous combustion, because the weight of the locomotive and the loaded cars breaks up the coal and produces fine material in the center of the pile, while the lumps roll to the bottom of the outside slope and thus afford a flue through which the air may reach the fine material.

Figs. 6 and 7 show the effect of dumping side dump cars which were evidently so loaded that the lumps were on one side and the fine coal on the other. The north side of the pile, Fig. 7, contained fine coal and the south side, Fig. 6, the lumps. This pile took fire in several places and always on the fine coal side.

21. *Trestle Storage.*—Trestle storage consists of storing coal by dumping it from railroad cars run upon a trestle underneath which is located a storage bin. This method of storage is extensively used by large retail dealers and by factories and power plants. The coal is reclaimed by hand, by steam shovels, by locomotive cranes, by washing with water into conveyors, or by other suitable mechanical means. Although simple in construction and low in cost of the equipment, trestle storage produces excessive breakage, and unless drop bottom cars are available the expense of unloading by hand is high. It also requires considerable space if coal cars are to be pushed up an incline by a locomotive, or if some hoisting device must be installed.

Fig. 9 shows such a trestle arrangement in connection with a locomotive coaling station as suggested by the International Railway Fuel Association.

Fig. 10 shows two systems of trestle storage and of reclaiming by means of a tunnel, which may be either above or below ground, the coal being fed by gravity into a car or into some form of a conveyor in the tunnel. The breakage is excessive, both in stocking and in removing coal.

The introduction of a reloading tunnel decreases the cost of handling considerably, but with the tunnel above ground only from 30 to 35 per cent of the coal may be reloaded by gravity. With the tunnel underground about 50 to 60 per cent may be thus loaded directly. The cost of an underground tunnel plant is estimated as from 65 to 75 cents per ton storage capacity, and if the trestles are built of timber the expense of repairs and maintenance is considerable. A trestle and tunnel system using buckets for unloading and reloading the coal has certain advantages over the car system, but is limited in its capacity to from 500 to 5000 tons.

The amount of storage space may be increased by the building of bulkheads along the sides of piles as shown in the upper sketch of Fig. 10.

According to Norris,*

“An early type of storage-plant consists of wooden bins (Fig. 11) traversed by railroad tracks, from which the various sizes and types of coal are dumped, each in its appropriate bin. Reloading is usually accomplished by cars passing under the bins, either on the surface or more frequently in tunnels.

* “The Storage of Anthracite Coal.” Trans., Am. Inst. of M. E., Vol. 38, p. 340.

“To reduce the danger from fire, the movement of the reloading-cars is usually by gravity or by rope-haulage. The individual bins are necessarily limited in capacity to from 50 to 100 tons each, and an extensive plant covers a very large area. One such plant at the seaboard has 384 bins, reloading into cars in nine tunnels, and covers approximately nine acres. Such a plant costs in excess of three dollars per ton of capacity to erect, requires an enormous amount of timber, with resulting large fire-hazard and high maintenance-charges, and the operating expenses approach ten cents per ton.

“A great advantage is the practicability of storing many sizes and kinds of coal, and keeping separate many small consignments.”

The following formulas and table are used by the Link-Belt Company for determining the horizontal pressure of a pile of bituminous coal against a retaining wall. This pressure depends upon

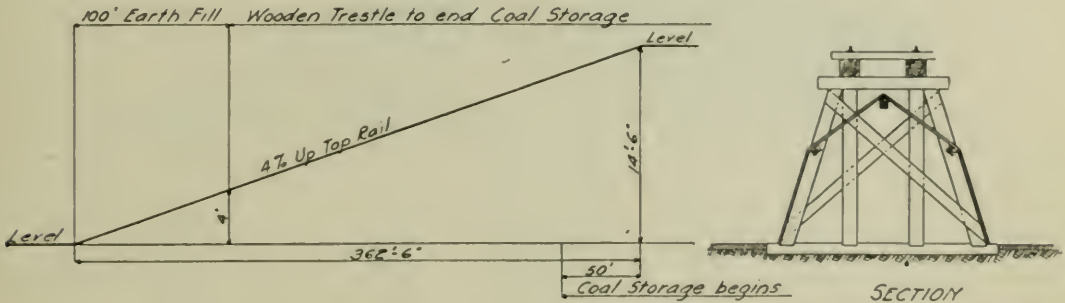


FIG. 9. TRESDLE FOR COAL STORAGE AS SUGGESTED BY FUEL STATION COMMITTEE OF THE INTERNATIONAL RAILWAY FUEL ASSOCIATION

the weight of the coal per cubic foot (assumed to be fifty pounds) and the depth of the material at the wall, and the slope of the surface of the pile.

For bituminous coal, let d represent the depth in feet. Then with surface of pile horizontal:

Total pressure in pounds on wall per foot of length $= 6.37d^2$

Pressure on wall on lowest foot of depth $= 6.37 (2d-1)$

With surface of pile sloping:

Total pressure in pounds on wall per foot of length $= 10d^2$

Pressure on wall on lowest foot of depth $= 10 (2d-1)$

Angle of repose $= 35$ degrees.

Table 3 gives these pressures in pounds for bituminous coal for every foot of depth up to 50 feet.

TABLE 3
HORIZONTAL PRESSURE EXERTED BY BITUMINOUS COAL AGAINST VERTICAL
RETAINING WALLS PER FOOT OF LENGTH

Depth in Feet	Horizontal Surface		Sloping Surface		Depth in Feet	Horizontal Surface		Sloping Surface	
	Total Pressure Pounds	Pressure Pounds Lowest Foot	Total Pressure Pounds	Pressure Pounds Lowest Foot		Total Pressure Pounds	Pressure Pounds Lowest Foot	Total Pressure Pounds	Pressure Pounds Lowest Foot
1	6.4	6.4	10	10	26	4,305	325	6,760	510
2	25.0	19.0	40	30	27	4,641	338	7,290	530
3	57.0	32.0	90	50	28	4,993	350	7,840	550
4	102.0	45.0	160	70	29	5,358	363	8,410	570
5	159.0	57.0	250	90	30	5,733	376	9,000	590
6	229.0	70.0	360	110	31	6,122	389	9,610	610
7	312.0	83.0	490	130	32	6,523	401	10,240	630
8	407.0	96.0	640	150	33	6,935	414	10,890	650
9	516.0	108.0	810	170	34	7,362	427	11,560	670
10	637.0	121.0	1,000	190	35	7,778	440	12,250	690
11	770.0	134.0	1,210	210	36	8,253	452	12,960	710
12	917.0	146.0	1,440	230	37	8,754	465	13,690	730
13	1,076.0	159.0	1,690	250	38	9,193	478	14,440	750
14	1,248.0	172.0	1,960	270	39	9,682	490	15,210	770
15	1,433.0	185.0	2,250	290	40	10,192	503	16,000	790
16	1,630.0	197.0	2,560	310	41	10,669	516	16,810	810
17	1,840.0	210.0	2,890	330	42	11,236	529	17,640	830
18	2,063.0	223.0	3,240	350	43	11,797	541	18,490	850
19	2,298.0	236.0	3,610	370	44	12,331	554	19,360	870
20	2,548.0	248.0	4,000	390	45	12,968	567	20,250	890
21	2,809.0	261.0	4,410	410	46	13,478	580	21,160	910
22	3,083.0	274.0	4,840	430	47	14,100	592	22,090	930
23	3,369.0	287.0	5,290	450	48	14,679	605	23,040	950
24	3,669.0	299.0	5,760	470	49	15,275	618	24,010	970
25	3,981.0	312.0	6,250	490	50	15,925	631	25,000	990

NOTE.—Weight of coal is taken as 50 pounds per cubic foot in calculating this table.

In the parallel trestle system (Fig. 12) two parallel trestles from ten to fifteen feet high are located about thirty feet apart. A locomotive crane runs on one of these trestles, unloads coal from railroad cars on the other trestle, and deposits it for storage in a pile the length of which is limited only by the length of the trestle. The crane and the cars are used interchangeably on the trestles so that the storage system consists of parallel piles. A pile of anthracite twenty-five feet high, whose angle of repose is 27 degrees, contains sixty-seven tons per running foot. For bituminous coal a pile of the same height, with an angle of repose of 40 degrees, contains 63 tons per running foot.

22. *Storage with Side Dump Cars.*—The Chicago, Wilmington and Franklin Coal Company at the Orient Mine, Franklin County, Illinois, has installed a very simple but effective storage system designed by G. B. Harrington, President of the Company, in which side air dump contractors' cars are used. These cars hold twelve cubic yards, but with sides built twice the usual height they have a capacity of about twenty-five tons of coal. In a level field located a short distance from the tippie, an elevated track was built, as shown in Fig. 13, from which the side dump cars empty the coal alongside the

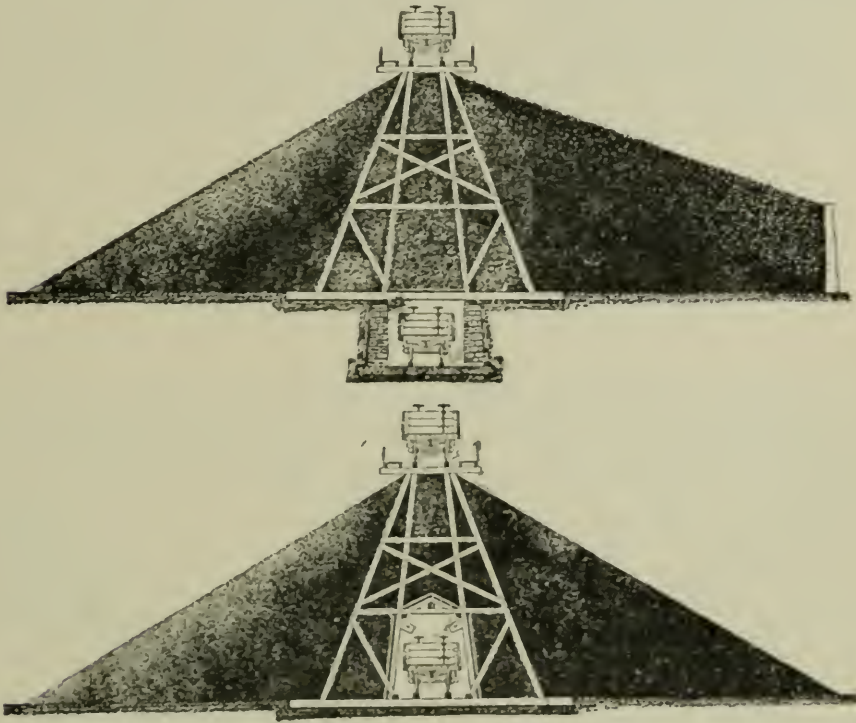


FIG. 10. TRESTLE AND TUNNEL SYSTEMS, SHOWING AN ABOVE-GROUND AND A BELOW-GROUND TUNNEL

track. Two locomotive cranes with two-yard clam-shell buckets, one on each side of the trestle, move the coal into storage piles parallel with this main elevated track. The dump cars are loaded at the tippie, as shown in Fig. 14.

The purpose of this plant is to permit continuous operation of the mine on days or parts of days when there is a failure in the railroad car supply and the mine would otherwise have to shut down. When railroad cars are scarce, the air dump cars take the output to the storage trestle. The cranes move the coal back into the storage piles



FIG. 13. STORAGE TRACK AND SIDE DUMP CARS AT THE ORIENT MINE OF THE CHICAGO, WILMINGTON AND FRANKLIN COAL COMPANY

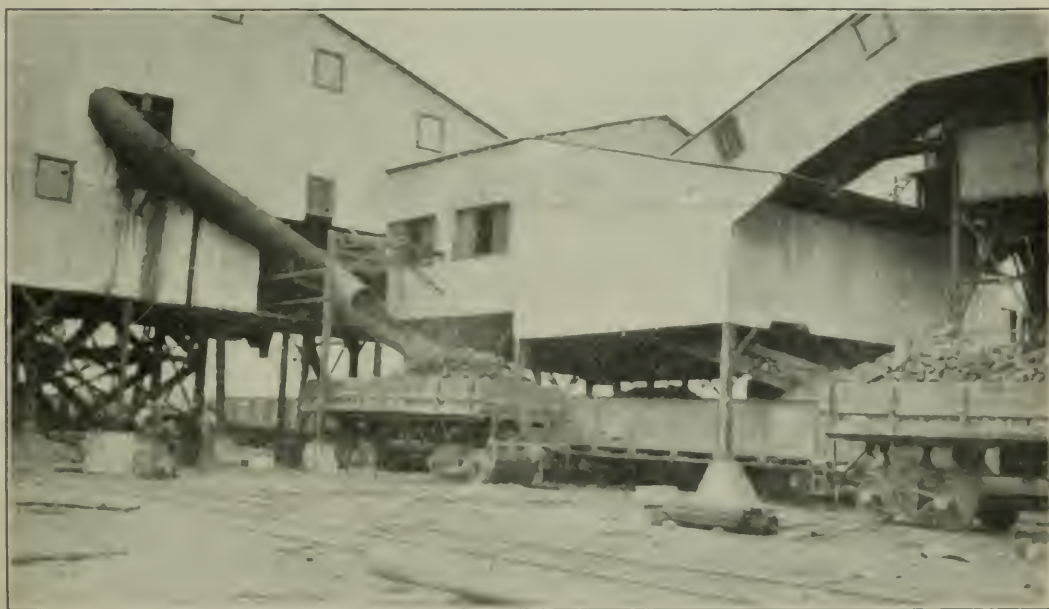


FIG. 14. LOADING DUMP CARS AT THE TIPPLE, ORIENT MINE OF THE CHICAGO, WILMINGTON AND FRANKLIN COAL COMPANY

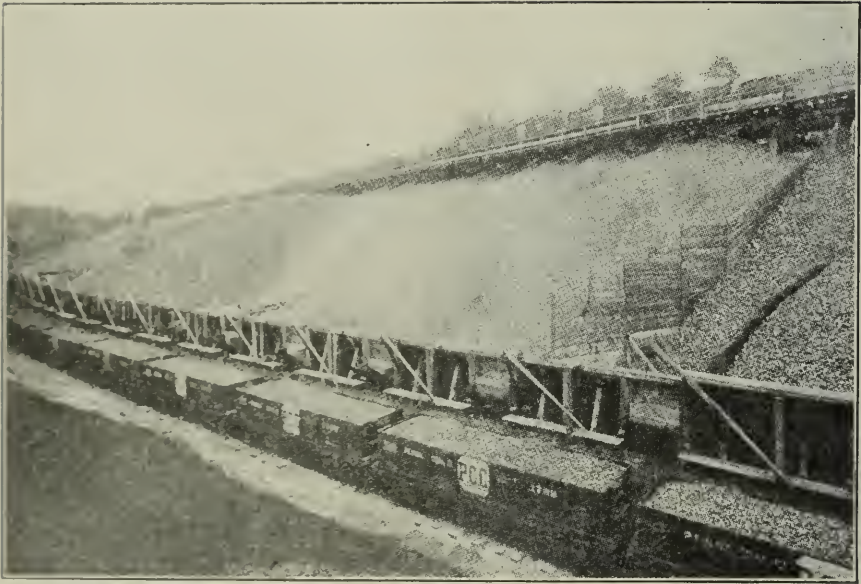


FIG. 15. SIDE HILL SYSTEM OF STORAGE

When not in use for storage the dump cars and cranes are useful in handling refuse, ashes, machinery and equipment, and for grading and other work at the mine.

The capacity of the Orient No. 1 Mine is from 4,500 to 5,000 tons a day, and the storage equipment described has proved effective in

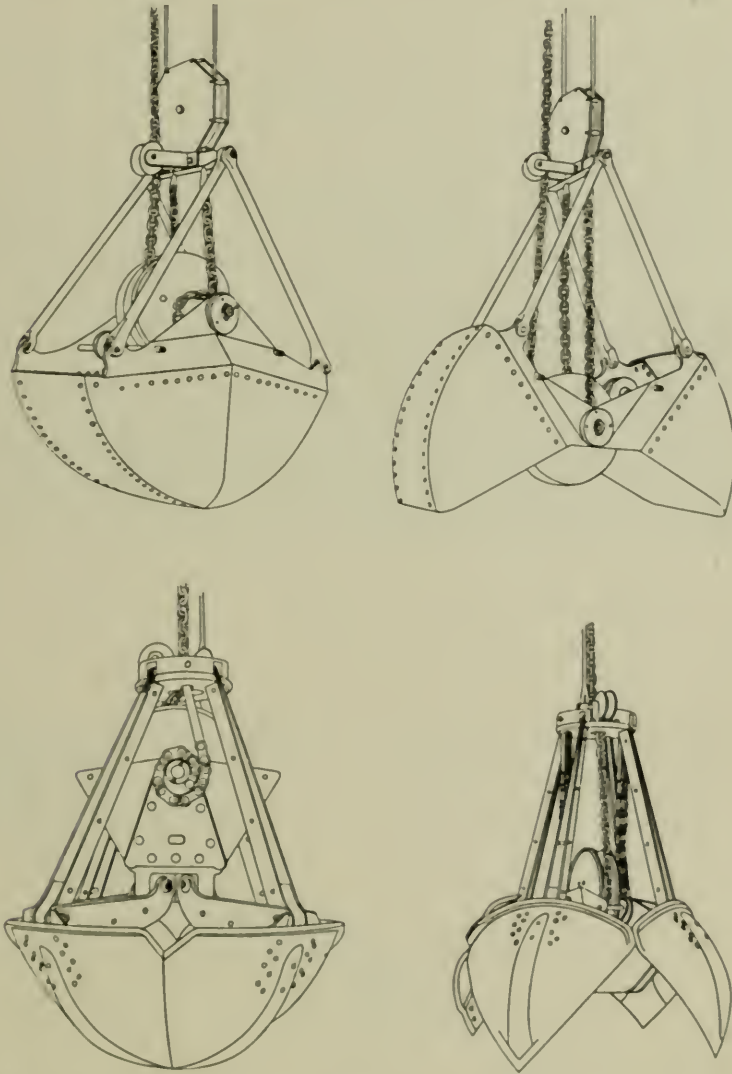


FIG. 16. CLAM-SHELL BUCKET (TOP) AND ORANGE-PEEL BUCKET

helping to maintain a fairly uniform output with an inadequate and irregular car supply. It takes about thirty minutes to load a string of fifteen dump cars at the tippie, to run them down to the storage trestle about a mile away, to dump them, and to return the empty cars to the tippie. The trestle and track hopper are designed to

save unnecessary breakage and the coal shows practically no degradation in the few days during which it is held in storage.

In normal times these dump cars cost about \$1,350.00 each and the cranes about \$5,800.00 each. The trestle used is about nine feet high and costs about nine dollars per running foot.

A number of other varieties of dump cars has recently been used for similar storage purposes at other plants. In one instance all steel cars of thirty cubic yards capacity have been substituted for the smaller cars.

23. *Side Hill Storage.*—In the side hill system of storage (Fig. 15) a steep hill side is utilized, and coal is unloaded by gravity from cars on a track or trestle at the upper side of the storage yard. At the bottom of the pile a retaining wall holds back the coal, and below this cars are loaded by chutes running into the pile. In order to increase the capacity of such a plant a level space is usually provided back of the retaining wall, but this produces a dead space from which the coal must be shoveled and in which when using the chutes a large amount of coal is left as dead stock. While side hill storage appears to be ideal in its arrangement and possibilities, there are comparatively few situations in which it can be applied conveniently and profitably.

24. *Self-filling Buckets.*—The type of bucket used for handling coal in connection with locomotive cranes, traveling bridges, and other mechanical storage devices is a matter of some importance. Coal must be handled quickly and in such manner as to prevent excessive breakage.

The two types of self-filling buckets most generally used in the handling of coal are the so-called clam-shell and orange-peel types (see Fig. 16). Both types are reported to be satisfactory.

25. *Use of Mast and Gaff Arrangement in Storage.*—In the simpler forms of coal handling machinery the ordinary mast and gaff arrangement (Fig. 17) is cheap and efficient when rapidity of handling and maximum storage are unnecessary.

The traveling cableway (Fig. 18) is more efficient so far as utilization of ground area is concerned but is slow in stocking and reclaiming where long lengths of travel are necessary.

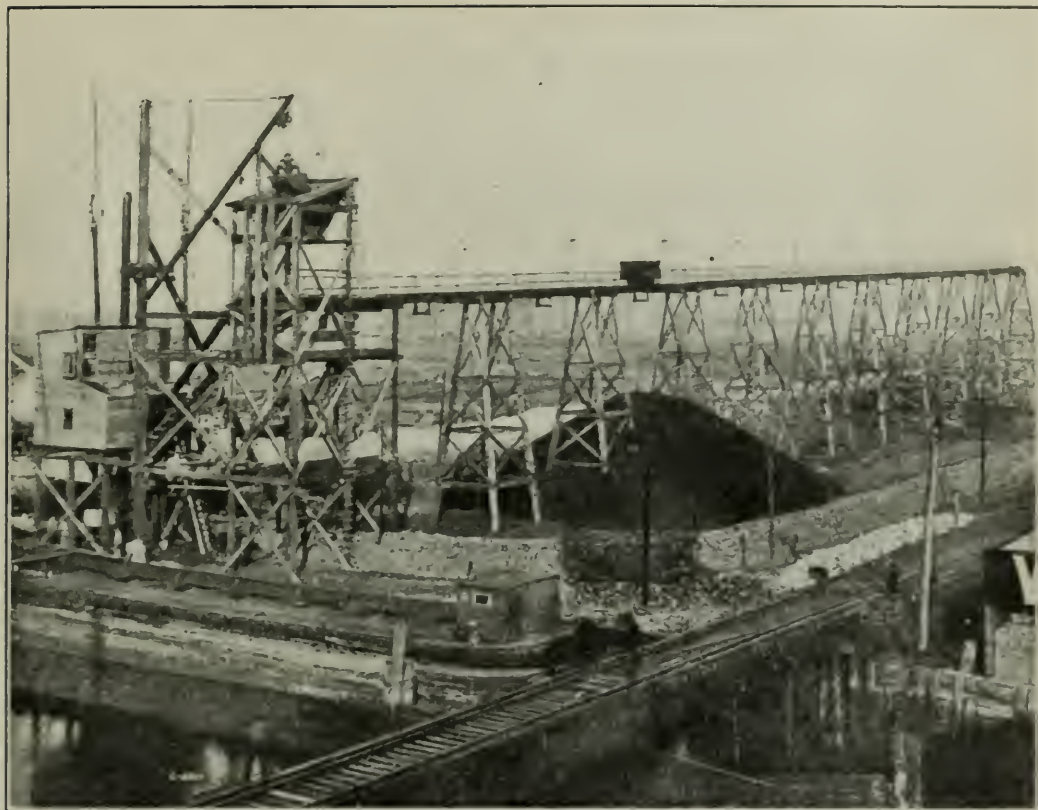


FIG. 17. AUTOMATIC GRAB ON MAST AND GAFF WITH SHUTTLE CABLE RAILWAY



FIG. 19. LOCOMOTIVE CRANE WITH CLAM-SHELL BUCKET UNLOADING COAL

26. *Locomotive Crane Storage.*—The device most generally used for storage except when very large special equipment is necessary, is the revolving locomotive crane equipped with a clam-shell or orange-peel bucket. Locomotive cranes are used particularly in large industrial plants.

Fig. 19 shows a locomotive crane operating from a low trestle and placing coal either into the bunker or on stock pile.

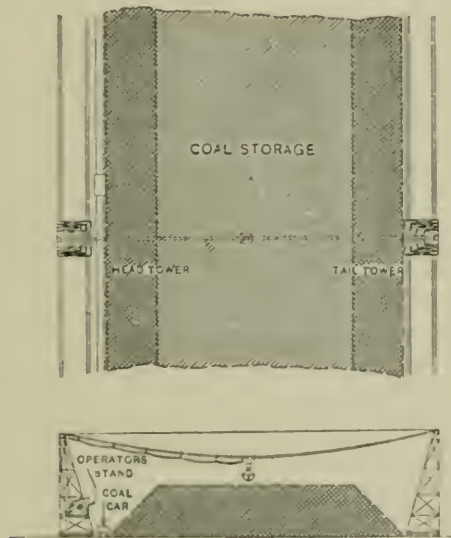


FIG. 18. TRAVELING CABLEWAY

These cranes are self-propelled at the rate of from four to eight miles per hour, and may also be used for shifting cars over small distances, thus eliminating the necessity of a locomotive. They are generally operated by steam, although sometimes electric power is employed. The revolving superstructure is supported on a base which rests on four to eight wheels, the eight wheeled car body being preferable for switching and for general work. The gage of the track on which these cranes run may have to suit existing conditions at any one plant, but when choice is possible the gage should be approximately as follows:

- 4 feet, 8½ inches for a maximum radius of swing of 30 feet.
- 14 feet, 6 inches for a maximum radius of swing of 45 feet.
- 16 feet for a maximum radius of swing of 60 feet.
- 20 feet for a maximum radius of swing of 100 feet.

The operator, who is on the covered platform, fills the bucket, and as he raises it the crane revolves to the unloading point. The speed of operation varies greatly with the operating conditions, but the speed generally averages about one bucket per minute.

Prices of machinery quoted during war times may be misleading, since they are generally higher than in normal times and they vary so rapidly that any quotation given by a manufacturer holds for only a short time. On account, however, of the extensive use of the locomotive crane for storing coal the following prices quoted by several firms in April, 1917 are presented:

(1) Standard 14-ton, 4-wheel steam locomotive crane with boom 35 feet, 4 inches long and double drums for operating a 54-cubic-foot grab bucket, f. o. b. Champaign, \$11,475. This crane will handle from $1\frac{1}{4}$ to $1\frac{1}{2}$ tons per minute, and under average operating conditions should easily make from sixty to ninety bucket trips per hour.

(2) Fourteen-ton, 4-wheel steam locomotive crane with boom 40 feet, $9\frac{1}{2}$ inches long, and double drums, for handling a 40-foot grab bucket, f. o. b. to Champaign, \$11,515. This crane will handle from 1800 to 2000 pounds per minute at about the same rate as crane (1).

(3) Standard 8-wheel crane with boom 40 feet, $9\frac{1}{2}$ inches, and double drums for handling a 54-cubic-foot grab bucket, \$14,600.

For a small amount of travel the 4-wheel crane will handle as much coal per day as the 8, but on account of its rigid base it is not as suitable for moving cars on curves or over general yard tracks.

(4) Fifteen-ton, 8-wheel crane with boom 40 feet, $9\frac{1}{2}$ inches to handle a 50-cubic-foot bucket at 40-foot radius, net shipping weight 100,000 pounds, \$14,600.

(5) Similar but heavier crane to handle 40 cubic feet at 50-foot radius, \$14,960.

(6) Standard gage steam operated revolving locomotive, 8-wheel double track crane with 50-foot boom, and two-cubic-yard clam-shell bucket, \$14,300.

(7) Standard gage steam operated revolving locomotive crane, 4-wheel 42-foot boom 2-cubic-yard clam-shell bucket, \$11,893.

J. S. Shearer, of the Industrial Works, Bay City, Michigan, says :

“In some cases a crane with a 45-foot boom is able to unload the far end of the car standing on the same track as the crane. At the present time, however, when cars are being made longer than before, we find it to be almost always the case that a boom giving a radius of fifty feet is necessary. If a crane is to work on a track parallel with one from which the coal is to be taken a shorter boom is, of course, possible.

“We formerly figured on the daily operating cost as approximately one dollar an hour. This contemplates using the crane ten hours a day and in the cost are included the services of an engineer, a helper on the ground, interest, depreciation, fuel, supplies, and repairs. At the present time when the first cost of equipment is higher and the cost of all supplies higher, we think you would have to take \$1.50 to \$2.00 an hour as the approximate charge. Under average operating conditions the crane would have no difficulty in handling from forty to seventy-five tons per hour. This would make the cost per ton somewhere around three cents. We formerly found that the cost averaged about two cents, sometimes being less and sometimes more.”

The Storage of Coal Committee of the International Railway Fuel Association* says with regard to storing with a locomotive crane :

“We would suggest the employment of a locomotive crane with clam-shell or similar device for unloading and reloading storage coal, where the amount to be stored is less than five thousand tons and where the daily issues are small enough to permit of its use. This arrangement requires no preliminary preparation and little or no additional expense. The cost of unloading from the road cars may be cheapened when dump bottom cars are available, by unloading them on tracks that have been blocked upon old car and bridge timbers, so placed and arranged that the structure can be raised bodily from time to time, by the use of a locomotive crane, working on an adjacent and parallel track.”

The ways in which a locomotive crane may be used in connection with storage of coal are numerous, and only some of the typical ones will be given. Catalogs of the manufacturers of these cranes are the best source of information regarding their uses.

At the coal yards of the Commonwealth Edison Company a locomotive crane with a two-yard clam-shell bucket will ordinarily unload from ten to fifteen cars per day of eight hours. One operator has loaded as many as twenty-five cars in ten hours. The amount is less in cold weather and in handling large coal. Two men are employed on the crane, and two shovel out the cars and clean the track. The crane can load faster than it can unload the cars.

According to W. L. Abbott, Chief Engineer of the Commonwealth Edison Company of Chicago, that company aims to keep in storage

* "Storage of Coal." Proceedings, Vol. VII, p. 152, 1915.

at its various city stations from 100,000 to 150,000 tons of coal and at the Glenn storage yards, outside the city limits, about 260,000 tons. All varieties of Illinois coal are stored, but those of southern Illinois are preferred. Coal is placed in storage and reclaimed by locomotive crane at an estimated expense of five cents per ton for unloading and five cents for reclaiming. This covers labor and materials only.

According to the storage scheme devised by Mr. Abbott, coal is stored on the ground in continuous pyramidal piles twenty-five feet high, each pile being between and parallel with two pairs of railroad tracks, as shown in the upper sketch, Fig. 20. The tracks bounding

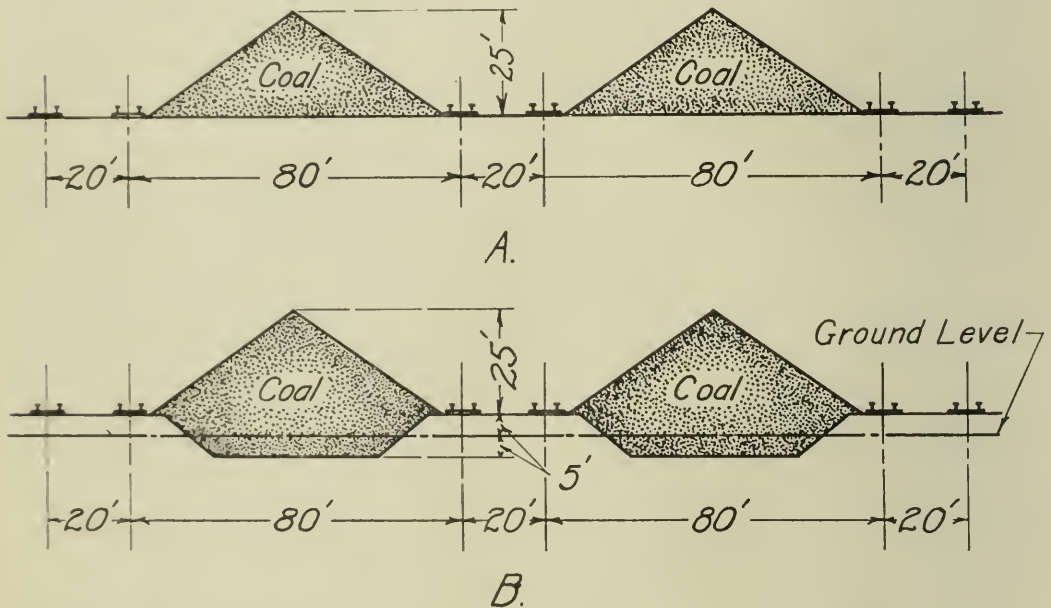


FIG. 20. ARRANGEMENT OF TRACKS AND STORAGE PILES EMPLOYED BY THE COMMONWEALTH EDISON COMPANY IN OPEN STORAGE

the piles are eighty feet, center to center, and the tracks between the piles twenty feet, center to center. The crane can operate from either track while loading or unloading cars on the parallel track. These piles contain about thirty tons per foot of length.

This company is now preparing a new storage yard for the accommodation of 120,000 tons of coal in which the general arrangement will be similar to that shown in the upper view, Fig. 20, with the difference that the ground under the coal space will be excavated to a depth of five feet below the surface, and the spoil will be used to raise the grade of the tracks five feet above the ground level, thus render-

ing possible a coal pile of a total depth of thirty-five feet and containing forty-five tons per foot of length, as shown in the lower sketch, Fig. 20.

27. *Parallel Track Storage.*—Fig. 21 shows a double track storage arrangement for a locomotive coaling station, the coal being piled between the parallel tracks, and loaded and unloaded with a locomotive crane.

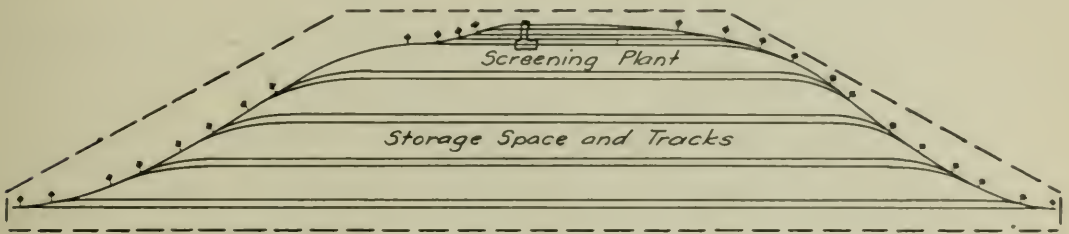


FIG. 21. PARALLEL TRACK STORAGE SYSTEM

(Reprinted from the Proceedings of the International Railway Fuel Association)

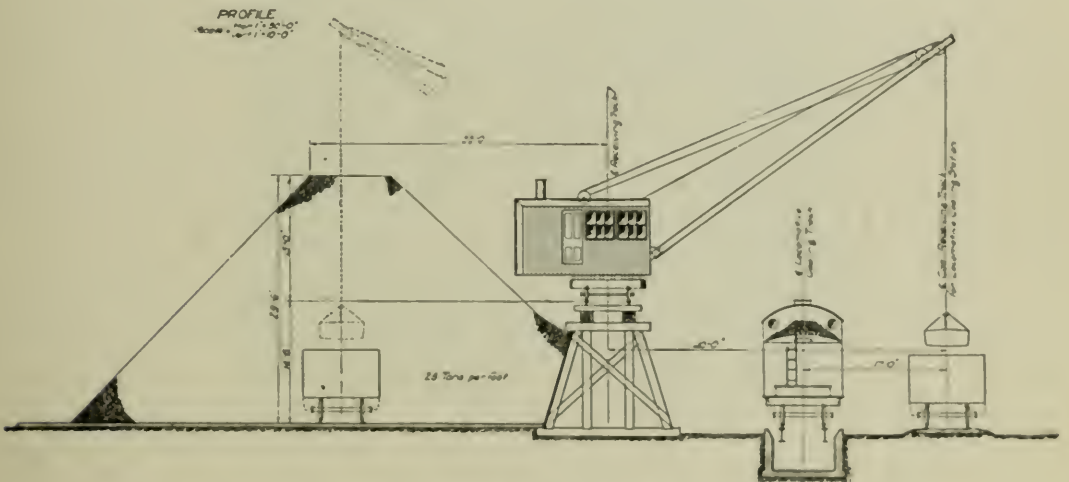


FIG. 22. RAILWAY TRESTLE AND CRANE SYSTEM OF STORAGE

(Reprinted from the Proceedings of the International Railway Fuel Association)

28. *The Trestle and Crane System.*—Fig. 22 shows a combined trestle and locomotive crane system suggested by the Fuel Station Committee of the International Railway Fuel Association. A timber trestle erected adjacent to the track on which locomotives are coaled and fireboxes cleaned provides a runway for a locomotive crane equipped with a clam-shell or grab bucket.

The Committee* states:

"It is advisable to place a plank wall or barrier along the side of the trestle contiguous to the storage pile, to prevent coal from accumulating under the trestle, thus obviating the necessity of reclaiming such accumulation by the employment of laborers.

"Adjoining the trestle, sufficient ground is reserved for the storage pile, which will require a strip of land about sixty feet wide, and varying in length in proportion to the capacity of the pile.

"The plan provides for a storage capacity of twenty-eight tons per lineal foot of coal pile.

"The operation of this plan is as follows:

"Coal will be delivered to the coal-receiving track in gondola cars of practically any type, from which it will be removed and transferred to the storage pile by the locomotive crane.

"When it is desired to reclaim the storage coal, the foregoing operation is reversed, the crane picking up the coal from the pile and reloading it in cars, which are then hauled away to be discharged at any of the coaling stations served by the storage plant. Or, if desired, this plant may be used for coaling engines either from cars on the receiving track or from the storage pile. It is not considered advisable to use this plant as a locomotive coaling station at terminals where a large number of engines are coaled, as delay might be caused in the rapid movement of engines, due to the comparative slowness of the crane as a coaling device. In the event of a serious breakdown in the coaling station, the crane would afford a very good emergency device for coaling engines while repairs were being made to the coaling station.

"From somewhat meager figures available it is estimated that coal can be stored from cars, or reloaded from storage at about two and one-half cents per ton."

Fig. 23 shows a McMyler locomotive crane placed on an elevated traveling platform.

The Clinchfield Fuel Company with mines at Dante, Va., has a storage plant at Spartanburg, S. C., which has a capacity of 150,000 tons and in which from 10,000 to 130,000 tons are kept in storage for periods of from six months to two years. The storage plant is 994 feet by 240 feet with a railway running through the center of the storage space. On each side of this trestle are tracks for a traveling crane. The trestle is approximately twenty feet high and incoming coal is dropped through this trestle from hopper bottom cars. A McMyler traveling crane with a 75-foot boom and a 2.5-ton grab bucket picks up the coal from the foot of the trestle and piles. The crane has a capacity of two hundred tons per hour.

The storage space is floored with two-inch planks nailed securely to timbers. The tops of these timbers are set flush with the ground level.

* "Storage of Coal." Proceedings, International Railway Fuel Association," Vol. VII, p. 156, 1915.

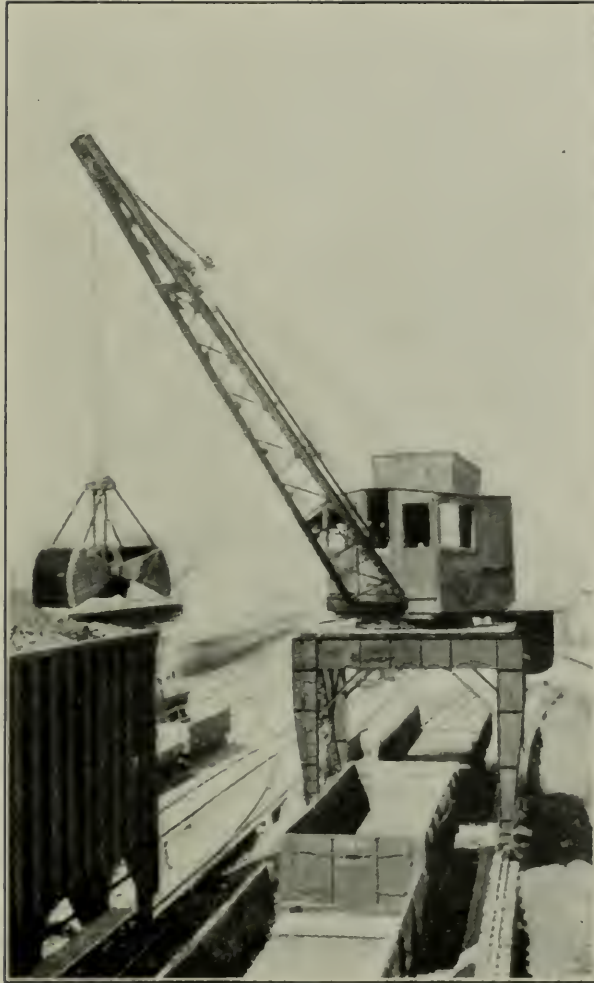


FIG. 23. LOCOMOTIVE CRANE PLACED ON AN
ELEVATED TRAVELING PLATFORM

The pile is continuous, and from thirty to forty feet high. The expense of storing is as follows:

Unloading or Storing Expense, June 24-October 4, 1912, including pay roll, repairs, supplies, power, depreciation at 10 per cent per annum and interest at 5 per cent.		\$8441.
Tons handled 125,696; expenditure per ton		\$0.0671
Reloading expense, October 8, 1912-January 31, 1913		\$8496.
Tons handled 129,778; expenditure per ton		0.0655
Total average expense of storing and reclaiming at 13.26 cents per ton. Unloading expenses May 1—August 1, 1914, tons handled 131,949; expenditure per ton		0.0618

The coal is from Russell County, Va., and is run of mine with about sixty per cent lump over a two-inch screen. No attempt is made to remove slack. It has the following composition:

	Per cent
Moisture	2.8
Volatile Matter	34.9
Fixed Carbon	56.6
Ash	5.8
Sulphur	.6
B. t. u.	.14150

It is placed in storage about six to ten days after being mined and during the summer time; thus the surplus from the mine is stored when the demand is below the output of the mine, provides for extra demand in winter, and insures customers against strikes and other contingencies.

There is no difference in the price of stored and fresh coal. In fact storage coal is sometimes higher because of "spot" demand. No material decrease in heating value is noted, but there is a decrease of ten per cent in lumps. Slight heating occurs after three or four months, and in some cases reaches 170 degrees F. The temperature is tested by tubes placed in each pile. No means, however, are taken to prevent heating, but if it occurs the coal is loaded out at once and shipped.

29. *Circular Storage.*—The peculiar adaptability of the locomotive crane to operation in a circle and its easy portability have led to a number of so-called circular storage systems. Some of these have been patented by the J. M. Dodge Company. Fig 24 illustrates a

Dodge type of circular system with a capacity of from 6,000 to 40,000 tons. The coal is dumped from a railroad car into a track hopper from which it is taken by a long radius locomotive crane and self-filling bucket and dumped upon piles, the circular crane track being finally completely covered. A number of piles may be joined, as shown. In reloading, the crane takes the coal from the pile and loads it directly into the car. The rate of handling in such a storage system

	Stocking	Reclaiming
Labor and supplies per ton	\$0.025	\$0.014
Power and superintendence	0.016	0.014
	\$0.041	\$0.028

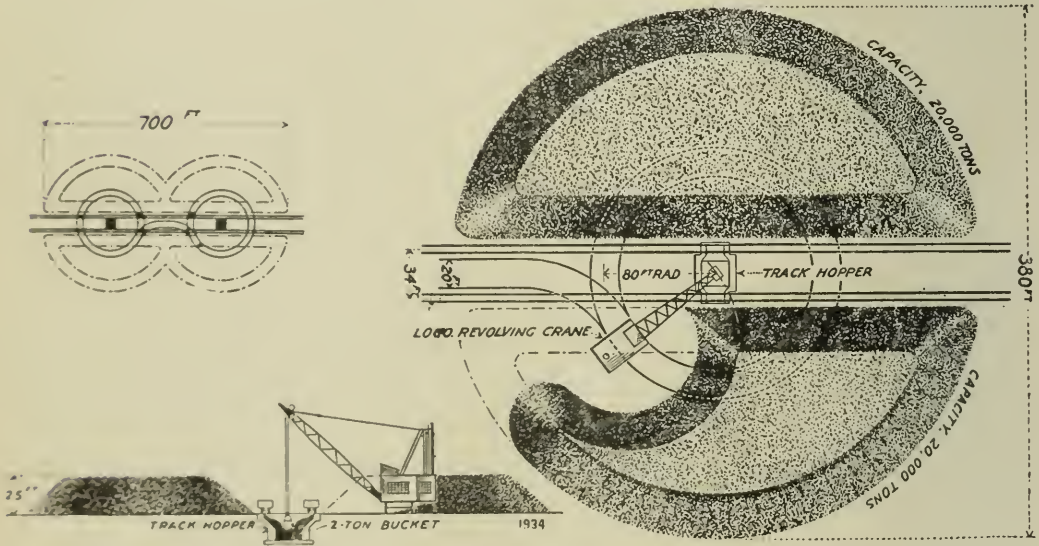


FIG. 24. DODGE TYPE OF CIRCULAR STORAGE SYSTEM

varies from 40 to 200 tons per hour, according to the size of the bucket and crane used. The length of the boom depends upon the storage capacity required. The estimated expense of operation is as follows:

An elaboration of this system by which concentric storage piles may be built up is shown by Fig. 25. By rehandling the coal in such a system from one pile to another an almost unlimited capacity may be secured.

Table 4 gives the storage capacity with this system for cranes of different radii and for the two conditions, first where the crane tracks are covered, and secondly, where the crane trucks are left uncovered. The railroad tracks are assumed to be twenty-seven feet center to center and the coal to weigh fifty pounds per cubic foot.

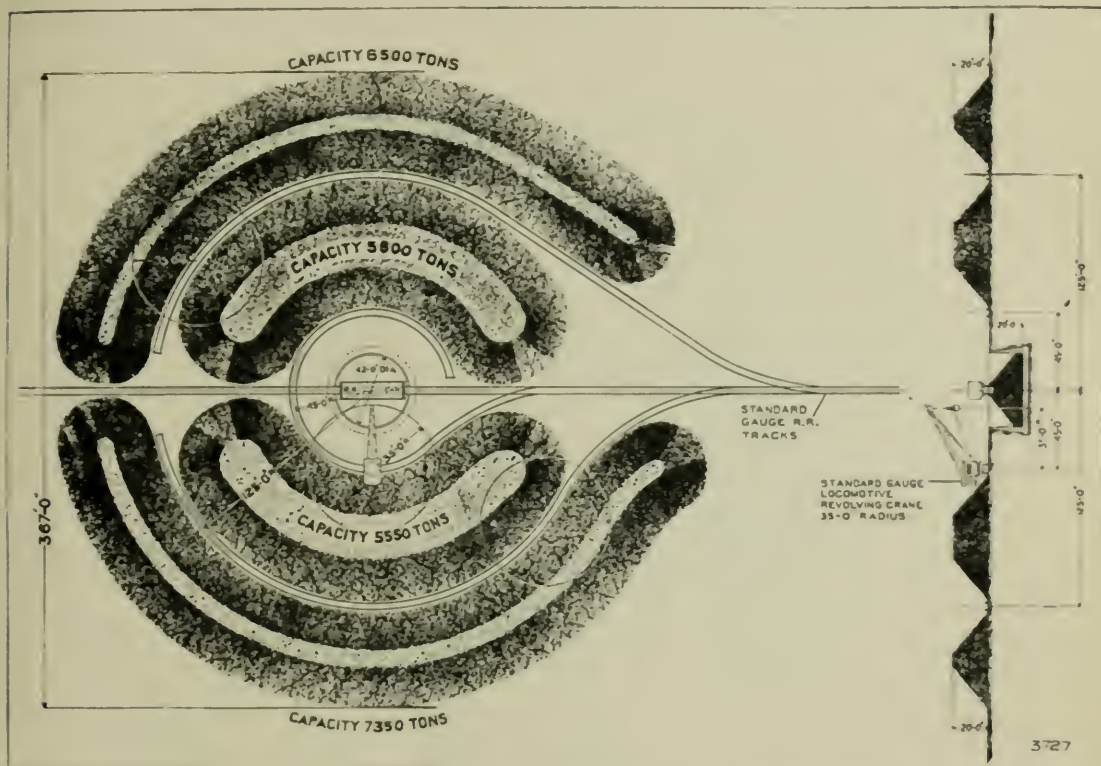


FIG. 25. TWENTY-FIVE-THOUSAND-TON STORAGE PLANT

The storage plant of the Old Ben Coal Corporation at its No. 9 Mine at West Frankfort, Franklin County, Illinois, (Fig. 26) is a modified circular storage. At this mine five sizes of coal are prepared for the market by means of spiralizers and seven sizes are shipped.

When there are no cars for direct shipment, the coal instead of being deposited in the shipping bins, *aa*, is carried by the apron conveyors, *bb*, delivered to a cross conveyor, *c*, and then deposited by the chute, *d*, upon the pile, *e*. It is then taken by a locomotive crane, *f*, which has a boom 110 feet long operating over a 100-foot radius. The capacity of the plant is about 300,000 tons. By means of an additional track outside the storage pile, the capacity could be greatly increased.

TABLE 4
CAPACITY OF CIRCULAR CRANE STORAGE
(Link-Belt Company)

Radius of Crane (Feet)	Crane Tracks Covered (Feet)						
	10	15	20	25	30	35	40
35			7,000				
40			9,600	12,400			
45			11,500	16,000			
50			15,000	19,500	25,300		
55			17,600	24,000	29,200		
60			22,700	28,500	35,400	42,100	
65			25,600	32,800	39,100	46,900	
70			30,500	39,100	46,300	54,300	
75			32,600	44,300	55,600	63,800	
80			39,440	49,700	60,800	69,600	83,200
85			43,600	56,100	68,300	82,300	94,400
90		34,500	46,300	58,100	70,100	84,250	96,200
95		39,130	53,220	66,540	78,800	92,300	106,200
100		43,500	57,700	74,000	89,300	103,900	123,000
110		53,500	71,300	89,000	107,700	123,800	142,300

Radius of Crane (Feet)	Crane Tracks Uncovered (Feet)						
	10	15	20	25	30	35	40
35			6,050				
40			6,850	8,875			
45			9,280	11,720			
50			11,200	14,000	17,600		
55			13,500	17,100	21,000		
60			15,900	20,400	25,100	29,600	
65			19,400	23,700	29,100	34,500	
70			22,300	29,600	34,200	39,300	
75			25,300	30,800	39,500	45,800	
80			30,100	37,600	45,200	53,600	61,500
85			34,400	41,900	50,700	60,500	71,100
90		31,200	41,500	52,100	60,800	70,800	80,600
95		32,900	43,860	54,500	63,000	72,100	82,500
100		36,200	47,000	60,000	69,000	79,100	90,600
110		45,500	60,100	74,000	85,000	94,800	106,500

In reclaiming, the crane delivers the coal upon the pile, *e*, which feeds into the boot, *g*. Thence by means of a gate and feeder the coal is delivered to the elevator, *h*, which in turn delivers it to the apron conveyor, *i*, which returns it to the screens and the spiralizers. If possible, coal of different sizes in storage is kept in separate piles, but if the plant is crowded all sizes are stocked together.

According to D. W. Buchanan, President of the Company, the cost of such a plant erected under war conditions would be approximately \$100,000. The expense of operation covering labor and supplies for stocking and reclaiming is about five cents per ton.

At the No. 8 mine of the same company, coal is stored by unloading cars with locomotive cranes upon piles parallel with the track and reclaiming it by means of the cranes.

At the mines of the Old Ben Corporation located in and near Christopher, a method of storage similar to that described as being in use at the Orient Mine of the Chicago, Wilmington and Franklin Coal Company (p. 51) has been established. This company has two mines at Christopher and two at Buckner. One mine of each pair has

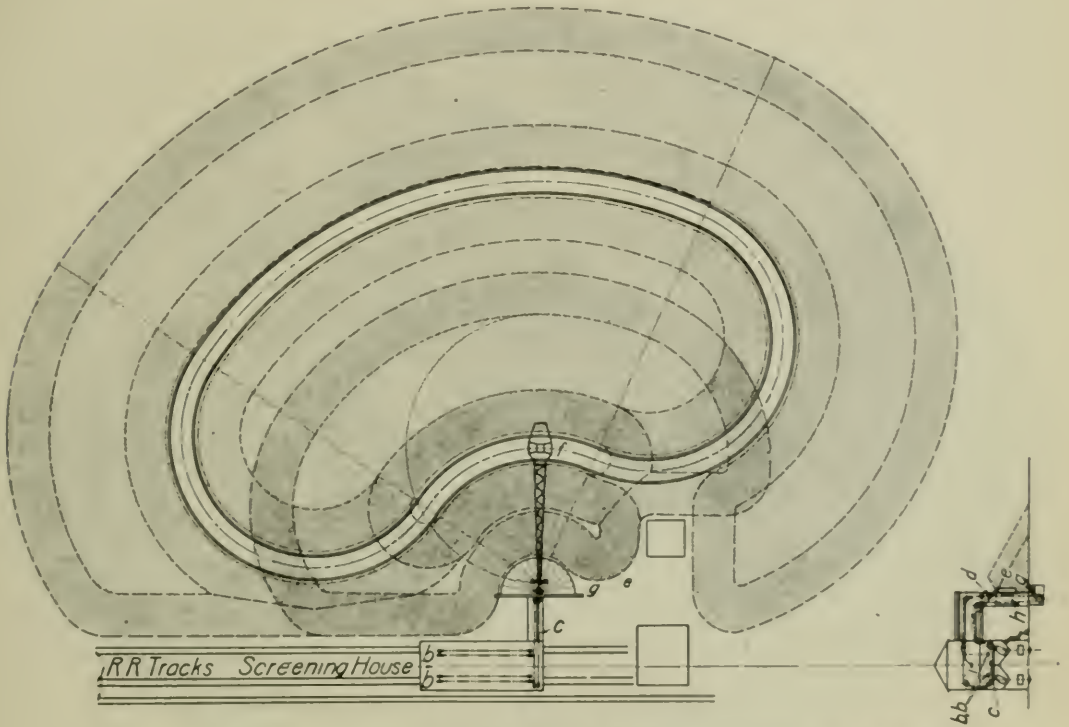


FIG. 26. ARRANGEMENT OF THE STORAGE PLANT OF THE OLD BEN COAL CORPORATION AT WEST FRANKFORT, ILLINOIS

been chosen as a storage point and the system of storage employing side dump cars and a locomotive crane has been established.

Many other adaptations of the semicircular or circular pile may be made, as, for instance, by placing the pile near a power house or a coaling station so that the crane in reclaiming can deposit the coal in a depressed hopper from which it is elevated into the power house, or coaling station (Fig. 27). For locomotive coaling the crane may deposit the coal into an overhead bin spanning the track as shown in Fig. 28.

According to the Storage of Coal Committee,*

* Proceedings, International Railway Fuel Association, Vol. VII, p. 159, 1915.

"The locomotive crane has been adopted by the Louisville & Nashville R. R., at three locations, where large mechanical coaling stations of reinforced concrete and steel construction have been erected (Fig. 29).

"The receiving hopper is enlarged at the back to form a pit of sufficient size to accommodate the grab bucket of the crane. The crane is located on a circular track back of this hopper, this track centering on the hopper. Coal is dumped into the hopper from the receiving track, and handled to storage by the crane. In reclaiming from the storage pile, coal is picked up by the crane and discharged into the hopper, from which it is handled to the overhead coal pocket in the usual manner.

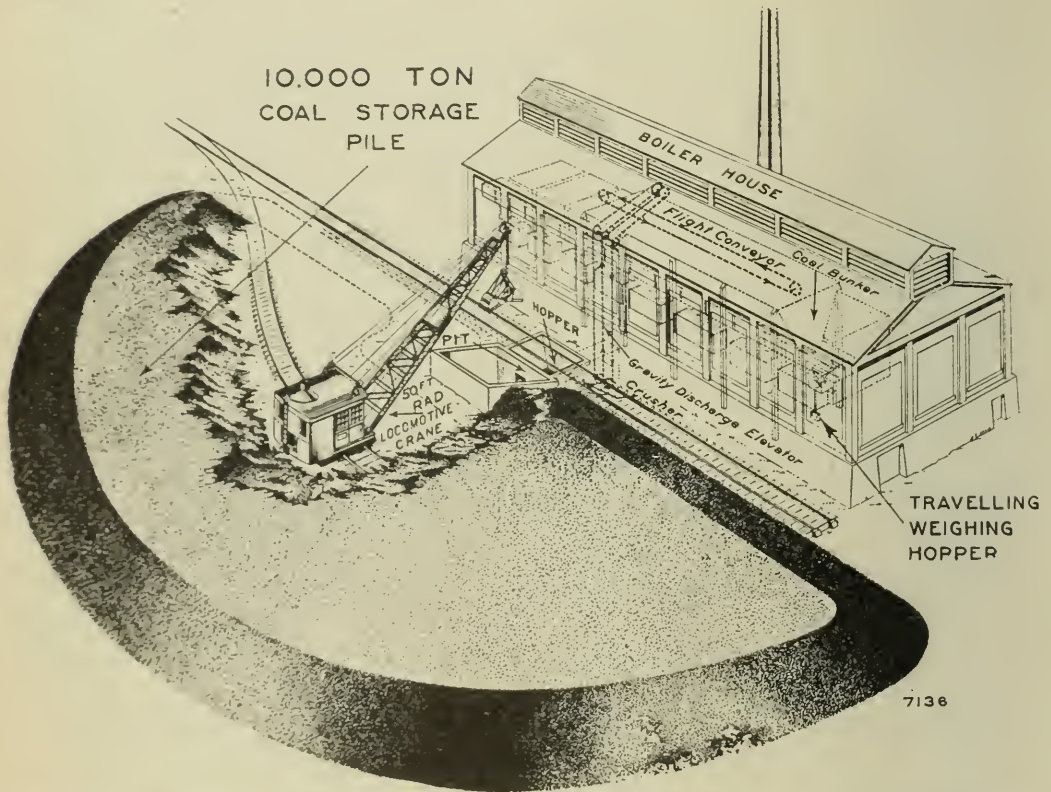


FIG. 27. SEMICIRCULAR STORAGE ADAPTED TO LOCATIONS NEAR POWER HOUSE OR COALING STATIONS

"A special crane of 110 feet radius is used by the Louisville & Nashville R. R., the crane track gage being 20 feet. In this way they secure a 22,000 ton storage with a 12-foot depth of pile. The crane uses a $3\frac{1}{2}$ -ton bucket, and has a working capacity of 150 tons per hour.

"An ordinary crane with a 40 or 60-foot boom could be used as well, but of course the size of storage pile would be reduced accordingly. The smaller crane would have some advantage in that it might be used for other purposes when not required to handle storage coal.

"For the most economical operation, such a crane should be electrically operated. Such a plant is estimated to cost \$23,000.00."



FIG. 28. LOCOMOTIVE CRANE RECLAIMING COAL AND DUMPING INTO BINS
ABOVE RAILROAD TRACKS

30. *Steeple Towers*.—Instead of a locomotive crane a steeple tower of steel or wood (Fig. 30) may be used. From such a tower a boom projects over the vessel or car into which the bucket is lowered. Upon being hoisted it is drawn back and dumped into a pocket from which the coal passes into a car, or is transferred by some other method to the point at which it is to be stored or used. These towers may be either fixed or movable. They usually run on tracks parallel with the water front. The booms are generally made so that they may be drawn back when not in use. With a $11\frac{1}{2}$ -ton bucket such a tower will unload and deliver to conveyors about 150 tons per hour.

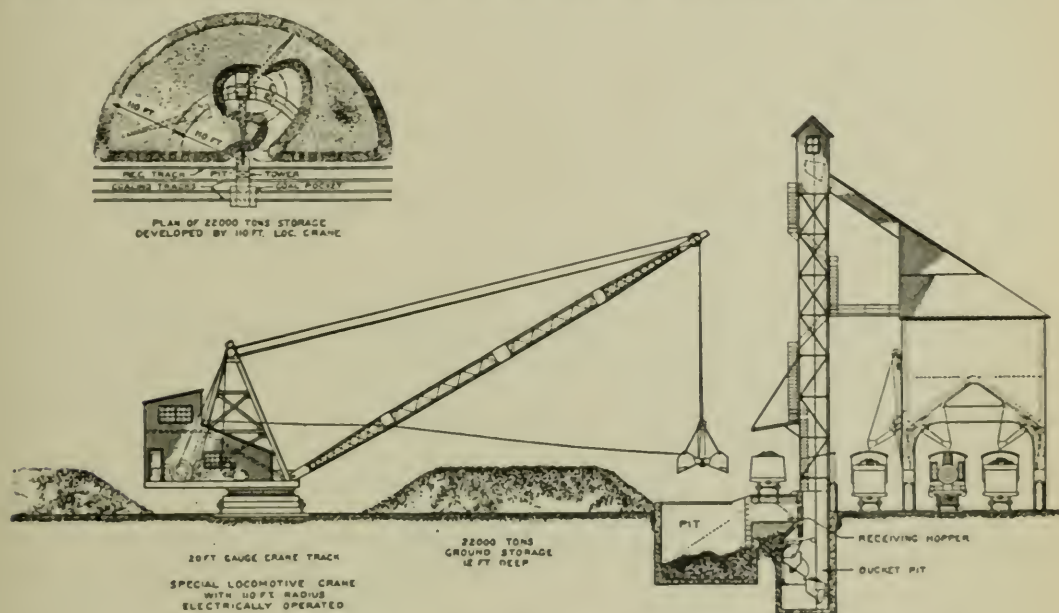


FIG. 29. L. & N. R. R. TYPE OF MECHANICAL COALING STATION
(Reprinted from Proceedings of the International Railway Fuel Association)

The storage yard of the Western Coal and Dock Company, Waukegan, Illinois, (Fig. 30) has three towers which deliver coal into cars running on a trestle about the storage yard.

By extending the boom backward through the tower, (Fig. 31) the buckets may be discharged into cars or upon a small stock pile. The tower may carry a self-contained hoisting unit or man trolley which moves along the boom and contains the drums and motors required to operate the bucket.

31. *The Hunt System*.—A common method of transferring coal to a pile is to dump it from a bucket into a bin from which it is discharged into a car which runs by gravity upon a trestle and as it

advances raises a counterweight. Such a system is illustrated by Fig. 32. The car is dumped automatically by a catch which may be set at any point along the trestle. As soon as the car is empty it is drawn back by the counterweight to its starting point and is then ready for another load. By means of a series of radiating trestles a large storage area may be reached by this system.

32. *Bridge Storage.*—A common form of transfer and storage is by means of a steel bridge which is usually movable about the storage yard and which serves as a support for a grab bucket, a belt, or other conveying devices. When both ends of the bridge are movable in a straight line, it becomes a gantry crane and may be made to cover any desired length of dock or storage yard, although because of the necessity for loading or unloading a cargo quickly these bridges do not generally travel over great distances. Fig. 33 shows a simple form of McMyler bridge gantry crane fitted with an electrically operated man trolley which serves a storage plant of about 50,000 tons.

A combination of unloading tower and traveling bridge utilizes to the fullest extent the ground area available, gives maximum storage, and usually provides maximum speed with minimum labor for both stocking and reclaiming. Steam or electricity is used for motive power, the choice being governed by the expense of operation.

Bridges may be divided into four classes according to the method of handling the coal on the bridge, as follows:—

(1) Rope trolley in which the bucket is operated from a stationary house on the bridge. Fig. 34 shows a bridge of this type built by Heyl and Patterson for the Reiss Coal Company at Superior, Wisconsin. The span of the bridge is 380 feet, the length of travel at present 1650 feet, and the storage capacity of the plant 450,000 to 600,000 tons.

(2) Man trolley in which the bucket is operated directly from a moving cab. Buckets holding from ten to twelve tons are used on this type of bridge. Fig. 35 shows two man trolley coal storage bridges built by the Wellman-Seaver-Morgan Company for the Indiana Steel Company, and equipped with 7½-ton buckets.

(3) Belt conveyor type in which the bridge serves as a support for a belt conveyor which, by means of an automatic trip, dumps the coal at any desired point.

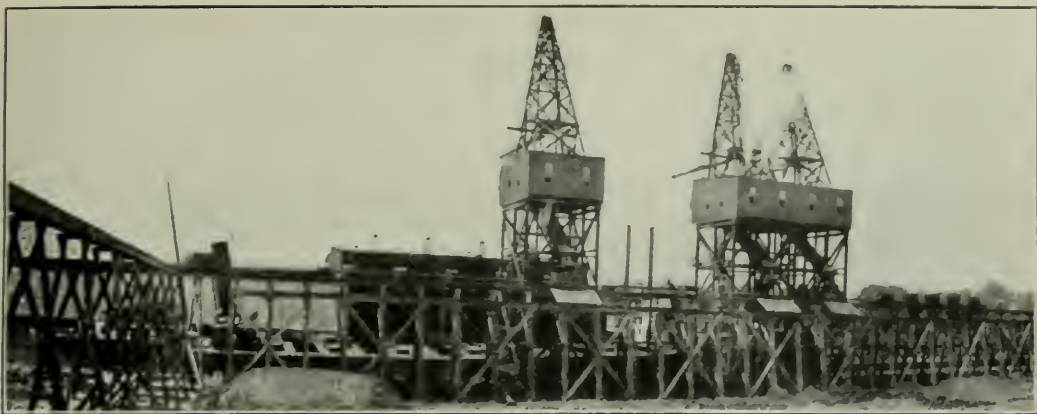


FIG. 30. TOWERS EMPLOYED IN HANDLING COAL, WESTERN COAL AND DOCK COMPANY, WAUKEGAN, ILLINOIS

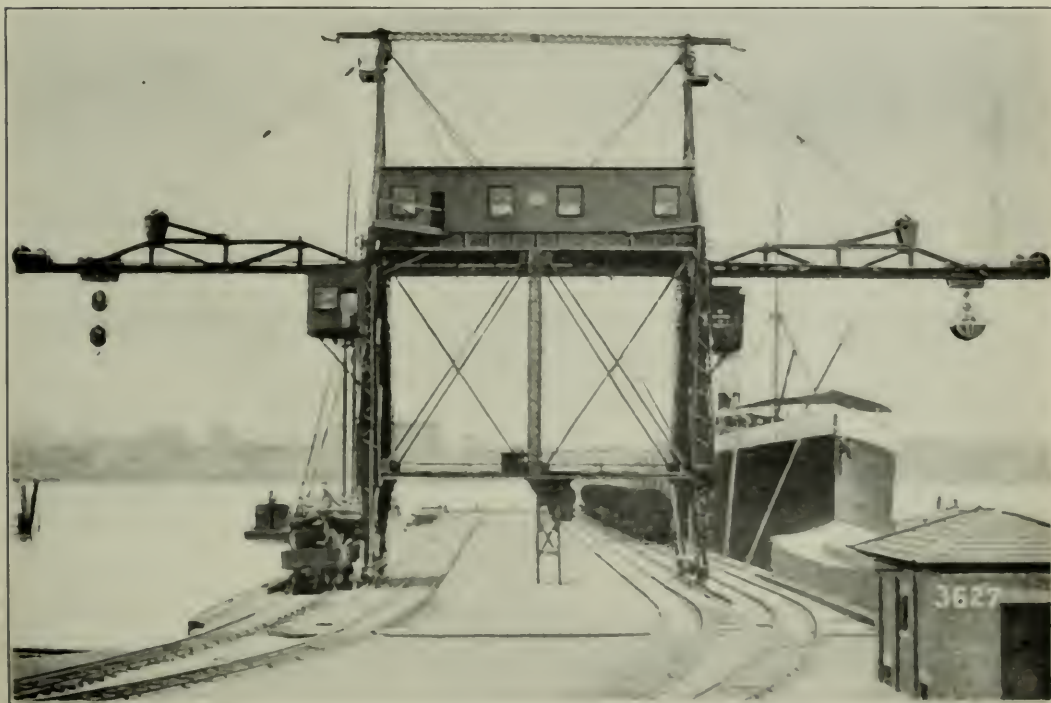


FIG. 31. TRAVELING TOWER OR DIRECT UNLOADER



FIG. 32. HUNT SYSTEM OF HANDLING COAL

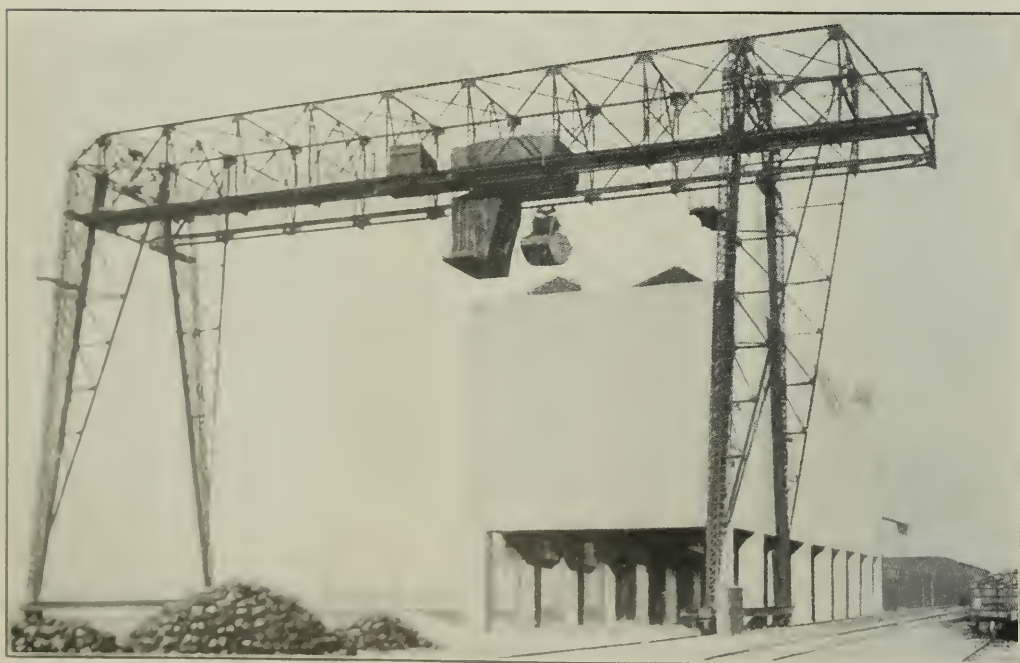


FIG. 33. BRIDGE GANTRY CRANE WITH ELECTRICALLY OPERATED MAN TROLLEY



FIG. 34. COAL HANDLING PLANT OF THE REISS COAL COMPANY AT
SUPERIOR, WISCONSIN

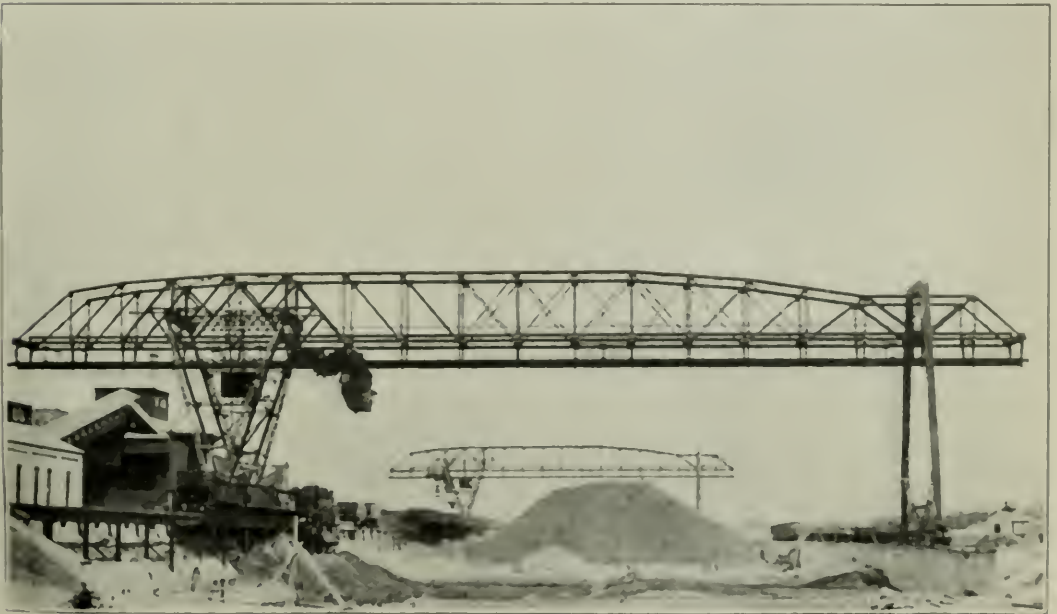


FIG. 35. MAN TROLLEY COAL STORAGE BRIDGES OF THE INDIANA STEEL COMPANY

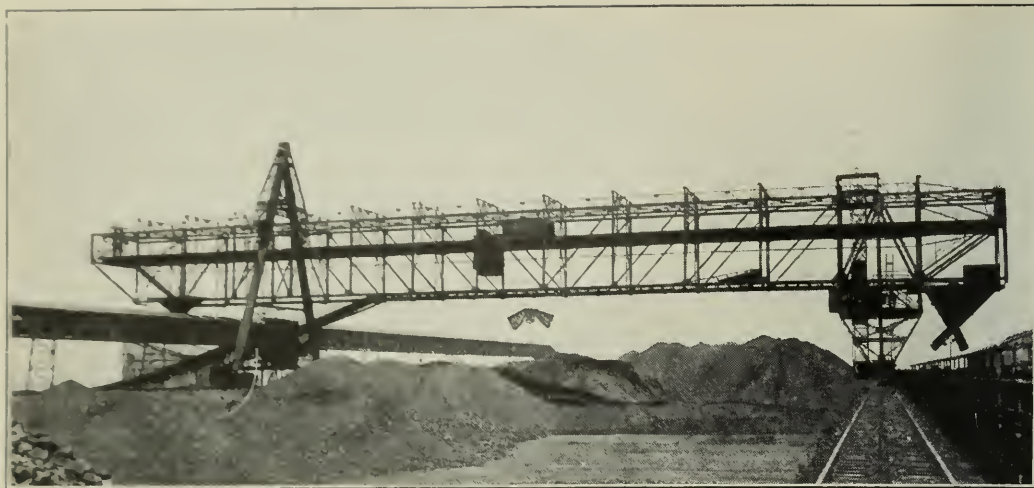


FIG. 36. BELT CONVEYOR TYPE OF BRIDGE OF THE INLAND STEEL COMPANY

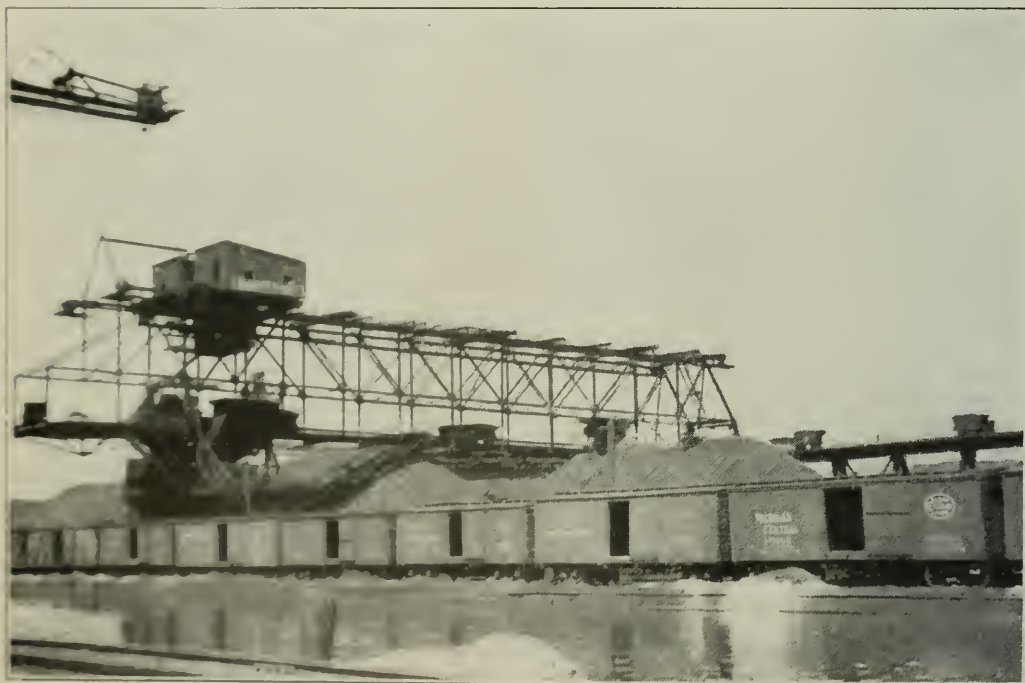


FIG. 37. MOVABLE BRIDGE WITH SIDE DUMP CARS

The belt conveyor type of bridge is illustrated by Fig. 36 which shows a large coal handling plant at the works of the Inland Steel Company at Indiana Harbor, Indiana, built by the Robins Conveying Belt Company.

(4) Cable road bridge on which a car runs. Fig. 37 shows a system in which the bridge moves over the length of the storage yard. Small side dump cars pass from a track running lengthwise of the storage yard upon the bridge from which they are dumped. The coal is reclaimed by a clam-shell bucket operated from the same bridge.

According to C. K. Baldwin* the capacity of storage bridges varies from 100 to 500 tons per hour, according to the size and speed of operation of the grab buckets or other carrying devices employed. The capacity of buckets varies from two to ten tons, the smaller sizes of from two to five tons being the more common. The span of the bridge usually depends upon the available storage space and may be as much as five hundred feet. The speed of the bridge varies from 50 to 200 feet per minute and that of the man trolley from 500 to 1500 feet per minute.

Fig. 38 shows a system of traveling bridges designed by Roberts and Schaefer Company, of Chicago, for the Clarkson Coal and Dock Company at Duluth, Minnesota. These bridges have a capacity of three hundred tons per hour from the vessel to the dock six hundred feet distant.

Fig. 39 shows a parallel track storage yard for a locomotive coal-ing station as suggested by the International Railway Fuel Association Committee†. The coal is unloaded and also reclaimed by means of a clam-shell bucket operated from a traveling bridge. The reloaded coal is delivered into a hopper at a stationary screening plant and elevated to the screens for preparation.

Fig. 40 shows the ground plan of a storage yard which is designed to handle as much as 20,000 tons of storage coal at a very low cost, by the use of a bridge and a traveling bucket. The coal is taken out of cars and placed in storage by means of the bucket, the labor being performed by the regular coal chute force during the dull seasons of the year, thus reducing greatly the expense of labor. The coal is recovered by picking it up with the bucket and delivering it to the con-

* Marks, L. S., "Mechanical Engineers' Handbook," p. 1131, 1916.

† Proceedings, Vol. VI, p. 115, 1914.

veyor chain which discharges it directly into the receiving hopper of the chute.

The following estimate for the expense of stocking with a bridge tramway is given by the Link-Belt Company (cost, interest, and depreciation not included) :

Labor and Supplies for stocking per ton	\$.041
Power and Superintendent's expenses	.014
Total	\$.055
Labor and Supplies for reclaiming per ton	.0385
Power and Superintendent's expenses per ton	.0130
Total	\$.0515

It is often desirable to screen the coal before it is reshipped from storage. The screening plant may be carried by the bridge, or it may be a separate movable structure. Fig. 41 shows a movable screening plant, built by the Link-Belt Company for the Berwind Coal Company, at Duluth, Minnesota.

A bridge is also applicable to storing in a circular pile. In such a case one end of the bridge is pivoted and the other end moves on a circular track about the pivot as a center. Instead of covering a complete circle this bridge may be used to cover any arc of a circle desired. A swivel bridge designed to operate in a semi-circle is shown by Fig. 42. Coal dumped into the pit at the side of the receiving hopper is picked up by the grab bucket on the bridge, and placed in storage. In reclaiming, the coal is picked up by the bucket, brought back to the receiving hopper, and elevated to an overhead storage bin.

This type of equipment is suited to a much larger storage pile and greater handling capacity than the locomotive crane, but is more expensive to operate.

The swivel bridge would probably be less economical than the locomotive crane for the handling of quantities within the capacity of the latter apparatus but can be advantageously employed for larger quantities. Coal may be handled at very little expense by a bridge where electric power is available.

The cost * of swivel bridges above the rails was given in 1915 as from \$25,000, for a bridge of 100-foot span, to \$50,000, for a bridge of 250-foot span; the capacities ranging from one hundred to three hundred tons of coal per hour.

* Proceedings, International Railway Fuel Association, Vol. VII, pp. 159-160, 1915.



FIG. 38. TRAVELING BRIDGES OF THE CLARKSON COAL AND DOCK COMPANY,
DULUTH, MINN.



FIG. 41. MOVABLE SCREENING PLANT OF BERWIND COAL COMPANY

Figs. 43 and 44 show a pivoted bridge built for the Milwaukee Coke and Gas Company by the Robins Conveying Belt Company. The semicircular concrete track on which the movable end runs has a 500-foot radius. The coal is elevated by an enclosed inclined belt conveyor which delivers it into a bin in the pivoted tower (Fig. 43). The coal is then put into storage by a belt conveyor, a tripper on the bridge depositing it at any desired point in the storage field. It is reclaimed by means of a six-ton bucket.

In order to get as much coal as possible under the bridge, a slightly inclined bulkhead, (Fig. 45) from twenty to twenty-five feet high, was built on both sides of the semicircular track and also around the outer edge of the pile. This bulkhead is made of boards and timbers projecting into the coal pile as shown in Fig. 46. When this method of bulkheading was first attempted there was a number of serious fires in the pile, probably caused by the circulation of slow currents of air through cracks between the planks. This difficulty has been overcome by studying the matter of sizes suited to storage and by closing the cracks, with reference to which J. F. Blackie, superintendent of the plant, says:

“It has been our experience that a coal from two inches up in size allowed a flow of air which cooled the pile rather than promoted combustion, and that fine coal prevented air circulation and did not, therefore, fire, but wherever the smaller lumps accumulated fires started. When all the cracks in the bulkhead were plastered with nine parts of coal dust and one part cement, fires were eliminated.”

The Milwaukee Coke and Gas Company has been storing each year for the past ten years 400,000 tons in one pile and 250,000 tons in another, the coal mixture consisting of 65 per cent West Virginia and Kentucky coals averaging 33 per cent volatile and 35 per cent Pocahontas averaging 18 per cent volatile. During the summer of 1917 it also stored Illinois coal in the same manner.

A method of unloading coal employed by the Canadian Pacific Railway at Fort William, Ontario, is illustrated by Fig. 47. The equipment consists of two Hulett unloaders with buckets of 8 tons capacity. The coal is rehandled from the stock pile under the unloaders by means of a large bridge, and stored in piles for future use. The bridge has a span of 285 feet with cantilevers approximately 150 feet long, is equipped with a man trolley which handles a nine-ton bucket, and is used both for stocking the coal and for reclaiming.

33. *Deep Reinforced Concrete Storage Bins.*—A storage plant of unusual design is that of F. W. Stock and Sons of Hillsdale, Michigan. It consists of reinforced concrete bins 28 feet in diameter and 70 feet deep built in pairs. Hopper bottom cars are unloaded through a track grating, the coal is carried by belt conveyor to a V-bucket elevator and dumped from the elevator head into the bins. A small pocket on the side opposite the elevator leg is used for anthracite coal for domestic use. The bins are roofed and the construction is such as to prevent the free circulation of air (Fig. 48).

As a precaution against fire each bin has six 1-inch pipes set vertically, each pile being open at the top and having three or four side openings at intervals. These pipes are connected to a pressure water line so that it is possible to flood the bins if necessary to extinguish a fire. The bins are designed to withstand the pressure of coal having the interstices filled with water.

The estimated cost of the plant, complete with machinery, is \$18,000. The capacity is about 2000 tons. Records covering the expense of operation are not available, but the opinion is expressed that the expense will be less than that of open pile storage, even when the interest on the investment is included.

34. *Under-water Storage.*—Because of the liability to spontaneous combustion of coal storage piles exposed to the air, under-water storage has been used to some extent. Thus far this method has been applied exclusively to the storage of screenings.

The consensus of opinion is that coal stored under water deteriorates little, if any, in its chemical properties and in heat value. Professor Parr* says:

“Under-water storage prevents loss of heat value and is not accompanied by deterioration in physical properties such as slacking. The water retained by the coal upon removal is substantially only that held by adhesion or capillarity.”

This opinion is concurred in by a number of other writers,† and the opinions expressed in the questionnaire are unanimous upon this point.

At the present time there is a number of under-water storage plants and others are under consideration. The general adoption of

* “Effects of Storage upon the Properties of Coal.” Univ. of Ill. Eng. Exp. Sta., Bul. 97, p. 7, 1917.

† Hall, R. G., Mining and Scientific Press, March 15, 1913; Bennet, A., The Black Diamond, April 14, 1917; Palmerberg, W., Proceedings, International Railway Fuel Association, May, 1917.

under-water storage, however, seems to be influenced by the cost. One of the largest elements of cost is that of a suitable storage pit which may be an excavation made in solid ground, a dammed depression, an old quarry, or an abandoned clay pit. An abandoned quarry or clay pit has the advantage of saving the expense of excavation but there may be a considerable item of expense for cleaning out the old excavation. A quarry has solid but irregular walls, but if these are made even and the quarry cleaned out, it forms an ex-

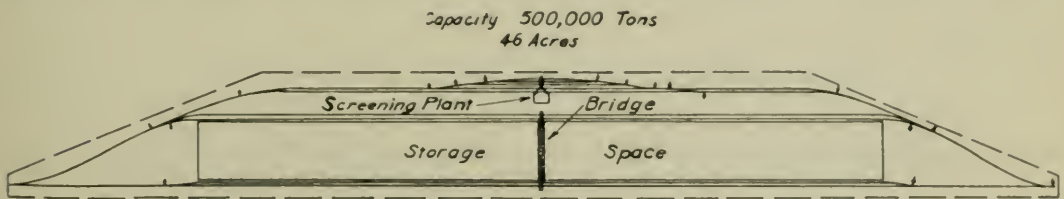


FIG. 39. PARALLEL TRACK STORAGE YARD FOR A LOCOMOTIVE COALING STATION
(Reprinted from the Proceedings of the International Railway Fuel Association)

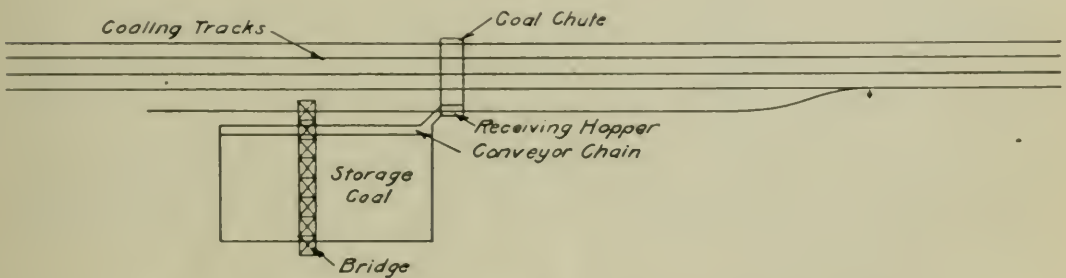


FIG. 40. LAYOUT OF STORAGE YARD FOR LOCOMOTIVE COAL STATION
(Reprinted from the Proceedings of the International Railway Fuel Association)

cellent place for storage. A clay pit has the disadvantage of soft walls, unless it is lined with timber or concrete. Several quarries have been used for storage purposes, and apparently with success. These old quarries are frequently fed by springs, so that the water supply is assured but is generally not sufficient to cause an overflow. One of the latest plants has utilized a pit excavated in an old slough which has been dredged out. In this case the cost of excavation was low and a permanent water supply assured. One drawback to under-water storage lies in the freezing of the water during winter, or if the surface does not freeze the wet coal may freeze while handling. So far as reported, however, freezing has not been found troublesome

in the vicinity of Chicago and other Middle West cities near which many of the under-water storage plants are located.

A pioneer in under-water storage was the Western Electric Company, Chicago, Illinois. This company, after a number of fires in its bunkers at the Clinton Street plant, built in 1902 a concrete pit with a capacity of 3000 tons. It was not water-proof, but the water level in the pit was maintained by an intake from the Chicago River. Coal was stored in this pit during the winter months and used during the early summer without appreciable loss in weight or trouble in burning the coal.

In connection with its extensive new plant at Hawthorne, Illinois, the Western Electric Company in 1906 provided a concrete-lined pit having a capacity of 10,000 tons. This pit, shown in Figs. 49 and 50, occupies a ground area of 310 by 114 feet, and is divided into three parallel sections by concrete arch partitions. It is about fifteen feet deep, and the bottom is inclined since rock was encountered in sinking the pit. Water is provided from the roofs of the adjacent factory buildings. Three railroad tracks are carried on the arched piers and extend the entire length of the pit, and there are two railroad tracks on each side. Coal is unloaded into the pit by hand at an expense of five cents per ton. A locomotive crane with a grab bucket reclaims and reloads the coal at an expense of about four cents per ton. Loaded cars stand on the tracks from twenty-four to forty-eight hours to allow the water to drain off into the pit. Illinois screenings of one inch or one and one-half inch size are stored.

According to *The Colliery Engineer*,* the original construction cost of a pit of this description was about \$7,000, for each thousand tons capacity. Later, when the company considered extending the plant by building a pit which would hold about 22,000 tons, the cost of the extension was found to be about \$60,000, including the crane and necessary railroad tracks.

Another pioneer in under-water storage is the Illinois Traction Company which has two plants, one at Riverton and another at Mackinaw.† The one at Riverton holds approximately 9,000 tons and the one at Mackinaw approximately 16,000 tons. The construction of these pits is shown in Fig. 51. The floor of the pit illustrated is 275 feet by 80 feet and the tank is 319 feet by 128 feet.

* January, 1915.

† Detailed descriptions of these plants will be found in *Electric Railway Journal*, April 1, 1911, and *The Colliery Engineer*, January, 1915.

The water is pumped from the Sangamon River by a motor-driven centrifugal pump, and the water in the pit may be drained out when desired through a ten-inch valve. The center trestle is of yellow pine twenty-four feet high, extends the full length of the pit, and rests on concrete with steel rail reinforcing. The floor between the trestle bents is also reinforced with rails placed four feet apart. The Riverton pit, similar in construction, is twenty feet deep, the bottom being 80 feet by 185 feet and the top 124 feet by 225 feet. The concrete bottom has triangular mesh reinforcement instead of rails and underneath it are concrete foundations for the bents. The con-

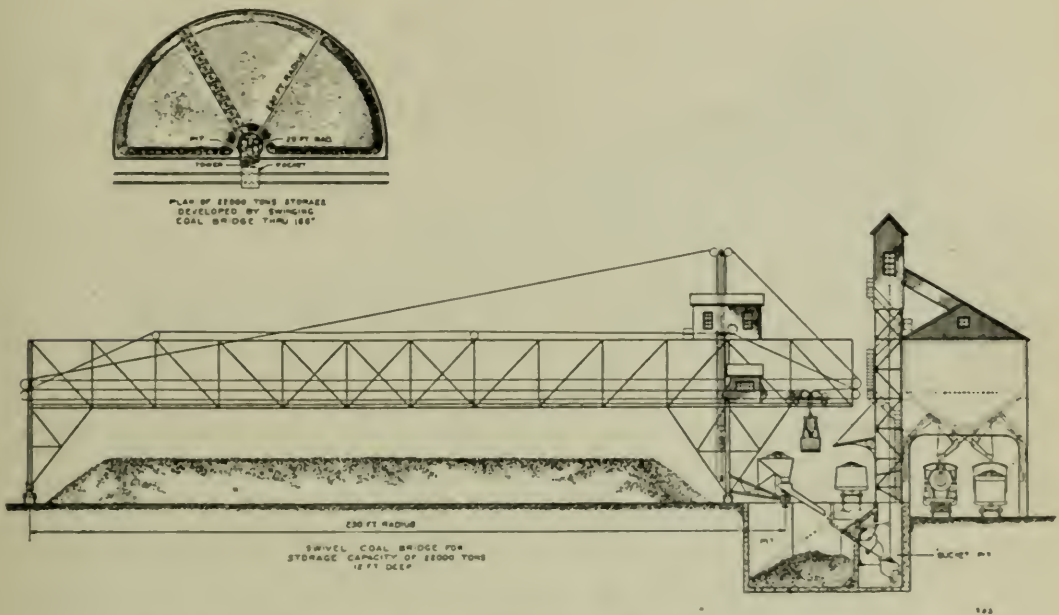


FIG. 42. SWIVEL BRIDGE DESIGNED FOR SEMICIRCULAR PILE

(Reprinted from the Proceedings of the International Railway Fuel Association)

crete has a one-inch wearing surface, two per cent of which is a water-proofing compound. R. J. Carley, chief operating engineer of the Illinois Traction System, reported in 1913 that little trouble was experienced in holding the water in the Riverton pit and that there was little deterioration of the coal. There was considerable trouble in holding the water in the pit at Mackinaw, and coal in storage there for one period of about eighteen months slacked considerably because of the lack of water. The difference in tightness was thought to be due to the better water-proofing at the Riverton pit. The Mackinaw pit required a small amount of excavation and cost \$14,300 or about \$900 per 1000-ton capacity. The Riverton pit cost \$11,000 or about

\$1.225 per 1000 tons. The Riverton pit was situated so that it was necessary to build only earth embankments to hold the concrete retaining walls.

The Illinois Traction Company stores coal of all sizes from the Worden district, Illinois, and keeps it in storage for from six months to three years. There is no appreciable loss in heat value, and there is no appreciable breakage. The total expense of operating these plants is 14.6 cents per ton for storing and 7.8 cents for reclaiming. The Company also stores coal upon the ground in continuous piles fifteen feet deep and in case of heating removes the coal from the affected spot.

The American Zinc Company of Illinois stores 35,000 tons in a lake at its Hillsboro plant, the coal being dropped into the water through a plate girder bridge, and reclaimed by a centrifugal pump which delivers it into a dewatering elevator. The coal keeps in excellent condition under water and contains practically no more moisture when reclaimed than when freshly mined. The company also carries a stock of from thirty to forty cars of coal which lies on the ground for not more than thirty days. According to A. Ives, Superintendent, coal stored in the open deteriorates so rapidly that it is almost useless for gas making.

The East St. Louis Light and Power Company cleaned out an old rock quarry adjacent to its power station in which about 7,000 tons of coal are stored under water. This plant has been operating satisfactorily except for some slight difficulty due to freezing in the winter. There is also some difficulty due to freezing in the bunkers.

The National Zinc Company of Springfield, Illinois, utilizes an old clay pit about 250 feet wide, 450 feet long, and 45 feet deep which has a capacity of about 30 car loads.* This pit formed a natural sink in the surrounding prairie, and was, therefore, usually filled with water to a point about five feet from the top. The sides and bottom are of comparatively hard shale. A railroad track parallels the longer side. The track is carried on a low trestle over an unloading chute, sufficiently wide to accommodate one car. The bottom of the chute has a slope of ten per cent toward the pond, and the outlet into the pond is about four feet wide. At the top end of the chute is a ten-inch pipe having four-inch holes in the side. This pipe is attached to a ten-inch centrifugal pump which delivers about 3,500

* Detailed description of this plant will be found in *Mining and Scientific Press*, p. 406, March 15, 1913 and in *The Colliery Engineer*, p. 299, January, 1915.



FIG. 43. PIVOTED BRIDGE OF THE MILWAUKEE COKE AND GAS COMPANY

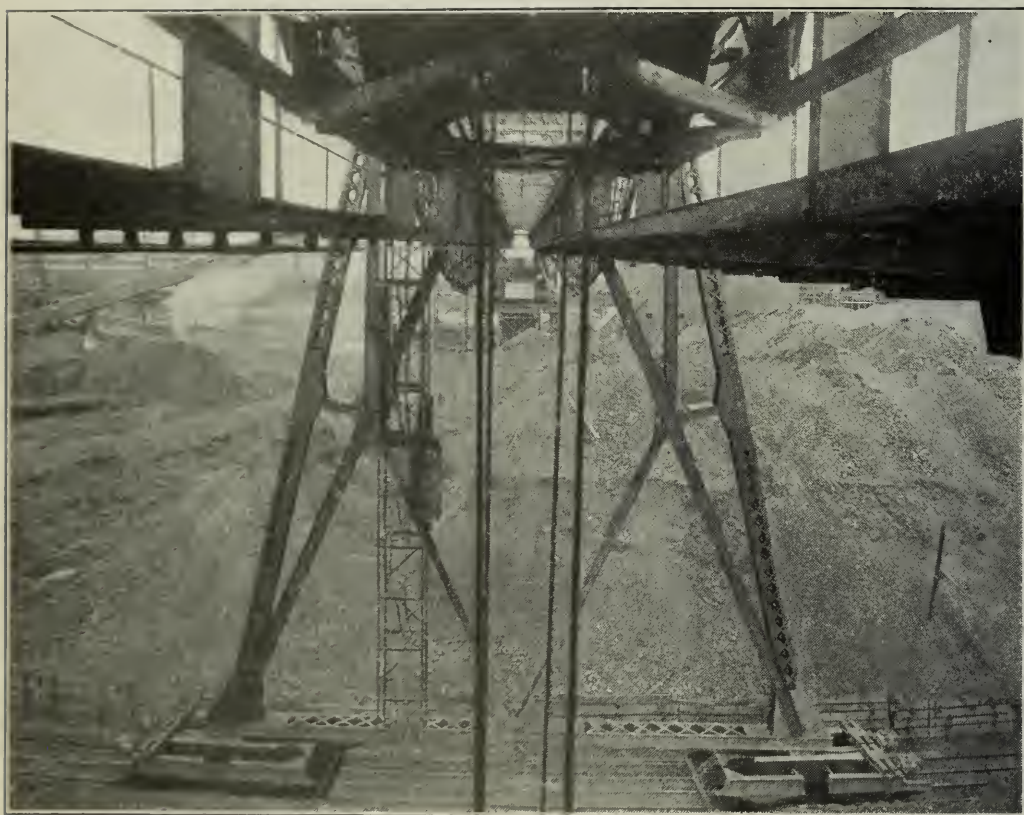


FIG. 44. VIEW LOOKING TOWARD PIVOTED END, BRIDGE OF THE MILWAUKEE
COKE AND GAS COMPANY



FIG. 45. BULKHEAD AT THE STORAGE PLANT OF THE MILWAUKEE COKE AND GAS COMPANY



FIG. 46. VIEW SHOWING CONSTRUCTION OF THE BULKHEAD AT THE PLANT OF THE MILWAUKEE COKE AND GAS COMPANY

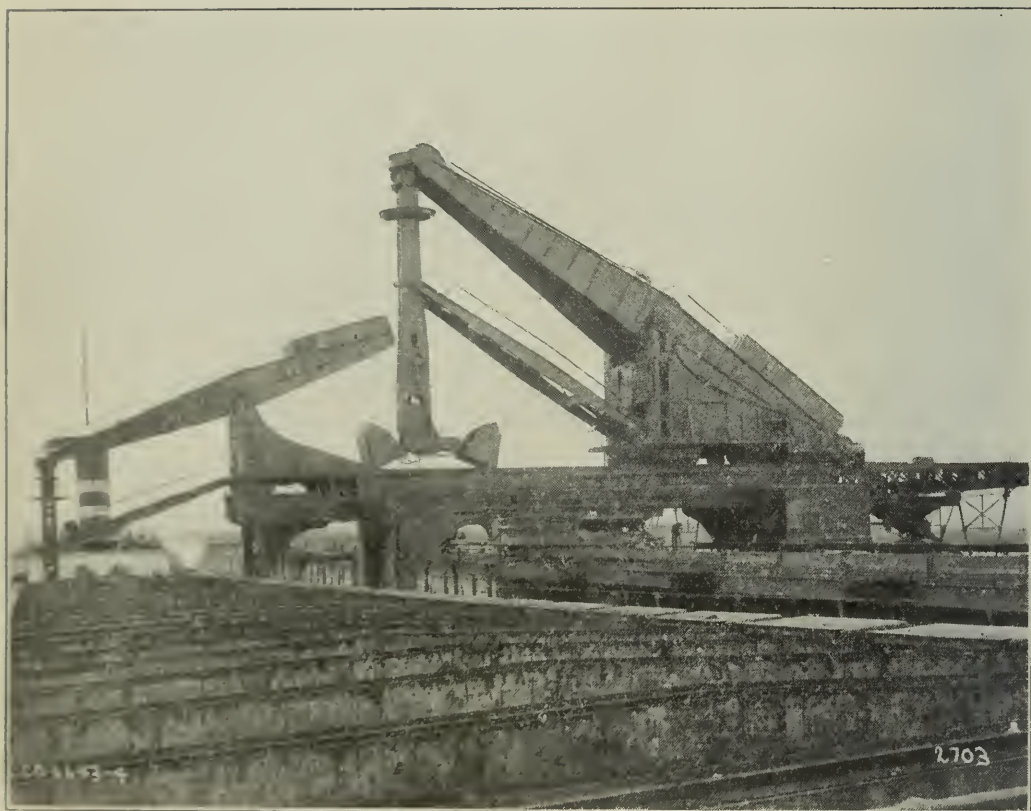


FIG. 47. HULETT UNLOADERS OF THE CANADIAN PACIFIC RAILWAY AT
FORT WILLIAM, ONTARIO



FIG. 48. REINFORCED CONCRETE STORAGE BINS OF F. W. STOCK AND SONS,
HILLSDALE, MICHIGAN, DESIGNED AND BUILT BY MACDONALD ENGINEERING
COMPANY, CHICAGO

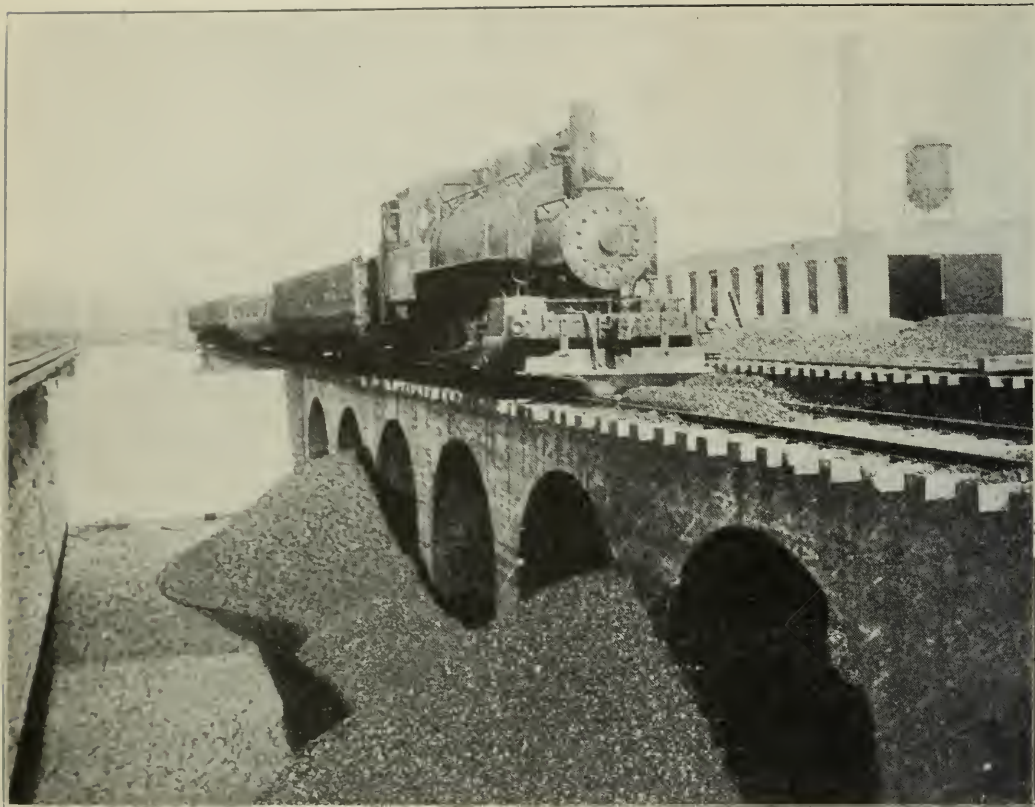


FIG. 49. VIEW OF THE UNDER-WATER STORAGE PIT OF THE WESTERN ELECTRIC
COMPANY AT HAWTHORNE, ILLINOIS



FIG. 50. VIEW OF THE UNDER-WATER STORAGE PIT OF THE WESTERN ELECTRIC
COMPANY AT HAWTHORNE, ILLINOIS



FIG. 52. STORAGE PIT OF THE NEW KENTUCKY COAL COMPANY, KANKAKEE,
ILLINOIS



FIG. 53. CONCRETE STORAGE PIT OF THE INDIANAPOLIS LIGHT AND HEAT
COMPANY



FIG. 54. STORAGE PIT OF THE METROPOLITAN WATER DISTRICT OF OMAHA

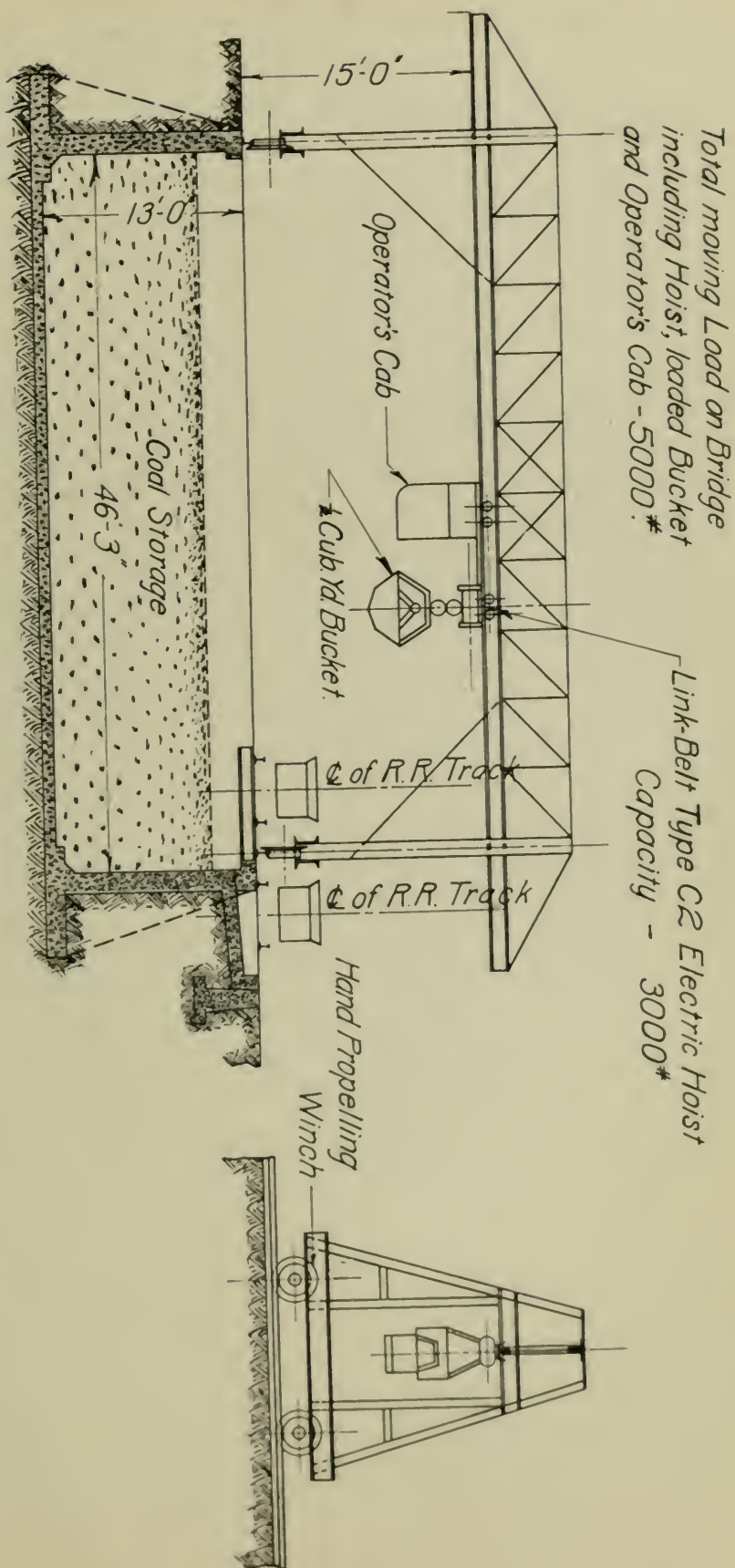


FIG. 55. STORAGE PIT AND BRIDGE OF THE WESTERN CLOCK WORKS

gallons per minute. When a loaded car is placed on the trestle, two men open the drops, section by section. This action throws most of the coal into the chute, and the water backs into the chute until the accumulation is sufficient to move the whole mass of material. When the bulk of the material has moved out, the men close a valve in the pipe to the chute, and throw the whole stream of water directly into the car to clear it. The whole operation of unloading a car requires on the average about ten minutes.

In reloading, the coal and water are pumped directly into the elevator boot which is placed in a hopper-shaped hole excavated in the ground and lined with plank, and the water is forced to travel slowly around the hopper before finding its way back into the pond.

The coal pump is placed on a barge 15 feet by 40 feet and is belted to a 50-horse-power motor. Both the suction and discharge are light spiral riveted iron pipes, which though short lived provide ease of its manipulation. The discharge pipe is carried on pontoons made from oil barrels. One operator on the barge finds no difficulty in moving the pipe around as the ground is worked out.

J. Kaercher, of the National Zinc Company, reports that the expense of storing at present is ten cents per ton and the expense of reclaiming ten cents per ton. The coal is kept in storage indefinitely and different sizes are stored without being separated. The coal stored is to be used for the production of producer-gas, and there has been no appreciable deterioration.

The New Kentucky Coal Company at Kankakee, Illinois, has a complete under-water storage plant, built according to the plans of Mr. Harwood, its vice-president and general manager. An old limestone quarry was utilized as the storage pit, the water being supplied from springs which keep the quarry full. The shape of the basin is elliptical, being about 700 feet long, 310 feet wide, and 30 feet deep. Since the rock was channeled, the sides are in excellent shape and are regular. A view of this pit is shown as Fig. 52. Water obtained from pipes along the track is used to flush the coal out of the cars and down the chute into the pond. As the coal piles up around the bottom of the chute, it is pumped out into the quarry by means of a ten-inch centrifugal pump operated by a 75-horse-power motor, the pump and the motor being mounted on a barge. The pipe line consists of a sectional pipe, ten inches in diameter, which rests on pontoons.

To reclaim the coal the process is reversed and the coal is pumped from the quarry into the concrete pit at the base of the inclined elevator. When this pit is filled with coal, the water overflows back into the quarry. Coal from the pit is elevated by a flight conveyor of perforated buckets through which much of the water drains and is delivered into cars on the railroad track. In the small house at the left of the elevator (Fig. 52) is a six-inch centrifugal pump operated by a 25-horse-power motor which pumps the water needed for flushing the coal from the cars and down the chutes. The storage capacity is estimated to be from 200,000 to 250,000 tons of coal and it is stated that about 2,000 tons per 8-hour day can be unloaded and 1500 tons reclaimed. Coal of sizes under three inches can be handled by the equipment. Unfortunately, the plant has not been operated owing to lack of coal supply. Three men are required to operate the plant and the cost is estimated to be from seven to ten cents per ton for unloading and reclaiming. The equipment cost about \$20,000, including the switch track and trestle, but at the present time the same equipment would cost from \$28,000 to \$30,000.

The Indianapolis Light and Heat Company (T. H. Wynne, Superintendent of Power)* stores 20,000 tons of Indiana run of mine coal from the Linton district under water in concrete tanks twenty-four feet deep (Fig. 53). The coal is dumped directly from railroad cars and is reclaimed by a locomotive crane. A deterioration of less than three per cent in heating value is said to take place. The stored coal is used after the first of January, when the price of coal is, normally, highest, and the pit is refilled during March, April, and May.

The Metropolitan Water District of Omaha, Nebraska, stores for steam purposes 4,000 tons of Iowa, Kansas, and Illinois coals. Most of this is slack below three-fourths inches, although some nut and washed coal is used. The storage pit (Fig. 54) is of concrete ten feet deep. The period of storage is about six months. The coal is piled from fifteen to twenty feet above the water in order to get an additional storage. This coal may heat in from three days to two months and if so it is then turned over by means of a locomotive crane which is used to stock and unload the coal from the bins.

*See *Electrical World*, Vol. 62, p. 592, Sept. 20, 1913, *Electric Railway Journal*, Vol. 42, p. 391, Sept. 6, 1913, *Engineering News*, Vol. 70, p. 527, Sept. 1, 1913, *Power*, Vol. 38, p. 485, Sept. 30, 1913.

The expense of this storage is given as follows:

	Storing	Reclaiming
Overhead	\$0.003	\$0.003
Labor	.012	.012
Supplies	.005	.005
Depreciation on mechanical equipment	.024	.024
Interest on investment	.019	.019
Total	\$0.063	\$0.063

The Company also stores some coal on the ground, but because of fires is now planning to store 30,000 additional tons in abandoned concreted water basins, utilizing a hydraulic ejector similar to those used for years in transporting sand through a pipe for filtration purposes. G. T. Prince, Chief Engineer, says:

“Any coal stored in the air, whether supported by coal in a pit under water or by planking on the ground is subject to spontaneous combustion. This is particularly true of coal received in a damp, wet state. When such coal reaches us we exercise great care in watching it, as it is sure to catch fire within a few weeks and possibly within a few days. Coal received during the dry summer days will be free from combustion for several months.”

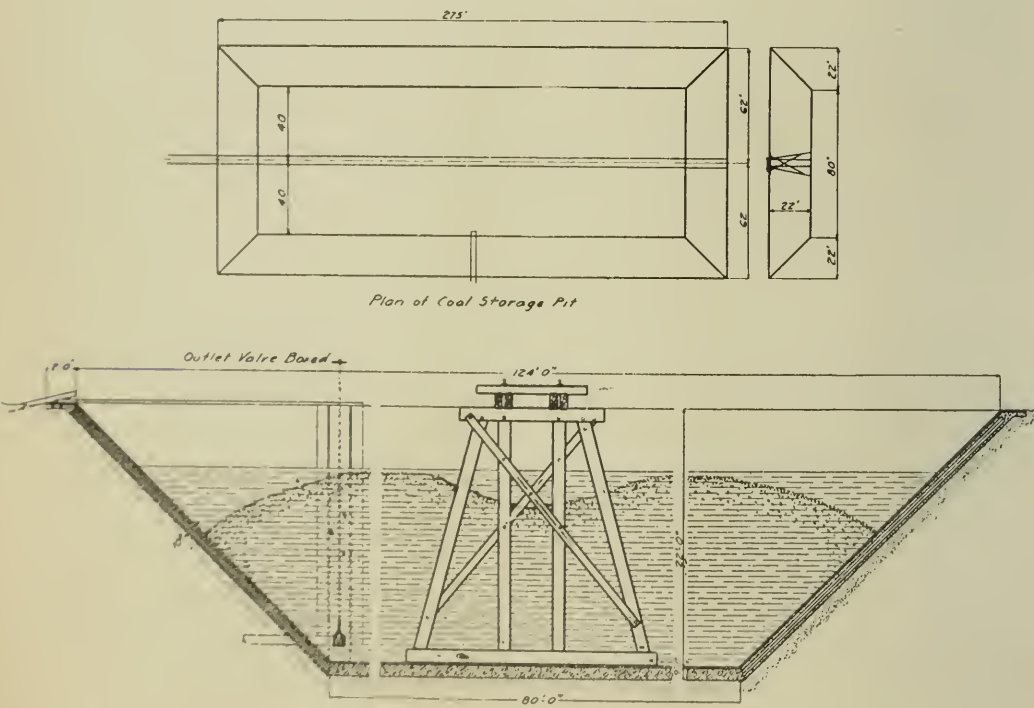


FIG. 51. COAL STORAGE PIT OF THE ILLINOIS TRACTION COMPANY AT MACKINAW, ILLINOIS

Fig. 55 shows a cross-section of a storage pit recently built by the Western Clock Works in Peru, Illinois. The pit is 87 feet long, 46 feet, 3 inches wide, and 13 feet deep and has a capacity of 1200 tons under water. The cost of the plant was \$15,300. The coal stored comes from a mine, about two hundred feet distant. The traveling bridge and bucket are used to distribute the coal in the pit and to reclaim it. The reclaimed coal is placed in the small cars on alternate days, the cars being allowed to drain for a day before the coal is delivered to the boiler room.

One of the largest under-water storage plants now under construction is being built by the Great Lakes Dredge and Dock Company for the Standard Oil Company of Indiana at Whiting, Indiana. The pit will be 1000 feet long, 202 feet wide below the ground water level, and 26 feet below the yard rail level. The system is to be practically the same as that of the Western Electric Company at its Hawthorne plant but instead of the heavy concrete construction for walls and trestles, wood piles will be used below water level and will be capped with a coping of concrete above water level. The bottom of the pit will be roughly graded and lined with concrete one foot thick. The only part of the construction which will be subject to deterioration will be the ties, stringers, and caps of the railroad trestle which are above water.

Four trestles, on which will be a standard gage track, will extend from end to end of the pit. Alongside the pit will be two railroad tracks, so that locomotive cranes to be used for reclaiming may operate from the trestles above the pit or from the side tracks. It is estimated that the cost of construction will be much less than that of the concrete types used in other plants. By means of the four tracks placed on the trestles and the two on the sides, six trains may be unloaded at one time.

The Peabody Coal Company of Chicago has three under-water storage plants, one holding about 80,000 tons at Kankakee, one holding 60,000 tons at Lemont, and another at Momence, Illinois. The one at Momence, however, has not been placed in operation. In each case an old quarry has been used. The coal is dumped into the quarry from a track running near the edge and distributed by means of a centrifugal pump. In reclaiming, the coal is pumped into an elevator and delivered directly into railroad cars. The Company reports that where the pile of coal projected above the water level along the bank it took fire.

An estimate given by the engineers of the Peabody Company for the cost of equipment and the expense of operation of the Lemont plant is as follows:

ESTIMATED COST OF EQUIPMENT AT LEMONT STORAGE

(BASED UPON HANDLING 700 TONS OF COAL PER DAY—ALLOWANCE MADE FOR STOPPAGE AND BREAKDOWNS)

Shooting rock and building unloading chute	\$350.00
Building concrete piers, laying stringers, and installing track	780.00
Rearrangement of track to get greater capacity	800.00
12-inch centrifugal pump, directly connected to 100-horse-power steam engine	2250.00
Boat to float pump and engine	600.00
300-foot spiral riveted pipe with bolts and gaskets	800.00
2- to 5-foot sections of rough bore suction and	
2- to 5-foot sections of smooth bore discharge hose	800.00
125-horse-power self-continued steam boilers	1000.00
Priming pump, connected to pump engine, 12-inch check valve	125.00
Steam injector, boiler fittings, etc.	225.00
100 feet of 3-inch steam pipe and 3-to 5-foot sections of 3-inch steam hose	110.00
Elevator of dewatering type, to elevate 120 tons of coal per hour at a speed of 50 feet per minute, connected to steam engine	2000.00
Wood receiving boat at foot elevator 20 by 20 by 8 feet, also wood support and erection of elevator	700.00
Self-contained car puller and engine with 1500 feet of ½-inch steel cable, with head sheaves, etc.	650.00
Ropes, blocks, lars, etc.	200.00
10 per cent for supervision	1140.00
Total	<u>\$12,530.00</u>

To clean quarry:

Pumping dry, removing debris from bottom, and additional expenditure estimated	\$1500.00
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ESTIMATED EXPENSE OF OPERATION AT LEMONT STORAGE
(BASED UPON HANDLING 700 TONS OF COAL PER TEN-HOUR DAY)

Unloading	Per Day	Expense Per Ton
Superintendent (1)	\$5.50	
Pumpman (1)	3.50	
Carpuller (1)	3.00	
Dumpers and Pipemen (2)	6.00	
Fuel for boilers and lubricating oil	6.00	
Miscellaneous repairs and supplies	2.50	
Night Watchman (1)	3.00	
Total	\$29.50	\$0.042
Reloading		
Superintendent (1)	\$5.50	
Pumpman and Helper (1)	6.50	
Pipeman (2)	6.00	
Trimmer (1)	3.00	
Carpuller (1)	3.00	
Fuel for boiler, oil, etc.	6.00	
Miscellaneous supplies and repairs	2.00	
Night Watchman and Trimmer (1)	3.00	
Total	\$35.00	\$0.0500
6 per cent per annum on investment*		
\$15,000 spread over 60,000 tons = \$900*		0.0150
Unforeseen delays, etc.*		0.0100
Losses one per cent on 600 tons at \$1.50		
per ton = \$900*		0.0150
Depreciation on equipment*		0.0200
Accident insurance, etc.*		0.0100
Total Expense of Storing		\$0.1620

If electric power is used, the cost will be five cents per ton higher.

The following are comparative estimates of the expense of under-water and surface storage made by C. G. Hall, formerly fuel agent of the C. & E. I. Railroad.

* Losses at one per cent are estimated upon the basis of operation over a period of years. The interest, delays, depreciation, and similar expenses are based upon loading and unloading 60,000 tons per annum. A smaller tonnage will materially increase these items of expense.

STORAGE OF SCREENINGS (INDIANA COALS)

Subaqueous Storage (3rd,

5th, and 6th vein coal)	100,000 Tons	
Pump outfit	\$12,000.00	
Interest and depreciation	20 per cent	\$2,400.00
Tracks (estimated)	2,000.00	
Interest and depreciation	15 per cent	300.00
Labor, supplies and power, Unloading and reloading		10,000.00
Rental of storage pit		250.00
Value of 100,000 tons at 50 cents per ton and freight charges at 82 cents	\$132,000.00	
Interest on \$132,000.00 at 6 per cent for 8 months		5,280.00
		\$18,230.00
Average per ton		18.23 cents

Surface Storage (4th Vein Coal) 25,000 Tons

Locomotive crane	\$8,000.00	
Interest and depreciation	20 per cent	\$1,600.00
Tracks	5,000.00	
Interest and depreciation	15 per cent	750.00
Labor and supplies, Unloading and reloading		2,500.00
Rental of ground, etc.		200.00
Value of 25,000 tons at 75 cents per ton and freight charges at 77 cents per ton	\$38,000.00	
Interest on \$38,000.00 for 8 months at 6 per cent		1,520.00
		\$6,570.00
Average per ton on 25,000 tons		26.28 cents

VI. EFFECTS OF STORAGE UPON THE PROPERTIES OF COAL

The effects upon the properties of coal may be considered under the following heads:

- (1) Appearance.
- (2) Loss of heating value.
- (3) Firing qualities.
- (4) Spontaneous combustion.
- (5) Coking and gas making properties.
- (6) Degradation, or the increase in the amount of fine coal and dust due to breakage from handling, and to slacking or weathering.
- (7) Loss in weight.

35. *Appearance*.—Many coals upon exposure to the air become covered with a coating of white iron sulphate which causes a pile to look as though covered with frost. This white coating is thought by many to signify deterioration of the coal, but as shown by Professor Parr it merely signifies that some of the sulphur has oxidized to the white sulphate, which is soluble in water, and is easily washed away by rain. The ash content of such coal is also slightly lower than that of fresh coal. In any case, the white coating is only a covering, and the coal below the surface of the pile is not affected.

36. *Loss of Heating Value*.—The loss of heating value resulting from storage is comparatively small and Parr* says with regard to this: "Bituminous coal can be stocked without appreciable loss of heat values provided the temperature is not allowed to rise above 180 degrees F. In fact, there is no appreciable evolution of CO_2 at temperatures below 260 degrees F. The indicated heat loss per pound of coal is due more largely to an increase in weight of a unit mass of coal resulting from the absorption of oxygen rather than from an actual deterioration or loss of heat units. . . . Under-water storage prevents loss of heat value." According to detailed values given by Parr† and summarized in Table 5, the indicated loss of heating value

* "Effects of Storage Upon the Properties of Coal." Univ. of Ill. Eng. Exp. Sta., Bul. 97, pp. 7, 38, 1917.

† Ibid., pp. 15-22.

TABLE 5
DECREASE IN HEATING VALUE (B. t. u.) OF ILLINOIS COALS

Coal Tested and Length of Time Stored	Nut		Screenings		Nut	Screenings
	Exposed Bins	Covered Bins	Exposed Bins	Covered Bins	Under- water	Under- water
	Per cent	Per cent	Per cent	Per cent	Per cent	Per cent
<i>Stored 1 year</i>						
Williamson County.....	0.84	1.64	1.37	1.35	0.93	0.57
Vermilion County.....	2.13	2.77	3.92	4.46	2.00	2.72
Sangamon County.....	3.15	4.12	5.14	4.52	1.83	0.86
<i>Stored 3 years</i>						
Williamson County.....	2.09	3.60	2.11	3.73		
Vermilion County.....	4.98	6.23	8.48	10.47		
Sangamon County.....	4.54		7.11	10.48		
<i>Stored 6 years</i>						
Williamson County.....	3.04	3.60	3.96	4.31		
Vermilion County.....	5.49	6.39	10.83			
Sangamon County.....	6.49	6.81	6.85	10.81		

after a period of one year in storage is relatively low, averaging for nut coals and screenings from Williamson, Sangamon, and Vermilion counties only about 3 or 3½ per cent. Coals vary in this respect, those from southern Illinois showing less change than those from central Illinois and this difference increases with the length of time in storage, that is, the coals which show a small decrease in heating value at first continue to show a relatively small decrease as time goes on.

Decrease in heating value is consistently greater with screenings than with screened nut, according to Parr's tests of Illinois coals.

Coal stored in open bins shows consistently a lower percentage of loss of heating value than coal stored under cover, due no doubt to the oxidation of the sulphur when exposed to the air and its subsequent leaching out.

Experiments made by the Bureau of Mines upon large samples of coal gave the following results:*

“The amount of deterioration of coal in heating value during storage has commonly been overestimated. Except for the subbituminous Wyoming coal, no loss was observed in outdoor weathering greater than 1.2 per cent in the first year, or 2.1 per cent in two years. The Wyoming coal suffered somewhat more loss; 2 to 3 per cent in the first year and as much as 5.5 per cent in three years.”

* Porter, H. C., and Ovitz, F. K., "Deterioration in the Heating Value of Coal During Storage." U. S. Bureau of Mines, Bul. 136, p. 8, 1917.

New River coal tested at Portsmouth, N. H., Pittsburgh, Pa., and Norfolk, Va., gave the following results:

“In general, the conclusion to be drawn from these tests is that New River coal, under severe conditions of outdoor exposure to the weather, deteriorates in heating value approximately one per cent in the first year, two per cent in the first two years, and not over three per cent in five years. Storage under water prevents practically all deterioration during one year, and no more than 0.5 per cent has been found in any test for two years or less. Salt water possesses no advantage over fresh water in preventing deterioration. Intermittent exposure and partial drying of the submerged coal probably causes deterioration in some degree, although very small.

“Submergence storage of New River coal is not to be recommended for the sake of preventing deterioration in heat value. Its advantage lies only in insuring against spontaneous combustion.” *

Tests of Pittsburgh coal at Ann Arbor, Mich., gave the following results:

“The amount of deterioration in one year’s open air storage was practically negligible, even in the upper six inches of the exposed coal. During the second, third, fourth, and fifth years the deterioration proceeded very slowly and did not reach an amount greater than 1.1 per cent in five years. The submerged portions may be said to have suffered practically no loss measurable by the degree of accuracy used.” †

Pocahontas coal tested on the Isthmus of Panama gave the following results:

“During one year’s outdoor exposure this coal deteriorated very slightly (less than 0.4 per cent) in heating value, and that the deterioration took place entirely during the first months (June 15 to December 15). There was a further deterioration of 0.4 per cent during the second year.” ‡

Sheridan, Wyoming, lignitic coal stored at Sheridan gave the following results:

“Coal under the conditions of these tests loses 3 to 5.5 per cent of its heat value in about three years’ storage, the greater part (70 to 80 per cent) of this loss being in the first nine months. During the period of $2\frac{3}{4}$ years, the deep bins suffered the greatest loss, probably because their sides offered greater surface for access of air than those of the small bins. The latter became covered with a 12-inch layer of fine slack that helped to protect the layers beneath from oxidation. In the deep bins, the lumps became badly cracked, but retained their form sufficiently to give more ready access of air, and thus permit greater oxidation.

“In storage of Sheridan coal for more than three months, covering the bins is

* Ibid., p. 22.

† Ibid., p. 25.

‡ Ibid., p. 29.

not as advantageous as the use of air-tight bottoms and sides (of concrete, for example), and the accumulation of a protecting layer of fine slack on the surface. The deterioration of Sheridan coal in heat value can probably in this manner be kept below three per cent in one year, and will probably not increase to more than four per cent in two or three years if the coal remains undisturbed. Physical deterioration (slacking) is also largely prevented in the under portions by the formation of a closely packed layer of slack, at least twelve inches thick on the surface.

“Although no indications of spontaneous heating were noted in the tests herein described, it is found in practice to be dangerous, on account of dangerous heating, to store Sheridan coal in piles greater than about ten feet in depth or width. In large masses of coal radiation of spontaneously developed heat is restricted to a dangerous degree. Submerged under water would probably prove particularly advantageous as a means of safely storing subbituminous coal of the Sheridan type.”*

The Coal Storage Committee of the International Railway Fuel Association in 1915 reported that for Ohio, Pennsylvania, and West Virginia the loss was very slight and that no difference was noted by customers of a large coal company between the fresh coal and coal stored from six to twelve months.

The Boston and Main Railroad has had coal in storage from fifteen to twenty years with no signs of heating, and although there was surface deterioration, lumps were found within the pile which looked like freshly mined coal and showed a heat value of 13,000 B. t. u.

Kansas coal showed a shrinkage of 5.26 per cent in weight due no doubt to drying out.

Oklahoma, Kansas, Missouri, and Northern Arkansas coals lose from two to eight per cent in heating value and Texas coals much more due to spontaneous combustion.

J. G. Crawford, Fuel Engineer of the C. B. & Q. R. R., says:

“In general, the deterioration of coal stored in piles will be less as the distance from the surface increases. Thus the total amount of deterioration per ton of coal depends upon the width and depth of the pile. If we have a pile only five or six feet deep, as sometimes happens when coal is stored hastily without any defined system, the total amount of the deterioration per ton is going to be a great deal more than if the pile were twenty (20) feet deep, because in the smaller pile the area of exposed surface per ton is a great deal more than in the case of the larger pile.

“We stored a Wyoming bituminous coal, which was 10 per cent moisture, 16 per cent ash, 5 per cent sulphur and 11,000 B. t. u. This coal was in a pile twenty feet deep and fifteen feet broad on top. At the end of four years it was found that

* Ibid., pp. 33-34.

there had been no deterioration in this coal when a point about three feet below the surface was reached. The major portion of the deterioration was in the coal forming the flat top, which allowed the rain and snow to collect, no means being provided to shed the rain and snow.”

The Consolidation Coal Company Fairmont, W. Va., reports no changes in the chemical properties of coal and less than $\frac{1}{2}$ to 1 per cent loss of calorific value after storing a 200-pound sample for a period of eighteen months.

The Westmoreland Coal Company of Irwin, Pa., says:

“This company has made some elaborate tests of the storage coal and weathering, and the results have shown that there is a very unexpectedly slight deterioration; that is to say, at least, in our coal. The coal pile oxidizes rapidly on the outside and puts on an overcoat of slack or fine coal, which seems to protect the interior of the pile and this process of greater or less oxidation and slacking goes on to a depth of, say, twelve inches or thereabouts, and underneath that we have generally found all its original value. Indeed, in some tests we have found that there has been no deterioration, even in the much more delicate tests of gassification and production of illuminating gas. We have had some coal exposed in piles, and some under cover, for some ten or fifteen years, and the general reply to your inquiry would be that there is no material depreciation.”

It is difficult to differentiate between losses in storage piles due merely to natural weathering and those due to incipient or actual spontaneous combustion of the coal, also between losses in heating value and losses in coking and gas making or other properties. An effort was made to obtain information upon these points through the questionnaire, but it has not always been possible to interpret correctly the exact meaning of the answers upon these closely related subjects. The answers received to Question 13 of the Questionnaire, “Does the coal decrease in heating value as a result of storage?” were, “Yes,” 21, and “No” or “Not appreciably,” 20 and “Very little,” 2. The percentages of depreciation given were less than 5 per cent in six cases, between 0 and 10 per cent in three, from 10 to 20 per cent in one, and from 20 to 50 per cent in one. The lower percentages probably express the same general opinion as “Very little” or “Not appreciably,” and it is evident that the higher percentages mean after heating takes place.

These opinions, based mainly upon observation and experience in storing large amounts of coal under various conditions though not upon calorimetric or boiler tests, agree in general with the results of the tests of Parr, Porter, and Ovitz upon comparatively small sam-

ples or test lots, that is, that the loss in heating value due to storage alone is small and should not deter any one from storing coal through fear of any financial loss due to a decrease in heating value.

The most adverse opinion, but an interesting one, was given by A. Geschwindt, General Manager of the Rockford Electric Company, who says:

“Our experience has been that through the heating of the coal and therefore drying out, we lose considerable efficiency. If there is some method used whereby the moisture could be returned to the coal that is dried through heating, we could obtain results from this coal with less loss than we do at the present time. This is aside from that part of the coal which is actually burned while in storage pile..

“I have not been in a position to make actual tests other than laboratory tests on the value of coal that has been heated thoroughly in storage piles, but we find that in the ordinary course of burning the coal under boilers we get only half the efficiency of the boiler therefrom. While on the other hand, from testing in the laboratory we find that the coal shows not to exceed ten per cent loss.

“This is the difference between actual test of coal through the laboratory method and that of burning in the ordinary course of firing for steam.”

37. *Firing Qualities.*—There is a general opinion among firemen that stored coal is dead, and burns less freely than fresh coal. It is often condemned by them as being “no good.” Others state just as strongly that it burns better than fresh coal, but it seems to be the opinion of those who have given most careful thought to the matter that although there is no material depreciation in heating value, some coals at least, probably due to the loss of a small amount of volatile material, do not burn so freely after storage and must be fired in a slightly different manner.

Melvin Patterson, of the Brown Hoisting Machinery Company, Cleveland, Ohio, says:

“Ohio, Pennsylvania, and West Virginia bituminous coal stand the weather conditions best. The coals are piled forty and fifty feet high along the lakes. The danger due to spontaneous combustion, provided the coal is placed in the pile properly, is very slight. With the modern large grab bucket machines deterioration due to spontaneous combustion is cut to a minimum as the grab bucket can dig around the fire in less than twenty-four hours. Large operators say if they can get the coal pile off the dock in one year deterioration is negligible.”

The Fuel Station Committee of the International Railway Fuel Association concludes with regard to different coals:

“No serious consideration need be given to the question of deterioration, but each coal should be carefully studied to determine the best methods for preventing

spontaneous combustion and it is advisable to store coal from different fields in separate piles.”

In connection with the six-year weathering tests* of southern and central Illinois coal, samples of nut coal from Sangamon, Vermilion, and Williamson counties and screenings from Sangamon and Williamson counties were tested under a boiler by A. P. Kratz.† He states:

“On the first test the coal banked slightly at the water-back, and the whole amount on the grate became clinkered. It immediately became evident that in order to run at all, the coal had to be kept away from the water-back. After the clinker had been removed, a fresh start was made and care was taken to keep the fuel bed from four to six inches away from the water-back. When this was done no further trouble was experienced.”

The tests showed that weathered coal requires a stronger draft than fresh coal, indicating that although the weathered and fresh coals have approximately the same B. t. u. values, weathered coal is more sluggish in action.

The general summary covering the behavior of the coal in steam generation after six years of storage, according to Kratz,‡ is as follows:

“1. Burning weathered coal is largely a question of correct handling and ignition. Under these circumstances it gives as good results as fresh screenings.

“2. Weathered coal requires a little thinner fire and more draft than fresh screenings.

“3. When using weathered coal the fuel bed should not approach any nearer to the water-back than from four to six inches, otherwise trouble with clinker is experienced.

“4. Practically as high capacity was obtained with weathered coal as with other coals used, and, if anything, the fuel bed requires less attention.”

These conclusions refer to burning stored coal under a stationary boiler where the draft can be much more easily regulated than in locomotive firing. If coal has become broken in storage and is burned in a locomotive, there will be an additional loss through the fine coal being carried out the stack.

Interesting information about coal exposed to the air for a long period has been furnished by John J. Davies, Commissioner of the

* Parr, Op. Cit., p. 36.

† “A Study of Boiler Losses.” Univ. of Ill. Eng. Exp. Sta., Bul. 78, p. 50, 1915.

‡ Op. Cit., pp. 53-54. Note.—Attention is particularly called to the opinions upon this subject given in Appendix II, p. 132.

Saline County Operators Association, and A. W. Helmholtz, of the Continuous Process Coke Company of Harrisburg, Illinois. In De-Koven, Union County, Kentucky, there is a pile of approximately 30,000 tons of coal, mostly slack and supposed to have been dumped where it now lies between the hills some forty or fifty years ago. When the samples were taken, the lower part of the pile was saturated with water, but the pile had previously been subjected to weather conditions. Tests of this coal made by Helmholtz gave 9.4 pounds of water evaporated per pound of raw coal as compared with 10.3 pounds of water per pound of Saline County slack. The B. t. u. values calculated from analyses gave 11,000 as compared with about 13,500 for fresh coal presumably from the same seam and locality.

38. *Spontaneous Combustion*.—The greatest difficulty in the storage of coal is undoubtedly the tendency of many coals to fire spontaneously. As previously mentioned it is the deterioration due to this cause which is often confused with a supposed decrease of heating value as a result of storage.

The gradual heating of a coal pile is due mainly to slow oxidation of the carbon in the coal and, to a less extent, to the oxidation of the sulphur in the iron pyrites contained in the coal. If the air supply is sufficient to permit oxidation but not sufficient to carry away the heat as rapidly as it is formed, the temperature in the pile will rise gradually and finally the coal will fire. Any method of storage to be successful must be so designed that the heat generated in the pile will not exceed the heat lost by radiation. Freshly mined coal has a special tendency to oxidize and thus to heat. While this property varies with different varieties of coal, it is generally true of all coals. The greater the time elapsing between the mining and storing of any coal, the less is the liability to firing; hence, if possible, coal should be exposed to the air, that is, allowed to become seasoned before it is put in storage. This is, of course, frequently impracticable.

Oxidation of both the carbon and the sulphur takes place more rapidly as the temperature increases; hence coal stored during hot weather is more likely to heat than that stored on cool days. In spite of this fact, however, many prefer to store coal during July and August not only because of the lower price which usually prevails during these months, but because, as they say, "the coal is drier and less apt to fire."

The smaller the coal the greater is the surface area exposed to the air, the more rapid is the oxidation and the greater the tendency to heat; hence coal in lumps is not so likely to fire as fine coal, slack, or run of mine coal. If possible, the slack should be removed before storing. Great care, therefore, should be taken in handling the coal to minimize the breakage. In handling large quantities of coal this precaution cannot be given the same attention as when handling small quantities, but whenever it is impossible to screen the coal before storing and whenever sizes and kinds must be mixed, means should be provided to open the pile rapidly since heating may then be expected. Many persons believe that washed coal will not fire so readily as unwashed, because the dust has been removed.

Air currents through a pile, unless ample to carry off the heat, should be avoided since many fires seem to be due to sluggish air currents in contact with fine coal. Coal should, when possible, be piled to avoid alternate stratification of coarse and fine coal.

The opinion is wide-spread that a mixture of different varieties of coals is more liable to spontaneous combustion than a pile of a single variety, and while no explanation of this opinion has been given it is possible that the most easily combustible coal in a pile merely starts the heating and thus lowers the safety point of the less combustible coal. J. B. Porter says:

“With reference to the prevailing opinion that a pile of mixed coal is more liable to heat than one of uniform material: I can only say that my observations so far as they go bear out this opinion, but do not throw any clear light on the case. I can see no reason why a mixture of coals should be any worse than the worst coal in the lot, and I think it quite likely that this will prove to be true. There are, however, so many authenticated cases of mixed coals heating when apparently either of them would have kept safely by itself that I think we should keep on the safe side by avoiding mixtures whenever possible. I may add that the objection to mixtures is by no means confined to this continent; in fact if my memory serves me the danger of coal mixtures was first pointed out in England, and I believe in connection with East Yorkshire coals.”

A high percentage of volatile matter in coal does not of itself increase its liability to spontaneous combustion. As a result of 1200 replies received from large consumers of coal, Porter and Ovitz* concluded that there is no reason to place special confidence in smokeless coals for safety in storage. In the large stock piles at Panama, Appalachian coals, with seventeen to twenty-one per cent of volatile

* “Deterioration and Spontaneous Heating of Coal in Storage.” U. S. Bureau of Mines, Technical Paper 16, pp. 9-10, 1912.

matter, give a great deal of trouble from spontaneous fires. Several large works report, moreover, that their low-volatile coals are more troublesome with respect to spontaneous fires than their high-volatile gas coals.

The high-volatile coals of the West, are, to be sure, usually very liable to spontaneous heating, but owe this property to the chemical nature of their constituent substances rather than to the amount of volatile matter that they contain. Strange as it may seem, the oxygen content of coal appears to bear a direct relation to the avidity with which coal absorbs oxygen; high oxygen coals absorb oxygen readily, and, therefore, have a marked tendency to spontaneous combustion.

Formerly the gradual heating of the coal and its final spontaneous combustion was thought to be entirely or largely due to the presence of sulphur. The chemical studies of Parr, Ovitz, Porter, and others have shown that a coal may heat as a result of the oxidation of the carbon, even when pyritic sulphur is absent. If pyritic sulphur is present, however, oxidation of this sulphur takes place, thereby supplying a definite additional source of heat and assisting in the oxidation of the carbon. The oxidation of the sulphur also acts mechanically to break up the coal; thus by increasing the amount of small coal, the tendency to fire is increased.

Although sulphur has been shown to be only one of the factors producing heat, it is still thought by many who store coal to be the determining element in the heating of coal.

J. G. Crawford* noted piles of coal containing from four to five per cent of sulphur which had been stored on the ground from four to six years without firing.

Stansfield† says:

“Pyrites, by weathering, could hardly heat itself up to ignition point, much less the coal surrounding it; in fact, heaps of pyrite free from carbonaceous material are never known to fire spontaneously.

“In considering these arguments we must remember that there are different varieties of pyrites, that known as marcasite weathering much more rapidly than the ordinary variety. The fine flakes of pyrites, sometimes scattered throughout the coal, are probably marcasite and certainly weather faster than the larger lumps.

* Proceedings, International Railway Fuel Association, Vol. VII, p. 295, 1915.

† “Spontaneous Combustion of Coals,” An Investigation of the Coals of Canada, Vol. VI, pp. 110-111, 1912.

“A single match may ignite a pile of shavings, a single flake of marcasite might cause a warm spot that might result in a coal pile firing. This might explain the fact that the tendency for a coal pile to fire is not proportional to the percentage of sulphur or pyrites present.

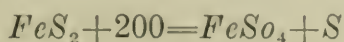
“We are not at present looking for something to heat the coal to its ignition point, but only to give it a little initial heating. If sulphur is liberated from pyrites and oxidizes (both actions being accompanied by an evolution of heat) at low temperatures, it might easily act as a starter, although the sulphur did not remain as such to lower the ignition point.

“The crucial question seems to be, does the coal or the pyrites generate heat the faster by its normal oxidation at low temperatures? The evidence seems to be strongly in favor of the answer: the coal. This would thus rule pyrites out of account as the usual causes of fires. It is, of course, always conceivable that under certain circumstances this might be reversed and the pyrites blamed; but this probably seldom or never occurs.

“On one point all parties are agreed. When flakes of pyrites weather they expand and fracture the coal, and thus expose more and fresh surfaces for oxidation and indirectly increase the danger of heating.”

Near Danville, Illinois, the Missionfield Mining Company has a plant for separating pyrite from the coal and preparing the pyrite for market. According to the superintendent of the plant, even the fine pyrite passing through a 7/16-inch screen has never fired in the bin, except when there has been found in the pile a piece of wood or other carbonaceous substance which has acted as a match to start the combustion.

In spite of the results of tests the opinion prevails very generally that a high sulphur coal is more likely to fire than one containing a small amount of pyrite, and an investigation should be undertaken to show the exact influence in storage piles of the sulphur content. It has been suggested that the pyrite may oxidize and set the sulphur free according to the reaction.



and that the free sulphur having a low ignition point may act as an igniter as it does in a match or in gunpowder. It is certainly true that on many weathered coal dumps there is a deposit of yellowish white material which very closely resembles free sulphur.

The effect of moisture upon the heating of coal is a much disputed question, although those who store coal are practically unanimous in the opinion that water stimulates spontaneous combustion. On the other hand scientific investigations seem to indicate that coal

oxidizes less when wet than when dry.* The effect of water in disintegrating coal high in sulphur is undoubted. Vivian Lewes, of England, draws a sharp distinction between wet coal and damp coal, the latter being considered dangerous. The undoubted fact that the top of a coal pile is warm after a rain is explained by the New South Wales Commission as due not to the fact that the water causes the coal to heat, but the pile being already hot inside heats the water which runs down into the pile and then returns as steam to the surface and heats it.

Stansfield† says: "It is probable that when our knowledge of the air circulation and rate of oxidation in coal piles is increased, water will be a good servant; at present it is a dangerous ally."

Coal immersed in water does not deteriorate to any extent chemically nor does it disintegrate except as a result of handling. The amount of water absorbed does not affect the burning qualities of the coal.

Instances in which water seemed to assist in stimulating spontaneous combustion are quoted in connection with reports regarding coal stored by railroads. A pile is reported to have fired under conditions in which seepage from the water tank kept part of the pile damp. Another instance occurred in a low corner of a coal pile where water collected, but this corner was shut in by a side hill which excluded the air so that it is a question which agency caused the firing, the presence of the water or an insufficient amount of air.

Most of the answers to the questionnaire agree in advising against storing on the ground on account of the presence of moisture, but an equally strong reason lies in the desirability of having a clean surface from which the coal may be taken up.

39. *Coking and Gas Making Properties.*—There are few recorded data concerning the effect of storage upon the coking properties of coal, although the replies to the questionnaire agree generally in the opinion that heating injures the coal for this purpose. H. D. Hall, Superintendent of the By-product Plant of the Inland Steel Company, says that the gas made from stored coal does not differ materially from that made from fresh coal. H. C. Porter, of the H. Koppers Company, says that the weathering of coal in the open has

* A most comprehensive discussion of this subject by Edgar Stansfield, of McGill University, will be found in *An Investigation of the Coals of Canada*, Vol. VI, pp. 95-120, 1912.

† *Ibid.*, p. 115.

a considerable effect upon the coking qualities. A. Ives, Superintendent of the American Zinc Company of Illinois, says that coal which is kept on the ground for more than thirty days deteriorates to an extent which renders it almost useless for gas making.

Tests to determine the effects of the age of coal on coking qualities, made by J. B. Porter in Canada on coals varying in age from one week to twenty-one months, showed that while in the majority of cases the difference in the coke was very slight, in some cases there was a marked deterioration of the coking quality of the stored coal.

J. B. Porter* says: "The coking quality of coal, even when stored carefully in sacks, does deteriorate somewhat with age, although not as much as is sometimes supposed, a fact which was confirmed by later tests on comparable coals. It was, however, shown that different coals varied very much in their susceptibility to aging."

"White has found," according to Porter,† "that weathering materially lowers the hydrogen-oxygen ratio, and as suitability for coking decreases rapidly as this ratio falls below fifty-nine per cent, it is easy to see that a relatively small amount of weathering may produce a profound effect on the coking qualities of a coal in which this ratio is close to this limit."

A. R. DeHoll, Superintendent of the By-product Coke Department of the Inland Steel Company, says:

"We know by experience that if the coal is heated to such an extent that smoke appears, the coke resulting from this coal is merely a conglomerate of small pieces. The coal seems to have lost its cementing qualities and at the same time the heating value of the gas will decrease from forty to fifty B. t. u. In the cases wherein we were troubled with coal thus affected I have found that the yield of ammonia was only slightly affected, but the yield of tar more so.

"In order to eliminate this trouble, I always use up the oldest coal, and when the shipping season stops at the first of December we reclaim the coal for our ovens on day turn in such a manner as to leave in our storage place room for coal taken from the opposite end of the pile. Thus by turning and airing the coal we have effectively eliminated coal fires in our operation."

Experiments made under a coöperative agreement between the U. S. Bureau of Mines‡ and the Gas Experiment Station of the University of Michigan upon West Virginia and Pennsylvania gas coals showed that coals, after being kept in storage for periods up to five

* An Investigation of the Coals of Canada, Vol. VI, p. 27, 1912.

† An Investigation of the Coals of Canada, Extra Vol., p. 172, 1915.

‡ White, A. H., and Baker, Perry, "Coals Available for the Manufacture of Illuminating Gas." U. S. Bureau of Mines, Bul. 6, 1911.

years, showed changes in their gas making properties too small to be detected by the present methods of testing and analysis. Prof. A. H. White, of the University of Michigan, says:

“The coal was piled seven to nine feet deep in bins with separate compartments so that the coal was not moved until the time came for its particular compartment to be tested. The temperature of the coal in the pile was measured during the first few months and no spontaneous heating was detected. This is important because it is known that coal which has heated is inferior as a gas coal. The coal was exposed freely to all the effects of the climate of Southern Michigan, which is rather damp and with rather severe winters.

“The main body of the coals suffered little physical change during the five years but the top six inches crumbled considerably. Chemical analyses of the coal failed to reveal any material change, even in this top six inches, after weathering five years. Neither was there any material change in the absolute weight of the coal. Samples of the coal immersed in running water showed rather more change in chemical composition than did the coals exposed to the weather.

“Gas coals of this type can be exposed to the weather in a rather severe climate for five years (provided they do not heat in storage) and at the end of the five years still be entirely satisfactory gas coals.”

There is little evidence to support the opinion that the value of middle west coals for gas making decreases as a result of storage, since these coals have not been used for gas making until recently.

40. *Degradation or Breakage.*—In connection with the storage of coal, degradation, i. e., the breakage of large lumps into smaller sizes, is the result of:

(1) A physical change due to weathering which causes the lumps to crack and break.

(2) Breakage due to handling.

Coals in storage alternately dry out and absorb moisture. This process tends to break up the lumps; also, flakes of pyrites oxidize and assist in the breaking up process. Coals of high moisture content in general deteriorate from breakage due to weathering.

Professor Parr* presents results with reference to the degradation of coal stored one and one-half and six years. These results are given in Table 6.

* “Effect of Storage upon the Properties of Coal.” Univ. of Ill. Eng. Exp. Sta., Bul. 97, p. 35, 1917.

TABLE 6

INCREASE IN FINE MATERIAL AFTER ONE AND ONE-HALF AND SIX YEARS OF STORAGE

(Basis of Reference, the Total Coarse Material in the Original Coal Passing Over $\frac{1}{4}$ inch Screen)

Table No.	Coal and County	How Stored	Initial Storage	After 1½ Years		After 6 Years	
			Dust Passing $\frac{1}{4}$ -inch Screen	Dust Passing $\frac{1}{4}$ -inch Screen	Increase in Per cent of Original Coal over $\frac{1}{4}$ -inch	Dust Passing $\frac{1}{4}$ -inch Screen	Increase in Per cent of Original Coal over $\frac{1}{4}$ -inch
1	Nut: Sangamon.....	Open	1.2	13.2	12.1	31.9	31.0
2	Nut: Vermilion.....	Open	13.6	24.0	12.0	35.0	24.7
3	Nut: Williamson.....	Covered	1.4	11.0	11.1	13.9	12.6
4	Screenings: Sangamon.....	Covered	26.3	38.5	16.5	45.1	25.5
5	Screenings: Vermilion.....	Open	37.7	48.4	17.1	49.6	19.1
6	Screenings: Williamson.....	Covered	38.8	45.4	10.7	50.6	19.2

Parr concludes from the results of these tests that the rate of disintegration is consistent for a given variety of coal and suggests that the oxidation of the organic material may be as largely responsible for this breaking down of the particles as the oxidation of the finely divided pyritic sulphur. The sulphur may or may not be distributed through the texture of the coal; this characteristic has not been determined for Illinois coal.

The amount of degradation depends upon the variety of coal, the method of handling, and to some extent upon the time the coal is in storage. The effect of time depends somewhat upon the size of the storage pile, since the depth to which the deterioration extends into any pile is not very great.

Breakage of coal in handling is a matter of great importance where close sizing is practiced for domestic use, not only because it increases the amount of fines and thus the liability to firing, but also because it means rescreening and additional cost. There is very little reliable information on this subject at present.

In experiments* to determine the extent of breakage of Illinois coal falling through different heights and upon different materials, such as steel, concrete, and wood, L. A. Mylius found that wetting the coal thoroughly before dropping seems to decrease breakage materially. Each coal has in itself a tendency to break which does not vary so long as the moisture content is the same. As a given coal gradually dries out its friability increases; thus a coal dried in storage with a decrease of from fifteen to eight per cent in moisture will break more readily on rehandling, owing to the change in moisture content alone. A coal which is wet before being handled seems to suffer less breakage from handling than one which has not been wet, but these statements should be more thoroughly tested before being presented as conclusive. In one of the discussions on coal storage at the International Railway Fuel Association it was stated that there was less breakage when the coal was handled wet.

The apparatus used for Mylius' tests, as shown in Fig. 56, consists of a steel framework inside of which a box is raised and lowered by means of ropes passing over pulleys at the top and attached to a small hand operated roller. The hinged bottom of the box can be dropped very quickly by means of strong springs.

Tests on the degradation of coal, made by Porter and Ovitz† are described as follows:

"Fig. 57 shows the results of some comparative breakage tests made on lump coals to bring out this point. A 50-pound sample of screened lump (over 2 inches) was dropped four times from a height of six feet and the coal then sized by screening into three portions, $\frac{1}{8}$ inch, $\frac{1}{8}$ inch— $\frac{1}{2}$ inch and $\frac{1}{2}$ inch—2 inches. The great friability of the low-volatile Appalachian coals is clearly shown and the wide variation between different coal types. It is found in practice that dust and fine coal, when mixed with some larger coal, add greatly to the danger of spontaneous heating. A single instance from commercial practice will serve to illustrate this. The Calumet and Hecla Mining Company has three large coal storage piles of 125,000 to 200,000 tons each, in which they store lump coal passed over $1\frac{3}{4}$ -inch bar screen just before placing on the piles. No spontaneous fires have occurred since they began this practice, although several fires had been experienced in the same kind of coal before that time."

The following values given by R. V. Norris‡ for anthracite coal

* These experiments have been carried on in the laboratory of the Department of Mining Engineering at the University of Illinois, but they have been interrupted by the withdrawal of Mr. Mylius to join the Canadian Army, and no conclusive results are available for publication at this time.

† "The Spontaneous Combustion of Coal." Original Communications, Eighth International Congress of Applied Chemistry, Vol. X, pp. 264-266, 1912.

‡ "The Storage of Anthracite Coal." Trans., Am. Inst. of M. E., Vol. 38, p. 314

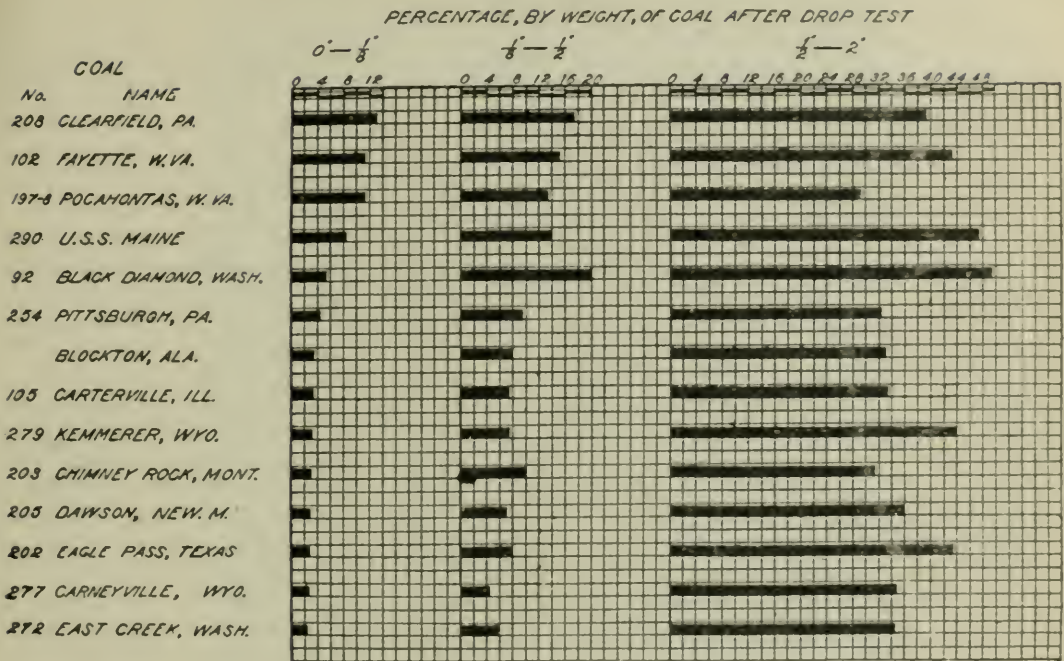


FIG. 57. CHART SHOWING EXTENT OF BREAKAGE IN A 50-POUND SAMPLE OF SCREENED LUMP COAL (OVER 2 INCHES) DROPPED FROM A HEIGHT OF SIX FEET IN PER CENT OF ORIGINAL SAMPLE

(From "The Spontaneous Combustion of Coal" by Porter and Ovitz)

show the commercial importance of degradation from handling, because with practically all bituminous coals the amount of degradation would be much greater than for anthracite although the depreciation in value for bituminous would not be so marked, except when bituminous coals are intended for domestic purposes.

"LOSS IN UNDERSIZE IN PASSING THROUGH STORAGE BINS

Size of Coal	Breakage into Smaller Prepared Sizes Per cent	Breakage into Small Sizes Per cent	Total Breakage Per cent
Broken.....	19.57	6.60	26.17
Egg.....	10.18	8.50	18.68
Stove.....	4.92	8.14	13.06
Nut.....		7.65	7.65
Pea.....		10.83	10.83
Buckwheat.....		4.06	4.06

“Even taking half these figures, which would be most conservative, and assuming perfect rescreening, the loss at seaboard on 1,000,000 tons of prepared and pea-coal in about the usual proportions would amount to \$545,000, as shown in the following table:

Size	Original			Final		
	Quantity	Price per Ton	Total Value	Quantity	Price Per Ton	Total Value
	Tons			Tons		
Broken.....	130,000	\$4.75	\$ 617,000	96,040	\$4.75	\$ 456,190.00
Egg.....	225,000	5.00	1,125,000	191,770	5.00	958,850.00
Stove.....	195,000	5.00	975,000	192,410	5.00	962,050.00
Nut.....	200,000	5.00	1,000,000	215,040	5.00	1,075,200.00
Pea.....	250,000	3.25	812,500	248,630	3.25	808,047.50
Buckwheat.....		2.50		35,515	2.50	88,787.50
Rice }		1.75		20,595	1.75	36,041.25
Barley }						
Total.....	1,000,000		\$4,529,500	1,000,000		\$4,385,166.25

“The loss in breakage, from this calculation, is 54.5 cents per ton, in addition to the cost of storage.

“Many attempts have been made to reduce the breakage involved in handling through pockets, and this is often minimized by the use of shallow pockets, with resultant loss of storage; counter-chutes, spirals, and shelf-chutes in the deeper pockets, and the use of feeding-shafts, which, when properly maintained and intelligently used, keeping them full, feeding in at the top as the coal is discharged from the bottom, certainly greatly reduce the losses by dropping.”

Breakage may be diminished by sliding coal along chutes or along the side of the pile rather than by dropping it. Coal moving in a large mass breaks less than single lumps. Drawing coal from the bottom of a pile under pressure results in heavy breakage. Handling prepared coal by scraping conveyors produces a breakage loss of from two to four per cent, according to the method of feeding and discharging, there being very little breakage during transit. Bucket elevators cause breakage of from two to five per cent according to the method of feeding and the discharging.

The commercial importance of the breakage problem in connection with the handling of coal is discussed very fully by Prof. E. A. Holbrook.*

* “Dry Preparation of Bituminous Coal at Illinois Mines.” Univ. of Ill. Eng. Exp. Sta., Bul. 83, 1916.



FIG. 56. APPARATUS USED TO DETERMINE THE EXTENT OF BREAKAGE IN COAL CAUSED BY DROPPING IN TESTS AT THE UNIVERSITY OF ILLINOIS

41. *Loss in Weight.*—The loss in weight due to exposure to the air is a matter which seems to be generally misunderstood. Such loss undoubtedly occurs whenever the water evaporates from the coal. Since water, however, is not a heat producing element, but rather is a heat absorbing one, the loss in weight is an advantage, because pound for pound the coal has a higher heating value after the water has been evaporated than before. It has been shown by M. L. Nebel* that coal as it occurs in the ground is probably saturated with moisture and, although the moisture gradually dries out, it may be returned to the coal by immersing it in water. This fact of course has no practical application in the utilization of the coal, except in connection with the determination of its specific gravity.

VII. EXPENSE OF STORING COAL

Statements and estimates of the expense of storing and reclaiming coal have been submitted by many public service corporations, industries, coal dealers, and railroads. These estimates vary widely according to the conditions under which the storing is done and, for this reason, are not directly comparable. The reports were submitted in response to the questionnaire, the form of which is shown on page 130, but in only a few cases did the replies contain detailed facts such as, for instance, the expense due to depreciation, interest, rental, insurance and overhead. Many stated that it was impossible to report these items separately. The estimates are summarized in the following list in which are also given page references indicating the number of the page in this circular on which more detailed information regarding the conditions of storage may be found. A study of these conditions is essential to a correct interpretation of items of expense reported.

Other statements of the expense of storage may be found in the Proceedings of the International Railway Fuel Association.†

* "Specific Gravity Studies of Illinois Coals." Univ. of Ill. Eng. Exp. Sta., Bul. 89, p. 6, 1916.

† Vol. VI, 123, Vol. VII, 263-264, 265-266, 267-268.

EXPENSE OF STORING AND RECLAIMING COAL

<i>Firm</i>	<i>Storing</i>	<i>Reclaiming</i>	<i>Total</i>
<i>Hand Storage</i>			
<i>Bohmer Coal Company, St. Louis, stored on ground and reclaimed by hand shoveling</i>			\$.15-.30
<i>St. Bernard Mining Company, Nashville, Tenn., dumped from wagons, reclaimed directly into wagons, p. 163</i>	\$.22		
<i>Polar Wave Ice and Fuel Company, St. Louis, hand labor, p. 161</i>	.48	\$.16	.64
<i>Crystal Ice and Fuel Company, Danville, Ill., stored in bins by dumping, reclaimed by shoveling, p. 159</i>			.17
<i>Ebner Ice and Cold Storage Company, Vincennes, Ind., by hand labor and conveyors, p. 161, for depreciation and interest, .154 cents, also under water .214 cents</i>			.368
<i>Rock Island Fuel Company, Rock Island, Ill., labor only, p. 159</i>	.08	.10	.18
<i>University of Illinois Storage with motor truck and by hand, reclaimed by wagon loader, p. 42 and 47</i>	.21-.34	.20	.41-.54
<i>Side Hill Storage, estimated in 1907, p. 56</i>			.19
<i>Locomotive Crane Storage</i>			
For cost of locomotive cranes, see page 60. Estimated expense of operating locomotive crane, \$1.50 per hour, or 3 cents per ton.			
<i>A large wholesale and retail coal company</i>	.015	.02	.095*
<i>Commonwealth Edison Company, Chicago, labor and materials only, p. 61</i>	.05	.05	.10
<i>Clinchfield Fuel Company, Dante, Va., Crane and trestle, p. 64 and 67</i>	.0671	.0655	.1326
<i>A large wholesale and retail company</i>			.04-.20
<i>Pittsburgh Plate Glass Company, Crystal City, Mo., p. 165</i>	.25	.25	.50
<i>Rockford Electric Company, Rockford, Ill., p. 165</i>	.10	.10	.20
<i>American Zinc Company, E. St. Louis, Ill., p. 171</i>	.20	.15	.35
<i>Crerar-Clinch and Company, Chicago, hand and locomotive crane, p. 161</i>			.25

*This total includes \$0.04 for interest and \$0.02 for depreciation.

<i>Firm</i>	<i>Storing</i>	<i>Reclaiming</i>	<i>Total</i>
<i>Locomotive Crane Storage—Cont'd</i>			
<i>Mineral Point Zinc Company, Depue, Ill.,</i> p. 171	\$.05-.06	\$.05-.06	\$.10-.12
Estimate by C. G. Hall			.2628
<i>Missouri, Kansas and Texas Ry., including cost</i> <i>of track, p. 183</i>	.035	.035	.07
<i>Chicago, Lake Shore and South Bend Ry., p. 181</i>			.12
<i>Grand Trunk Pacific Ry., p. 184</i>	.08	.062	.142
<i>Atlantic Coast Line, p. 183</i>			.04-.05
<i>Central of Georgia Railroad, p. 180</i>	.0258	.0209	.0467
<i>Steam Shovel Storage</i>			
<i>Union Light and Power Company of St. Louis,</i> <i>dumped by hand, reloaded by steam</i> <i>shovel, p. 167</i>	.10	.10	.20
<i>Bridge Storage</i>			
<i>Wisconsin Gas and Electric Company, Racine,</i> <i>Wis., p. 166</i>	.08	.22	.30
<i>Berwind Fuel Company, Duluth, Minn., p. 82.</i>			.60
<i>Link-Belt Company, estimate for bridge stor-</i> <i>age, p. 82</i>	.056	.0515	.1075
<i>Calumet and Hecla Mining Company, Calumet,</i> <i>Mich., Hunt system, steam shovel, p. 174</i>			
Summer	.15	.07½	.22½
Winter	.15	.11¼	.26¼
<i>Large swivel bridge, p. 82</i>		.06	
<i>Under-water Storage</i>			
<i>New Kentucky Coal Company, Kankakee, Ill.,</i> <i>Storage of 250,000 tons, \$20,000-\$30,000,</i> <i>pp. 100 and 101</i>			
<i>Metropolitan Water District, Omaha, Nebr.,</i> p. 102	.063	.063	.126
Estimate by C. G. Hall, p. 106			.1823
<i>Peabody Coal Company, Lemont, Ill., pp. 104</i> <i>and 105</i>			
Estimated expense of operation			.162
Estimated cost of equipment, \$13,830			
<i>Illinois Traction System, see pp. 88 and 89, for</i> <i>cost of plant</i>			
<i>Western Electric Company, Chicago, p. 88</i>	.05	.04	.09
<i>National Zinc Company, Springfield, p. 90 1913</i>			.225
" " " " 1917	.10	.10	.20

APPENDIX I

QUESTIONNAIRE ON COAL SHORTAGE DATA

1. Kind of Coal Stored	State and District Where Coal was Mined	Sizes Stored	Amount Kept in Storage
.....			
Semibituminous			
(Pocahontas, Central Pa., W. Md.)			
Bituminous			
Note: Please give definitely the district in each state from which the coal comes.			
2. How long is coal kept in storage?			
3. Are sizes kept separate in storage?			
4. Is slack removed before storage?			
5. How long time elapses between the mining and storing?			
6. What do you consider the best months to store?			
7. Why do you consider this the best time to store?			
8. Why do you store coal?			
9. What do you consider the financial advantages of storing?			
.....			
10. For what purpose is the stored coal to be used?			
11. Is there any difference in the selling price of fresh and stored coal?			
.....			
12. Does the coal decrease in heating value as a result of storage?			
..... If so, to what extent?			
13. What loss is there through breakage as a result of storing and reclaiming the coal?			
14. Does the coal heat up after being stored?			
If so, how soon?			
15. What measures are taken to prevent heating?			
16. In case of heating, how is coal handled?			
17. To what extent is coal injured by such heating?			

- | | | | |
|-----|---|---------|------------|
| 18. | What method of storing is used? | | |
| 19. | What method is used for reclaiming the stored coal? | | |
| 20. | Is coal stored on the ground? | | |
| 21. | If not on what is the pile built up? | | |
| 22. | Is pile continuous or divided by partition wall? | | |
| 23. | Is storage under cover? | | |
| 24. | What is the shape of the storage piles? | | |
| 25. | What is the height of the storage piles? | | |
| 26. | What machines are used for storing? | | |
| 27. | What machines are used for reclaiming? | | |
| 28. | Are these machines satisfactory? | | |
| 29. | If stored under water what percentage of water is retained by coal when taken from storage? | | |
| 30. | In storing are lumps allowed to roll down and accumulate at bottom of pile? | | |
| 31. | What percentage of fines below $\frac{3}{4}$ inches are present when stocked? | | |
| 32. | Cost of | Storing | Reclaiming |
| | Overhead | | |
| | Labor | | |
| | Supplies | | |
| | Depreciation on mechanical equipment | | |
| | Interest on investment | | |
| | Rental on land on which coal is stored | | |
| | Insurance on equipment | | |
| | Insurance on coal | | |
| | Totals— | | |

Note: If itemized costs are not available please give totals and state what totals include.

33. Please give reference to any articles which have appeared descriptive of your storage.
34. Information furnished by
- Official position
- Company

APPENDIX II

SUMMARY OF CONCLUSIONS AND SUGGESTIONS

REGARDING COAL STORAGE

THE questionnaire, presented on page 130, was sent out to a large number of firms and individuals and on the basis of replies a summary of conclusions and suggestions was prepared. This summary included as nearly as possible all the suggestions regarding coal storage contained in the replies. It was then sent to those who had replied to the questionnaire for an expression of opinion and the request was made that reasons for disagreement be given. The responses were most gratifying, not only because there was a general agreement with the conclusions, but because so much care was evidently taken in studying the summary. In many cases no answers were given to certain questions, possibly because the persons replying had had no experience with the particular phase of the subject in question and did not, therefore, wish to express an opinion. Certain answers of "NO" also do not indicate disagreement from the conclusions, but merely that the conclusion outlined does not represent the practice of the person answering the question or that the person has had no experience with that phase of the question.

The answers have been tabulated under three heads: YES, NO, and DOUBTFUL (?). The question mark does not necessarily imply that the conclusion is questioned, but may mean merely that the person replying had no data upon which to base an opinion. The tabulation of replies is presented, in the form in which the summary was submitted to firms and individuals, as follows:

CONCLUSIONS AND SUGGESTIONS REGARDING COAL STORAGE

(Submitted to 175 corporations or individuals storing coal)

It is impossible to give definite rules and regulations regarding the storage of coal which will be universal in their application and each storage problem must be solved in accordance with such local conditions as the kind of coal to be stored, the space and appliances available, the capital obtainable, etc. The following are some suggestions which seem fairly to represent the results of present storage practice and of certain theoretical considerations based upon experimentation.

Coal is stored:	Yes	No	?
1. To assure a regular supply when the mines are shut down or when coal is not delivered regularly by transportation companies	84		
2. To take advantage of low water freight rates	78	6	
3. Because the price is often less at certain seasons of the year, generally in the summer	87	6	
4. To equalize the prices on the different sizes of coal	65	11	5
5. To avoid the maintenance by the railroads of equipment which is used for only a part of the year	74	2	1
6. Coal storage should not be considered only as a war expedient but as an essential part of all large coal using operations	81	1	
7. Coal should be stored as near as possible at a point where it is to be used so as to avoid rehandling and to equalize transportation facilities	79	2	
8. There is a considerable difference in the ease with which different coals oxidize and this must be considered in storing coal	78		
9. Coal exposed to the air seems to lose some of the volatile ingredients which assist in producing spontaneous combustion; hence the greater the time elapsing between the mining and storing of the coal, the less is the liability to spontaneous combustion	64	4	5
10. The opinion formerly held that the chief source of spontaneous combustion is the pyrites (sulphur) in the coal has not been substantiated by experiments. There is, however, a wide-spread opinion that coals high in sulphur are much more liable to spontaneous combustion than those low in sulphur and an effort is made by many in choosing a coal for storage to get one low in sulphur	65	3	5
11. Although the oxidation of the sulphur is not the principal heat producing element, it tends to break up the lumps and thus increases the fine coal which fires easily	71		7
12. Some coals and particularly those high in sulphur heat more readily if damp, and alternate wetting and drying is undoubtedly harmful.	76		2

	Yes	No	?
13. All varieties of bituminous coal may be stored if of a proper size, if the fine coal and dust have been removed, and the coal is so handled that dust and small coal are not produced in the storing in excessive amounts . . .	76	4	2
14. Most varieties of bituminous coal are liable to spontaneous combustion if the fine coal is not removed before storage	79	1	3
15. In storing coal handle it so as to produce a minimum of dust and fine coal	78	1	
16. Owing to the cost of screening, to the difficulty of arranging for it in large storage piles and to the difficulty of disposing of the screenings, it is often not practicable to store lump or screened coal	74	5	1
17. Unless lump and screened coal are piled carefully and in low piles, crushing is likely to take place and produce dust near the bottom of the pile and thus to start spontaneous combustion	69	6	3
18. Pile as uniformly as possible and avoid stratification of large and small lumps	75	4	
19. As liability to spontaneous combustion increases with the temperature, the cooler the coal is when stored, the less is the liability to spontaneous combustion	72	5	2
20. Coal stored in cold weather is less likely to give trouble than that stored in summer and where summer storage is necessary the liability to heating is much less if the storage is carried on on cloudy or cool days	70	7	3
21. Coal of different varieties should not be mixed in storage if this can be helped, since such mixture seems to increase the liability to spontaneous combustion	72	6	4
22. All storage appliances and arrangements should be so designed as to make it possible to load out the coal quickly	83		
23. Coal should not be stored in large piles unless provision is made for loading it out quickly in case of spontaneous combustion	72	3	
24. Since moisture from below assists spontaneous combustion the storage pile should rest on a dry base	74	2	1

	Yes	No	?
25. Since a small amount of water seems to aid spontaneous combustion, the drier the coal is when stored and during storage, the less is the liability to spontaneous combustion	72	1	5
26. Do not allow pieces of wood, greasy waste or other easily combustible material to be mixed with the coal during storage since they may form a starting point for a fire	78	2	
27. Avoid contact between the coal and all such external sources of heat as steam pipes	82		
28. Pile coal as uniformly as possible and do not allow the lumps to roll to the bottom or to one side of the pile and thus form a flue for the entrance of air to the interior of the pile	72	2	2
29. Avoid air channels in a coal pile such as occur about imbedded pipes, the bents of trestles, cracks through bulkheads, etc., since fires frequently originate near such air channels	67	3	2
30. Unless coal can be piled so that air can thoroughly circulate through the pile and thus carry off the heat as rapidly as it is generated (this is very difficult to accomplish), the more closely fine coal is piled to exclude air the better	65	3	3
31. Ventilation of coal piles by means of pipes running through the pile has usually not been successful in preventing spontaneous combustion, because the air does not circulate freely throughout the pile and in some cases the air currents through these pipes seem to be starting points for fires	63	2	3
32. The great objection to ventilating systems is that they interfere with the handling of the coal	75	1	
33. There is great difference of opinion in regard to the height of piles in which coal may be stored safely. Coal seems to fire almost as quickly in low piles as in high and as often near the top or outside of a high pile as near the bottom. The principal disadvantage of high piles is that coal in them cannot be so readily removed as from a low pile	65	5	
34. Water is not generally successfully applied in extinguishing fire in a coal pile, because it is impossible thoroughly to saturate the pile. A small amount seems to aid spontaneous combustion	73	2	

	No	Yes	?
35. The deterioration in coal stored under water is negligible. Such coal absorbs little extra moisture	74	1	
36. With under-water storage if the entire coal pile is not covered, the part of the pile above the water line is liable to spontaneous combustion	64		3
37. The white coating on the outside of some coal piles is very thin and does not affect the value of the stored coal	66	5	2
38. The best method of handling coal in danger of firing is to load it out and scatter it so that it will be thoroughly cooled off	75		1
39. Coal which has a tendency to spontaneous combustion should be turned over frequently to keep down the temperature, although this causes an increase in breakage	71	3	3
40. If possible, stored coal should not be moved until it is to be used, unless it is heating, since piles which have apparently been safe have taken fire when opened to the air and the coal moved elsewhere, probably because of the opening up of new faces for oxidation	75	1	1
41. Coal which has once heated and cooled is not subject to the same oxidizing processes as fresh coal	64	3	2
42. Coal which has been in storage is considered by many to burn less freely when fired in a furnace, but this is largely prejudice and certainly can be overcome by keeping a thinner bed on the grate than with fresh coal and by regulating the draft	60	8	2
43. The decrease in heating value as a result of storing coal is small unless the coal has heated up sufficiently to decompose the coal or to drive off the volatile ingredients and the loss of moisture from the coal may increase the heating power per pound of coal	70	1	1
44. The coking properties of eastern bituminous coals are not materially affected by storing unless the coal heats, but the coking properties of Illinois and other Middle West coals are seriously affected or destroyed by storage	53	1	4
45. The best preventive of loss in coal storage is the constant inspection and watching of the coal for incipient heating and immediate removal of coal from the spot affected	73		
46. Other suggestions			

The following is a digest of the answers given to the summary of conclusions, listed on pages 133-136, which could not be included under the headings listed as "Yes," "No," and "?". These opinions show that the experiences of those storing coal are by no means the same and that local conditions evidently have a great deal to do with the opinions expressed.

Reasons for Storing Coal

1. To assure a regular supply when the mines are shut down or when coal is not delivered regularly by transportation companies.
2. To take advantage of low water freight rates.
3. Because the price is often less at certain seasons of the year, generally in the summer or spring.
4. To equalize the prices on the different sizes of coal.
5. To avoid the maintenance by the railroads of equipment that is used for only a part of the year.

Comments:

"To have a supply on hand for special seasons, such as during the grain rush."

"To reduce cost of production at the mine by giving more uniform running time and thus utilizing labor and equipment to best advantage."

"To provide against labor unrest at the mines."

"To equalize distribution of coal."

"Make 3 to read, because the producers will often make price concessions on sized coal at certain seasons of the year (generally in the summer) which will offset the storage expense incurred by the consumer."

"Amend 4 to read: From the standpoint of the producer it is advantageous to store some sizes at the mines or at distributing centers to equalize more nearly the prices on different sizes of coal."

"Most of the large dock companies contract their tonnage at a fixed price for the season."

"Screenings may be higher in summer than in spring and fall on account of decreased operations of mines and lack of demand for screened sizes."

Advisability of Storage

6. Coal storage should not be considered only as a war expedient but as an essential part of all large coal using operations.

Comments:

"May be true at some industrial plants, never true on a railroad. The storage of coal on a railroad is done only to meet a possible shortage of coal due to mine strikes or other causes. It is always more expensive for a railroad to store coal than it is to rely upon a current supply the year aroundSeasonable use of equipment, or traffic conditions, would not in-

fluence the storage of coal on a railroad as the expense of handling and storing is great enough to overcome any or all these conditions.”

“Should be more zealously practiced in peace times than during war. Should be strictly limited in war times.”

Place of Storing

7. Coal should be stored as near as possible to a point where it is to be used so as to avoid rehandling and to equalize transportation facilities.

Comments:

“Rehandling is a preventive against fires and is an advantage where coal is to be used for coke and is pulverized anyhow.”

“Should be stored at point of use and at mines.”

“Preferably so, but in order to avoid transportation difficulties, rather than for the reasons given. Coal can be unloaded into storage at some intermediate point and later loaded up and forwarded to its destination on the original freight rate.”

“All modern mines should have storage at the mines as well as at point of consumption.”

“Railroads should store heavily in spring months, February 1 to May 1, so that they can serve the public and keep their locomotives and cars busy in the dull season.”

Liability to Spontaneous Combustion

8. There is a considerable difference in the ease with which different coals oxidize and this must be considered in storing coal.

Comments:

“Broadly eastern and western coal should never be stored together.”

“Not of great practical importance among the high grade Appalachian coals; applies to essentially different types of coal.”

“You can store western coals without loss and danger from heating and it becomes a question of going as close as you can to the danger line to get the storage.”

9. Coal exposed to the air seems to lose some of the volatile ingredients which assist in producing spontaneous combustion; hence the greater the time elapsing between the mining and storing of the coal, the less is the liability to spontaneous combustion.

Comments:

“The general opinion is that it is not the loss of volatile ingredients which lessens the liability to spontaneous combustion of coal which has been exposed to air for a period of time before storing, but rather it is believed that during this preliminary period of exposure to air, the surfaces of the coal have become somewhat satisfied in their capacity for absorbing oxygen and the rate of absorption becomes less. Since the oxygen combines chem-

ically with generation of heat, the faster the rate of absorption, the greater will be the tendency to spontaneous combustion."

"Think that this is due to completed oxidation and degradation of all sulphur compounds. I have never heard that the extension of time between mining and storage minimized chances of combustion. It is very interesting."

"Seems reasonable, but as car or boat must be unloaded on receipt, do not see much value in the suggestion."

"Coal which has passed through the heating process successfully, that is, without ignition may lay for years without further heating."

"Depends on whether the decreased liability of fire is more essential than the decreased coking qualities of the coal."

"Our experience has been that coal obtained from western Kentucky coal fields is much more likely to fire in storage than coal from eastern Kentucky coal fields."

"Cannot confirm conclusion, as coal that had been in storage for years took fire after it had been moved and restored."

"Do not notice any difference between storage at mines and distant points."

"Probably but not confirmed by our practice."

"The loss of volatile matter, if any, is irrelevant and immaterial; it has nothing to do with the liability to spontaneous heating which is caused rather by surface oxidation."

"I doubt this difference being enough to give it commercial consideration."

"The reason for this is probably that moisture is less."

"Is there not some ambiguity possible in this paragraph? Evidently refers to storage of coal in room at mines, not storage of coal in large quantities by mining companies at surface. Public might interpret this to mean feasibility of operators storing large quantities of coal at surface during periods of slack demand."

"Since coal has a limited absorbing power for oxygen, after the surfaces are exposed to the air for some time there is less liability to spontaneous combustion."

Effect of Sulphur on Spontaneous Combustion

10. The opinion formerly held that the chief source of spontaneous combustion is the pyrites (sulphur) in the coal has not been substantiated by experiments. There is, however, a wide-spread opinion that coals high in sulphur are much more liable to spontaneous combustion than those low in sulphur and an effort is made by many in choosing a coal for storage to get one low in sulphur.

Comments:

"There seems to be a rather general impression that sulphur is the determining factor of spontaneous combustion. In some analyses of samples which I took, the coal analyzed anywhere from four to five per cent sulphur, and this pile has been on the ground for over four years and another pile of

the same coal for a period of six years without firing. This seems to be conclusive evidence that sulphur is not the determining factor."

"Pocahontas, New River and other high grade smokeless coals, which are low in sulphur, are extremely liable to fire, undoubtedly caused by oxidation, as we have frequently found small fires within two feet of the top of a forty-foot pile and again at the floor in the very center."

"I have wondered whether pyrite and iron nail and water might not start a fire. See Watt's Dictionary of Chemistry on the Result of mixing pyrite and wet nails under a clay pot. An explosion takes place the steam blowing off the clay cap."

"We have never had trouble in storing West Virginia coal which is low in sulphur. All the coal which we stored this year and which caught on fire was high in sulphur."

"Pocahontas with lower sulphur gives more trouble than Pittsburgh with higher sulphur."

"All cases of spontaneous combustion observed have been high or moderately high sulphur coals."

"The sulphur is generally associated with elements that promote spontaneous combustion."

"This is the general practice and opinion. I cannot vouch for its correctness."

"This belief is losing strength."

"The fact that coal is low in sulphur is, in my opinion, no evidence that it is a suitable coal for storage. The coal on the Milwaukee and Western Fuel Company docks at Milwaukee have been on fire for a number of years. The coal is unloaded in the summer, immediately begins to heat and they never succeed in extinguishing the fire until the next cargo comes in the following year. This coal is $\frac{3}{4}$ -inch Youghiogheny slack containing not to exceed .75 sulphur. On the other hand I have personally stored in Louisville, Kentucky, a large tonnage of a straight creek coal containing about the same amount of sulphur and have had no trouble whatever with the same heating although the coal was piled twenty feet high and remained in storage two years."

11. Although the oxidation of the sulphur is not the principal heat producing element, it tends to break up the lumps and thus increases the fine coal which fires easily.

Comments:

"Therefore the presence of the sulphur facilitates."

"Do not concur in the conclusion, as in warm weather the heating takes place before there is time for such disintegration."

"I have stored considerable lump coal which was high in sulphur and have taken the same out of storage four or five months later and have not found that the high sulphur coal caused any more degradation than any other class of coal. I have stored the majestic lump coal from the mine in Prairie County, which is extremely low in sulphur, and have found that it breaks up fully as much as high sulphured coal."

Effect of Moisture on Spontaneous Combustion

12. Some coals and particularly those high in sulphur heat more readily if damp, and alternate wetting and drying is undoubtedly harmful.

Comments:

“Make to read most instead of some coals.”

“Moisture is a cause of spontaneous combustion.”

“Not true of the hard bituminous coal, like McAlester, Oklahoma, Colorado and Pennsylvania bituminous.”

“All the coal that I have ever stored has been stored in the open so that same has been exposed to the weather and the alternate wetting and drying but have never noticed that this had anything to do with the coal firing, although some of the coal stored was high in sulphur and did not fire.”

Relation of Size of Coal to Spontaneous Combustion

13. All varieties of bituminous coal may be stored if of a proper size, if the fine coal and dust have been removed, and the coal is so handled that dust and small coal are not produced in the storing in excessive amounts.

Comments:

“Do not think many coals can be handled in this way successfully.”

“Yes, but impracticable.”

“All is a sweeping word. Should prefer nearly all or probably all.”

“We have never had any trouble with lump coal, but have had trouble with mine run.”

“And the pile is not too high.”

“Excepting unwashed screenings.”

“Impracticable and almost impossible.”

“Desirable; not always practicable.”

“Cleaned coal undoubtedly has less liability to fire than coal with slack.”

“Fine coal should be removed thoroughly or else run of mine stored. Partial ventilation worse than none.”

“Some bituminous coal will slack and then is no longer sized and is likely to burn.”

“Hard to do and expensive.”

“Simply a matter of ventilation.”

“So far I have had no experience in storing coal with the fine dust removed. In ordinary commercial undertakings where large volumes of coal are to be stored, it seldom happens that the fine dust is removed. I, therefore, do not know what effect it would have if such could be accomplished.”

“All kinds may be kept if stored properly. We store dust and slack under water.”

“Not practicable for railroads.”

“Can store low sulphur mine run coal with slack in it if properly handled.”

"I believe that the character of coal has a great deal to do with spontaneous combustion, in that a soft, porous coal which fires much more quickly than a more solid one, possibly due to additional breakage and water, and moisture absorption by the porous coal."

"As a result of our experience this year we have decided to store only lump coal in the future, getting coal that is as free from sulphur as possible. We will store it in a number of small piles of not over 1500 to 2000 tons each separating the piles by a distance sufficient to prevent the spread of fire."

"Heating in storage coal is proportional to the amount of coal surface (not pile surface) in contact with air. If the sizes of the lumps are large, the amount of surface exposed is relatively small, and so too is the amount of heat evolved. If, however, the pieces of coal are small, including a considerable amount of coal dust, the amount of contact surface is enormously increased, the amount of heat evolved is also increased, and the danger of the temperature of the coal rising to a point where active combustion will take place is much greater."

"This would indicate two principal ways of storing coal so that it will not take fire:

(1) Store large lumps only, thus reducing the amount of surface exposed to the air.

(2) Store fine coal only, packed so tightly that there will be no voids through which air can circulate."

"The friability or ease of breakage of coal is a very important factor affecting spontaneous combustion and is the cause of the prevalence of this trouble with semibituminous coals."

"Use screened coal for storage. Size of piles, kind of weather conditions, conditions of storage and sulphur content are then of minor importance."

"We cover our coal pile with fine raw dust when we discover the top or outside of the pile burning, thus preventing the air having access to coal and increasing the fire. We are able to successfully check a fire in this manner for a few weeks until we are ready to move the pile."

14. Most varieties of bituminous coal are liable to spontaneous combustion if the fine coal is not removed before storage.

Comments:

"Not high grade hard bituminous."

"It is best to store mine run coal in small piles to lessen danger of spontaneous combustion."

"To a certain extent."

"More liable."

"We have never been successful in storing either screenings or mine run coal in large piles above ground. In our storage operations at the present time we are confining ourselves to selected mine run. In making this selection we choose coal having a very small percentage of screenings in it. We find that even this selected coal when piled in large piles will start heating and eventually fire."

“Choose the right coal to store. Store screened coal if in doubt.”

“I think that the quality and structure of the coal placed in storage is of more importance than the manner in which the lump or screening coal is piled. If the coal is soft, it will undoubtedly break up and in these days when labor is so scarce, it is necessary to handle more of the coal with some mechanical device, particularly by a clam-shell. This manner of handling is very rough but we have a number of piles fourteen to sixteen feet high of lump, egg and nut which have been handled in this manner and we have had no trouble with spontaneous combustion.”

15. In storing coal handle it so as to produce a minimum of dust and fine coal.

Comments:

“Not practicable for railroads.”

“Yes, where possible; depends on machinery, quantity, quality of coal and ground area.”

16. Owing to the cost of screening, to the difficulty of arranging for it in large storage piles and to the difficulty of disposing of the screenings, it is often not practicable to store lump or screened coal.

Comments:

“Lump coal that is practically free from fines or dust is to a large extent self ventilating and therefore safe for a reasonable time if other conditions are favorable. On the other hand, fine coal entirely free from lumps which would form air pockets and chimneys may be stocked almost as safely as the lump. A combination of the two, such as run of mine, is dangerous.”

“We store as much crushed coal as possible.”

“There is no difficulty in disposing of the screenings. When we buy coal for storage we have it screened at the mine, and the screenings are used for our daily service.”

“It is perfectly practicable to store lump or screened coal but such coal costs more per ton.”

“This was true until the last few years. During 1916-17 there was a very large demand for screenings.”

“Storage of screenings is dangerous under any conditions.”

“Depends upon intended use of coal.”

“Depends on coal.”

“Coal should be screened at the mine and only the sizes not readily marketable should be stored. This probably would mean in normal times that there is a supply of lump and egg coal in the summer which would be the grade that the ordinary householder should store. At the present time it is clearly developed that the distribution of domestic coal burdens the railroads to a greater extent for the amount of tonnage involved than any other class of coal business. The movement, switching and placing in single cars results in greater delay to equipment, greater loss of efficiency from

motive power and engine crews and in every way increases the operating difficulties."

"I would say sometimes instead of often. It is generally practicable and the expense of it is generally a reasonable insurance."

Height of Pile

17. Unless lump and screened coal are piled carefully and in low piles, crushing is likely to take place and produce dust near the bottom of the pile and thus to start spontaneous combustion.

Comments:

"We have piled lump coal in piles thirty feet high and have never had trouble with spontaneous combustion."

"Crushing has not been noticed in piles twenty-five feet high. Fires start at a depth between four and eight feet."

"We do not find it so. Pile as high as sixty feet with little crushing."

"Don't store in piles over eight feet high."

"Depends on the character of the coal."

"Where coal is placed in storage, it should not be scattered over a considerable area in thin layers, but should be piled in one mass as much as possible."

"Weight is a big factor; so restrict open air storage to piles eight feet high. Under-water storage is the best."

Uniform Piling

18. Pile as uniformly as possible and avoid stratification of large and small lumps.

Comments:

"I have replied 'no' as we store in the neighborhood of 1,000,000 tons coals per year and it is absolutely impossible to avoid stratification of the large and small lumps."

"Impracticable unless the coal is put into a pit or a bin. When piled in a pyramidal pile, fine coal will stay on the ridge and the larger sizes roll to the bottom and edge."

"Impracticable to follow out this suggestion."

"Not practicable for railroads."

"True for some coals, not for others."

"It is further important that the tops of all piles be finished off as nearly level as possible, avoiding to as large an extent as possible peaks and valleys over the surface. In the unloading of coal from buckets into storage the larger lumps tend to roll to the bottom of the pile along the surface of the coal already in storage, so that unless care is taken in the dumping a coal pile will consist of fine coal in the center with a fringe of large and intermediate lumps along its edges. Fires very frequently start along these edges, for obvious reasons, and if the coal is to be kept in storage any great length of time the lumps should be either removed or the whole covered with

fine coal so as to remove from contact with the air the mixture of lumps and fines.”

“Very important.”

Temperature at Time of Storage

19. As liability to spontaneous combustion increases with the temperature, the cooler the coal is when stored, the less is the liability to spontaneous combustion.

Comments:

“We commence storing coal at the head of Lake Superior at the opening of navigation, but have never had fires during the summer time. They usually make their first appearance during the month of September and we have the situation well in hand prior to December 1.”

“The variation in the temperature of mined coal is so slight that I question whether this would have any bearing.”

“Theoretically but would not make much difference in practice.”

“Believe this will effect but short delay.”

“Do not believe this applies to large and deep storage. Outside temperature of no account in such cases.”

“Yes, for the time being, but the pile will not be permanently safe until it has gone through a summer.”

“Doubt if there is anything in this suggestion, as the majority of our fires occur after the coal has been stored for some time and bear no relation to the temperature of the coal at the time it was placed in storage, as all our coal is stored in the open.”

“Possibly slight effect.”

“Yes; but negligible compared with item of fine, fresh coal dust.”

“In cold countries storage in the open is less liable to spontaneous combustion than storage under cover.”

20. Coal stored in cold weather is less likely to give trouble than that stored in summer and where summer storage is necessary the liability to heating is much less if the storage is carried on on cloudy or cool days.

Comments:

“Difference in temperature of the coal at the time of storing may have some influence, but under ordinary conditions where large quantities of coal are stored, there is very little choice as to the time the coal can be put down.”

“Naturally, we have to store our coal regardless of weather conditions.”

“Perhaps at the time but doubt importance.”

“Not important.”

“Believe this is of little benefit.”

“Do not find any difference in summer and winter storage.”

“Rather a fine distinction.”

“Would omit this. Impracticable.”

“Yes, but how can such days be chosen?”

“Yes; but negligible compared with item of fine, fresh coal dust.”

Effect of Mixing Coals in Storage

21. Coal of different varieties should not be mixed in storage if this can be helped, since such mixture seems to increase the liability to spontaneous combustion.

Comments:

“Extremely important. We have experienced no trouble with the heating of stored coal, and the only time we have ever had a fire on our dock was when we bought a number of different kinds of coal. We have our coal dock situated close to the brewery and we generally purchase our coal before the opening of navigation, and have such a capacity that the coal can come in almost as fast as the companies desire to send it to us, all, of course, coming in during the season of navigation. Our usual method is to purchase our entire season’s supply from one company and secure all the coal from one mine. Mixed coals are not only objectionable on account of the danger of spontaneous combustion, but different coals require different methods of firing.”

“If coal that is known to give trouble from spontaneous combustion is mixed with coal that does not, it will undoubtedly increase the liability for the spontaneous combustion of that given pile. However, if two coals are mixed together in which there is very little difficulty of spontaneous combustion, I doubt if it makes very much difference if they are mixed or not.”

“Believe this effect is more a result of stratification than of difference of quality.”

“Probably true and vital to docks storing large quantities of coal.”

“We mix our coals.”

“Not borne out by our experience.”

“We have not found this to be true.”

“Should never be mixed in pile.”

“Highly important.”

“Do not believe this.”

“Coal stored from different mines together if not submerged appears inadvisable.”

“Depends on meaning of varieties; in general, yes.”

“Know case where two coals, which in themselves will not fire, yet fired when stored together, but a few tons of coal which fires readily will set a pile on fire. May be likened to a hot cigar ash or match thrown into a waste basket.”

“Have not heard of any evidence of this; do not see any reason to believe it.”

Precautions Which Prevent Fires

22. All storage appliances and arrangements should be so designed as to make it possible to load out the coal quickly.
23. Coal should not be stored in large piles unless provision is made for loading it out quickly in case of spontaneous combustion.

Comments:

“Yes; very important.”

“Would suggest that something be said in regard to breaks in piles where large quantities of coal are stored, so that if one section of the pile fires the entire tonnage would not have to be moved.”

“A pile of coal will give several weeks’ warning of an impending fire.”

“We always leave sufficient room near our storage piles to enable us by means of a scraper run by a motor to turn over such parts of the pile as show to be heating. This is the only way we have been able to successfully fight heating in coal piles.”

24. Since moisture from below assists spontaneous combustion the storage pile should rest on a dry base.

Comments:

“Yes, in case of dry storage.”

“Snow or rain on the pile and then covered is very dangerous, particularly snow.”

“Have not observed this, but if true, moisture is driven off before ignition.”

“Remove all refuse and provide drainage.”

“Fires do not originate at the base.”

“With some coal.”

“Do not find that moisture at base changes situation.”

“Our conditions are such that we cannot determine the effect of water on the bottom of the storage pile. We have found an equal number of fires twenty feet from the water and this would make it approximately thirty feet from the top of the pile. We do not believe that the fire originates on account of water at base of piles.”

“Questionable; some store part under water, part above.”

“Becomes saturated at base, evidently where fire always starts.”

25. Since a small amount of water seems to aid spontaneous combustion, the drier the coal is when stored and during storage, the less is the liability to spontaneous combustion.

Comments:

“I do not have the experience permitting me to question this but it looks doubtful as the chemical actions of moistening cannot take place once the hydration is completed. However, dehydration may later enable oxidation to act on the coal.”

“Generally yes.”

“Is this based on theory or experiment?” (An opinion expressed generally in the discussions of the International Railway Fuel Association).

“Doubtful.”

“Possibly.”

“When I was fleet engineer of the Pacific Station, about ten years ago, I used to have the colliers take on their loads with several lines of hose playing into the holds. I acted on the principle that, while a small quantity of moisture might

do harm, a liberal wetting would be a good thing. The scheme appeared to work all right."

Experiments by the U. S. Bureau of Mines show a lack of uniformity in the action of coals toward oxygen in the presence of moisture. In one case the dry coal oxidizes faster and in another case the moist coal oxidizes faster; also conflicting results have been obtained by different investigators.

Cause of Fires Other Than Spontaneous Combustion

26. Do not allow pieces of wood, greasy waste or other easily combustible material to be mixed with the coal during storage since they may form a starting point for a fire.

Comments:

"Unable to control usually."

"I have never observed this. Pieces of wood have no apparent effect, and I doubt that waste saturated with mineral oil would make any difference."

"Also avoid irregularities in structure of walls and floors such as angles and pockets."

"Almost invariably have found this the cause of our fires in the coal piles. In other cases the fires have started near trestle post or stubbing piles."

"Greasy waste buried in coal will invariably fire."

"Very important. Consider this most important."

"From my observation this is the most frequent cause of fires if the facts were known. Should be made clear that when a fire is once started it cannot be classed as spontaneous combustion. Subject to be divided: (1) Prevention of cause of spontaneous combustion. (2) Handling of fire when once started."

27. Avoid contact between the coal and all such external sources of heat as steam pipes.

Comments:

"My several experiences with fires in bunkers on board naval vessels were in each case associated with a hot steam pipe or bulkhead heated by proximity to steam boilers."

"Yes; either live or exhaust or over poorly installed exhaust lines which rust out and allow moisture and heat to reach pile."

"Most emphatically."

28. Pile coal as uniformly as possible and do not allow the lumps to roll to the bottom or to one side of the pile and thus form a flue for the entrance of air to the interior of the pile.

Comments:

"Impossible to prevent the lumps rolling to the bottom of the pile."

"The coarser the coal, the greater the voids through which air may enter."

"Seems desirable. Not practicable in our case."

"Hardly practicable."

"Impracticable."

"We aim to keep the bulkheads fairly high."

"Cannot be avoided."

"Cannot be helped in high piles."

"Inconsistent. Universal ventilation is a good thing."

"Rather impracticable for a railroad to do this. Let the air in, so that any heat generated will be carried off."

"Very important."

"Difficult to comply with, as regardless of the manner in which the coal is unloaded lumps will roll to the bottom and sides of the pile. If lump or egg coal is stored, this is not likely to cause trouble."

"Most important."

29. Avoid air channels in a coal pile such as occur about inbedded pipes, the bents of trestles, cracks through bulkheads, etc., since fires frequently originate near such air channels.

Comments:

"Never had this trouble."

"Believe ventilating the thing after removing the heating."

"Are you sure that nails in the bents are without blame?"

"Have used pipes to permit circulation of air. Feel that this has done some good toward preventing spontaneous combustion."

"We have always used old boiler flues in storage piles in order to ascertain the temperature by hanging thermometers in them or by noting the haze which is visible on cold mornings and indicates heating."

"Inconsistent. Universal ventilation is a good thing."

"This also provides ventilation which will prevent firing."

"Wooden bents of trestles and other wood, partitions, etc., furnish an easily oxidizable substance."

"Our methods of preventing spontaneous combustion are principally as follows: The maintaining of a 25-foot bulkhead all around the two piles; one containing 250,000 tons of high volatile and the other 150,000 tons of low volatile coals. When coal is received early in the season it is stored so that it can be used first; or in other words, our object is to use the oldest coal first."

"Protect the pile by bulkheads or walls. My storage contains December 1, generally 200,000 tons, 225 feet wide, 40 feet high. On the east and west sides concrete walls 13 feet high protect the pile. The north and south ends are open. Our fires, when we had them, a few years ago always were on the north and south sides of the piles, never on the east and west sides."

Method of Piling Coal

30. Unless coal can be piled so that air can thoroughly circulate through the pile and thus carry off the heat as rapidly as it is generated (this is very difficult to accomplish), the more closely fine coal is piled to exclude air the better.

Comments:

“ I have always believed that tight packing of coal through pressure produced heat.”

“It is impossible to carry off the heat generated in fine coal by any circulation of air.”

“I think it is impossible to pile coal so closely as to exclude air to any beneficial extent.”

“Inconsistent. Universal ventilation is a good thing.”

“Coal must be piled low enough so that the generated heat can be carried off without reaching the firing temperature.”

“To be dispensed with by favoring the subaqueous storage and is the only manner in which fine coal can be stored without hazard from spontaneous combustion.”

“Since the voids in fine coal of uniform size are about the same as the voids in course coal of uniform size, would it not be better to say mixtures of various sizes tend to reduce the void spaces and exclude air? ”

*Ventilation of Coal Piles**

31. Ventilation of coal piles by means of pipes running through the pile has usually not been successful in preventing spontaneous combustion, because the air does not circulate freely throughout the pile and in some cases the air currents through these pipes seem to be starting points for fires.”

Comments:

“Several years ago, we were storing quite a large amount of Oklahoma mine run coal at Smithville, a coaling station which is located not far from San Antonio. I experimented with one pile of coal by placing a six-inch glazed tile on the bottom and running it the entire length of the pile. This tile was the ordinary bell shaped and in putting it together we used no cement, but just laid the ends together. Along the side of this pile of coal I stored an equal quantity without using any tile. The result was that in the pile where I had placed the tiling, we had no fire whatever, but in the other pile where we had no tile we experienced considerable amount of trouble by reason of spontaneous combustion. I am of opinion that pipes or flues do not always produce the desired results.”

“Ventilation by means of pipes has not been successful. Do not think these pipes are the starting points for fires.”

“Helps some.”

“Heartily agree; think there has been a lot of misinformation put forth on this point.”

“Have had no trouble with pipes. We think tendency is to carry away heat by radiation.”

“Such pipes are probably useless except to indicate or follow rise of temperature.”

“In two plants we have put in iron pipes to admit air but in the other where at least 12,000 tons are under storage, we have not taken measures to prevent

* See page 32, for extended discussion of coal pile ventilation by J. B. Porter.

heating. I am of the opinion that we have as good results where no measures are taken as where we have the air pipes."

"My personal opinion is that a cooling system employing extra heavy iron pipe and circulating water would be more satisfactory and certainly much safer, than any ventilating system that could be devised."

32. The great objection to ventilating systems is that they interfere with the handling of the coal.

Comments:

"Very expensive."

"They do not pay for the efforts."

"Unless thorough not of much benefit."

"Great objection is that they do not work as intended."

33. There is great difference of opinion in regard to the height of piles in which coal may be stored safely. Coal seems to fire almost as quickly in low piles as in high and as often near the top or outside of a high pile as near the bottom. The principal disadvantage of high piles is that coal in them cannot be so readily removed.

Comments:

"I have always believed that the higher the coal is piled the more liability to fire through pressure."

"Height undoubtedly varies for every coal but for any coal low piles are safer and definite height can be established for any coal. Low piles lessen heat from pressure and make for safety."

"It is not our experience that coal fires as quickly in low piles as in high. It has been our experience that if the depth of the coal is limited to twelve feet no fires will occur."

"After eight or ten feet there is no additional hazard because of increased height, and any increase of height within reach of the crane is not attended by an inconvenience in reloading the coal."

"If piled by hand piles should not be over ten to twenty feet wide, fifty feet long, though bottom of piles may touch. More space is required but it is safer. Piles may be much larger if mechanical handling is used."

"High piles due to the greater pressure fired more readily than low piles."

"Depends on the storage system."

"Think height should be limited to ten feet."

"We pile from twelve to fourteen feet."

"Believe in low piles about eighteen feet high."

"The lower the pile the less liability to fire."

"High piles give more trouble."

"Think height is important factor, the greater pressure inducing heat."

"Coal almost always fires near the bottom."

"Height for safety depends on the kind of coal."

"More fires from higher piles."

“If space is available pile low.”

“Height of pile of no consequence, excepting crushing effect.”

“Height of pile is one of determining factors. Have proved this, this summer.”

“Depends entirely on method of storing. Low piles for hand, high for crane bucket.”

“Might be amended to the effect that the height to which coal is piled should be regulated to conform to the facilities for removing or reloading the coals from the piles.”

“Coal fires at the depth where enough air cannot penetrate and all of heat is not lost by convection to the atmosphere. This depth varies with the character of the coal.”

“We limit the height of piles to twelve feet and have had less difficulty from spontaneous combustion in low piles. Furthermore if piles are narrow, they are easier to handle in case of spontaneous combustion.

Use of Water in Extinguishing Fires

34. Water is not generally successfully applied in extinguishing fire in a coal pile, because it is impossible thoroughly to saturate the pile. A small amount seems to aid spontaneous combustion.

Comments:

“Nearly all bituminous coals contain considerable tarry matter which is distilled from the coals during the heating process before the coal actually ignites. These tar vapors rise and condense in the cooler coal above forming a crust of mixed tar, pitch and coal which water will not penetrate and therefore the fire cannot be reached by water from the top except possibly in spots.”

“We have extinguished several fires by applying water, cooling the coal for a distance of four or five feet, shoveling out the wet coal, then applying more water, cool another layer and shoveling that out and so on until we reach the point where the coal is either on fire or very hot.”

“This is not our experience. Have an 8-inch water line along the field 1300 feet long and extinguish fire successfully with water.”

“Water is effective when applied directly to the fire after removal of super-imposed coal.”

“We expose the heated coal to the air which reduces the temperature and extinguishes fire.”

“This agrees with our experience.”

“Depends on size of fire.”

“After getting to spot that is on fire water has to be used.”

“Fire must be dug out.”

“Whenever I had trouble with a bunker fire on board ship I simply opened one of the deck scuttles and let in a hose until the compartment was fairly well filled with sea water. This appeared to be an easy and sure way of overcoming the fire.”

“Heated portion of pile can be cut off frequently by driving spray pipes into pile and flooding pile with water from fire hose connections which will retard fire and put it out sometimes.”

Under-water Storage

35. The deterioration in coal stored under water is negligible. Such coal absorbs little extra moisture.

Comments:

“Coal when stored under water absorbs considerable moisture, and in fact, becomes so wet that it is necessary for us to lengthen the coking time.”

“Only place to store slack or nut.”

36. With under-water storage if the entire coal pile is not covered, the part of the pile above the water line is liable to spontaneous combustion.

Comments:

“We would not suppose that submerging part of a pile would render the remainder immune, neither would we suppose that the exposed part would be more liable to fire.”

“This liability is less than that with dry storage of depth equal to depth above water line. Believe the only practicable and reliable plan of storage to be with partial submergence. By this plan coal may be successfully piled twenty-three feet above the water line.”

“Very readily. I assume capillary action and wetting of the pile.”

37. The white coating on the outside of some coal piles is very thin and does not affect the value of the stored coal.

Comments:

“If not sold on looks as is often the case.”

“Not appreciably.”

38. The best method of handling coal in danger of firing is to load it out and scatter it so that it will be thoroughly cooled off.

Comments:

“If handy to point of consumption it can be put on the engine tank or boiler house coal bin for immediate use after being thoroughly wet down. This will save the expense of rehandling.”

“A uniform depth of four feet would probably not fire, and if a pile which contains fire were spread out to this depth, the fire could be extinguished with hose after which the coal, if not piled up again, would be safe.”

“Does not always work with Mid-West coal.”

“Turning pile over slowly will answer.”

“If the scattered coal is well thinned out, water should be put on it. Handled 7,000 ton pile of screenings this way this summer.”

39. Coal which has a tendency to spontaneous combustion should be turned over to keep down the temperature, although this causes an increase in breakage.

Comments:

"Impracticable for railroads."

"This would probably be effective, although expensive. It need not be resorted to until the coal begins to gas.

"Yes, but makes the expense of storing coal prohibitive."

"Do not disturb a pile at all once a pile has heated. It cannot be again stored."

"Not necessary until heating commences."

"Impracticable."

"Bad practice for Mid-West coal."

"Does not seem practicable for if the pile is a large one, say of 50,000 tons, the cost of turning it over frequently would be prohibitive."

"This conclusion and the one following seem to be inconsistent with each other."

"Pile it low enough this will not need to be done. No experience turning coal over."

"And soaked with water."

"Questionable."

"Yes, but once in six weeks or two months is often sufficient."

"Seems to indicate unusual difficulties in connection with storing coal and I am sure no one would undertake to store coal if it had to be turned over or moved frequently."

"On the Lake docks it is a common practice to move to another part of the dock, or to turn over a pile that begins to heat and which cannot be shipped out at once. This practice is diametrically contrary to the conclusion reached at one of the largest known storage plants for fuel coal.

40. If possible, stored coal should not be moved until it is to be used, unless it is heating, since piles which have apparently been safe have taken fire when opened to the air and the coal moved elsewhere, probably because of the opening up of new faces for oxidation.

Comments:

"Doubtful."

"Cut out unless it is heating."

"This looks doubtful; still I would leave it alone unless the temperature reaches the danger point about 150 degrees F."

- 41 Coal which has once heated and cooled is not subject to the same oxidizing processes as fresh coal.

Comments:

"Not if it has heated long enough."

"Never."

"Not to the same extent, but nevertheless coal that has been scattered and thought to be cooled has fired."

"One trouble is disintegration of shale seam or thin laminations of dirt and the pieces break up with handling or in firebox."

"Coal once heated has very little value for fuel purposes."

"One of our mechanical engineers, W. L. DeBaufre, for about five successive years tested Pocahontas coal which had been stored at the New London, Conn., Naval Station in three different ways, namely, one part was stored under water, one part was stored on shore under cover, and the remainder was stored on shore not under cover. The test showed about the same evaporative value for each of these coals. If there was any difference it was in favor of the coal stored in the open, and the coal stored under water ranked next. Out at Cavite, in the Philippine Islands, the Navy Department has had considerable trouble with spontaneous ignition of its coal piles; but of course out there the sun is very hot and the days are long. Here at the Experiment Station we store our coal about twelve feet high on a concrete pavement, and so far we have had no trouble."

"Practical railroad man hates storage coal. It is usually used the coldest season of the year when mines will barely produce enough steam; therefore, an 85 per cent storage results in only 50 per cent good service."

"Our trouble is disintegration of shale or their laminations of dirt and the pieces of coal break up with such handling and in the firebox."

42. Coal which has been in storage is considered by many to burn less freely when fired in a furnace, but this is largely prejudice and can be overcome by keeping a thinner bed on the grate than with fresh coal and by regulating the draft.

Comments:

"It seems to us that the ingredients which would promote spontaneous combustion must also act as agents to promote free burning."

"Have found fresh-mined coal always gives best results. It is proved that even coal mined from the pillars in mines is very inferior to coal taken from the rooms."

"It is usually claimed that heat units are less. Never heard of it burning less freely."

"We like a slow-burning coking coal."

"No material effect."

"Nothing in this."

"Coal frequently loses twenty per cent efficiency."

"Should be saturated with water."

"Actual experience indicates otherwise."

"Due to absence of volatile."

"Would burn less freely according to previous conclusion."

"If volatile matter has diminished coal will not burn as freely."

"We have more trouble the longer stored."

"Not entirely prejudice. I believe it does not have the affinity for oxygen that fresh coal has."

"We see no apparent difference."

43. The decrease in heating value as a result of storing coal is small unless the coal has heated up sufficiently to decompose the coal or to drive off the volatile ingredients and the loss of moisture from the coal may increase the heating power per pound of coal.

Comments:

"Yes, depending on the length of time coal has laid in storage."

"Never had this experience. Degradation first, then loss of volatile pulls down the value."

"Our coal takes on moisture from storage."

"But not of the original pound."

"High burning rate on locomotives makes loss greater."

"Disintegration disturbs firing method on account of varying percentages of sizes in boiler house, coal bin and in firebox."

"This is a very debatable question and while tests show very little decrease in the thermal value in the coal during storage, the fact remains that for a given pumpage of water, our coal consumption has increased by amounts varying from ten to sixteen per cent. The coal has lost its shiny appearance and as a considerable portion of the volatile is lost, it does not act the same in our stokers as when freshly mined."

"We find that by wetting the coal that has been in storage for some time, its burning properties have been enhanced, especially so in the case of smokeless coals."

44. The coking properties of eastern bituminous coals are not materially affected by storing unless the coal heats, but the coking properties of Illinois and other Middle West coals are seriously affected or destroyed by storage.

Comments:

"As to the deterioration of coking properties in eastern coals, will say that some of them deteriorate rapidly while others do not. Eastern Kentucky is a marked instance of a coal that does not deteriorate very rapidly. The coking properties of Illinois and middle west coals are seriously affected."

"From conversation with by-product coking men, I gathered the opinion that the coking properties of all bituminous coals, including the Eastern, was affected by storage. Some coals to a greater extent than others."

"Have not noticed coking properties of Illinois coal to be affected by storage. We have about 15,000 tons in stock now and are using some that has been in stock over six months with no bad effect."

"Omit the phrase 'or destroyed by storage.'"

"Yes, for eastern coals. No experience Illinois coal."

"Gradual absorption of oxygen by coal does slightly decrease its coking ability, even though the coal does not heat seriously, particularly true with western coals."

"It is said by many practical coke men that coking quality is generally impaired by long storage of the coal. It is claimed by many coke oven operators that weathering of some coal affects the contracting qualities of the coal during the coking and, therefore, a stored coal is more likely to stick in the coke oven. By-product yields, especially that of the gas, are probably lowered in a high volatile coal by long storage."

45. The best preventive of loss in coal storage is the constant inspection and

watching of the coal for incipient heating and immediate removal of coal from the spot affected.

Comments:

“Isolated hot spots sometimes occur which can be readily dug out with a crane, leaving the remainder of the coal in good condition. The temperature of every pile will go up and down, influenced in part by the temperature of the surrounding air and in part by internal heating, but it would be inexpedient to disturb the pile unless the temperature rises to near 200 degrees and is accompanied by the evolution of gas.”

“Best preventive is properly storing it. Know your coal and store it low enough so that it will not fire. An inverted V-shape pile is preferable.”

“Proper selection of storage space, sizes stored, properly seasoned coal, manner of handling. Do not store mine run under any conditions and especially do not build up pile by carrying track upon pile. Recent experience of the Big Four shows that fire is sure to follow such methods.”

“The real successful method of storing coal in this locality, Springfield, Illinois, has been under water. When piled on the ground should be in long piles with quantity limited as much as practicable. Would recommend only storage of screenings of fine coal and when this is done the only successful method has been under water.”

“Place 1½-inch pipes every sixty feet in pile and take interior temperature every 36-48 hours by lowering thermometer floating in mercury down the pipe, leaving it in the pile for about ten minutes. On large storage a mercury electric alarm system will pay.”

“Give suggestions as to manner of “inspection and watching; i. e., should thermometer holes be built in piles for inserting thermometers in center of piles? What rise in temperature is considered dangerous? Examination of gases? Odors from pile?”

46. Other Suggestions.

Comments:

“Use coal from the newer piles.”

“Coal successfully stored for a year is usually past danger.”

“No chemicals applied as for extinguishing other fires have any more effect than water (which is very little). The thing to do is dig into the fire and move the coal as quickly as possible.”

“Successful storage depends on proper selection of storage space, sizes stored, storage of properly seasoned coal, and manner of handling. Do not store mine run under any conditions and especially do not build up pile by carrying track on pile, as recent experience of railroads shows that fire is sure to follow such practice.”

Through the courtesy of Frank Haas, Consulting Engineer, Consolidation Coal Company, the following conclusions of that company regarding coal storage are given:

(1) “Spontaneous combustion is not limited nor is the tendency greater

with any one class of coal—referring now only to the bituminous coals ranging from sixteen to forty in volatile matter.

(2) “The tendency is not in proportion to the quantity of sulphide present.

(3) “In coals of the same kind (graded according to volatile matter) those which have the highest per cent of impurity are more liable to spontaneous combustion.

(4) “In coals of the same kind, heating is more liable to occur in the fines.

(5) “Weather conditions apparently have no effect (in this it should be explained that our records do show more on cold and wet days, but we have assumed for good reason that heating is more readily observed on such days).

(6) “The coals with which we are familiar have never been known to fire inside the mines, either in the solid or in accumulations inside the mine.

(7) “In stored coal quantity or depth of pile have evidently not much to do with it as we have found local heating to occur near the top as well as in the center or bottom of the pile.

(8) “In our opinion, and we have numerous cases to substantiate it, a great number of local heating cases have been due to waste material brought out with the coal, such as pieces of wood, discarded clothing saturated with miners oil, waste, brattice cloth, etc. We do not hold exclusively to this for there still remains to be explained the slow propagation at a temperature far below that of normal combustion temperature of coal.

(9) “We have many instances showing that more local heatings have occurred than were discovered, showing very small areas of activity and were spontaneously extinguished, indicating that propagation of heat is not inherent in average coal, but localized by some particular, perhaps foreign, condition.”

APPENDIX III

EXPERIENCES OF FIRMS AND INDIVIDUALS STORING COAL

MANY of the answers to the questionnaire sent to coal dealers and users have been incorporated in the preceding pages of general discussion, but there still remain a few specific instances of successful and unsuccessful attempts to store coals of many different kinds under various conditions which have not been discussed. Available information relating to these cases is presented in the following paragraphs. In each case a brief statement of the conditions is given and while a certain amount of repetition is thus unavoidable, this method of presentation seems desirable for purposes of comparison.

42. *Storage for Domestic Use and by Coal Dealers.*—Personal interviews with coal dealers in Urbana and Champaign and answers received to the questionnaire indicate that there is no trouble in storing coal under the conditions which prevail in the ordinary coal yards and in dwelling houses. The loss from breakage in storing and reclaiming bituminous coal is 5 per cent or more, according to the coal and the method of handling. There is no loss in fuel value and consequently stored coal sells for the same price as fresh coal, although there may be a slight loss of weight due to drying out.

The Crystal Ice and Fuel Company* of Danville, Illinois, stores coal from Harrisburg, Illinois, and from the No. 4 vein in Indiana during July and August, when the coal is cheapest and driest, in 65-ton covered bins with plank floors. The period of storage is three months. The sizes are kept separate and the slack is not removed before storing. The coal is stocked by dumping and reclaimed by shoveling at a total cost of seventeen cents per ton.

D. G. Porter, of the Rock Island Fuel Company, stores about 8,000 tons of 1¼-inch washed coal from Springfield, Illinois. The coal is placed on the ground by hand and in continuous flat top piles eight feet high, at a cost of eight cents per ton for stocking and ten cents for reclaiming. It is kept in storage for six months with no decrease in heating value unless the pile heats. The coal is stored in November and December to take advantage of cold weather so that the snow and ice will stay in the coal until late in the spring. The coal thus stored does not ordinarily heat for six months, but if heating does occur, it is loaded out and burned.

The Beck Coal and Lumber Company of Harvey, Illinois, stores about 500 tons of Pocohontas coal and 700 tons of Illinois coal for five or six months, the sizes being kept separate and slack being removed before storing. It is stored from fourteen to thirty days after mining and preferably in June, July, and August. There is no difference in the selling price of stored and fresh coal and no depreciation from heating occurs, but there is a loss of from two to eight per cent from breakage. If heating occurs the coal is moved. It is stored on the ground in bins from eight to ten feet high and reloaded by hand shoveling. The expense of unloading, storing, and reclaiming is 27.82 cents per ton.

* Thomas Corsey, Secretary.

O. S. Dodge and Sons, Incorporated, of Monroe, Wisconsin, store 1,200 tons which are mostly two inches by three inches nut coal from Franklin County, Illinois. It is stored in bins by elevators and run out by gravity. The different sizes are kept separate but the slack is not removed. There is no decrease in heating value, but a considerable loss through breakage.

The Davenport Locomotive Works, Davenport, Iowa, report that 300 tons of 2-inch screenings from Franklin County, Illinois, are kept in storage on the ground in a continuous pile ten feet high, from three to six months with slight undetermined loss in heating value and no firing.

French and Hecht, of Davenport, Iowa, stored washed Illinois pea coal in large quantities, either under cover or in the open without any firing, but they report that they always have trouble from fires in attempting to store unwashed screenings.

The H. A. Hillmer Company of Freeport, Illinois, stores from 1500 to 2000 tons of West Virginia egg, and Illinois and Kentucky lump and nut sizes on cement floors in a coal elevator in which there are five compartments thirty feet by thirty feet by fifteen feet, the coal being piled from ten to thirty feet high. It is stored and reclaimed by wheelbarrow and conveyor and if carefully handled there is little breakage. From five to fifteen days elapse between mining and storing and it is kept in storage six months, the sizes being kept separate. The slack is not removed and coal is preferably stored in April and May, because then the price is usually fifty cents less per ton than in other months, and since the year's domestic business is about closed a force of men is available which may be employed in unloading the coal. The weather, moreover, is not too hot, whereas in July and August it is almost impossible to get men to unload coal. There is no difference in the selling price and no deterioration, for if the coal is kept under cover and dry there will be no heating, but if heating occurs the coal is removed and wet down. The expense of storage is not kept.

I. C. Cuviller, publisher of *The Coal Trade*, Minneapolis, Minnesota, summarizes the storage problem for the city and coal trade as follows:

In the domestic trade lump, egg and No. 1 nut from Williamson, Franklin and Saline counties are stored out of doors mainly on the ground in round pyramidal piles of varying heights, about three weeks after being mined, and the coal usually is kept in storage from July to October. The sizes are separated and the duff removed. May, June, July, and August are considered the best months in which to store since the price is from 50 to 75 cents lower than at other seasons. There is no decrease in value and no difference in price of stored and fresh coal. The loss in breakage is thought to be about five per cent. Small amounts are usually stored in bins and fires are not common. When eastern coal is stored in large piles, it fires and the piles must be worked over and spread out. This is done usually by means of wheelbarrows but conveyors are being installed.

The Joseph Schlitz Brewing Company of Milwaukee stores West Virginia and Western Pennsylvania coals in continuous pyramidal piles fifteen feet high on a wooden floor with concrete walls during the season of lake navigation. There is little loss in heating value, and breakage is negligible since the coal is crushed

for use in stokers. The company does not buy high sulphur nor friable coals because of the tendency of such coal to heat.

Crerar Clinch and Company* of Chicago keeps in storage, if possible, about 15,000 tons of egg and nut, Nos. 2, 3, 4, and 5, washed and screened coal from Macoupin, Perry, and Jackson counties, Illinois. The coal is screened before storing and is placed in storage from one to two weeks after mining. Most of it is stored in July and August. The coal is used for steam purposes and there is no financial advantage in storing, the only difference in selling price being the added expense of storing. The coal is stored and reclaimed by hand or by a crane. It is stored on a plank floor in conical piles about sixteen feet high, and the sizes are kept separate. The coal is thought to decrease in heating value about five per cent and the large sizes to lose about ten per cent through breakage. The stored coal which has considerable sulphur in it heats within two or three weeks. No precautions against heating are taken except to store in dry weather, and if heating occurs to turn the coal over or use it. It is estimated to cost twenty-five cents per ton to store and reclaim.

The Ebner Ice and Cold Storage Company†, Vincennes, Indiana, keeps in storage several thousand tons of 1¼-inch screenings from Fort Branch, Indiana, Wheatland, Knox County, Indiana, and Clinton County, Illinois. This coal is placed in a concrete pit arranged so that it may be flooded in a short time if it heats. A small amount of coal is also kept in storage on the ground. The coal is dumped from a trestle into a pit and is reclaimed by a bucket conveyor. The expenses are as follows:

	Storing	Reclaiming
Labor	\$0.04	\$0.10
Depreciation on mechanical equipment	0.03	0.03
Interest on investment	0.084	0.084
Total	\$0.154	\$0.214

The Polar Wave Ice and Fuel Company‡, St. Louis, Missouri, stores 20,000 tons of screened lump coal from St. Clair and Williamson counties, Illinois. All the lump coal is forked into storage, the sizes being kept separate. It is stored within one or two weeks after being mined and is kept in storage from a few weeks to several years. It is stored during the dry summer months, when possible, and there is a slight decrease in heating value and a loss of from ten to fifteen per cent from breakage. Heating is more likely to occur in the open than under cover. Dried lump coal forked into a shed seldom heats, but wet coal will heat in time. It is stored by means of especially constructed chutes traveling on a trolley and by wheelbarrows, and is reclaimed by hand. It is usually placed on a brick or concrete floor in piles, about 10 feet high and from 30 to 60 feet wide, parallel with a railroad track.

* A. R. Stooper, Yard Superintendent.

† C. B. Wissing, Superintendent.

‡ John C. Muckerman, Vice President.

The expense of storage is as follows:

	Storing	Reclaiming
Overhead	\$0.05	\$0.05
Labor	.20	.10
Supplies	.02	.01
Depreciation on mechanical equipment and S. S.	.07	
Interest on investment and coal	.12	
Insurance or equipment and sheds	.01	
Insurance on coal	.01	
Total	\$0.48	\$0.16

In September, 1914, run of mine coal in the bin of the new high school in Urbana, Illinois, ignited and considerable difficulty was experienced in extinguishing the fire. The bin is 56 feet long, 12 feet wide, and 12 feet deep fed through two top holes. The walls are of brick, 8 inches thick, laid in plaster with about eight inches of air space between the storage bin and the wall enclosing the boilers. The coal stored was from Seam No. 6 from central Illinois and 200 tons of run of mine were placed in the bin within a few days after it was mined, the coal being wet down sufficiently to lay the dust as it was stored. The only separation of sizes was that due to the large lumps rolling down the sides of the cones while the fine coal formed the centers. The last load was placed in the bins June 29 and five weeks later the coal was discovered to be on fire. The top charging holes and the bin door were covered as tightly as possible with tar paper, and CO₂ gas was run into the bin which was then not opened until November 5. Two days after opening fire was again discovered. After partially flooding of the bin proved ineffectual, part of the coal was removed so that a hose could be played on the fire from the inside, but this plan did not prove effective and the heated coal was shoveled out. Similar trouble was experienced during the following winter, but since that time the practice of wetting down the coal when placed in the bin has been discontinued and an effort made to store only lump coal. Since this practice was adopted no trouble has been experienced.

A similar fire occurred in the bunkers of the Champaign, Illinois, high school, where 500 tons of central Illinois coal were stored in May, 1917, the coal containing considerable fines and the upper part of the pile being wet down when it was loaded into the wagons to be hauled to the bins. Toward the end of October smoke was discovered coming from the pile, which was then wet thoroughly by the fire department. About 20 tons were loaded out from the top of the pile and a hot spot found from four to five feet from the top. The surrounding coal was then taken out and no further trouble has been experienced.

The Northwestern Fuel Company*, St. Paul, Minnesota, stores, 50,000 tons of West Virginia and Maryland lump, egg, nut, and run of mine and one million tons, or more, of Pennsylvania, Ohio, West Virginia, and Kentucky lump, run of mine and screenings. Slack is sometimes removed at the mines, but not at the

* G. H. Hutchinson, Chief Engineer.

dock, and sizes are kept separate in storage when they are so delivered to the dock. There is thought to be a slight decrease in heating value and a loss from breakage which varies widely with the conditions of handling. If the bituminous coal heats, it is usually within one to six months, according to the kind, character, and size of coal. To prevent heating, the coal is shipped out as promptly as possible, or if heating occurs, it is moved by a clam-shell bucket and bridge. The loss through heating varies from slight to complete destruction according to the extent of the heating. The coal is stored on a plank floor, which is laid on sand filling. It is placed in continuous piles from 33 to 55 feet high. If possible, the coal is piled to prevent large sizes rolling to the bottom. There are from 20 to 75 per cent fines below $\frac{3}{4}$ inches when the coal is stored.

The Consumers' Company* of Chicago stores 250,000 tons or more of bituminous coal and a similar amount of anthracite, the bituminous being mainly Illinois and Indiana coal larger than the No. 4 washed. Some Pocahontas mine run is also stored. At times, portions of the stored coal are not moved for several years. In the larger bituminous storage yards, the coal is stored by a locomotive crane and grab bucket. The plat of Yard 150 (Fig. 58) shows a typical storage yard. The coal is stored about 26 feet high in piles, the shape of which depends upon the shape of the available storage space. The expense of handling depends upon conditions. Ordinarily, the coal is not insured against fire.

The experience of the Consumers' Company is that screenings below $1\frac{1}{4}$ inches in size will fire and that size No. 5 washed coal will fire even more readily than screenings, but that sized coal above $1\frac{1}{4}$ inches from which the duff has been removed will, in general, stock without firing. Firing will start more quickly if wood is present. The leg of a trestle or a similar piece of wood will act as a flue and firing is likely to start near any such wood imbedded in the coal pile. High sulphur coals are thought to fire more readily than others. Ample flooding with water will put out a fire, but a small amount will accelerate the fire, and the best way is to turn over the pile.

The St. Bernard Mining Company† of Nashville, Tennessee, stocks a great deal of $2\frac{1}{2}$ -inch bituminous lump coal in square piles on the ground to a height of 20 feet. The coal is stocked by dumping from wagons and is reclaimed by forking. The expense is ten cents per ton for labor and twelve cents for all other items, a total of twenty-two cents per ton. The coal is placed in storage from three to six days after being mined and remains in storage for about nine months. The best time for storing is said to be September and October, because then it is dry and cool. The coal is stored to protect contracts of the company and to insure a domestic supply. Storage is not profitable, but it is done as a precautionary measure. The coal stocked is to be used for steam and domestic purposes and there is no difference in the selling price of the fresh and stored coal. It is estimated that a loss of twenty-five per cent in breakage has resulted in storing and reclaiming. In 1900 the Company had a bad fire in its storage pile. The next year enough air shafts, which were 12 inches by 12 inches and of 2-inch oak, were placed around the posts, which supported the shed, and connected at the bottom by troughs to give circulation of air through the pile. Since that time,

* F. J. Hibbs, Superintendent of Construction.

† J. B. Love, Manager.

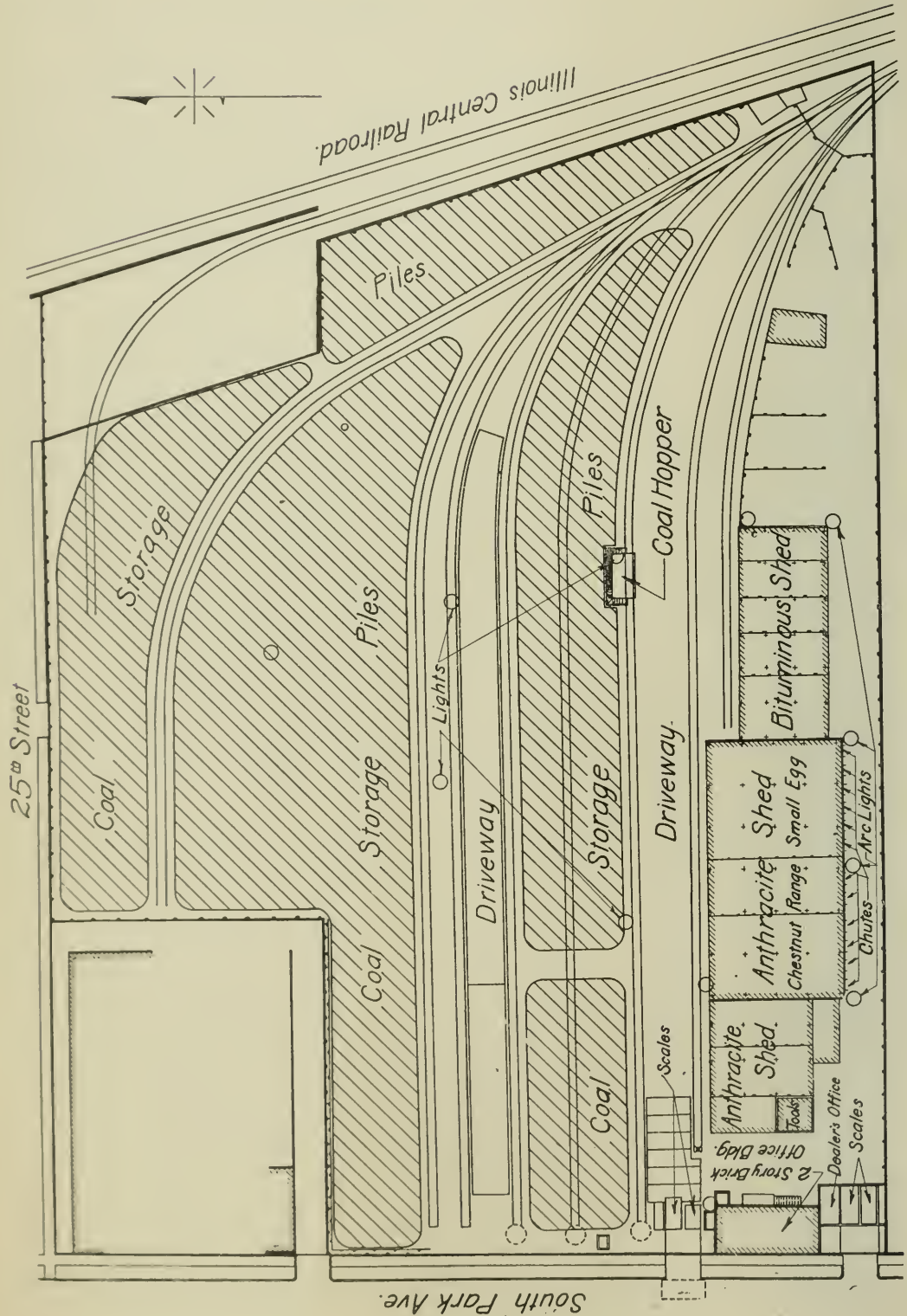


FIG. 58. TYPICAL STORAGE YARD OF THE CONSUMERS' COMPANY, CHICAGO

there have been no fires. Where the troughs rest flat on the ground, auger holes are bored to permit the air to circulate, but sometimes they are raised by blocks from two to four inches high to give circulation. As much as 8000 tons have been stored in this shed, but usually not more than 6000 tons. No fires have occurred since the flues were installed, but J. R. Love says:

“There might be some other reason for the fire that we had at the beginning of our operation as it started where we dumped the first coal, which had about a 20-foot drop and was badly broken and made a large amount of slack or dust. The next year, in addition to putting in the air shafts, we were very careful to build up the coal from the ground about 15 feet, so as to break the fall of the first coal that was dumped into the shed. I am not in a position to say for certain, whether the different method of handling the coal, or the air shafts, has kept us from having any hot coal after the first year.”

W. D. Langtry says in this connection:

“Recently I investigated a storage pile in Michigan at a plant which had the idea of putting down pipes for ventilation. In one instance there was one pipe that was a little taller than the balance, and I found by blowing some cigarette smoke over the top of the short pipe that this smoke was carried into the pipe by the draft, and the taller pipe was conducting the gases out of the pile and any smoke that was blown across its top was naturally conveyed up. It was plain to be seen that this was a current of air passing through the coal. If this was sufficient to keep the coal cool no trouble would occur, but from the gases being given off, I am afraid the air current was deficient.”

The Pittsburgh Plate Glass Company,* Crystal City, Missouri, stores 40,000 tons of mine run coal from Franklin and Williamson counties, Illinois, on the ground in continuous pyramidal piles twenty feet high without having the slack removed. A Browning clam-shell locomotive crane is used to store and reclaim the coal and the expense is said to be twenty-five cents to store and twenty-five cents to reclaim. Storage during the summer is considered the best. The coal is kept in storage from two to three years, with a slight decrease in heating value. The coal may heat at any time after storage, and if so it is moved to another place.

43. *Light, Heat and Power Companies.*—Public utility companies operate under the necessity of maintaining continuous service and are, therefore, vitally interested in the storage of coal. Many of these companies have already given considerable attention to the subject.

The Rockford Electric Company†, Rockford, Illinois, stores about 8,000 tons of 1½-inch screenings from Harrisburg, Illinois, on the ground in continuous pyramidal piles twenty feet high. The coal is placed in storage within about two weeks after being mined and is kept there for a period of six months. The coal may heat after two months and if so, it is turned over with a crane, but an effort is made to keep it from heating by ventilating with pipes. If the coal dries out and heats to any considerable extent, it is thought to deteriorate from twenty to fifty per cent. It is reported that it costs ten cents per ton to place it in storage and ten cents to reclaim it.

* E. C. Taylor, Superintendent.

† Adam Geschwindt, General Manager.

The Wisconsin Gas and Electric Company,* Racine, Wisconsin, stores 30,000 tons of washed mine run Youghiogheny coal in a continuous pyramidal pile thirty feet high. The coal is stored on the ground by means of a bridge and bucket at an expense of eight cents per ton for storing and twenty-two cents per ton for reclaiming. This includes labor, supplies, depreciation on mechanical equipment, interest on investment, and rental of the land on which coal is stored. An interval of three or four weeks elapses between mining and storing. The coal is used for making city gas and it is thought to suffer a slight deterioration in value for that purpose. It sometimes heats in from two to four months in which event it is quenched with water and worked over.

The United Railways Company† of St. Louis keeps in storage as much as 5,000 tons of central and western Illinois screenings and mine run. These sizes are kept separate in storage but the slack is not removed. The coal is stored in bins and in piles on the ground about ten feet high with a locomotive crane. It is placed in storage about three days after leaving the mines and is kept there about three months in hot weather. The financial advantage of storing is regarded as doubtful. The coal is used for power purposes and little deterioration in heating value is noted unless firing takes place. The lumps are said to break up, but at this plant the coal is crushed before firing. The coal frequently heats and the time of heating is said to depend on the nature of the coal and on the weather. When heating begins, the coal is, if possible, removed and used.

The Aurora, Elgin and Chicago Railroad Company‡ keeps in storage from 2,000 to 15,000 tons of coal from Franklin and Williamson counties, Illinois. The coal is stored on the ground in piles eighteen feet high, each pile containing about six hundred tons. This has been decided upon as a safe size of pile to avoid heating. The coal is stored with a crane and clam-shell bucket and is placed in storage about one week after being mined. The slack is not removed but the sizes are kept separate. The coal is kept in storage for periods as long as one year. Winter is regarded as the best time in which to store, but because of the availability and price of coal in the summer most of it is stored between July and September. The reason for storage is mainly to prevent a shortage, but there is also said to be a saving of from thirty to fifty cents per ton by storing in the summer. There is considered to be a differential of fifteen cents per ton in the selling price against stored coal, which is thought to decrease in heating value from ten to twenty per cent. It has been the experience of this company that coal high in sulphur is not suited to storing, but, if stored, should have the slack removed and should be screened. It is reported that low sulphur coal, such as the best from Franklin and Williamson counties, stores well in any size and does not heat or deteriorate to any considerable extent for a period of several months. Certain coals from the same field, however, have heated after less than a week in the bin. Gillette is convinced that the amount of sulphur in the coal is a determining factor in heating. He says:

“Some three years ago we had about 10,000 tons of Cuba screenings on the ground. This coal started to fire from the inside and before we were

* D. E. Callender, General Manager.

† W. E. Bryan, Superintendent.

‡ E. S. Gillette, Electrical and Mechanical Engineer.

able to pick it off the ground the majority of it had deteriorated about twenty to forty per cent in heat value, leaving some three cars of ashes which could not be run through the plant at all and consequently were a total loss. This coal had a high sulphur content.

“Since then we have piled our coal in small separate piles. We aim to put from twelve to sixteen cars in one pile. If we notice a fire starting in one of these piles, we immediately pick it up with a crane. Since we have started this system we have had practically no trouble from firing of our coal.”

John Hunter, chief engineer of power plants, Union Light and Power Company, St. Louis, Missouri, says:

“For the past ten years the miners’ and operators’ agreement expired every two years, and anticipating a strike at such time our local industrial and central station plants usually protected themselves by storing coal for at least thirty days. In 1912 our company stored about 30,000 tons of coal, and I am enclosing herewith a photograph (Fig. 59) showing one of the piles of 25,000 tons of mine run from the Mt. Olive and Staunton mines in Madison County. The coal was stored in February and March and was entirely removed and used by June 1. It was laid on the pile by hand and the track raised as the height of the pile increased, and was reloaded with a steam shovel from the pile into cars. We paid the Litchfield and Madison Railroad Company ten cents per ton to pile the coal and an equal amount to reload it, and it is my understanding that this figure just about covered the actual expense. After the coal was reloaded it was immediately shipped to our power station at Ashley street in St. Louis, placed in our bunker and used up in fifteen days.

“We experienced two fires in this pile after it had been on the ground for two months. These fires were checked by working into the pile with our steam shovel and loading the coal in steel cars; about twenty cars in all were removed in this way.

“The coal generally used by our company is from the mines in Macoupin, Madison and St. Clair counties, ninety-five per cent of it being screenings. The only regular storage we have is in the coal bins directly over the boilers in the main central station. These bins have a capacity of 12,000 tons with an average operating storage of 7,000 tons. The bins are divided into eight sections, and it is necessary to have one of these bins working down for cleaning and reloading, otherwise extensive fires are very troublesome. The average temperature below these bunkers is about 125 degrees, and we find it impossible to carry the coal longer than two months, in other words, it is necessary to renew the coal in each of our bins every two months.

“In the fall of 1911 I personally knew of one of the mine operators in St. Clair County laying on the ground the screenings from their mines, about 30,000 tons in all, with the view of holding them in storage for the anticipated strike on April 1, 1912. However, in February of that year I examined this coal and found that over eighty per cent of it had burned, the

high percentage of sulphur, clay and other impurities being responsible for these fires.

"It is a common expression among the engineers in our locality who are handling coal that it is impossible to store any of the standard coals from the inner group of mines in Madison and St. Clair counties, this being my personal views, unless the coal is specially selected. By this I mean that no coal of less than six-inch sizing be laid on the ground."

W. E. Bryan, of the United Railways Company of St. Louis, says:

"We try to decrease our storage in hot weather on account of the dangerible to firing than other coals. It has also been our experience that coal which has been cleaned by washing, say of nut or pea size, will not fire so readily as the screenings containing so much dust. The hotter the weather, the more is the tendency to heat. Also wet weather in connection with sulphurous coal exposed to the air will cause heating especially when followed by hot weather.

"Three months is the average length of time coal is in storage at one plant, which has a regular storage house. We do not make any tests for heating and as a rule have no trouble if the coal is not stored more than three months. We have stored the coal longer than this without trouble, but we do not like to carry more coal than can be moved in three months.

"We try to decrease our storage in hot weather on account of the danger of firing and in the past have been stocking up to some extent in the fall when screenings are comparatively cheap. Under these conditions we may at times hold the coal in storage longer than three months.

"It is hard to give any definite figures because our results have varied quite a good deal."

The Laclede Gas Light Company* of St. Louis stores from 5,000 to 15,000 tons of Pocahontas screenings and from 40,000 to 85,000 tons of run of mine coal from Elkhorn, Kentucky. The sizes are not kept separate and slack is not removed before storing. It is stocked from seven to ten days after being mined and preferably in the spring or summer. No decrease in heating value is noted and there is no loss from breakage. The Pocahontas coal heats in about six months. To prevent heating the piles are turned over, but if heating occurs the pile is quenched with water and used as soon as possible. Heating causes a loss in gas and in by-products. The coal is stored by means of a locomotive crane and clam-shell bucket. It is similarly reclaimed directly from the pile. The coal is stored directly on the ground in long, continuous piles from twenty to forty feet high.

The Cleveland Illuminating Company keeps about 15,000 tons of slack, 85,000 tons of run of mine Ohio No. 8, western Pennsylvania, and Fairmont, West Virginia, coals in storage, the sizes being kept separate. The coal is stored by means of a gantry crane hoist in rectangular, continuous piles to a height of thirty-five feet and is recovered by means of a locomotive crane which handles about 500 tons in ten hours, the hoist handling about 1200 tons in ten hours.

The East St. Louis Light and Power Company stores from 5,000 to 12,000 tons of screenings under water from St. Clair and Madison counties, Illinois, as an

* J. F. Bulfin, Auditor.



FIG. 59. TWENTY-FIVE-THOUSAND-TON PILE OF RUN OF MINE COAL STORED BY THE
UNION LIGHT AND POWER COMPANY, ST. LOUIS, MISSOURI

emergency supply. There is no loss from breakage or depreciation in heating value.

44. *Storage at Metallurgical Plants.*—According to the latest reports of the U. S. Geological Survey, there are more retorts for smelting zinc in Illinois than in any other state. This number has gradually increased and much of the smelting of zinc formerly carried on in Kansas, owing to the presence of natural gas, has within a few years been transferred to Illinois on account of the adequate coal supply. It is of especial interest, therefore, to note to what extent coal is being stored at the zinc plants. The following is a resume of the answers given in response to the questionnaire sent to the ten zinc smelters in the State.

The Illinois Zinc Company at Peru and the Matthiessen and Hegeler Zinc Company at LaSalle operate their own mines contiguous to the smelters and do not store.

The Sandoval Zinc Company at Sandoval and the Missouri Zinc Company at Beckemeyer do not store coal.

The Collinsville Zinc Company at Collinsville stores No. 4 and No. 5 mixed Franklin County coal on the ground in continuous piles from ten to twelve feet high, in order to assure a supply for continuous operation. The coal is thought to decrease in heating value, but the amount of this decrease is not known. Coal is stored and reclaimed by hand, and no heating has been observed.

The American Zinc Lead and Smelting Company of St. Louis, at its plant in East St. Louis, keeps in storage on the ground from thirty to sixty days 200 cars of St. Clair County, Illinois, run of mine coal without removing the slack. Coal is unloaded by locomotive cranes upon conical piles ten feet high. Cranes are also used for reclaiming the coal and while the storage costs are not segregated, they are estimated to be twenty cents per ton for placing the coal in storage, and fifteen cents for reclaiming. These figures include interest on the investment, repairs of unloading machinery, and all charges. The coal is used for making producer-gas and is stored to insure a supply at all times, but no financial advantage is secured. The coal is thought to decrease in heating value from about five to ten per cent. No data are available concerning breakage. Some kinds of coal fire, the time of firing after storage depending on the size and the sulphur content of the coal. As soon as heating is noticed the coal is picked up and used. Little time elapses between mining and storing. Winter is considered the best time to store.

The Mineral Point Zinc Company at Depue, Illinois, stores coal in continuous piles from 600 to 700 feet long, 60 feet wide, 12 feet high, the piles being parallel with two tracks on which the locomotive crane and cars operate. The storage yard has a capacity of about 60,000 tons. Grab buckets are used and the expense of storing is from five to six cents and for reclaiming from five to six cents per ton for labor, materials, maintenance, and overhead. Coals principally from Harrisburg, Saline County, Illinois, are stored and they are kept in storage from six to twelve months. Lump or mine run is preferred. Screenings are kept in storage from one to three months, the different sizes being kept separate, but slack is not removed. The coal is stored in from five to ten days after being mined, mainly in the fall in anticipation of the spring needs but some coal is kept

in storage at all times to insure against strikes and transportation trouble. Coal used for fuel and gas making purposes is thought to deteriorate in heating value, but no definite data are available. Screenings from southern Illinois heat in about ninety days, those from the Peoria district in about twenty-five days. From 20,000 to 25,000 tons of mine run, lump, egg, and washed nut from the Peoria district are kept in storage at all times and no firing has occurred, although the piles are of the same size as those of the screenings. It is reported that a pile of Peoria run of mine has been in storage for a year without firing, the pile being twelve feet high, thirty-five feet at the base and containing about 3,000 tons.

J. H. Brooks, Superintendent of the Depue plant, says that the slack is sometimes received hot in the cars and he believes that while a shallow pile will help to prevent the firing of unwashed Illinois screenings to a certain extent, his experience is that this class of coal cannot be stored for an indefinite period on the ground. As soon as heating is discovered the coal is loaded out.

The Robert Lanyon Zinc and Acid Company* of Hillsboro, Illinois, stores about 2,000 tons of run of mine Montgomery County, Illinois, coal from which the slack has been removed. It is stored in five to seven days after being mined and is kept in storage from three to six months. The coal is used for steam and gas making purposes and is stored to insure supply, with no financial advantage resulting. It is thought to decrease two per cent in heating value and it sometimes fires in from one to nine months, the injury from firing depending upon the extent of the fire. Piles are continuous, fifty feet wide and five feet high, and are placed on the ground or preferably on cinders. To prevent firing the piles should be kept low.

45. *Storage for Coke Ovens and Blast Furnace Plants.*—The coal required for coke and blast furnace plants is so great that a considerable quantity is always kept in storage to meet emergencies. Storage also makes it possible to take advantage of lake transportation.

A number of coke and steel companies operating in the Chicago and St. Louis districts, and in other districts in the Middle West stores coals both from the Middle West and from the East.

The By-Products Coke Corporation† of South Chicago keeps in storage a maximum of about 400,000 tons of miscellaneous sizes of West Virginia coals made up of about 160,000 tons McDowell Big Sandy coal and 240,000 tons of Kingston coal from Fayette County. Coals of different varieties are separated, but no attempt is made to separate sizes. The coal is piled on a 4-inch timber flooring in pyramidal piles from forty to fifty feet high by means of a belt conveyor operated on movable bridges. It is reclaimed by buckets. The costs are not available since they are not segregated. Although the company stores large quantities of coal in the summer when lake transportation is open, it is considered best to store in the winter so that there may be less initial heat in the pile. In stocking an effort is made to avoid segregation of lumps in the pile. The coal is believed to deteriorate in heating value, but no data are available. After the coal is in storage four months, it is turned over to prevent heating. The coal some-

* R. M. Roosevelt, Manager.

† W. H. Allen, Superintendent of the Coke Plant.

times heats in from three weeks to three or four months in which event it is used up promptly or is turned over upon fresh piles.

The Inland Steel Company* stores at Indiana Harbor 70,000 tons of run of mine and slack coal from a district north of the Pocahontas region in West Virginia, and 140,000 tons of $\frac{3}{4}$ -inch Youghiogheny lump coal. The coal is stored from ten to fourteen days after mining. The slack is not removed and the sizes are not kept separate. The greater part is kept in storage from December 1 to April 15, while lake transportation is closed, and some remains in storage two or three months longer. It is stored on a 3-inch maple floor in a continuous pile forty feet high by means of a traveling bridge and belt conveyor. When the storage season opens, the floor is cleaned and coal is stored gradually from one end to the other of the pile. Coal is reclaimed by means of buckets, the coal longest in storage always being taken out first. August and September are considered the best months in which to store since the coal received then is dry. The heating value of the coal is not determined, but the gas made from the stored coal does not vary materially in heating value from that made from fresh coal. Coal sometimes heats and fires in from a few weeks and to prevent heating it is turned over occasionally. If heating occurs, the affected part is removed. Coal thus heating loses its coking qualities.

The Illinois Steel Company† keeps in storage at the Joliet Works from 10,000 to 30,000 tons run of mine Pocahontas and from 10,000 to 30,000 tons of coals from Letcher and Harlan counties, Kentucky. Coal is piled twelve feet deep by means of bridge and conveyor in concrete pits having concrete floors. It is stored within one or two weeks after mining and is kept in storage from one to twelve months. The coal may heat to a variable degree in from sixty to ninety days and and if heating occurs it is taken out of stock.

The Illinois Steel Company‡ also stores at South Chicago, Illinois, quantities as great as 100,000 tons of coal from McDowell County, West Virginia, and 100,000 tons from Letcher County, Kentucky. The grade is run of mine and the slack is not removed. The coal is stored in the summer and within two weeks after being mined. It is used to make by-product coke. Coal is stored in a concrete bin in continuous piles fifty feet high. It is stored and reclaimed by means of overhead bridges.

The Wisconsin Steel Company of South Chicago does not stock in large quantities and keeps only from 4,000 to 10,000 tons in piles which are being constantly drawn upon.

The Iroquois Iron Company¶ of South Chicago keeps about 1,000 tons of run of mine coal from Christian County, Illinois, in storage for about eight months. The coal is stored within two or three weeks after being mined and the slack is removed before storing. It is placed on the ground in a continuous pile twenty-five feet high by means of a locomotive crane at an expense of ten cents per ton for storing and reclaiming. The coal is used for steam purposes and it does not

* H. D. DeHoll, Superintendent of the By-Product Coke Ovens.

† D. R. Mathias, General Superintendent.

‡ W. P. Gleason, General Superintendent.

¶ Perrin Rule, Superintendent.

depreciate materially in heating value, but suffers a loss of five per cent in breakage.

The Coal Products Manufacturing Company* of Joliet, Illinois, stores about 5,000 tons of run of mine from the Elkhorn and Logan Coal Company, West Virginia, and 3,000 tons of run of mine from Franklin and Williamson counties, Illinois, for a period of from 60 days to 90 days. The coal is stored, within a week or two after mining, on the ground in separate piles, thirty feet high by means of a bridge which is also used for reclaiming. The coal is used for the manufacture of by-product coke and no decrease in heating value has been noted. The Pocahontas coal heats within sixty days, but the Logan County coal has not done so. Heating is said to impair its coking qualities.

The Jones and Laughlin Steel Company† of Woodlawn, Pennsylvania, keeps in storage 100,000 tons of raw Pennsylvania No. 5 coal all of which is passed through a three-inch crusher, also 200,000 tons of washed Pennsylvania No. 5 which is passed through a $\frac{3}{4}$ -inch crusher. The coal is stored in pyramidal piles 300 feet wide and 350 feet long by means of a bridge and conveyor and is reclaimed by buckets from the bridge. The coal is stocked within three days after being mined and most of the washed coal is kept separate and left in storage for a period varying from four to eighteen months. The washed coal is stored at any time during the year, and the raw coal in November and December because of river navigation. The raw coal will fire spontaneously after about four months, starting about eight feet from the bottom of the pile, but while the washed coal heats up it does not fire. The raw coal is now piled only twenty feet deep instead of forty-five feet as formerly, and the washed coal is stored to a depth of forty-five feet.

Horace C. Porter of the H. Koppers Company of Pittsburgh says: "We have noted especially in the plant of the Lehigh Coke Company, South Bethlehem, Pennsylvania, now owned by the Bethlehem Steel Company the fact that the weathering of coal in the open has a considerable effect on its coking qualities after a certain period and affecting the degree of shrinking or of expanding of the coal during coking."

G. F. Downs, Superintendent of the Lackawanna Steel Company, Buffalo, New York, says:

"Our experience in stocking coal has been that any low sulphur coal, such as Pocahontas coal, can be piled to a height of thirty to sixty feet and will stand indefinitely without heating or firing.

"It is our experience that coals containing one per cent of sulphur and more cannot be piled higher than twelve to fifteen feet without heating and firing inside two months. We have tried piling such coals twenty to thirty feet high and using ventilating pipes, well distributed over the surface of the pile, but this gave no relief from the heating and we believe made the fire more rapid after it had once started as the pipes acted as flues."

The Calumet and Hecla Mining Company,‡ Calumet, Michigan, stored a max-

* A. H. Harris, Superintendent.

† A. W. Belden, Superintendent, Coke Department.

‡ James McNaughton, Vice-President and General Manager.

imum of 500,000 tons of coal from Thacker, West Virginia, and Mansfield, Pennsylvania, for a period of eight months, the coal being received within about two weeks after being mined and during the open navigation season from May to November. There is some depreciation in heating value but no material loss from breakage. If screened, there is no trouble from heating. The coal is stored both in sheds and in the open in continuous rectangular piles about twenty-five feet high. A tower system of storage is now being installed. About ten per cent of the coal is below three-fourths inches. Sizes are not kept separate in storage, but slack is removed. The expense of unloading from boats and storing is 15 cents per ton and the total reclaiming expense is $7\frac{1}{2}$ cents per ton in summer and $11\frac{1}{4}$ in winter. These amounts do not include rent, interest, or depreciation.

46. *Mine Storage.*—The simplest method of storage at the mine is by means of bins which form a part of the tipple structure or are located in a detached shed or building. Such bins provide a small storage capacity which will tide over any short stoppage of the mine and at mines where there are coke ovens and where it is necessary to insure a steady supply of oven coal they are extensively used. In the Pocahontas region of West Virginia, bins were at first built to take care of the slack only, but the system has gradually been extended to include all commercial sizes. In the Connellsville district of Pennsylvania the storage bins hold usually from 1000 to 2000 tons and permit the mines to be idle for a period of one day without interfering with the operation of the ovens. At some of the mines of the Middle West, particularly where coal is rescreened or washed, similar bins are used, but they are generally placed in a separate rescreening structure. The capacity of bins varies from 30 to 250 tons each and while they furnish small temporary storage for the washing or rescreening, they do not materially effect the operation of the large modern mines.

The Federal Coal Company of Straight Creek, Kentucky, recently built at the Barker mine a storage bin which holds 150 tons of run of mine coal. The purpose of this bin is to furnish storage for at least a part of a day's run in case cars are not placed at the mine, or in case there are not enough cars for the day's supply. With regard to this bin R. R. Atkins, Superintendent, says:

“We selected for the site for the storage bin, an old slate dump which was near our tipple and when dumping into the bin cars are shoved up a short grade to the top of this dump, about 15 feet above the dump at the tipple, and straight run of mine is placed in the bins, which have a gate at the bottom for discharging into railroad gondolas. This has proved to be so successful that we have plans under consideration for erecting bins at our other mines to be used in a similar way, but we do not contemplate the storage of coal in any quantities. It is our idea to build bins of such capacity that the size of the bin, together with the mine cars at each mine, will hold about a day's run. We believe that by doing this, we will be able to run our mines when we have no cars and also to load out a greater number of cars on days when the railroad places them, thus increasing our car allotment.

“While it is rather difficult for us to figure on the exact price of dumping the coal in this bin, and discharging it, we figure that the additional cost

amounts to about ten cents per ton. This could be considerably lowered by the erection of a bin of a different type but this would necessitate a greater expenditure of money in the erection of a bin as this bin was an experiment, we did not at the time feel justified in going to too heavy an expense."

Detailed descriptions of two plants for storing coal at the mines in Franklin County, Illinois, are given on pages 51 and 71, and for the large plant of the Clinchfield Fuel Company on page 64.

The Bell and Zoller Company at Zeigler, Franklin County, Illinois, during the winter of 1916-17 stored 45,000 to 50,000 tons of Nos. 3, 4, and 5 coal by means of a 1½-ton clam-shell bucket operated by a locomotive crane. The coal piles were from forty to fifty feet long and thirty feet high, and the three sizes were kept separate. No difficulty was experienced from firing, although the coal was kept in storage until the following winter. This coal was stored for the purchaser who paid the actual cost which was found to be slightly over three cents for unloading and from seven to eight cents for loading.

At the Security and Majestic mines in southern Illinois, coal has been stored on the ground for the Illinois Central Railroad by means of side dump cars furnished by the road. Whenever the coal companies had no cars for regular shipments, these cars were loaded and dumped on the ground near the tippie. The coal was reclaimed when needed with locomotive cranes.

Coal has been similarly stored at Buckner and Christopher, Illinois, by the Old Ben Mining Company. At one mine in central Illinois coal was stored in a small pond near the tippie and reclaimed with a locomotive crane.

In the report of the International Railway Fuel Association for 1915* occur the following suggestions for storage at the mines:

"At strip pit mines practical storage could be accomplished by leaving a small covering on the coal. If areas operated were large, from 50,000 to 100,000 tons could be partially uncovered in the summer and thus carried in storage without using the water covering method, and this, I believe, could be done at only small additional cost to reimburse the steam shovel operator for returning and uncovering and loading coal when season requires. A strip pit could be made to double or treble its daily production with only the one loading cost, and this would also reduce breakage."

This suggestion applies to coal which weathers very quickly, but for coal which stands the weather for a reasonable period of time a considerable storage capacity may be made available at stripping mines by taking off the surface and leaving the coal until it is needed; in fact, it is one of the advantages claimed for strip mining.

A. H. Davies,† Fuel Inspector of the Grand Trunk Pacific Railway, Pocatonton, Alberta, Canada, suggests the following system of storage for a mine in a hilly country (see Fig. 60):

"In mountainous districts, where seams are pitching, operators always enter the hillside at a point that gives them elevation for their tippie above the railroad track, generally about 25 to 30 feet.

* Proceedings, Vol. VII, p. 265.

† Ibid. pp. 273-275.

“At this depth, at foot of sloping hill, adjacent to tippie, a side-cutting about eight feet deep, and as long as capacity requires, at one per cent grade, dipping away from tippie, should be made. In this cutting construct tunnels of ordinary heavy timber, boxed in with round lugging or planking. Then bring rock dump at tippie level; parallel with tunnel and opposite the hill. The apex of rock dump to be 50 feet from center of tunnel-timber-collar, thus forming a V-shaped bin above tunnel. Trim or shear side of rock dump

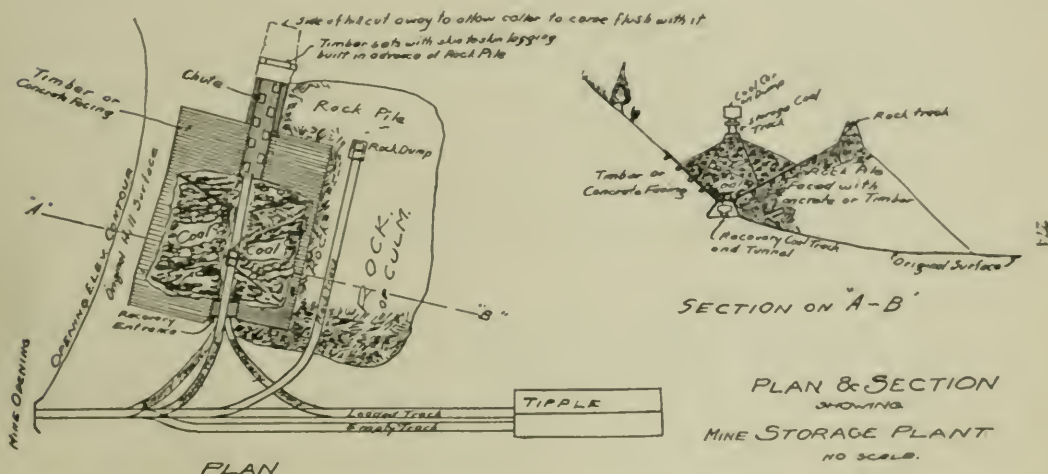


FIG. 60. SUGGESTED SYSTEM OF STORAGE FOR A MINE IN A HILLY COUNTRY
(Reproduced from the Proceedings of the International Railway Fuel Association)

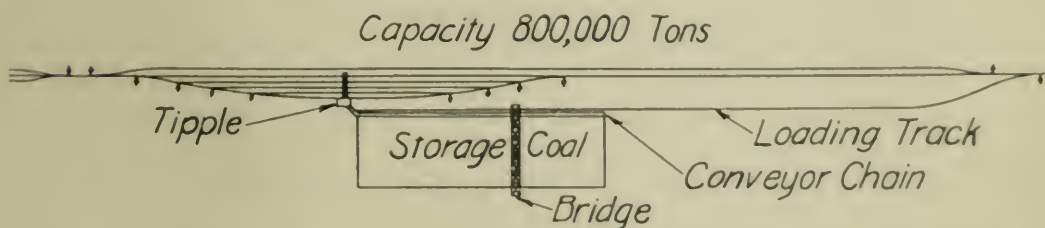


FIG. 61. SUGGESTED SYSTEM OF STORAGE FOR SCREENINGS AT THE TIPPY
(Reproduced from the Proceedings of the International Railway Fuel Association)

and hillside to an angle of not less than 30 degrees. Cover over with layer of ashes, then corduroy, plank, or face with concrete, as desired. Construct double track on to storage bin on tippie elevation, one for loads and one for empties. From commencement of bin onwards, over and above whole length of bin, construct single track. To run on this track, build a detachable traveling carriage dump With 25 feet elevation from track in tunnel to track on tippie, the capacity would be approximately 2,000 tons per one hundred lineal feet of bin. If mine situated on level ground, prairie,

elevation is still available in tipple. The only difference would be, that another rock pile would have to be constructed, to replace natural hillside.''

Fig. 61 shows a plan suggested for storing screenings at the tipple.* The coal is carried from the tipple by a chain bucket conveyor which runs parallel with one side of the storage yard, the conveyor being supported on concrete posts, and provided with doors every ten feet through which the coal may be dropped to the ground, when the bridge bucket places the coal directly in the car standing on the loading track independent of the other mine tracks. Such a plan would cost more and seems to offer few advantages over the locomotive crane storage plant.

The Consolidation Coal Company†, Fairmont, West Virginia, stores coal in its mines in the Miller's Creek district of eastern Kentucky. The coal is stored during the summer time to supply the winter demand and is placed on the ground in long, narrow piles about thirty feet high. It is reclaimed by shoveling into mine cars. No heating has occurred but the amount of slack increases from thirty to fifty per cent. The coal contains about fifty per cent below three-fourths inches when stocked.

47. *Storage by Railroads.*—The subject of railroad storage has been discussed so fully by the International Railway Fuel Association since 1914 that it is very difficult to present any data not already contained in the Proceedings of the Association.

The replies to the questionnaire have brought out the following points:

The practice of the Illinois Central Railroad‡ may be summarized as follows:

The place where coal is to be stored should be free from all rubbish and special attention should be paid to drainage to avoid trouble from excessive moisture. The coal in storage should not be in contact with steam pipes.

It is advisable not to store the coal to a depth exceeding sixteen or eighteen feet, nor to a width exceeding thirty feet. Alleyways are left between the storage piles, thus separating the coal in storage into piles about thirty feet long.

After the storage piles have been completed, pipes, preferably old flues removed from locomotives, are placed at intervals, and staggered about the piles. In inserting the pipe into the coal pile a pointed wooded plug is placed in the end of the flue, so that it may be driven to the required depth without excessive labor. The flue should be driven through the coal to within about twenty-four to thirty inches of the soil and it should then be withdrawn sufficiently to permit the plug to be driven out of the pipe. By inserting a $\frac{7}{8}$ -inch rod through the top flue and using a medium hammer the wooden plug can be driven out through the bottom, thus creating an air chamber at the bottom of the flue in which temperature reading may be taken by means of a thermometer fastened to a line. Note of the atmospheric temperature should be made before inserting the thermometer into the flue. The thermometer should remain in the hole fifteen or twenty minutes. If the temperature of the pile is thirty degrees

* Ibid, Vol. VI, p. 124, 1914.

† Frank Haas, Consulting Engineer.

‡ H. B. Brown, General Fuel Inspector.

above that of the atmosphere, that part is immediately taken out and used. Since a rapid increase in temperature indicates excessive heat, arrange to remove the coal by the use of a locomotive crane or similar mechanical device.

Do not place fine coal on the bottom of the pile, and keep the bottom dry. Mixed screenings will fire more readily than unmixed.

The alleyways or spaces between the piles give an opportunity to save the coal with little expense and provide an easy means of locating trouble from heating.

Deterioration of coal in storage may be greatly reduced by a covering of small coal an inch or less in size.

Do not mix coals from different mines if it can be avoided, particularly when the action of a given coal in storage has been determined. Screenings particularly should not be mixed before storing. Instances are cited of screenings intended for power plant use which fired in the car within a short time after having been rained on. Locomotive coal from the same mining region which had been stored twenty-five years had not fired.

The following instructions regarding storage of coal on the Rock Island Railroad were issued during the spring of 1917 by Carl Scholz, then manager of the mining and fuel department, in connection with the contemplated storage of 465,000 tons of coal at various points, 165,000 tons were to be used to effect car economies and to be reloaded after the peak-load of railroad transportation, and the remainder to insure an ample supply in case of labor trouble.

Instructions Governing Storage of Coal

“Engine coal is stored for the purpose of providing fuel at a time when the demand for it is greater than the current orders can provide or guard against shortage in case of suspension at the mines.

“In unloading coal, the reloading cost must always be considered, and the place selected for storing should be convenient to fuel chutes so as to handle coal with the minimum mileage from the storage pile to the place of ultimate consumption.

“In selecting the ground for storage pile, drainings should be fully considered, because the coal should not be stored on wet ground or filled material liable to cause spontaneous combustion. Hard clay is preferable as a foundation, with drainage on both sides, and if necessary, drainage tiles or ditches through the center.

“The 2-inch track unit has been accepted as standard. Under this arrangement, two tracks will be built at sixty-five foot centers. These tracks will be raised as the coal is unloaded, so as to rest on the storage pile itself until the piles reach their height limit. Then they will be removed and laid at fifteen-foot centers between the piles, thereby enabling the reloading of coal, either by hand or clam-shell, the clam-shell to travel on one track and the cars to be loaded to occupy the other track.

“Where Illinois and Iowa coals are stored, the coal piles should not exceed a maximum height of fifteen feet, but with Oklahoma coal the height may go to twenty or twenty-two feet.

“A unit built of this plan of forty cars lengths will store approximately

35,000 tons of coal. As far as possible, drop bottom cars will be furnished for the handling of storage coal.

“Immediately after heavy rains, it is desirable not to pile fresh coal on the stored coal until the moisture has had a chance to evaporate. Care should be taken to reject cars containing an unusual amount of slack, and such cars should be switched and sent to fuel chutes for immediate consumption, so that coarse coal is stored as far as possible.

“The reloading track which remains on the ground should not be covered more than three or four feet above the rails. For the purpose of observing any rising temperature, flues or old pipes should be driven in the storage pile every 300 feet so that a thermometer can be suspended therein to note any rise in temperature.”

The Central of Georgia Railway* stores 31,500 tons of Alabama coal four inches and under in size, without separating it into sizes or taking out the fine coal, about five days after it is mined and keeps it in storage six months. To insure a regular supply and for winter use, coal is stored in July, August, and September, because the railroad business is dull and necessary equipment is available for storage; also the price of coal is usually lower.

It is placed on the ground in continuous, long, narrow piles limited to fifteen feet in height and is reclaimed with a locomotive crane at an expense of 2.58 cents per ton for storing, and 2.09 cents per ton for reclaiming, including labor and supplies. Rental of tracks, depreciation, interest, and insurance are not included in these amounts. Coal is thought to decrease in heating value not more than five per cent. There was no loss from breakage and no heating. Sixty thousand tons stored for six months burned very well.

The Louisville & Nashville Railroad† has forty-six coaling stations and at some of them coal is stored in amounts varying from 50 to 25,000 tons on the ground or more generally on a timber floor. It is placed in small bins by hand or in large bins by gravity from a trestle or by a locomotive crane from cars. A circular system is also used. It is reclaimed by locomotive crane or by hand.

The height of the pile depends upon the coal used, being limited in some cases to twelve feet and in others reaching to more than twenty feet without heating. In case of heating, the coal is removed or sprinkled with water. Coal is kept in storage indefinitely and is stored during the summer to take advantage of the low price, to relieve car service in winter, and to insure supply. Sizes are not kept separate and fine coal is not removed. The heating value is thought to decrease and the extent to which coal will heat depends upon the height of the pile and the condition of the coal when received.

The Baltimore & Ohio Railroad‡ is using industrial tracks, which are elevated or are raised on cribs, to store on the ground run of mine coal from Somerset, Pennsylvania. In this way cars are released promptly. The coal is reclaimed with a locomotive crane and there is only a slight loss in heating value and a slight breakage loss.

* A. P. Wells, Engineer of Tests.

† Engineering Record, 1914-15, Engineering News, 1914-15.

‡ W. H. Averell, General Manager.

The Chicago & Northwestern Railroad* stores from 12,000 to 40,000 tons of coal from Macoupin County, Illinois, generally storing the six-inch lump, and using the fine coal at once. Run of mine has also been stored. Continuous piles not over twelve feet high are used with tubes placed for temperature observations and the piles are very closely watched. This method has been tried for four years and if deterioration occurs it is mostly during the first few months and not after one year.

The Southern Railway System† stores about 10,000 tons of Illinois and Kentucky screened coal at Danville, Kentucky, 18,000 tons at Huntington, Indiana, and also from 60,000 to 180,000 tons of coals from Middlesboro, Kentucky, and southwest Virginia. These are stored within two to twenty days after being mined and are kept in storage from four to eight months. The coal is stored on the ground in continuous piles from ten to fourteen feet high. The slack is not removed and the sizes are not kept separate. There is thought to be a ten per cent decrease in heating value and a breakage loss of fifteen per cent. The coal sometimes heats but it has never fired. The piles are limited to a height of fourteen feet. If heating occurs, the coal is picked up and used immediately, or is spread out and allowed to cool. Goodwin says: "Theoretically, there is little decrease in heating value, but practically, on account of the slack present, we do not get the same results from stored coal as from fresh."

The Michigan Central Railroad‡ had in storage at Gary, Indiana, during the summer of 1917, 35,000 tons of 1¼-inch lump and run of mine coal from West Clinton, Indiana, and from Harrisburg and Freeburg, Illinois. The sizes were not kept separate and the slack was not removed. The coal was stored from five to fifteen days after being mined. It was placed on the ground by hand or by clam-shell buckets in long piles eighteen feet deep. This coal had been in storage for four or five months when it took fire, although at other times coal had been kept from eight to eighteen months without firing. Where the fire started, the pile was not more than twelve feet high. At the point of firing there was found to be a quantity of very dry fine slack. The moisture had evidently been dried out by the heat from below, since the coal above this spot was damp and this dampness evidently came from the moisture being driven from the coal below. Where the coal fired it was lumpy or coarse, and covered with very fine slack. Apparently the slack retained the heat and did not permit it to radiate, so that the temperature gradually rose to the ignition point. After the fires had been dug out the remaining coal cooled and since then no fires or extreme heat have been noticed.

The Chicago, Lake Shore & South Bend Railroad¶ stores about 3,000 tons of coal from Clinton, Indiana, on the ground in continuous piles about fourteen feet high. The sizes are kept separate in storage and slack is removed. The coal is stored two weeks after being mined and, if possible, is placed in storage between the first of September and the first of December because of the prevailing favorable

*W. E. Dunham, Supervisor of Motive Power.

†C. G. Goodwin, Fuel Agent.

‡J. C. Dougherty, Coal Inspector.

¶W. E. Bolston, Superintendent of Power and Equipment.

weather conditions during that period. It is kept in storage about one year. There is a slight decrease in heating value and heating is likely to occur in from one to six months. In case of heating, the coal is removed at once. To avoid heating, the piles are kept small and low in height. The coal is handled by means of a locomotive crane at a cost of twelve cents per ton for storing and reclaiming.

Chicago, Burlington & Quincy Railroad* had in storage October 1, 1917, 562,000 tons of coal, which were stored between April and September. This coal was intended for use in November and December. The experience of the Burlington as given by Crawford is summarized as follows:

At Havelock, Nebraska, there were stored 8,000 tons of coal from the Virden, Illinois, district in long piles from twelve to fifteen feet high and forty-two feet wide. The coal was stored early in June and about July 30 was reported to be heating throughout. Although the company began to load out the coal at once, heating continued to spread rapidly and on August 16 a pipe was laid to the pile which was spread out so that the coal was only from three to five feet thick and water was put on it day and night; consequently it is stated that only five per cent of the coal was lost. In other piles at Galesburg and Savanna, Illinois, there was similar trouble from firing.

At Clyde, Illinois, southern Illinois screenings were stored and where fires occurred, Crawford attributes them to excessive depth of piles. The general policy of the Burlington is to store from April to September, and where two coals from different districts are stored at the same point, to keep the different coals in separate piles. Egg and lump coals are not separated, but where the storage coal is to be handled with a clam-shell directly upon an engine, it is advisable to store only egg coal. Coal should be stored in as dry weather as possible. Superintendents are advised to store coal on ground reasonably level and as high as possible from which rubbish has been removed, and not over steam lines.

Crawford states:

"I think every one should figure that there is going to be heating in storage coal and should provide full means for dissipating that heat. The only way I know of to stop this heating in coal storage is to store the coal low enough, and be careful to store lump and egg coals so that there will be reasonably free circulation.

"It is impracticable for a railroad to allow coal to season in a car, and a better way to let it season is to store it in long piles, building it up gradually. Here, however, we come to the point of the advisability of putting freshly mined coal upon a pile of coal already stored which has been subjected to the rays of the sun when the temperature is around 100 degrees. In other words, I think the fresh coal will pick up more heat from a pile of storage coal than it would from the ground.

"I am satisfied that by carefully inspecting the Belleville district coal going into the North St. Louis pile which is seventeen feet high that we saved a fire at this point. The mines started to ship us mine run billed lump and fortunately we discovered this in time and placed an inspector in the district."

* J. G. Crawford, Fuel Engineer.

The Missouri, Kansas & Texas Railroad* stores Illinois 2-inch lump coal from the Belleville district, coal from the Pittsburgh district, Kansas, with twenty-five per cent of the slack removed, called modified lump, mine run coal from the McAlester district, Oklahoma, and also coal from Thurber, Texas. The coal is kept in storage from six months to one year. Different varieties and different sizes are piled separately. It has been found, that coal from the McAlester district, Oklahoma, will fire when mixed with coal from the Lehigh district, Oklahoma. The slack is removed except where Oklahoma mine run is stored. As much time as possible is recommended between the mining and storing, and March, April, May, and September are considered the best months in which to store. There is thought to be a slight loss in heating value and a slight breakage loss. Hibben summarizes his practice as follows:

“Heating depends upon the manner in which the coal is stored and the kinds of coal. It is not wise to store fine coal and lump should be used, if possible. Do not pile over fifteen feet high. The cost of storing with clam-shell buckets has been 3½ cents per ton which includes the cost of construction of new track when necessary, the salary of the operator and other necessary employees to remove the coal left in the car. The estimated cost for reclaiming is the same.”

C. M. Butler, of the Atlantic Coast Line, said at the 1917 meeting of the Railway Fuel Association:

“There will be a week or ten days when from certain coal supplies we do not receive a single car. That is not due to the fact that the mines have not been shipping it, but it is due to the fact that railroad companies handling the coal from the mines to us have been unable to get it to us. Then there is another week when we will get three times as much coal as we need. The result is that we have a surplus of coal loaded in cars, and in order to help the situation we have made some arrangements to release that coal regardless of consumption. We have made places all over our line to store coal, and to store coal this week and use it next and restore it in the same place and pick it up, simply to release the cars. I feel sure that our own little supply has been very much helped by our prompt handling of the cars; not only from the fact that we have put those cars back to the mines to be reloaded, but we have created a feeling at the mines where our coal is loaded that any coal allotted to us will receive prompt attention and the cars will not be held indefinitely.”

The Atlantic Coast Line keeps Virginia and West Virginia run of mine, high volatile coals in storage for six months without separating the sizes and without removing the slack, there being from fifty to sixty per cent slack below ¾ inches in the coal when stored. Some of the piles are continuous and some are partitioned into sections. The coal is stored in August and September, before the heavy fall traffic, and is placed in semicircular piles, twenty-five feet high, by means of a locomotive crane which takes the coal from a pit. If stored in a wet place or in a high pile, the coal heats very soon after being stored. The expenditure

* R. R. Hibben, Assistant Fuel Agent.

for labor, operation, repairs, and maintenance varies from four to five cents per ton, depending upon the appliances used.

The Grand Trunk Pacific Railroad* stored 150,000 tons of Ohio and Pittsburgh No. 8 coal over $\frac{3}{4}$ inches in size and 40,000 tons of Jasper Park Alberta mine run coal containing 70 per cent below $\frac{3}{4}$ inches on the ground in continuous piles twenty-four feet high by laying a track on top of the pile as it is gradually built up. It is reclaimed by hand, by a clam-shell bucket, and occasionally also by a ditcher. It is stored during the dry months when the coal is available and usually about three weeks after mining. It has been in storage from one to five years with little firing, no appreciable deterioration in heating value, and little degradation, but this is stored in a cold climate. The expenditure for labor and supplies is eight cents per ton for stocking, and 6.2 cents for reclaiming.

The accompanying illustrations (Figs. 62 and 63) show a coal pile stored by the C. C. C. & St. L. R. R., near Hillary, Illinois, west of Danville. About 30,000 tons of run of mine, No. 7 coal were stored, in from one to three days after being mined, in two piles and to a depth of from fifteen to twenty feet by means of tracks on top of the piles. (See also Figs. 6 and 7 on page 45). No attempt was made to remove the fine coal. Most of the fine coal fell on the north side of the pile and the lump on the south side. In the east pile pipes were placed and temperature readings taken daily at points, 5, 10, 15, and 20 feet from the top. The log of these readings as given by P. T. White, Superintendent of the C. C. C. & St. L. R. R., is presented as Table 7. A temperature of 170 degrees was recorded shortly before the fire broke out in the pile, the highest temperature being recorded usually five feet below the top of the pile. Fire broke out after several days of intensely hot weather, which were followed by heavy thunder storm and then by more days of hot humid weather. The starting of the fire in the west pile is shown in Fig. 62 and the same fire a few days afterwards is shown in Fig. 63.

The appearance of the pile at one point after the burning coal had been loaded out is shown in Fig. 63, which shows that there was a large amount of very fine coal. As the hot and steaming coal was loaded out, it was quenched with water and used immediately. P. T. White, Superintendent of the Big Four, says:

“One place where the storage pile was burning we smothered with dirt and two days later on opening this place up with the steam shovel, we found the coal at an intense white heat, fire extending for about six feet into the pile. I am inclined to believe that our trouble this year is due to the great quantities of impurities and slack in the coal and to the peculiar weather conditions that prevailed just previous to the fire. I am of the opinion that successful storing of coal demands that coal be reasonably free from impurities and slack.”

The C. C. C. & St. L. R. R. also had fire in coal at Beech Grove, Springfield, and Bellefontaine, Ohio, and Mattoon, Illinois, stored similarly to that at Hillary.

* T. Duff Smith, Fuel Agent.



FIG. 62. COAL PILE OF THE C. C. C. AND ST. L. R. R. NEAR HILLARY, ILLINOIS
THIS PILE CONTAINED ABOUT 30,000 TONS OF NO. 7 RUN OF MINE COAL
WHICH FIRED IN THE SUMMER OF 1917



FIG. 63. ANOTHER VIEW OF THE BURNING COAL PILE OF THE C. C. C. AND ST. L.
R. R. NEAR HILLARY, ILLINOIS, IN THE SUMMER OF 1917

TABLE 7
TEMPERATURE OF COAL STORED AT HILLARY, ILLINOIS, BY THE C. C. C. & ST. L. R. R.

Date	Pipe No. 1				Pipe No. 2				Pipe No. 3				Pipe No. 4			
	5 ft.	10 ft.	15 ft.	20 ft.	5 ft.	10 ft.	15 ft.	20 ft.	5 ft.	10 ft.	15 ft.	20 ft.	5 ft.	10 ft.	15 ft.	20 ft.
7-13	150°	152°	150°	110°	116°	108°	100°	94°	150°	126°	90°	84°	120°	110°	88°	74°
7-14	160	160	140	100	110	108	98	92	126	120	108	102	120	110	96	90
7-15	154	150	120	112	102	106	96	88	116	106	90	84	120	114	102	96
7-16	148	154	130	122	120	112	84	80	130	122	92	80	128	110	92	84
7-17	152	162	134	94	120	114	102	86	126	126	112	96	118	114	92	86
7-18	146	158	130	122	114	112	98	86	112	114	104	96	118	102	94	82
7-19	154	162	132	102	124	118	106	94	132	124	98	86	110	102	94	82
7-21	150	156	106	94	130	116	102	90	134	112	100	92	120	114	96	88
7-25	156	142	114	98	128	120	94	88	134	112	92	88	116	106	90	84
7-26	154	140	122	102	128	118	104	84	160	130	104	90	120	114	102	86
7-27	162	150	121	94	130	112	94	90	132	112	98	82	122	110	98	86
7-28	160	144	120	108	121	108	100	94	126	114	104	92	128	112	98	86
7-29	146	138	118	92	118	106	94	86	120	108	94	88	126	110	96	84
7-30	154	146	118	102	126	114	102	88	140	116	98	84	112	106	94	88
7-31	166	148	116	98	130	110	92	84	130	112	92	80	128	120	102	94
8-1	152	144	128	106	112	112	94	92	134	110	96	80	120	108	92	86
8-2	164	146	124	90	112	114	98	82	116	110	112	98	122	110	102	96
8-31	156	148	122	94	128	114	94	86	114	108	98	82	126	112	92	80
8-4	158	148	114	96	112	108	90	76	134	112	94	82	112	110	96	88
8-5	170	154	128	88	122	108	106	92	134	118	102	84	121	106	94	86
8-6	162	148	126	104	126	112	98	92	134	120	92	84	124	114	96	82
8-7	164	148	128	106	118	104	96	94	122	166	94	82	134	118	106	102
8-8	162	160	136	114	132	118	112	98	144	132	110	92	130	122	108	96
8-9	164	156	132	112	124	108	102	94	118	110	98	102	130	110	96	88
8-10	154	152	140	118	122	120	112	98	128	122	106	102	118	112	108	98

West pile fired. Note:—No temperature taken between July 19 and 21, on account of thermometer being broken.
East pile fired August 14.

LIST OF PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION

- Bulletin No. 1.* Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*
- Circular No. 1.* High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None available.*
- Bulletin No. 2.* Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*
- Circular No. 2.* Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*
- Circular No. 3.* Fuel Tests with Illinois Coal (Compiled from tests made by the Technological Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904-1907), by L. P. Breckenridge and Paul Diserens. 1908. *Thirty cents.*
- Bulletin No. 3.* The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*
- Bulletin No. 4.* Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot 1906. *Forty-five cents.*
- Bulletin No. 5.* Resistance of Tubes to Collapse, by Albert P. Carman and M. L. Carr. 1906. *None available.*
- Bulletin No. 6.* Holding Power of Railroad Spikes, by Roy I. Webber. 1906. *None available.*
- Bulletin No. 7.* Fuel Tests with Illinois Coals, by L. P. Breckenridge, S. W. Parr, and Henry B. Dirks. 1906. *None available.*
- Bulletin No. 8.* Tests of Concrete: I, Shear; II, Bond, by Arthur N. Talbot. 1906. *None available.*
- Bulletin No. 9.* An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Revised Edition 1912. *Fifty cents.*
- Bulletin No. 10.* Tests of Concrete and Reinforced Concrete Columns, Series of 1906, by Arthur N. Talbot. 1907. *None available.*
- Bulletin No. 11.* The Effect of Scale on the Transmission of Heat through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *None available.*
- Bulletin No. 12.* Tests of Reinforced Concrete T-Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*
- Bulletin No. 13.* An Extension of the Dewey Decimal System of Classification Applied to Architecture and Building, by N. Clifford Ricker. 1907. *None available.*
- Bulletin No. 14.* Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*
- Bulletin No. 15.* How to Burn Illinois Coal Without Smoke, by L. P. Breckenridge. 1908. *None available.*
- Bulletin No. 16.* A Study of Roof Trusses, by N. Clifford Ricker. 1908. *None available.*
- Bulletin No. 17.* The Weathering of Coal, by S. W. Parr, N. D. Hamilton, and W. F. Wheeler. 1908. *None available.*
- Bulletin No. 18.* The Strength of Chain Links, by G. A. Goodenough and L. E. Moore. 1908. *Forty cents.*
- Bulletin No. 19.* Comparative Tests of Carbon, Metallized Carbon and Tantalum Filament Lamps, by T. H. Amrine. 1908. *None available.*
- Bulletin No. 20.* Tests of Concrete and Reinforced Concrete Columns, Series of 1907, by Arthur N. Talbot. 1908. *None available.*
- Bulletin No. 21.* Tests of a Liquid Air Plant, by C. S. Hudson and C. M. Garland. 1908. *Fifteen cents.*
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- Bulletin No. 25.* Lighting Country Homes by Private Electric Plants, by T. H. Amrine. 1908. *Twenty cents.*

*A limited number of copies of bulletins starred is available for free distribution.

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Bulletin No. 29. Tests of Reinforced Concrete Beams: Resistance to Web Stresses, Series of 1907 and 1908, by Arthur N. Talbot. 1909. *Forty-five cents.*

**Bulletin No. 30.* On the Rate of Formation of Carbon Monoxide in Gas Producers, by J. K. Clement, L. H. Adams, and C. N. Haskins. 1909. *Twenty-five cents.*

**Bulletin No. 31.* Tests with House-Heating Boilers, by J. M. Snodgrass. 1909. *Fifty-five cents.*

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1.
UNIVERSITY OF ILLINOIS
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APRIL, 1918

FUEL ECONOMY IN THE OPERATION
OF HAND FIRED POWER PLANTS

PREPARED UNDER THE DIRECTION OF
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CONTENTS

	PAGE
I. INTRODUCTION	7
1. Purpose	7
2. Authorship	8
II. FUELS AVAILABLE FOR POWER PLANT USE IN THE MIDDLE WEST	9
3. Kinds of Fuel	9
4. Properties of the Central Bituminous Coals	11
5. Preparation, a Factor Affecting the Value of Coal	14
6. Storage of Coal	18
7. Storage Systems	22
III. THE COMBUSTION OF FUEL AND THE LOSSES ATTENDING IMPROPER FIRING	24
8. Principles of Combustion	24
9. Significance of Draft	26
10. Significance of CO ₂ in the Flue Gases	29
11. Losses of Heat Value	33
Excess Air and Air Leaks	34
Loss Due to the Presence of Combustible in the Ash	38
Loss Due to the Presence of Carbon Monoxide in the Flue Gases	39
Loss Due to Soot	39
Loss Due to Moisture in the Coal and Air	40
Loss Due to Heat in the Escaping Gases	40
Loss Due to Radiation	40
12. Significance of Smoke	40
13. Methods of Hand Firing	41
14. Stoker Firing	42

	PAGE
IV. FEATURES OF BOILER INSTALLATION IN RELATION TO	
FUEL ECONOMY	44
15. Boiler Settings	44
Foundation	44
Side and End Walls	44
Settings for Horizontal Return Tubular Boilers	45
Settings for Water Tube Boilers	48
Defects in Settings	50
V. INSTALLATION FEATURES AFFECTING DRAFT CONDITIONS	52
16. Stacks and Breechings	52
The Stack Damper and Its Use	52
"Draft" is in Reality a Pressure	54
Air Leaks Affect the Draft and Waste Coal	56
Breechings for a Battery of Two or More Boilers	57
Conditions Under Which a Stack will Operate Eco- nomically	57
VI. FEED WATER HEATING AND PURIFICATION AS FACTORS IN	
FUEL ECONOMY	60
17. Feed Water Purification	60
18. Treatment of Feed Waters	61
Chemical Treatment	61
Heat Treatment	61
Combined Chemical and Heat Treatment	62
19. Boiler Compounds	62
20. Feed Water Heaters	62
Exhaust Steam Heaters	63
Closed Heaters	63
Advantages and Disadvantages of Exhaust Steam Heaters	64
21. Economizers	65
22. Live Steam Heaters	65
23. Feeding Boilers	65

	PAGE
VII. STEAM PIPING REQUIREMENTS FOR FUEL ECONOMY IN SMALL PLANTS	67
24. Possibility of Fuel Loss in the Transmission of Steam .	67
25. Value of High Pressure Drips as Hot Feed Water . .	67
26. Leakage Losses at Valves and Fittings	67
27. Size of Steam and Exhaust Mains	68
28. Heat Insulating Materials Required on Piping, Boilers and Breechings	73
29. Requirements for a Good Covering	77
30. Bad Effects of Water of Condensation in Steam Lines .	78
31. Uncovered Pipes Waste Steam as Well as Coal . . .	78
VIII. RECORD OF OPERATION	81
32. Purpose of the Record	81
33. Character of the Record	81
34. Profit Sharing or Bonus Systems	84
IX. SUMMARY OF CONCLUSIONS	85
35. Conclusions	85
Coal	85
Principles to be Observed in Firing	85
Features of Boiler Installation	86
Stacks and Breechings	87
Feed Water and Fuel	87
Steam Piping Requirements	88
Record of Operation	88

LIST OF FIGURES

NO.		PAGE
1.	Map Showing the Locations of the Coal Fields of Illinois, Indiana, and Western Kentucky	15
2.	Chart Showing the Theoretical Value of Coals of Different Heating Values at Various Prices per Ton	19
3.	Manometer Tube for Showing the Difference in Pressure between the Outside and the Inside of Boiler Wall	27
4.	Sketch Showing the Correct Method of Connecting Draft Gages	28
5.	Apparatus for Determination of CO ₂ in Flue Gas	30
6.	Sketch Showing the Proper Location for Gas Sampling Tubes to Avoid Damper Pockets for Both Front and Rear Take-off	32
7.	Curve Showing Relation between Excess Air and CO ₂ in Flue Gas . . .	37
8.	Hartford Setting for Return Tubular Boilers	45
9.	Double Arch Bridge Wall Setting for Smokeless Combustion	46
10.	Sketch Showing Effects of Baffling and Dampers in Causing Pockets and Eddies in the Flue Gas Stream	50
11.	An Approved Form of Hinged Damper	52
12.	Isometric Sketch Illustrating the Principle that Light Fluids or Gases are Pushed Upward when in Contact with Heavier Fluids or Gases . . .	55
13.	Sketch Showing Variations in Draft at Different Points and Indicating Tendency Toward Air Leakage	56
14.	Stack and Breeching Connections for a Battery of Three Boilers . . .	58
15.	Diagrammatic Section and Energy Transformation Chart for Small Steam Power Plant	70
16.	Chart Showing Amount of Heat Transmitted by Steam Pipes Insulated with Commercial Coverings	75
17.	Chart Showing Heat Lost by Bare Steam Pipe and Saving which may be Secured by Using a Good Covering	76
18.	Diagram Showing Comparative Saving in Water and Steam to be Effected by Covering Live Steam Mains	79

LIST OF TABLES

NO.		PAGE
1.	Analyses of Coals of Illinois, Indiana, and Western Kentucky	12
2.	Sizes of Central Bituminous Coals	17
3.	Stack Sizes Based on Kent's Formula	54
4.	Impurities in Feed Waters, Their Effects and Remedies	60
5.	Coal and Steam Loss Based on 100 Feet of Uncovered Steel Pipe . . .	74

FUEL ECONOMY IN THE OPERATION OF HAND FIRED POWER PLANTS

I. INTRODUCTION

1. *Purpose.*—The need for greater economy in the use of coal is too apparent, under present conditions, to need emphasis. The demand for coal is unprecedented, and production is proceeding at a rate which is barely, or perhaps not quite, keeping pace with the demand. The U. S. Geological Survey reports that, during 1917, approximately 545,000,000 tons of bituminous coal were produced and used in the United States. The demand, moreover, is increasing at the rate of about ten per cent per year, so that at present the rate of consumption is about 600,000,000 tons per year. Illinois produces about $12\frac{1}{2}$ per cent of this amount, or 78,000,000 tons.*

Approximately 45,000,000 tons of bituminous coal are used within the state of Illinois, and of this amount about 6,000,000 tons are consumed in hand fired power plants. It is believed to be within the limits of practical attainment to effect a saving of from 12 to 15 per cent of this fuel. Expressed in tons and dollars, such a saving amounts to 750,000 tons, or \$3,500,000. The possible saving in the case of many individual plants is much greater than the percentage stated.

It is the purpose of this circular to present to owners, managers, superintendents, engineers, and firemen of hand fired power plants certain suggestions which, it is believed, will help them in effecting greater fuel economy in the operation of their plants, and in determining the properties and characteristics of the coal purchased. Features of installation essential to the proper combustion of fuel are discussed and their importance emphasized; the practice to be observed in the operation of the plant is outlined; and the employment of simple devices for indicating conditions of operation is prescribed.

Special attention is called to the fact that, to secure the greatest degree of success, coöperation between owners and managers, and the men who fire the coal is essential. Mechanical devices to increase

* It is estimated that Illinois will produce more than 85,000,000 tons of coal in 1918.

efficiency in the use of coal cannot produce satisfactory results unless the firemen who handle them are impressed with the importance of their duties. While the suggestions presented apply particularly to hand fired plants and no attempt is made to define practice for stoker fired plants, many of the factors affecting fuel economy are common to all power plants, and for this reason much of the information contained herein will, no doubt, be helpful to those interested in more economical operation of stoker fired power plants.

To the experienced engineer much that is presented here will seem elementary and inadequate. If, however, the plant owner who is not familiar with the extreme refinements of practice may obtain here the facts which will enable him to improve his results to the extent of the modest saving suggested, the purpose of the publication will have been fulfilled.

2. *Authorship.*—The information contained in this circular has been compiled under the direction of a committee consisting of A. C. WILLARD, Professor of Heating and Ventilation (Chairman), H. H. STOEK, Professor of Mining Engineering, O. A. LEUTWILER, Professor of Machine Design, C. S. SALE, Assistant Professor of Civil Engineering and Assistant to the Director of the Engineering Experiment Station, and A. P. KRATZ, Research Associate in Mechanical Engineering.

This committee has had the assistance of an advisory committee consisting of JOSEPH HARRINGTON, Advisory Engineer on Power Plant Design and Operation, Chicago, ARTHUR L. RICE, Editor, *Power Plant Engineering*, Chicago, JOHN C. WHITE, Chairman, Educational Committee, National Association of Stationary Engineers, Madison, Wis., O. P. HOOD, Chief Mechanical Engineer, Bureau of Mines, Washington, D. C., D. M. MYERS, Advisory Engineer on Fuel Conservation, United States Fuel Administration, Washington, D. C., and C. R. RICHARDS, Dean of the College of Engineering and Director of the Engineering Experiment Station of the University of Illinois. Each member of this Advisory Committee personally reviewed the original manuscript and a meeting was held at Urbana on March 21, 1918, at which the work was examined in detail. The authors gratefully acknowledge the valuable assistance and coöperation of the members of this committee and feel that the value of the publication has been greatly enhanced as a result of their efforts.

II. FUELS AVAILABLE FOR POWER PLANT USE IN THE MIDDLE WEST

3. *Kinds of Fuel.*—The varieties of fuel used by hand fired power plants in Illinois are:

Central bituminous coals as represented by those from the coal fields of Illinois, western Kentucky, and Indiana.

Eastern bituminous and semi-bituminous, or soft coals, from the Pennsylvania, West Virginia, and eastern Kentucky fields.

A classification of solid fuels available for this purpose will, of course, include anthracite and coke but none of these is used to any considerable extent for power purposes in Illinois. The liquid fuel, petroleum, is produced in large quantities in Illinois but is not used directly for fuel purposes to any great extent.

All these coals are composed of the following materials in varying proportions:

- (1) Solid or fixed carbon which burns with a glow and without flame.
- (2) Gases or volatile materials which escape from the coal when it is heated and which burn with a flame.
- (3) Gases or volatile matter and water which escape from the coal when it is heated and which do not burn.
- (4) Ash or mineral matter which does not burn and which remains as ashes after the coal is burned.

The relative proportions of these materials in different coals determine their value for particular purposes.* Fuels having a large amount of fixed carbon and a relatively small amount of volatile matter burn with a short flame and the whole process of combustion takes place at or near the surface of the fuel bed. Such fuels can be burned without developing visible smoke. On the other hand coals containing a relatively large amount of volatile matter and a lower proportion of fixed carbon burn with a longer flame and tend to produce more visible smoke than the high carbon coals because the volume of combustible gases distilled from them is greater.

The bituminous coals of the central field (Illinois type) contain

* The "fuel ratio," which is the quotient obtained by dividing the fixed carbon by the volatile matter, is often used as a means of classifying coals, and for bituminous coals it answers fairly well.

from 40 to 55 per cent of fixed carbon, 10 to 25 per cent of combustible gas, 5 to 15 per cent of non-combustible gas, 8 to 15 per cent of moisture, and 8 to 15 per cent of ash. When improperly fired or burned in furnaces not adapted to their use, central bituminous coals give off so large an amount of sooty material that flues are often quickly clogged. These unconsumed volatile products also represent a direct loss of heat value. Coals of the Illinois type ignite easily and burn freely.

Because the amount of solid carbon in most Illinois coal is lower and the percentage of ash and moisture higher its heating value is usually less than that of most eastern bituminous coals, but the cost is usually so much less that it is more economical to use local coals. At this time (March, 1918) the transportation of fuel over long distances is not only undesirable, but it is practically impossible, and bituminous coals of the central field constitute the only fuel available in quantities for use in Illinois.

The moisture and non-combustible gases present in all coals are detected only by chemical analysis. They not only do not produce heat, but represent a definite loss because they absorb and carry off heat which would otherwise be available for useful purposes. The term moisture in coal does not mean the water adhering to the surface of the lumps, but that contained within the pores of the coal. A coal containing a high percentage of moisture by analysis may appear perfectly dry.

The ash content of different coals varies greatly. Ash is non-combustible mineral matter which not only has no heating value and, therefore, represents a portion of the coal from which no return is received, but it may hinder the free burning of the combustible components of the coal. If the ash contains certain mineral substances, it may by clinkering greatly interfere with the process of firing and with the cleaning of grates. The ash normally is removed through the ashpit into which often passes also a certain amount of unburned coal. For this reason the amount of ashes removed from the pit usually represents a larger percentage of the fuel fired than the analysis of the ash content indicates. It should be clearly understood that ash will *not* burn and that no treatment with chemicals, or "secret processes," will cause it to burn. Likewise, it is not possible to increase the heat value of coal by treating it chemically or by adding a nostrum to it.

The ash in coal may be divided into two classes; first, that which is a definite part of the composition of the coal and which cannot be separated from the coal by hand or by mechanical process, and, secondly, that which is due to rock, slate, and shale which become mixed with the coal in mining and which can in a large measure be separated from the coal either in the mine or in the tipple.

Bituminous coal may be either of the coking or the non-coking variety. Coals vary widely with reference to their coking properties. A true coking coal when fired swells, becomes pasty and fuses into a mass of more or less porous coke. Such coke will burn without flame and will hold fire for a considerable period. This fusing or coking takes place without respect to the size of the piece of coal. A non-coking coal does not swell and become pasty but burns away gradually to ash, the pieces becoming gradually smaller and smaller. There is a gradual gradation from true coking to true non-coking coals and many coals cannot be distinctly placed in either class. Coal which will not coke on a furnace grate may, however, give good coke in by-product coke ovens, particularly when mixed with other more easily coking coals. This is the case with many Illinois coals.

The eastern bituminous coals contain from 5 to 10 per cent of ash, from 25 to 35 per cent of combustible gases, from 2 to 5 per cent of moisture and non-combustible gases, and from 55 to 65 per cent of solid carbon. They are more generally of the coking variety than are the Middle West coals. In general, they are higher in heating value and lower in ash. They are more friable and are not so well suited for transportation and repeated handling as are many of the central bituminous coals.

4. *Properties of the Central Bituminous Coals.*—Coals used in Illinois power plants come mainly from the Illinois, Indiana and western Kentucky fields. The properties of these coals as disclosed by analyses of samples from different localities are given in Table 1.

The average analyses of the important Illinois coals have been determined with great care. The averages for Kentucky were obtained by average analyses of composite samples from several mines. Average analyses are not available for Indiana coals and instead analyses are given of samples from three important Indiana coal counties, namely, Clay, Green and Sullivan counties.

TABLE 1

ANALYSES OF COALS OF ILLINOIS, INDIANA, AND WESTERN KENTUCKY
(Figures are for face samples and for coal "as received")¹

DISTRICT	Coal Bed	Moisture	Volatile Matter	Fixed Carbon	Ash	B. t. u. (Heating Value)
ILLINOIS (Average Analyses)						
La Salle.....	2	16.18	38.83	37.89	7.08	10,981
Murphysboro.....	2	9.28	33.98	51.02	5.72	12,488
Rock Island and Mercer Counties ..	1	13.46	38.16	39.75	8.63	11,036
Springfield-Peoria.....	5	15.10	36.79	37.59	10.53	10,514
Saline County.....	5	6.75	35.49	48.72	9.04	12,276
Franklin and Williamson Counties..	6	9.21	34.00	48.08	8.71	11,825
Southwestern Illinois.....	6	12.56	38.05	39.06	10.33	10,847
Danville: Grape Creek coal.....	6	14.45	35.88	40.33	9.34	10,919
Danville: Danville coal.....	7	12.99	38.29	38.75	9.98	11,143
INDIANA (Typical Analyses)						
Clay County.....	{ Brazil }	15.38	32.66	46.08	5.88	11,680
	{ block }					
Greene County.....	IV	13.53	33.54	45.38	7.55	11,738
Greene County.....	V	10.30	36.31	41.64	11.75	11,218
Sullivan County.....	IV	12.15	33.48	46.23	8.14	11,722
Sullivan County.....	V	12.14	35.17	43.73	8.96	11,516
Sullivan County.....	VI	14.86	31.65	46.14	7.35	11,324
KENTUCKY (Average of Composite Samples)						
	9	8.17	36.82	45.17	9.83	11,867
	11	7.33	38.28	45.28	9.11	12,056
	12	9.67	34.86	46.46	9.01	11,695

¹"As received" samples represent the coal as taken from the mine. It is probable that the values given are fairly representative of the coals as purchased from local dealers.

A study of the values presented in Table 1 reveals the following facts:

- (1) The amount of ash in the various coals as they exist in the mine varies within a range of about 6 per cent. With inadequate preparation of the coal for the market, however, the range of difference may be as much as 12 or 15 per cent.
- (2) There is a variation in the percentage values over a range of about 5 per cent in the volatile matter in the different coals, an amount which is negligible in view of the proportionately greater variations in heating value and in ash.

- (3) The variation in the amount of moisture present in the different coals is considerable, but this variation is reflected to some extent in the B. t. u.* values. If two coals have about the same amount of fixed carbon, volatile matter and ash, the coal having the higher moisture content has the lower B. t. u. value. Accordingly, if the B. t. u. value of a coal is known, the moisture content is not important. This statement is also true as regards ash, except that the ash represents a residue to be handled.

With regard to the B. t. u. values, the table shows that there are important and distinguishable differences in the heating quality of the different coals found in the three states, yet the extent of difference is not sufficient to justify extravagant statements in praise of certain coals or in disparagement of others. On the basis of heating value alone, the difference between the value of the poorest and that of the best coals, as they are found in the mine, amounts to about one-fifth of the value of the poorest coal.† As stated previously, however, the care with which coal is prepared affects its value as fuel (see section 5 "Preparation, a Factor Affecting the Value of Coal," p. 14).

The values given cover the most wide-spread and most important coal beds of Illinois, Indiana, and western Kentucky. It should be observed that the variations are as great between coals which come from the same bed in widely separated localities as between coals which come from different beds, for instance, the No. 6 coal of Franklin and Williamson counties differs nearly as much from the No. 6 coal of the Belleville region of southwestern Illinois as it does from the No. 5 coal of Saline County. For large areas, however, the characteristics of each bed are remarkably constant and variations in the character of the coal are regional rather than local. It is possible therefore, to subdivide the large coal fields of the three States into districts as shown on the accompanying map (Fig. 1).

The subdivisions of the Illinois field as shown on this map were based mainly upon geological conditions and upon the general sim-

* For a definition of B. t. u. see foot-note on page 17.

† Comprehensive tables giving analytical values for Illinois coals are contained in Bul. 29 of the State Geological Survey, Urbana, Ill., entitled "Purchase and Sale of Illinois Coal under Specifications," by S. W. Parr, and in Bul. 3 of the Illinois Coal Mining Investigations, Urbana, Ill., entitled "Chemical Study of Illinois Coals," by S. W. Parr. Professional Paper 100A, U. S. Geological Survey, Washington, D. C., contains analyses of coals from all parts of the United States.

ilarity in the methods of mining in each district rather than upon a difference in the quality of the coal. This fact should be understood in considering the analyses of coals from the different districts. For instance, the coals from the eastern part of Perry County are very similar to those from Franklin and Williamson counties, although classed by the map as being in a different district. The dividing line accepted by the Illinois Coal Mining Investigations between District 6 and the southern part of District 7 is the Duquoin anticline, a distinct geologic structural feature which has, however, not effected any distinct change in the character of the coal, that just west of and near the anticline being practically of the same quality as that east of the anticline in the same locality.

The several Illinois coals do not differ materially in appearance and it is often difficult to distinguish one from another without more careful tests than the ordinary purchaser can make. The apparent difference is frequently due to preparation rather than to actual differences in chemical composition and in heating quality.

5. *Preparation, a Factor Affecting the Value of Coal.*—Coal occurs in the earth in beds or seams, and usually in a solid mass as a rock. In mining it is blasted with powder, shoveled into cars, and conveyed to the surface. In the process of mining and handling it becomes broken up into pieces of all sizes. It may have some rock or dirt from the floor and roof of the mine mixed with it or there may be bands or layers of earthy matter in the coal seam itself. Coal as it comes from the mine is therefore not usually in condition for immediate delivery to the consumer, but ordinarily must first be “prepared” in order to remove these impurities and to separate it into the proper sizes for various purposes or markets. The impurities in the large sizes of coal are removed by picking them out by hand, and in the smaller sizes by treating the coal in cleaning machinery. Separation into different sizes is accomplished by sending the coal over screens having holes of the proper size.

Table 2 gives the customary sizes and the corresponding names of central bituminous coals as they are available in the market.

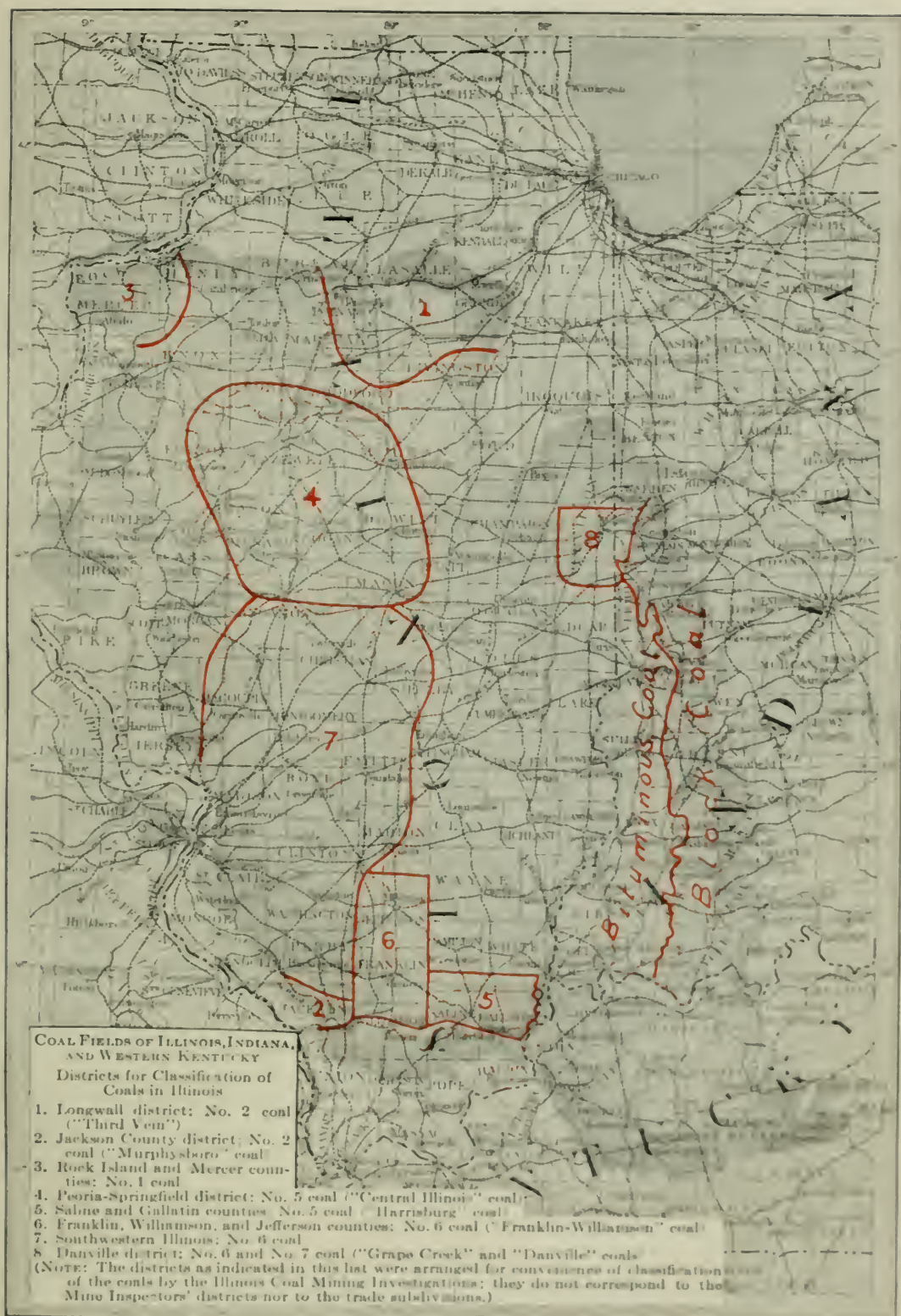


FIG. 1. MAP SHOWING THE LOCATIONS OF THE COAL FIELDS OF ILLINOIS, INDIANA, AND WESTERN KENTUCKY

TABLE 2
SIZES OF CENTRAL BITUMINOUS COALS

NAME	SIZE OF PIECES
Run of Mine	Mixture of all sizes
Lump	Large lumps separated from the finer sizes
Egg or Furnace	Lumps 3-6 inches
No. 1 Nut or Small Egg	2-3 inches
No. 2 Nut or Stove	1¼-2 inches
No. 3 Nut or Chestnut	¾-1¼ inches
No. 4 Nut or Pea or Buckwheat	¼-¾ inch
No. 5 Nut	Under ¼ inch
Screenings	A mixture of all sizes under 2 inches

For large power plants the custom of purchasing coal on the B. t. u.* basis is increasing and if the specifications for such purchase are properly drawn and understood it is the logical way to buy coal, because it is equivalent to buying so many heat units instead of so many tons of coal. Upon this basis a purchaser should be able to determine whether a low priced coal which gives less efficient boiler service and involves greater expense for handling ashes is really cheaper than a higher priced coal.

With reference to the selection of different Illinois coals, the B. t. u. value and the percentage of ash furnish a general guide to their relative values. If two coals are otherwise alike in composition, the ash content increases as the B. t. u. value decreases; hence their relative values may be expressed with fair accuracy by either the B. t. u. or the ash value alone, although the evaporative value of any coal drops off more rapidly than its B. t. u. value when the ash content exceeds 10 or 15 per cent. A close approximation of the percentage of actual heat producing material in Illinois coal may be obtained by dividing the B. t. u. value of the coal by 155. Thus, a 12,000 B. t. u. coal contains $12,000 \div 155$, or 77 per cent of heat-producing material. In order to enable the small consumer to judge the relative values of coals offered at different prices, the chart, Fig. 2, has been prepared

* B. t. u. is a term made use of by engineers to express a certain amount of heat. It is an abbreviation of "British thermal unit." One B. t. u. is the amount of heat required to raise the temperature of one pound of water one degree Fahrenheit. If a coal has a heating value of 14,000 B. t. u., there is sufficient heat in one pound of it to raise 14,000 pounds of water one degree Fahrenheit.

to show the theoretical value of coals of different heating or B. t. u. values at various prices per ton.

It should be understood that the purchase of coal on the B. t. u. basis does not insure a maximum evaporative value from the fuel, because a high-grade coal carelessly fired may give poorer results than a low-grade coal carefully fired. In other words, the B. t. u. value of a coal is simply an indication of what should be obtained with careful firing and the person who furnishes coal of a high B. t. u. value cannot be held responsible for poor results obtained from that coal through improper use. It should also be remembered that while the B. t. u. value shows the chemical composition, it indicates nothing with regard to the physical properties of the coal, and these properties may be equally as important as the chemical properties in their effects upon firing, storing, and transportation.

6. *Storage of Coal.*—The storage of a certain amount of coal by every power plant is both desirable and essential in order to insure continuous operation. Although there is some misapprehension with regard to the practicability of storing bituminous coal, a study of the subject based upon the reported experience of more than a hundred firms and individuals indicates that the difficulties attending storage are not serious.* These investigations have shown that:

- (1) It is practicable and advantageous to store coal, not only during war times, but also under normal conditions, near the point of consumption. The practice of storing coal has the advantage of (a) insuring the consumer a supply of coal at all times, (b) permitting the railroads to utilize their cars and equipment to the best advantage, and (c) permitting the mines to operate at a more nearly uniform rate of production throughout the year. The expense of storage may be regarded as the expense of insurance against shut-downs.
- (2) Certain requirements affecting the kinds and sizes of coal must be observed as follows:
 - (a) Most varieties of bituminous coal can be stored successfully if of proper size and if free of fine coal and dust. The coal must be so handled that dust and fine coal are not produced

* For a more nearly complete discussion of the problem of coal storage, see Bulletin 97 of the Engineering Experiment Station, University of Illinois, entitled "Effects of Storage Upon the Properties of Coal," by S. W. Parr, and Circular 6 of the Engineering Experiment Station, University of Illinois, entitled "The Storage of Bituminous Coal," by H. H. Stoek.

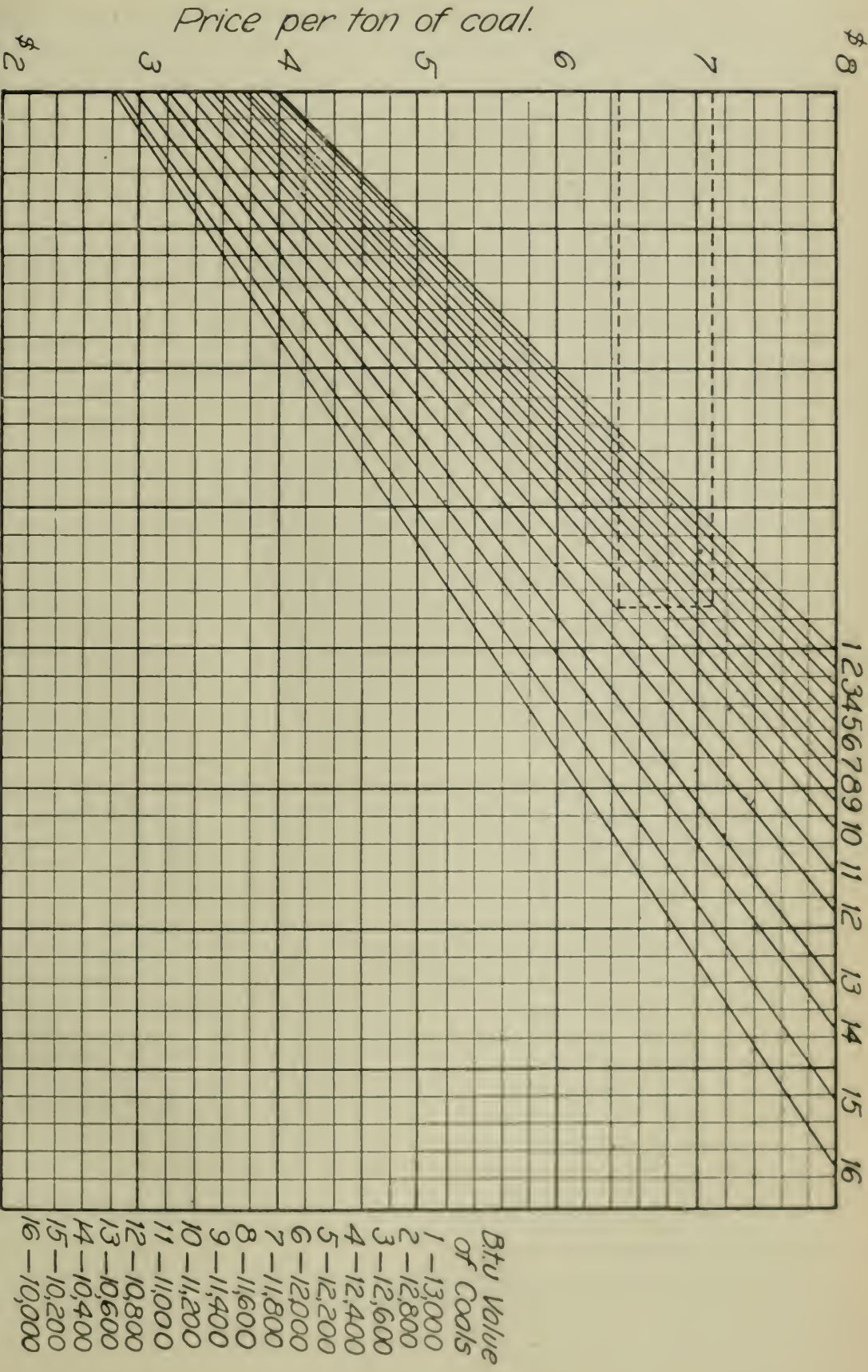


FIG. 2. CHART SHOWING THE THEORETICAL VALUE OF COALS OF DIFFERENT HEATING VALUES AT VARIOUS PRICES PER TON

To make a comparison between coals of different B. t. u. values, locate the point on the line representing the B. t. u. value of the coal in question directly opposite the price involved. Through this point draw a vertical line, and from the intersection of this with the diagonal lines representing any other B. t. u. value read the comparable price from the price scale at the left. For Example: If a 12,000 B. t. u. coal is offered at \$7.10 per ton, a coal having a heating value of 11,000 B. t. u. will be worth \$6.45.

in excessive amounts, and allowed to remain during storage. Although some coals can be stored with greater safety than others, the danger from spontaneous combustion is due more to improper piling of the coal than to the kind of coal stored. The danger of spontaneous combustion can be very greatly reduced if not entirely eliminated by storing only lump coal from which the dust and fine coal have been removed.

- (b) Fine coal or slack has sometimes been stored successfully in cases in which air has been excluded from the interior of the pile. Exclusion of air from the interior of a pile may be accomplished (a) by a closely sealed wall built around the pile, or (b) by very close packing of the fine coal. A pile of slack must be carefully watched to detect evidences of heating and means should be provided for moving the coal promptly if heat develops. The only absolutely safe way to store slack or fine coal is under water.
 - (c) Many varieties of mine run bituminous coal cannot be stored safely because of the presence of fine coal and dust.
 - (d) Coal exposed to the air for some time may become "seasoned" and thus may be less liable to spontaneous combustion because of the oxidation of the surfaces of the lumps. Experience covering this point, however, is by no means conclusive.
 - (e) It is believed by many that damp coal or coal stored on a damp base is peculiarly liable to spontaneous combustion, but the evidence on this point also is not conclusive. It is safest not to dampen coal when or after it is placed in storage.
- (3) The sulphur contained in coal in the form of pyrites is not the chief source of spontaneous combustion, as was formerly supposed, but the oxidation of the sulphur in the coal may assist in breaking up the lumps and thus may increase the amount of fine coal, which is particularly liable to rapid oxidation. The opinion is wide-spread that, if possible, it is well for storage purposes to choose a coal with a low sulphur content.
- (4) In piling coal for storage the following conditions should be observed:
- (a) To prevent spontaneous combustion, coal should be so piled

that air may circulate through it freely and thus may carry off the heat due to oxidation of the carbon, or it should be so closely packed that air cannot enter the pile and stimulate the oxidation of the fine coal.

- (b) Stratification, or segregation of fine and lump coal, should be avoided, since an open stratum of coarse lumps of coal may provide a passage or flue for air to enter and come in contact with the fine coal, and thus to oxidize it and start combustion.
- (c) Coal can be stored with greater safety in piles not more than six feet high than in piles of greater height since the coal is more fully exposed to the air in low piles, the superficial area of the pile in relation to its volume being greater. The coal pile should preferably be divided by alleyways so as to facilitate the rapid removal of the coal in case of necessity and so that an entire pile may not be endangered by a local fire.
- (d) The practice of ventilating coal piles by means of pipes inserted at intervals has not proved generally effective as a means of preventing spontaneous combustion in storage piles in the United States and it is not advised. Such practice is, however, reported to have been successful in certain parts of Canada.
- (e) Coals of different varieties should not be mixed in storage, because a single variety of coal which has a tendency toward spontaneous combustion may jeopardize the safety of the entire pile.
- (f) Storage appliances and arrangements should be designed so as to make it possible to load out the coal quickly if necessary. *Coal should positively not be stored in large piles unless provision is made for loading it out quickly.*
- (g) Pieces of wood, greasy waste, or other easily combustible material mixed in a coal pile may form the starting point of a fire, and every precaution should be taken to keep such material from the coal as it is being placed in storage.
- (h) It is very important that coal in storage should not be affected by external sources of heat such as steam pipes. The susceptibility of coal to spontaneous combustion increases rapidly as the temperature rises.

- (5) The effects of storage on the value and properties of coal may be summarized as follows:
- (a) The heating value of coal as expressed in B. t. u. is decreased very little by storage, but the opinion prevails that storage coal burns less freely than fresh coal. Experiments indicate that much of this apparent deficiency may be overcome by keeping a thin bed on the grate and by carefully regulating the draft to suit the fuel.
 - (b) The deterioration of coal when stored under water is negligible, and such coal absorbs very little extra moisture. If only part of a coal pile is submerged, the part exposed to the air is still liable to spontaneous combustion.
- (6) In order to guard against loss in the event of fire in a pile of stored coal the following facts should be understood:
- (a) The best means of preventing loss in stored coal is to inspect the pile regularly and if the temperature in any part of the pile rises to 150 degrees F. to prepare to remove the coal from the spot affected. If the temperature continues to rise and reaches 175 degrees F., the coal should be removed as promptly as possible. Temperature readings may be taken by lowering a thermometer into the interior of a pile through a pipe driven into it. The common methods of testing for fires in coal piles are:
 - (1) By watching for evidences of steaming.
 - (2) By noting the odor given off.
 - (3) By inserting an iron rod into the pile and when drawn out noting the temperature by applying the hand.
 - (4) By inserting maximum temperature thermometers into pipes driven into the pile.
 - (5) By noting spots of melted snow on the pile.
 - (b) Water is an effective agent in quenching fires in a coal pile only if it can be applied in sufficient quantities to extinguish the fire and to cool the mass, but unless there is an ample supply for this purpose it is dangerous to add any water to a coal pile.

7. *Storage Systems.*—Since coal is a comparatively cheap and bulky product, it must be handled as economically as possible, and also,

unless it is to be used in the form of screenings, in a way to produce a minimum of breakage.

The ordinary power plant is frequently limited in the choice of a storage system by a lack of available space and by the fact that expense must be kept at a minimum, but it should be recognized that provision for storage may be counted as an insurance against interrupted operation. The storage may be temporary or permanent, that is, the coal may be stored for use within a comparatively short time or it may be stored with the expectation that it will remain in storage for a considerable period to serve as a reserve in case of an emergency, the current daily supply being used as received.

At hand fired power plants coal is usually stored by dumping or shoveling from a car or cart upon a pile or into a bin or bunker, or merely by dumping upon the ground. From such storage piles coal is shoveled directly into the furnace or, if the pile is at some distance from the furnace, carried by wheelbarrow or conveyor to the furnace.

Trestle storage involves the dumping of the coal directly upon the ground or into a bin from cars on an elevated trestle. Although simple in construction and low in first cost, trestle storage produces excessive breakage and unless drop-bottom or dump cars are available the cost of unloading is high.

The cost of storing and reclaiming coal from storage by manual labor varies from 15 to 64 cents.*

WARNING:—Special emphasis is laid upon the fact that safety in the storage of coal depends upon a very careful and thorough consideration of and attention to the details referred to in the foregoing. Lack of attention to these details and lack of care in handling will in many cases result in losses due to dangerous fires. Do not undertake to store coal until you are sure you know how to do it properly and safely.

* Stoeck, H. H., "The Storage of Bituminous Coal." Univ. of Ill. Eng. Exp. Sta., Circ 6, 1918.

III. THE COMBUSTION OF FUEL AND THE LOSSES

ATTENDING IMPROPER FIRING

8. *Principles of Combustion.*—The combustion of coal in a furnace is essentially a chemical process. The combustible in coal consists of carbon,* hydrogen and sulphur. During the progress of combustion these elements unite with oxygen to form carbon dioxide, steam, and sulphur dioxide respectively. The air, which furnishes the oxygen for this process, consists of a mixture of 21 per cent by volume of oxygen and 79 per cent of nitrogen. Oxygen is the active element as affecting combustion, the nitrogen being inert and taking no part in the process.

When combustion takes place, heat is given off. For every pound of carbon burned to carbon dioxide, 14,600 B. t. u.† are released. In the same way, for every pound of hydrogen burned to water vapor 62,100 B. t. u. are liberated. One pound of sulphur in burning to sulphur dioxide gives up 4,000 B. t. u. The heat liberated serves to raise the temperature of the fuel bed, of the surrounding surfaces, and of the products of combustion. Part of the heat delivered to the water in the boiler is transmitted by direct radiation from the hot surfaces, and the rest is absorbed by conduction from the gases, thus lowering their temperature. The heat carried away by the gases after they have left the heating surfaces of the boiler represents the loss entailed in the process. The extent of this loss is, of course, indicated by the temperature of the gases leaving the heating surfaces. The nitrogen, as stated, takes no part in combustion, but on the contrary it absorbs a certain amount of heat in having its temperature raised from that of the air to that of the gases leaving the fire. Consequently, the temperature of the other gases does not reach so high a point as would be possible if oxygen alone could be introduced into the fuel bed. When the nitrogen leaves the heating surfaces with the rest of the gases, it carries away part of the heat released by the fuel, and,

* The chemical symbols used for these elements and compounds are as follows: Carbon (C), hydrogen (H), sulphur (S), oxygen (O), carbon dioxide (CO₂), steam (H₂O), sulphur dioxide (SO₂), nitrogen (N), and carbon monoxide (CO). Carbon dioxide is variously known as carbonic acid gas and as black damp, while carbon monoxide is known as carbonic oxide and as white damp.

† For definition of B. t. u. see foot-note on page 17.

therefore, represents loss. This loss amounts to about 0.24 B. t. u. per pound of nitrogen per degree F.

In the process of combustion, one pound of carbon unites with 2.67 pounds of oxygen to form 3.67 pounds of carbon dioxide. Since the composition of the air is 77 per cent nitrogen and 23 per cent oxygen by weight, 2.67 pounds of oxygen requires 11.6 pounds of air.

One pound of hydrogen in burning unites with eight pounds of oxygen to form nine pounds of water vapor. In this case the amount of air required is 34.8 pounds.

One pound of sulphur in burning unites with one pound of oxygen to form two pounds of sulphur dioxide. The air required is 4.35 pounds.

It is evident that if the weights of carbon, hydrogen, and sulphur in one pound of coal are known, the air necessary to burn completely one pound of coal amounts to 11.60 times the weight of carbon, plus 34.80 times the weight of hydrogen, plus 4.35 times the weight of sulphur. This is about 12 pounds of air per pound of coal.*

Every cubic foot of oxygen used in the combustion of carbon is replaced by one cubic foot of carbon dioxide. For this reason, the percentage by volume of CO_2 in the flue gas is an indication of the amount of excess air present in the furnace. A given amount of CO_2 will be formed for every pound of carbon burned. If just enough air is used for the complete combustion of the carbon, the oxygen will be replaced by the CO_2 formed and the latter will be the same percentage, by volume, of the mixture as the original oxygen. If twice as much air as necessary is used, the same volume of CO_2 will be formed as before, but this will replace only one half of the oxygen used, and hence its percentage of the mixture will be only one half as great as in the former case. These relations are somewhat affected by the fact that hydrogen and sulphur are present, but their amounts are too small to have an important bearing on the result.

If less than enough air is furnished for complete combustion, part of the carbon in the coal, instead of being burned to carbon dioxide, will form carbon monoxide. Under these circumstances the amount of heat liberated per pound of carbon, instead of being 14,600 B. t. u. will be only 4,500 B. t. u. The difference, 10,100 B. t. u., will represent the heat lost for every pound of carbon burned to carbon monoxide.

* Any air above the chemical requirement for complete combustion is known as excess air.

Since combustion is the result of the union of oxygen with the various elements in the coal, and with the combustible products formed in the fuel bed, it necessarily follows that in order to have complete combustion, each particle of these elements must come into contact with a sufficient amount of oxygen. To insure this contact between the particles of the combustible and oxygen, it is necessary to supply an amount of oxygen, and hence of air, somewhat in excess of the amount theoretically required; otherwise carbon monoxide will be found in the escaping gases. This excess acts as a further diluent, and represents loss, just as the nitrogen in the air represents loss. A compromise must, therefore, be made. The correct amount of air to be used is obtained when the loss due to heating the excess just balances the loss due to the carbon monoxide appearing if the excess is reduced. For best operating conditions it is found necessary to use between 30 and 40 per cent of excess air.

Before the union of oxygen and the elements in the coal can take place with sufficient rapidity to be of any practical use, it is necessary that the whole mass be brought to a temperature known as the ignition temperature. If, because of any condition, such as contact with the cold surfaces of the tubes or the inrush of an excessive amount of cold air, the temperature of the gases is lowered below the ignition point before combustion is complete, combustion will cease and part of the fuel will escape from the furnace unburned. This, of course, represents a loss.

The three fundamental conditions necessary for complete and smokeless combustion may now be stated as follows:

- (1) A sufficient amount of air must be supplied.
- (2) The air and fuel must be intimately mixed.
- (3) The mixture must be brought to the ignition temperature and maintained at this temperature until combustion is complete.

9. *Significance of Draft.*—The technical meaning of the term “draft” does not refer to the motion of the air or gases, but merely defines the difference in pressure existing between the air outside and the gases inside the furnace (See Figs. 3 and 13). If there is an opening into the furnace and the draft is maintained, air will be forced in from the outside. The amount of air which passes will depend upon the size of the opening and the resistance offered to the flow;

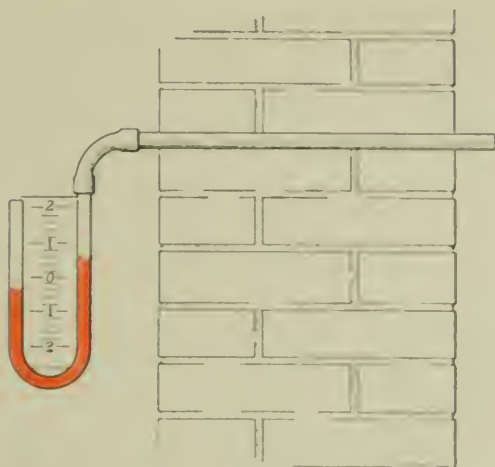


FIG. 3. MANOMETER TUBE FOR SHOWING THE DIFFERENCE IN PRESSURE BETWEEN THE OUTSIDE AND THE INSIDE OF A BOILER WALL

hence the weight of air passing through the fuel bed from the ash-pit for any given draft over the fire will depend upon the thickness of the bed, the size of the pieces of coal, and the condition of the bed. In any case, the combustion of a given amount of coal always requires a definite amount of air. Since for large pieces of coal the voids in the fuel bed are correspondingly large, a fuel bed of given thickness will present less resistance to the passage of air than a bed of finer coal of the same thickness. Hence it requires less draft with large coal than with fine coal to pass a given amount of air through the fuel bed. It is, however, advisable to use a thicker bed with large coal in order to close up the holes. This in turn will make it necessary to increase the draft to a point about equal to that used for fine coal, although the exact relation existing between thickness of bed, draft, and load on boilers must be determined by experiment in each case.

In view of facts developed in a recent investigation,* special attention should be given to the regulation of the overdraft in hand fired furnaces, since, contrary to the generally accepted belief, it is shown

* "Combustion in the Fuel Bed of Hand Fired Furnaces," by Henry Kreisinger, F. K. Ovitz, and C. E. Augustine, Tech. Paper No. 137 U. S. Bur. of Mines, Washington, D. C. The investigation reported in this publication discloses the following facts: "The current of air in passing through a uniform fuel bed without holes will have all its oxygen used within the first four inches from the grate. The rate of combustion therefore varies directly with the rate at which air is forced through a uniform fuel bed. The completeness of combustion is determined by mixing volatile gases with air in the space above the fuel bed. The reactions here are between two gases rather than between a solid and a gas, and the space required for this process is much greater. If only the theoretical amount of air is here available, the mixing may not be sufficiently perfected before the gases have passed out of the combustion space. Hence it is necessary to supply an excess amount of air over the fuel bed. This air must be introduced through openings above the fuel bed or come in through holes in the fire."

that most of the excess air is admitted into the combustion chamber above the fuel bed instead of through the fuel bed.

Every boiler should be equipped with two draft gages, one connected directly into the space over the fire (Fig. 4), and one connected

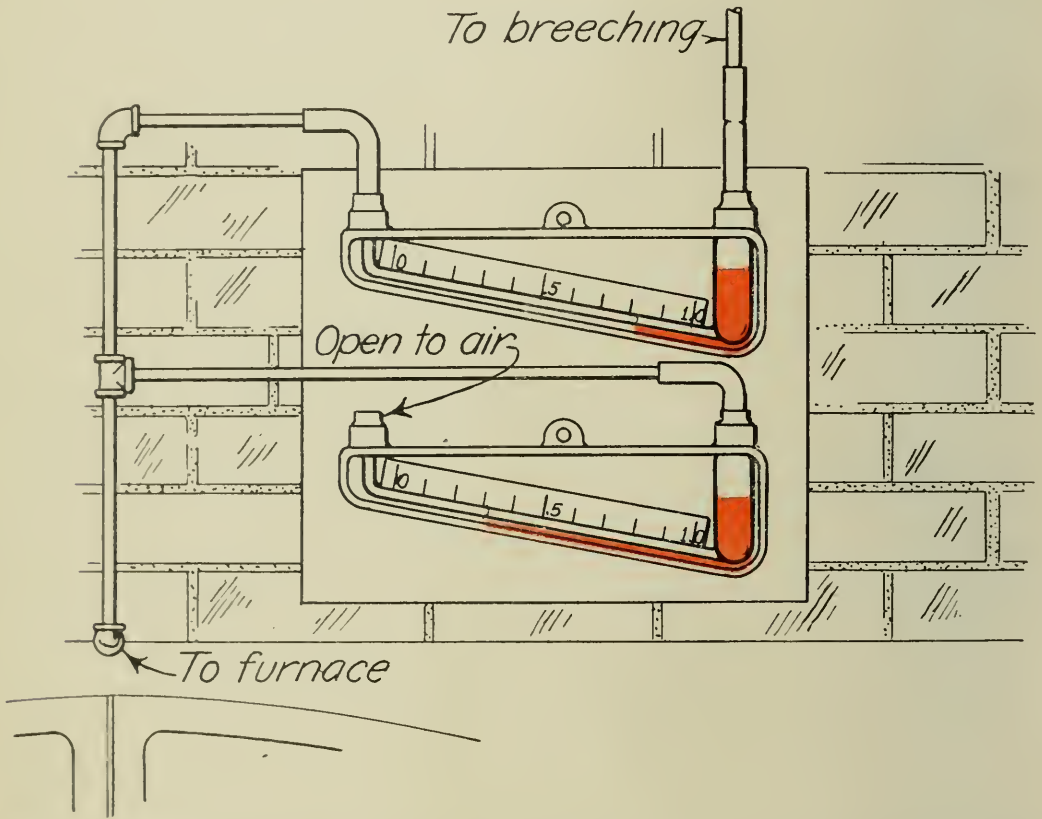


FIG. 4. SKETCH SHOWING THE CORRECT METHOD OF CONNECTING DRAFT GAGES

both into the space over the fire and into the gas passage below the damper, giving the drop in pressure through the tubes and baffling. The operation of the boilers should be controlled by means of the draft over the fire. The draft necessary to carry any given load and the corresponding proper thickness of fuel bed with the grade of coal used should be determined. With everything in good shape and no leaks in the setting and a given draft over the fire, there should be a definite loss of draft through the setting, or differential draft as it will hereinafter be called. When the damper is opened to increase capacity, both the furnace draft and the differential draft will increase. Assuming that the correct thickness of fuel bed is being used, an increase in the differential draft reading over the normal reading with the given

furnace draft indicates that there are holes in the fuel bed or that the tubes have become clogged with soot and ash. A decrease in the differential draft indicates that the fuel bed is dirty and that the resistance is greatly increased by ash or clinker, or that some of the baffling is down, causing a short circuit of the gases.

10. *Significance of CO_2 in the Flue Gases.*—A study of the amount of carbon dioxide (CO_2) in the flue gases affords the only practical means of obtaining a knowledge of conditions existing within the furnace on the basis of which correction or regulation to obtain the best results may be made. The importance of making CO_2 determinations, therefore, warrants a discussion of the methods by which these determinations may be made. Every plant should be equipped with some form of CO_2 analyzing apparatus and the fireman or other employe taught to use it. Since it is comparatively inexpensive, the outlay will be returned many times by the gain in efficiency and the consequent saving of fuel. For this purpose an Orsat apparatus or some of its modified forms should be used. The complete Orsat apparatus provides a means of analyzing for carbon dioxide, oxygen, and carbon monoxide, but since the CO_2 values give a sufficiently accurate indication of the amount of excess air passing through the fire, the analysis for the other two gases may be omitted and the apparatus used in its simplest form, as shown in Fig. 5. This consists merely of of a pipette, *h*, to hold the solution (potassium hydroxide), a measuring burette, *e*, of 100 cubic centimeters capacity, a leveling bottle, *f*, containing water, and an aspirating bulb, *m*. The solution may be made by mixing equal weights of potassium hydroxide (KOH) and water. In the absence of this chemical, concentrated lye may be used.

In using the CO_2 apparatus, the liquid in the pipette, *h*, is first brought to the mark, *o*, just below the cock, *d*. This can be done by lowering the leveling bottle, *f*, after which the cock, *d*, should be closed. The 3-way cock, *c*, is then opened to the burette, *e*, and to *b*, and by raising the leveling bottle, *f*, the water in the burette is brought to the mark, *g*, and the cock, *c*, closed to the burette, and opened through *a* and *b*. The aspirating bulb, *m*, is now worked, drawing gas from the sample tube, *n*, in the setting and forcing it out through *b*. When sufficient gas has been forced through to clean out the air and dead gas from the sample tube, the cock, *c*, is

turned so that *b* is closed, and *a* is in communication with the burette, *e*. The sample is then pumped into the burette, thus driving the water into the leveling bottle and more than filling the burette. The leveling bottle is then raised until the water in the

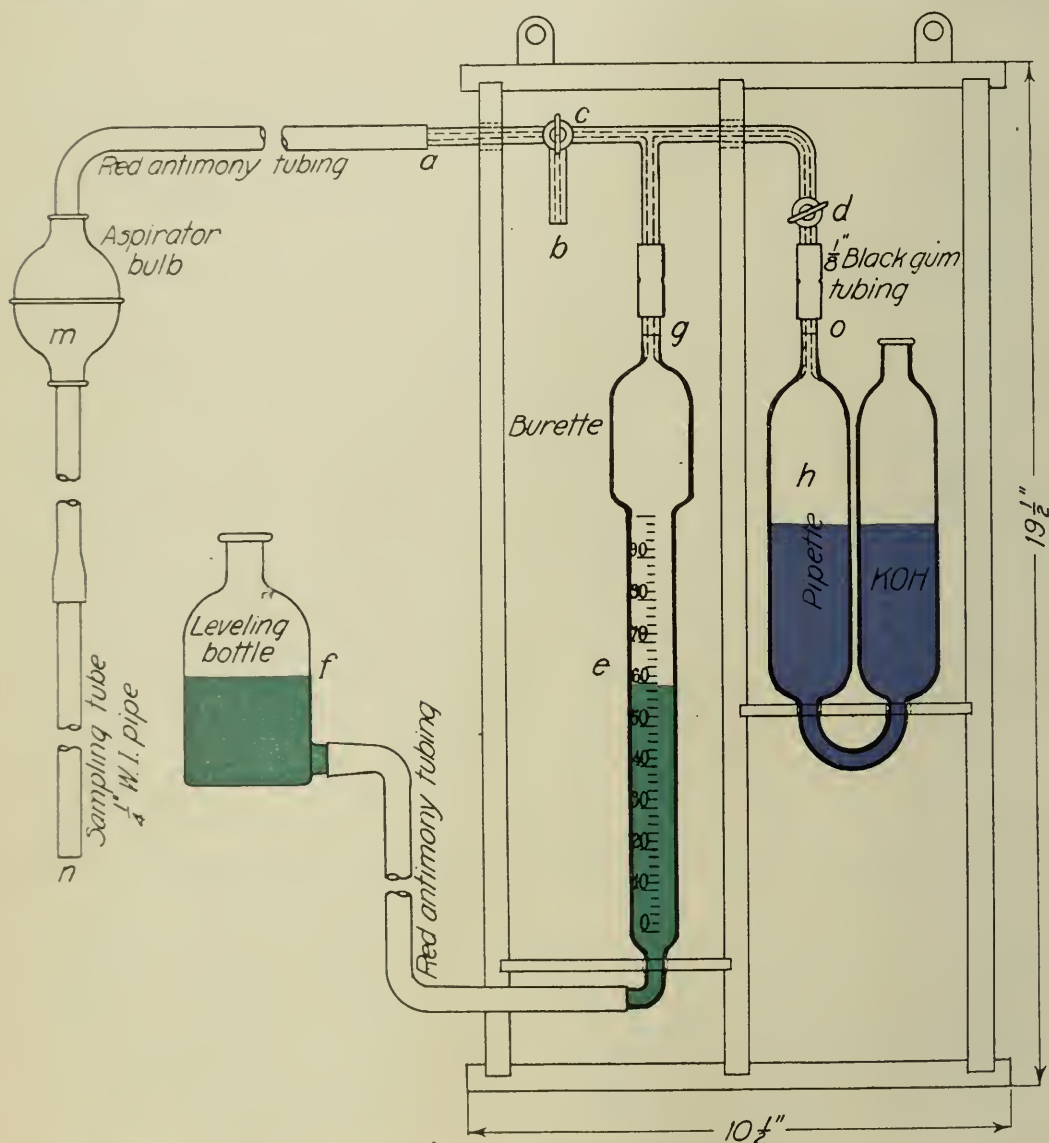


FIG. 5. APPARATUS FOR THE DETERMINATION OF CO₂ IN FLUE GAS

burette stands exactly at 100 cc., the rubber tubing between the leveling bottle and the burette is clamped between the thumb and finger so that no change in the level at 100 cc. can take place and the cock, *c*, momentarily opened to the atmosphere through *b* and then closed to the burette. If this has been done correctly, when the two

surfaces, *e* and *f*, are brought to the same level, *e* should stand at 100 cc. An alternate method of obtaining 100 cc. at atmospheric pressure is to have the 3-way cock open through *a* to *c* and closed to *b*. The gas may now be forced out through the liquid in the leveling bottle, *f*. The water at *f* and *e* may now be brought to the same level and the cock closed to the burette, *e*. The cock, *d*, is now opened and the gas driven into the pipette, *h*, by raising the leveling bottle. It should be driven back and forth between the burette and pipette several times, and then the liquid in the pipette brought back to the mark, *o*, and the cock, *d*, is closed. The surfaces, *f* and *e*, are again brought to the same level, and the amount of CO_2 in the gas sample is read from the burette at *e*. This operation is easily performed and a fireman of ordinary intelligence can analyze a sample in about two minutes.

There are several precautions which should be observed in taking samples. There must be no leaks in the rubber tubing or connections. If air leaks in during the analysis, it invalidates the result. The sole object in making an analysis is to determine what the fire is doing at the time the sample is taken; hence the apparatus should be hung on the setting at a point as near as possible to the point where the sample is taken in order to reduce the amount of piping and rubber tubing between the sampling tube and the analyzer, and to insure a sample representative of conditions at the time. If the sample is conveyed through tubes of considerable size and length, as is usually the case with a CO_2 recorder or even with a CO_2 indicator, the analysis is made from 5 to 15 minutes after the sample is taken. Thus a hole in the fire may be disclosed by the analyzer 5 or 10 minutes after its initial occurrence and even after its disappearance by filling up. The CO_2 recorder, therefore, is useful for giving an idea of the average operation over a long period, but is not satisfactory as a means of determining the proper relation between load, draft, fuel bed thickness, and other conditions. The determination of such relations involves simultaneous readings.

Precautions must be observed in inserting the sampling tube. An elaborate sampling apparatus used in the hope of obtaining an average sample is not to be recommended. Such apparatus consists mainly of a double tube arrangement having a series of small holes drilled into the tubes, the tubes extending across the gas passage. These do not accomplish the desired result, however, be-

cause the holes become clogged with soot and ash making it impossible to know the point in the flue from which the sample is drawn. Another reason why these tubes are not reliable for procuring an average sample lies in the fact that the gas stream varies across the flue in all directions, while the sampling tubes can at best give an average in only one direction. In order to obtain an exact average

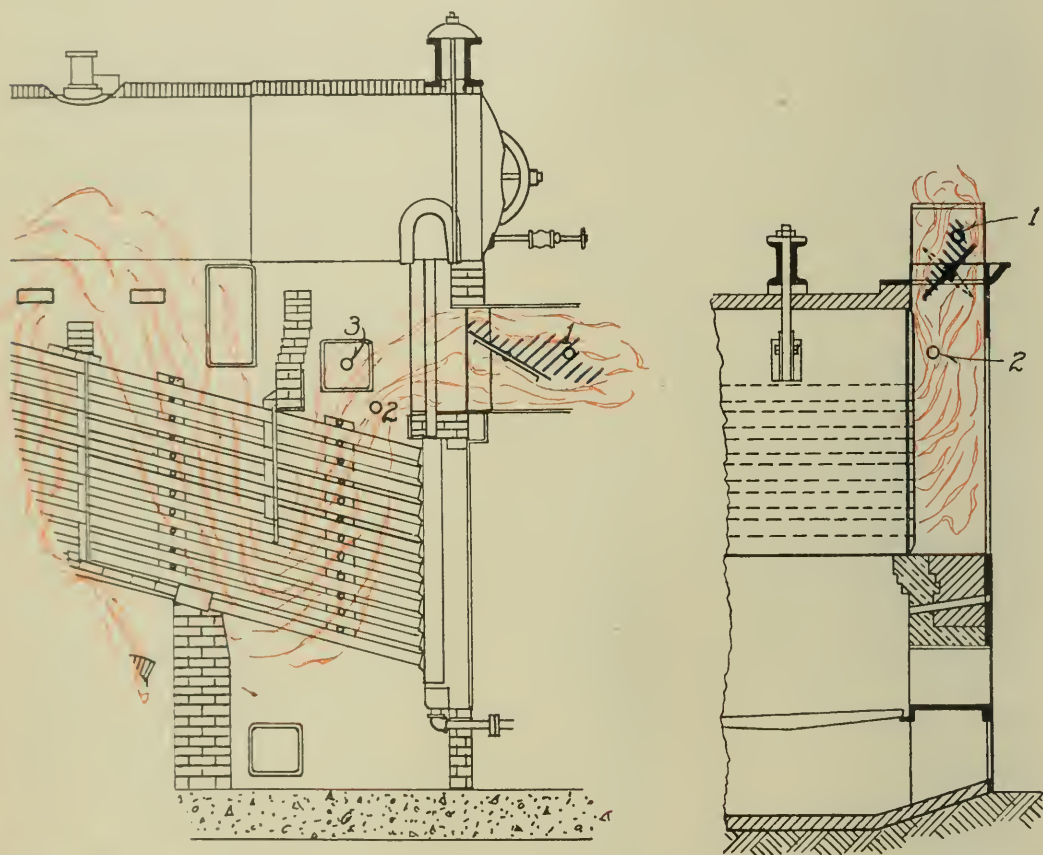


FIG. 6. SKETCH SHOWING THE PROPER LOCATION FOR GAS SAMPLING TUBES TO AVOID DAMPER POCKETS FOR BOTH FRONT AND REAR TAKE-OFF

Point 2 is in the center of the main gas stream and indicates correct position of the sampling tube. Points 1 and 3 show locations of pockets in which representative samples cannot be secured.

it would be necessary to fill the flue with a network of sampling tubes so arranged that each might take a quantity of gas proportional to the velocity of the stream at its point of sampling. For all practical purposes, therefore, it is best to take a sample through the end of a straight tube consisting of a piece of $\frac{1}{4}$ -inch pipe so that the point of sampling may be known with accuracy. A sample taken from the center of the main gas stream where the gas has

the greatest velocity has been found to yield an accurate indication of the condition of the fuel bed at the time of sampling, slight variations in the condition of the fire being reflected immediately in the sample. In placing the tube, care should be taken to have the end in the center of the main gas stream at point No. 2, Fig. 6, and not in any of the dead gas pockets as indicated by points 1 and 3. Otherwise low CO_2 values not representative of the actual conditions will be obtained. The tube should be inserted at the point where the gases leave the heating surfaces of the boiler for the last time as indicated by point 2 in Fig. 6, and not further out in the flue. An iron tube should not be used if the temperature at the point of sampling is sufficient to raise it to a red heat, because the character of the sample may be affected by part of the oxygen in the sample uniting with the red hot iron. A small cotton filter, contained in a glass tube, should be inserted between the sampling tube and the aspirator bulb. In using the apparatus shown on page 30, care should be taken to prevent a draft of cold air striking the burette during a reading. A material change in temperature during a reading will invalidate the result.

11. *Losses of Heat Value.*—The losses in the boiler plant may be divided into two classes:

- (1) Those due to the loss of green coal in handling.
- (2) Those resulting in the process of combustion. Losses of the first class are usually small and easily detected; hence they will not be discussed further.

The principal losses are those entailed in the process of combustion. These may be divided into the following classes:

- (1) Loss due to excess air and air leakage through the setting.
- (2) Loss due to combustible in ash.
- (3) Loss due to CO_2 formed.
- (4) Loss due to soot on the tubes.
- (5) Loss due to moisture carried in with the coal and air.
- (6) Loss due to heat carried out by the escaping gases.
- (7) Loss due to radiation.

Losses included under classes 1 to 4, inclusive, are largely preventable, while those under classes 5, 6 and 7 are more or less inevitable, although they may be reduced to a minimum with proper care.

Excess Air and Air Leaks

Losses due to excess air and to air leaks are discussed together because they may both be detected by the same means, *i. e.*, by analysis of the flue gas. Under the head of excess air may be included all air which goes through the combustion zone in excess of the amount required for perfect combustion. Air leakage includes all air going through holes in the setting and other places besides the fuel bed.

The space inside the average boiler setting is at less than atmospheric pressure; hence if there are any openings in the setting, air will leak through from the outside. This cold air not only takes no part in the combustion, but its temperature must be raised to that of the rest of the gas, a process which requires heat and lowers the temperature of the other gases. Some of this heat is given back to the water in the boiler, but all that indicated by the difference between the temperature of the flue gas and that of the air in the boiler room represents a dead loss. This is also true of the excess air carried through the fuel bed. While these losses cannot be detected with the naked eye, like that due to green coal in the ash, they are by far the most serious of all losses occurring in the average plant.

Leaks in the setting may occur in the metal work around doors and joints as well as in the brickwork. When leaks are found they should not only be stopped with asbestos or stove putty, but should be calked with waste or asbestos fiber soaked with fireclay in such manner as to prevent cracking off or falling out as soon as dry. The last of the leaks may best be found by building a smoky fire and shutting the damper. Smoke may then be seen to issue wherever there is a leak. When the setting has been made as tight as possible, air will still seep in because the bricks and the mortar are porous. This leakage may be reduced to a minimum by tacking metal lath to the setting and applying a coat of plaster one inch or so in thickness made of a mixture of about 80 per cent magnesia and 20 per cent old magnesia pipe covering. In order to secure a satisfactory surface 85 per cent magnesia should be mixed with cement to form a thin grout, spread on the surface, troweled to a smooth finish, and painted. This makes a good lagging not affected by temperature changes and also serves to reduce the radiation loss listed under class (7). If the setting is too hot to permit touching it with the hand without discomfort, the radiation loss is excessive.

After the setting has been made absolutely tight it will pay to

give attention to the excess air loss, but it is well to emphasize that the former should be done first. There exists a very definite relation between capacity, draft, fuel bed thickness, and air passing through the fuel bed with a given grade of coal. For Illinois coal a draft of approximately .01 inch of water is required to burn one pound of coal per square foot of grate surface per hour. This ratio is slightly increased for rates of combustion above twenty-five pounds of coal per square foot per hour.

In order to determine these relations in any given plant, a time should be chosen when the load on the boilers will remain constant for several hours. The fire should be clean, of uniform thickness, and free of holes, and the surfaces of the tubes should be free of soot. A draft and fuel bed thickness sufficient to maintain the load without loss of pressure should then be chosen. Simultaneous readings of the draft and analyses of the flue gas should now be made as rapidly as possible and repeated at brief intervals to insure permanence of conditions, and a watch should be kept on the fire to see that holes do not develop. Care must be taken not to open the furnace doors during a reading. Records should be kept of the drafts, CO_2 , and fuel bed thickness. Thickness of fuel bed should then be varied and the draft adjusted to carry the load without pressure drop, or without blowing the safety valves. When sufficient time has elapsed to allow conditions to become constant, another set of readings should be taken. If too thin a fuel bed were used at the start, it will be found on comparing the readings that as the thickness of the fuel bed is increased, the draft increases, and the percentage of CO_2 also increases. Finally a point will be reached at which the CO_2 does not increase further as the thickness of the fuel bed and the draft increase. The draft and fuel bed thickness to give this CO_2 reading represent the proper values for the given load on the boiler, under which the suggested changes in operating conditions have been made. This process should then be repeated for a number of different loads on the boiler. Upon doing this it will be found that with a given thickness of fuel bed, certain more or less well defined limits of draft over the fire will give a maximum CO_2 reading. The draft then becomes the key for controlling the whole situation. If the load on the boiler is such as to require drafts between certain limits, then the thickness of fire which should be used is immediately known, provided there are no holes in the fire and the tubes and fire are clean. These latter

conditions will be indicated by the differential draft gage, Fig. 4, readings of which should also have been taken during the tests when the tubes were known to be clean and the fire in good condition. A table or chart should be laid out for the use of the fireman, which, as soon as the approximate draft necessary to maintain boiler pressure is known, gives the thickness of fire to be carried and the corresponding differential draft gage reading. If the furnace draft and thickness of fire are correctly maintained and the differential is then too low, it indicates either that the fire is dirty or that some of the baffling has fallen. A too high reading of the differential indicates that there are holes in the fire, or that soot is clogging the passages through the tubes.

The thickness of the fire should not be left to the judgment of the fireman, but definite marks should be placed on the inside door liners, or at some points where they may be seen. In any case, it should be thoroughly understood that the coöperation of the fireman is necessary, and unless the fires are kept clean, and the firing is done in such manner as to maintain a uniform fuel bed without holes the other precautions suggested are useless.

Since air leakage through the setting tends to increase as the draft increases, it is good policy to run on the minimum draft which will carry the load without pressure drop. The CO_2 readings are a direct indication of the total loss due to both excess air and to air leakage when taken just below the damper. A curve * is presented in Fig. 7 which has been plotted from flue gas readings when burning Illinois slack on a chain grate. It gives the percentage of excess air represented by different percentages of CO_2 in the gas. From this curve it may be seen that 12 per cent of CO_2 represent about 35 per cent excess air. This is the maximum CO_2 reading obtainable with this coal when burned on grates of approximately 93 per cent grate efficiency without danger of incomplete combustion and a corresponding loss due to carbon monoxide. If this value is not exceeded, it will not be necessary to analyze for carbon monoxide and the determination for CO_2 is sufficient.

It has been mentioned in a preceding paragraph that the proper position for the sampling tube is at a point where the hot gases leave the heating surfaces for the last time. The reason for this may now

* Kratz, A. P., "A Study of Boiler Losses." Univ. of Ill. Eng. Exp. Sta., Bul. 78, p. 33, 1915.

be made clear. The CO_2 in the sample at this point is an indication of all the excess air and leakage which has diluted the gas and absorbed heat which should have gone into the water. After the gases leave the heating surfaces there is no longer any chance of heat being absorbed and it is not important, so far as efficiency is concerned whether it is lost in a small amount of gas at a high tem-

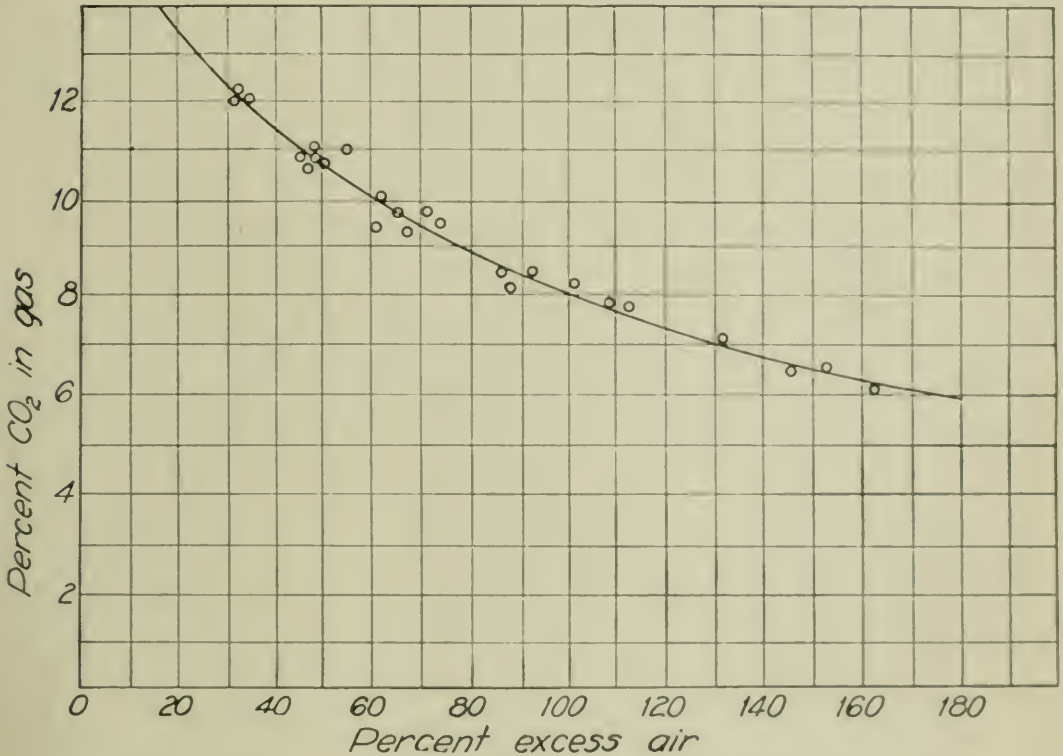


FIG. 7. CURVE SHOWING RELATION BETWEEN EXCESS AIR AND CO_2 IN FLUE GAS
(See "A Study of Boiler Losses," Univ. of Ill. Eng. Exp. Sta., Bul. 78, 1915.)

perature, or in a large amount of air and gas at a lower temperature. Any air leakage beyond this point, therefore, does not lower the efficiency. The harmful effect, however, is shown on the capacity. It not only adds its own bulk to the gases the chimney must carry, but also, due to its cooling effect on the hot gases, lessens the draft available to produce the flow. In many cases the mere stopping of the leaks in setting and breeching has enabled boilers to carry overload, while previously it had been impossible to obtain rated capacity.

The draft should be controlled by means of the dampers at the flue, and not by the ashpit doors. Closing the ashpit doors prevents air from going through the fuel bed, causes clinker and hot grates,

and also increases the air leakage loss. Each boiler should be equipped with a separate damper and the position of maximum and minimum damper opening should be determined. The damper should then be operated between these limits. The points of maximum and minimum opening should be found by noting the reading of the draft gage while the damper is moved from one extreme position to the other. These points of maximum and minimum draft gage reading may not coincide with the points at which the damper is mechanically open or closed. A position will usually be found at which the draft is maximum, and a further opening of the damper will not change the reading. It may also be found that the damper can be opened quite appreciably before the draft gage begins to read. In many plants of large and medium size automatic draft control has proved economical and it is also of advantage in maintaining constant steam pressure. If automatic control of the draft is used, it is important to have the damper adjusted for the range of travel determined by experiment as suggested, so that the draft will be proportional to the opening. An automatic damper regulator, when used, should preferably be of the type which responds to small decreases or increases in steam pressure by causing a corresponding movement of the damper, and not of the type which either completely opens or completely closes the damper in response to small decreases or increases of pressure. If there are several boilers in the plant, the best plan is to adjust the individual dampers so that each boiler is carrying its share of the load under the most economical draft, and then, if the total load changes, to regulate with a master damper in the main flue.

Loss Due to the Presence of Combustible in the Ash

The loss due to partly burned coal in the ash should not, with very careful handling of the fire, exceed more than about three per cent of the heat value of the coal. Excessive carbon in the ash with stoker fired furnaces usually indicates too rapid feed for the rate of combustion used. In hand fired furnaces it may indicate that the grate openings are too large for the size of coal used or that the fire is worked too much, or both. So far as possible the fire should be operated without much working except at times of cleaning. Too much working does two things; first, it shakes the green coal down into the ash and allows it to pass through the grate bars, and

secondly, it brings the ash up into the hot part of the fire where it fuses and causes clinker. Partly burned coal is fused in with the clinker and is lost when the clinker is removed, and the coal which is shaken through the grates during the additional working required to remove the clinker adds further to the loss. The possible loss due to firing green coal into holes in the fire and thus permitting it to pass through the grate has not been considered in detail because it is obvious that the holes should be filled up by leveling the fire before adding a fresh charge of green coal. In working a fire, it should be sliced from the bottom in such a manner as to avoid or minimize the possibility of forcing ash up into the fuel bed. This applies to stoker firing as well as to hand firing.

Loss Due to the Presence of Carbon Monoxide in the Flue Gases

Carbon monoxide is formed if too thick a fire or an insufficient draft is used. With central bituminous coals there is little possibility of large loss from this source if the CO_2 reading is not more than 12 per cent.

Loss Due to Soot

The largest part of the loss due to smoke does not result from the fact that the particles of carbon floating in the gas stream have passed out before giving up their heat value, but it comes from the deposit of soot on the tubes. The actual heat value of this deposit of soot is small when compared with the amount of coal fired in producing it, but its power of preventing the heat in the gases from reaching the tubes and being absorbed is a factor of considerable importance. Soot makes an excellent heat insulator, about five times as effective as asbestos. Under normal working conditions and with the normal amount of air, the temperature of the gases leaving the boiler should be somewhere near 550 degrees F. If they leave at a much higher temperature and the fire and drafts are normal, it signifies that the tubes need blowing. The soot deposited on the heating surfaces is keeping the heat in the gas from reaching the water and the gases consequently are not cooled. Where automatic blowers are installed the tubes should be blown every four or five hours. In all cases they should be blown at least once for every shift. A pyrometer placed at the point where the gases leave the tubes for the last time will give a fairly good indication of their condition, provided of course that low temperature is not due to excess air.

Loss Due to Moisture in the Coal and Air

The loss due to moisture in the air is very small and need not be considered. That due to moisture in the coal may be larger. The coal may carry 13 or 14 per cent of moisture, and the heat required to evaporate this must be furnished by the coal itself, thus decreasing the amount available to heat water in the boiler. Fine coal tends to pack if fired dry. This prevents the proper amount of air getting to the fuel, and results in the formation of carbon monoxide and in cold fires. Sometimes very dry coal burns out unevenly, and will not stay on the grates without allowing holes to form. For these reasons, it is advisable to wet down the smaller sizes of coal just before firing because the other losses mentioned are greater than that due to the water. With larger sizes wetting is not necessary and is not advisable. This, however, must be decided for each individual plant. In no case should more water be added than is absolutely necessary.

Loss Due to Heat in the Escaping Gases

Every pound of flue gas passing up the stack represents a loss of about .24 heat units per degree F. above the temperature of the steam in the boiler. This loss cannot be entirely eliminated. For plants operating on natural draft, a temperature of about 500 degrees F. is required in the stack to produce the draft necessary to operate the boilers at full capacity. An average of 550 degrees F. is good practice. For forced draft and four-pass boilers, it may run lower than this temperature. An indicating pyrometer should be used on each boiler and if the temperature in the flue becomes abnormally high it may be accepted as an indication of an excessive deposit of soot upon the tubes or of a draft greater than is necessary for the load.

Loss Due to Radiation

Uncovered surfaces and other surfaces which are too hot to touch without discomfort represent a serious loss of heat. This loss can be decreased by covering the setting as previously suggested.

12. *Significance of Smoke.*—Smoke, depending upon its cause, may or may not indicate a loss in efficiency. The loss is largely due to the soot deposit, and not to the heat value of the fine particles of floating carbon. The three principles of smokeless combustion

have already been stated. The question concerning whether there is sufficient air for combustion can be answered with the CO_2 analyzer. If the CO_2 reading is normal and smoke still appears, the trouble is due either to a faulty mixture of the air and combustible gases, or to a too small combustion chamber. Increasing the air supply may decrease the smoke, but it also decreases the efficiency. In this case smokeless combustion does not indicate high efficiency. If the fire is hot and there is much oxygen in the gas, together with high carbon monoxide, the trouble is due to poor mixing of the gas and air over the fuel bed. Mixing piers or arches will eliminate the smoke which in this case is an indication of loss of efficiency. If the fire is white hot, and the CO_2 normal, without any indication of carbon monoxide in the gas the trouble is due to a too small combustion chamber. In most plants this is the cause of smoke, and in such cases it does not indicate poor furnace efficiency. The loss is due to soot rather than to smoke.

13. *Methods of Hand Firing.*—There are two general methods advocated for hand firing, (1) Coking, (2) Spreading. The first involves the placing of a considerable amount of green coal on some convenient part of the fuel bed where the heat will pass into it and will slowly distil the volatile gases. These gases then mix with air above the bed and in passing over the white hot bed are burned before they reach the cold surfaces. The method usually adopted is to pile the green coal at the front of the bed. After 10 or 15 minutes, during which the coal has become well coked, this pile is broken up and spread over the back part of the fuel bed, and a fresh charge is piled at the front. This method accomplishes satisfactory results so far as smokeless combustion is concerned, but it does not promote efficiency. The keynote of efficiency lies in the maintenance of a uniform fuel bed, while with the coking method of firing the bed burns unevenly. The bed is usually too thick at the front, and burns out and develops holes at the rear, and although it is less liable to form clinker, this practice is not to be recommended as highly as some form of spreading.

In the spreading method, small quantities of coal are fired at frequent intervals. In alternate spreading, a thin layer of coal is spread on one side of the furnace. As the gases distil, they mix with the air, and the white hot surface on the other side maintains the

mixture at the ignition temperature. After a period of about five minutes, when the distillation is complete, another charge of fresh coal may be spread on the other side of the bed. By this method the fire may be kept in a more nearly uniform condition than by coking. Where the coal is spread over any considerable area, however, there is still a tendency for the resistance at different parts to vary, and for holes to develop.

The best method is to fire very often and in small quantities. Holes should not be permitted to develop; to prevent holes, small amounts of coal should be placed on the thin parts of the bed. Thin places may be recognized from the fact that they appear brighter and hotter than the rest of the bed. This method requires more attention on the part of the fireman, but it pays in the long run. It is possible to maintain a uniform bed for long periods without barring or working the fire. Where the coal is fired in small quantities, the volume of combustible distilled at one time is small and may be easily consumed over the hot part of the fuel bed without forming smoke.

In any case, in hand firing, it is necessary that some auxiliary air be taken in over the fuel bed for several minutes immediately after firing. This should be admitted across the fire close to the surface of the fuel bed, preferably from the front through auxiliary dampers in the fire door, which should have an area of at least four square inches per square foot of grate surface. This admission of air may be accomplished either automatically or under the control of the fireman. The automatic device opens small supplementary dampers when the fire door is opened, and then closes them gradually after the door is shut. The time of closing is usually about three minutes. The same result can be accomplished by the fireman regulating the dampers in the fire door by hand. In some cases the use of a steam jet immediately after firing has proved advantageous, since it not only carries in the air necessary for the combustion of the volatile gases, but also serves thoroughly to mix the air and gases.

14. *Stoker Firing*.—It is not within the scope of this discussion to give detailed instructions for the use of different types of stokers. All the statements, however, concerning tight settings, determination of proper fuel bed thickness, draft, soot, etc., apply to stoker firing as well as to hand firing. Most stokers accomplish one

prime requisite for good combustion, *i.e.*, a uniform supply of coal and air, but they require as intelligent attention as does hand firing. Stokers should be inspected regularly to see that all ledge plates, baffles, and other devices designed to decrease air leakage are performing their functions properly. The grate should be kept uniformly covered with fuel and the rate of feed adjusted so as to minimize the amount of unburned coal carried over into the ashpit. The chain grate stoker is probably more generally used for Illinois coal than any other type, and it has usually proved satisfactory. It is used largely for burning slack, or screenings, and for this purpose a fuel bed of about six inches gives the best results, particularly when natural draft is used. Detailed instructions for the operation of any particular type of stoker may be obtained from the company building it.

IV. FEATURES OF BOILER INSTALLATION IN RELATION TO FUEL ECONOMY

15. *Boiler Settings*.—The boiler setting consists of the foundation and such parts of the furnace and gas passages as are external to the boiler shell. The setting must furnish a proper support for the boiler and at the same time provide the necessary passages for the products of combustion as well as a pit for the ashes.

Foundation

A good solid foundation resting on a firm footing is absolutely necessary to insure a boiler setting which will remain tight and free from any tendency to crack. The depth of the foundation and the width of the footings necessary for a given installation depend upon the character of the soil. In the case of good solid soil capable of supporting heavy loads, the excavation need not be made very deep, but where the ground is soft it is well to excavate the entire space occupied by the setting and to construct a bed of concrete about two feet thick upon which the walls may be supported.

When boilers are supported on steel columns, as is usually the case with water tube boilers and as is desirable for fire tube boilers also, the footings at the base of the columns must be enlarged, since with such means of support the loads are concentrated and not distributed as in the case of horizontal return tubular boilers supported by a series of lugs resting on the walls of the setting. In general, the foundation for all types of boilers should be very rigid; a weak foundation will always cause the setting to crack no matter how well the brick in the walls may be set. Weak foundations may, furthermore, tend to produce severe stresses at pipe connections to the boiler which are likely to cause trouble.

Side and End Walls

The side and rear walls are supported upon the foundation and in the older designs of settings a two-inch air space is generally provided in these walls. In many of the newer types of setting, however, this air space has been omitted. As a result of a series of experiments made by the United States Geological Survey, it has been found that the two-inch air space provided in the walls of boiler settings has

practically no effect in preventing the flow of heat from the interior of the setting. As a matter of fact the radiation losses for a wall with an air space are greater than those for a solid wall. In order to strengthen the side walls, buck-stays held together by long bolts are used. The furnace, the bridge wall, and all parts of the side and rear walls including the back arch which are exposed to the hot gases must be lined with high grade fire brick capable of withstanding the high temperatures. The fire brick should be backed with hard, well burned, red bricks laid in a high grade mortar. All arches, piers, and wing walls which may form a part of the combustion chamber should be made entirely of fire brick.

Settings for Horizontal Return Tubular Boilers

In order to use soft coal economically it is desirable to obtain as complete combustion as possible. Complete combustion also means elimination of smoke. To obtain proper combustion sufficient air must be introduced into the furnace to supply the necessary oxygen. The air thus introduced must mix thoroughly with the gases given off by the burning coal and the mixture thus obtained must be kept at a high temperature until the process of combustion is completed. The old

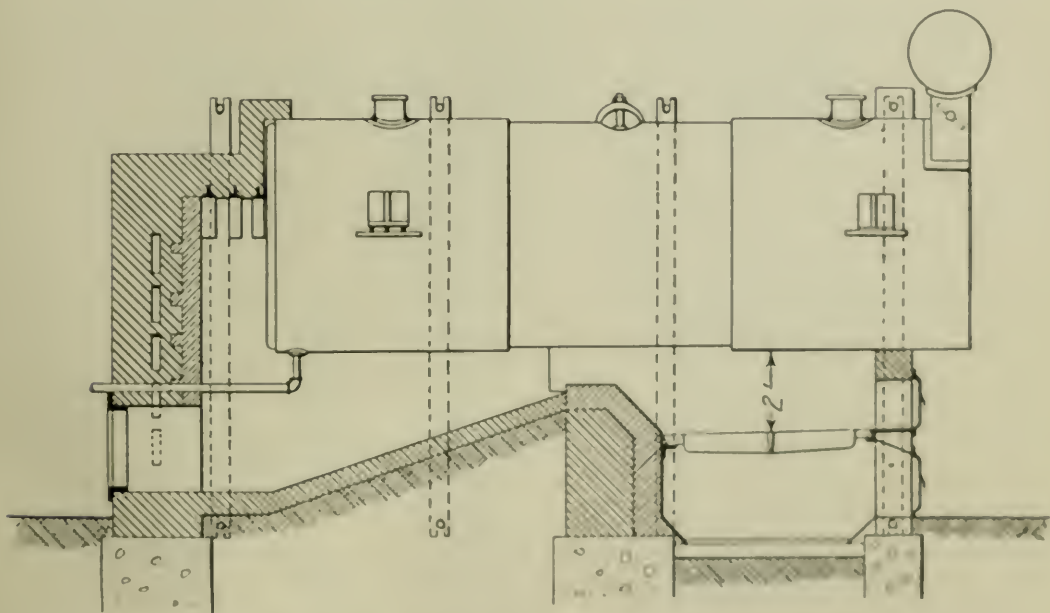
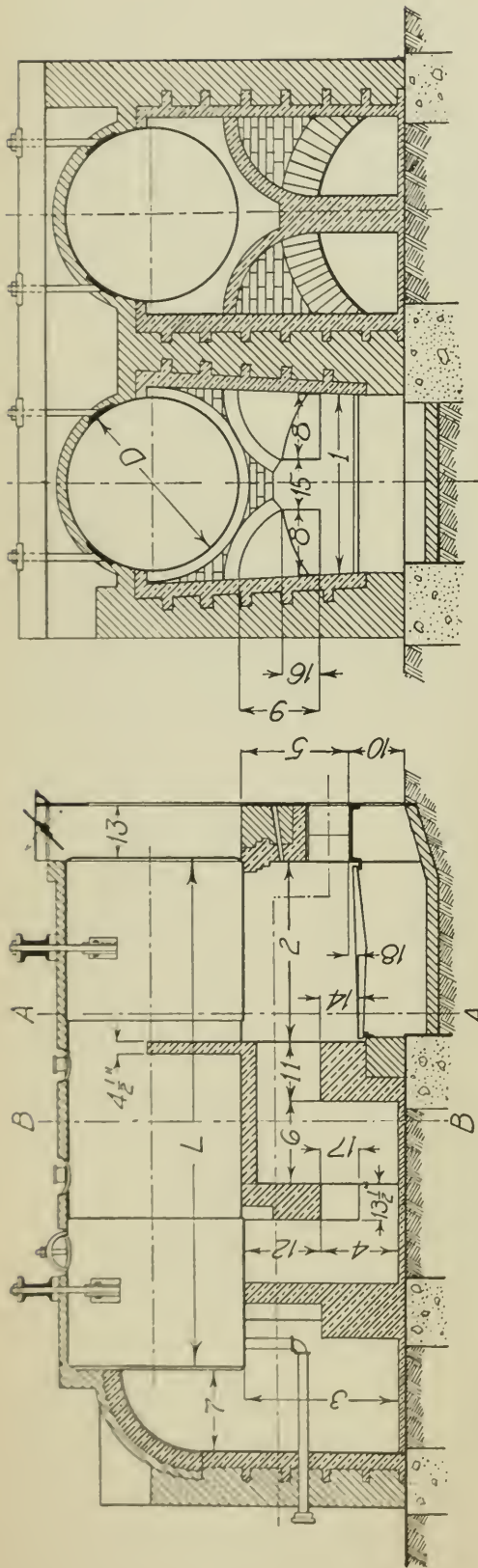


FIG. 8. HARTFORD SETTING FOR RETURN TUBULAR BOILERS

This setting does not satisfy the conditions for smokeless combustion. In fact it is, for Central Western coals, very unsatisfactory and should not be used.

Note for Fig. 8, page 45 of Circular 7

The setting shown in the above figure is no longer used or approved by the U. S. Bureau of Mines.



Section AA Section BB

Boiler		Dimensions of Double-arch Bridge-wall Setting for Horizontal Return Tubular Boilers.																				Dimensions in Inches	
Dia. V ₂ h ₁	h ₂	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
42	12	42	42 1/2	42 1/2	27 1/2	28	21	22	16 1/2	17	19	18	17	16	15 1/2	9	11	8	3 1/2				
48	14	48	48 1/2	48 1/2	30	30	24	24	19 1/2	17 1/2	20	18	17 1/2	18	16 1/2	9	9 1/2	8	3 1/2				
54	16	54	54 1/2	54 1/2	30	30	26	26	19 1/2	19 1/2	22	18	19 1/2	18	15 1/2	15	11 1/2	4	4				
60	18	60	60 1/2	60 1/2	30	32	24	24	20 1/2	20	22	22	22 1/2	22 1/2	17	13 1/2	13 1/2	9	4 1/2				
66	16	66	66 1/2	66 1/2	33 1/2	34	26	26	23 1/2	24 1/2	22 1/2	22 1/2	22 1/2	22 1/2	18 1/2	15 1/2	13 1/2	10	5				
72	18	72	72 1/2	72 1/2	36 1/2	36	28	28	26 1/2	24 1/2	24	20 1/2	22 1/2	22 1/2	17 1/2	13 1/2	13 1/2	11 1/2	5 1/2				
78	20	78	78 1/2	78 1/2	40	36	30	30	29 1/2	26 1/2	24 1/2	24 1/2	22 1/2	22 1/2	18 1/2	13 1/2	13 1/2	12	6				
84	18	84	84 1/2	84 1/2	43	38	32	32	30	27 1/2	26	26	25	21	19	18	15 1/2	12	6 1/2				
20	20	84	84 1/2	84 1/2	43	38	32	32	33	27 1/2	26	26	24	20 1/2	18 1/2	16 1/2	14	12	6 1/2				

FIG. 9. DOUBLE ARCH BRIDGE WALL SETTING FOR SMOKELESS COMBUSTION

standard setting for return tubular boilers (shown in Fig. 8) is frequently used, but it does not satisfy the conditions stated and for this reason should not be used when smokeless combustion is required. In recent years several types of settings have been devised which if properly fired make possible the satisfactory combustion of bituminous coal.

A type of horizontal return tubular boiler setting which has given good results with soft coals is shown in Fig. 9. It was originated and perfected by the engineers associated with the Department of Smoke Inspection of the City of Chicago and is generally recommended for boilers operating at a steam pressure of sixty pounds or more. This setting differs from that illustrated in Fig. 8 in that a series of arches are constructed over the bridge wall and in the combustion chamber. A double-arch rests on the bridge wall and on a suitable pier built up between the bridge wall and a single span deflection arch, the latter being located from two to three feet back of the bridge wall. The highest point of this deflection arch is at the elevation of the top of the bridge wall or somewhat below it. So as to prevent the gases from passing over the arches bulkheads extending up to the boiler are constructed above them.

It is evident from this brief description that the gases in passing from the grate over the bridge wall are divided into two separate streams by the central pier supporting the double-arch. In going through the retorts formed by the double-arch and the side walls of the setting the gases are thoroughly mixed and at the same time are subjected to high temperatures. Having passed through the retorts the gases are compelled to change their direction of travel so as to pass under the deflection arch back of the bridge wall, thus promoting further mixing and thereby insuring proper combustion. The setting is also provided with the usual panel door for the admission of air over the fire. The steam jets shown extending through the furnace front are generally considered standard equipment and it is recommended that they be freely used after each firing.

The arches used in connection with the setting shown in Fig. 9 produce a side pressure on the walls, and in order to prevent bulging and cracking of the walls additional buck-stays and tie rods should be installed. The floor of the combustion chamber is subjected to high temperatures and for this reason should be paved with second-grade fire brick laid on edge.

According to Osborne Monnett * the following general proportions should be satisfied in order to obtain satisfactory service with this type of setting.

- (1) The free area through the double-arch above the bridge wall should be made equivalent to at least 25 per cent of the grate area.
- (2) The area from the back of the bridge wall to the deflection arch should be at least 45 per cent of the grate area.
- (3) The area under the deflection arch should be at least 50 per cent of the grate area.

For convenience of reference the dimensions indicated in Fig. 9 for various sizes of boilers are given in the table presented with the illustration. These dimensions were obtained from a report submitted by the Standards Committee at the eleventh annual convention of the Smoke Prevention Association.

There are several patented settings in which the gases are maintained at a high temperature and are thoroughly mixed by the use of piers and wing walls in place of the arches shown in the setting of Fig. 9. Experience seems to indicate that the temperature in the combustion chamber of such settings is sufficiently high to promote proper combustion, and furthermore it is claimed that they burn coal with good economy and are cheaper to construct than the double-arch setting.

Settings for Water Tube Boilers

A large number of hand fired water tube boilers of the horizontal type designed for the use of soft coal are installed with what is generally called the standard vertical baffling. This form of baffling compels the gases to pass across the tubes, thus producing a rather short flame travel in the first pass and rendering complete combustion impossible. In order to improve combustion in hand fired water tube boilers of the horizontal type when burning soft coal, it has been demonstrated by the Chicago Department of Smoke Inspection that the setting should be so arranged as to fulfill the following specifications: †

* "Hand Fired Furnaces for Water Tube Boilers—I." *Power*, Vol. 40, p. 264, August 25, 1914.

† *Ibid.*

- (1) Some provision must be made so that the bottom row of tubes will absorb some heat directly from the fire. This is accomplished by the use of T-tiles, thus exposing the bottom row of tubes to the fire for a short distance from the front header.
- (2) Over the bridge wall and for some distance back of it a high temperature zone, through which the gases and air pass, must be provided. This zone is obtained by using box-tiles around the bottom tubes. The box-tiles extend from the end of the T-tiles to a point several feet in front of the back header. The area provided between the bridge wall and the box-tile should be made equivalent to 25 per cent of the grate area.
- (3) In order that the gases and air will thoroughly mix, a deflecting arch is provided a short distance back of the bridge wall. The distance between the arch and the bridge wall should be such that the area obtained between them is equivalent to 40 per cent of the grate area. The height of the arch must be sufficient so as to provide an area underneath it equivalent to 50 per cent of the grate area.
- (4) As in the case of the horizontal return tubular boiler settings, excess air is provided through the panel doors in addition to a siphon steam jet located above the fire doors.

When water tube boilers are very wide, it may be necessary to construct the deflection arch mentioned in (3) in two or three spans. These spans should be supported on suitable piers in order to relieve the strain on the side walls. In case the area over the bridge wall is in excess of 25 per cent of the grate area, the desired area may be obtained by introducing piers upon the bridge wall opposite the spans in the deflection arch. Due to these piers the gases and air will be more thoroughly mixed, thus promoting combustion.

In water tube boilers equipped with horizontal baffles it generally happens that there are parts of certain tubes which are not acted upon effectively by the gases. This is due to the fact that the gases become trapped in the corners as shown at A in Fig. 10 and become inert. Such a condition can be improved with a gain in the efficiency of the boiler, by providing openings approximately one inch wide in alternate rows of the tile B, Fig. 10.

One of the most serious defects found in brick settings is air leakage. The fire brick used on the interior of the setting must be selected with great care as the life of the setting very largely depends upon the quality of these bricks as well as upon the workmanship in laying them. Generally the arches used in settings cause more trouble than

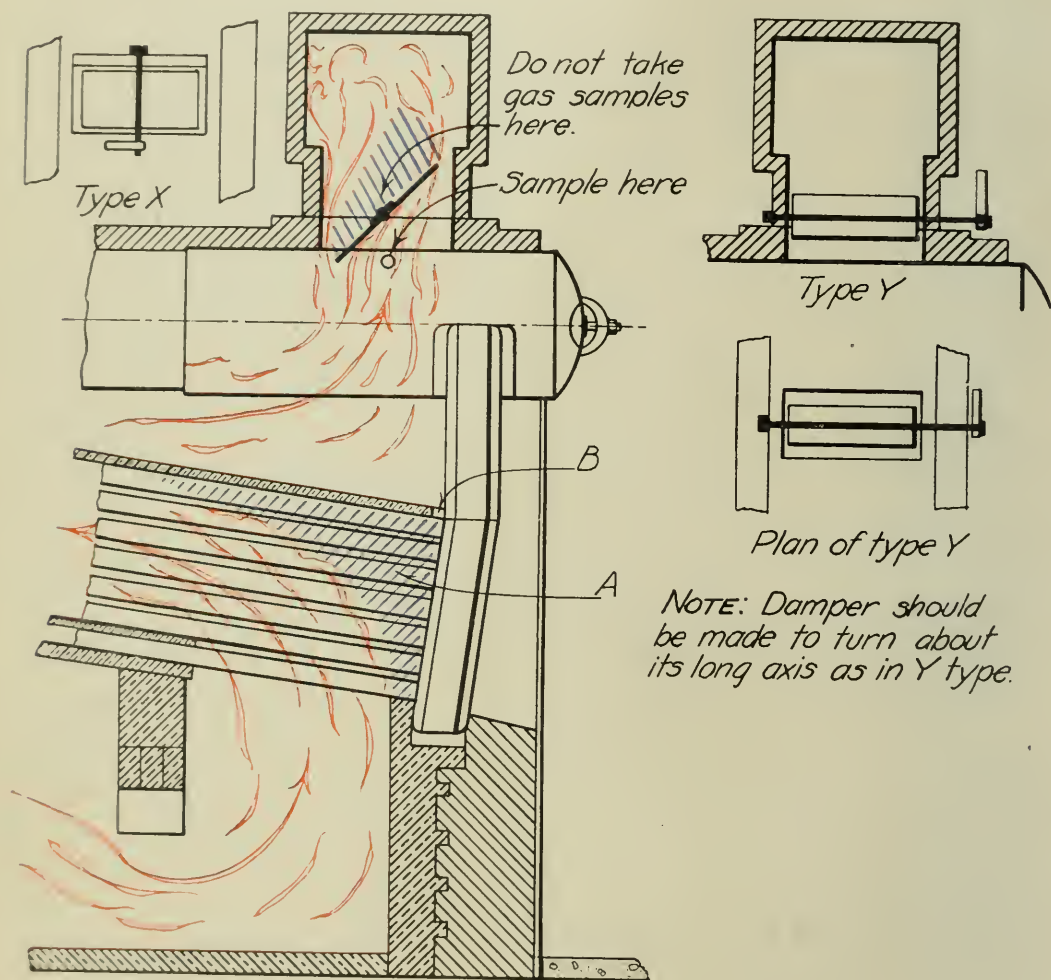


FIG. 10. SKETCH SHOWING EFFECTS OF BAFFLING AND DAMPERS IN CAUSING POCKETS AND EDDIES IN THE FLUE GAS STREAM

Defects in Settings

any other part of the setting. In some cases arches fail because of the use of a grade of brick not suited to the purpose, but more frequently failures are due to poor workmanship in laying the bricks. Air leakage through the setting can be reduced by pointing up the brick work in the proper manner and by covering the entire setting with an

insulating material, as for example a high grade of asbestos or magnesia covering. The exposed parts of the shell of horizontal return tubular boilers and of the steam drums of water tube boilers should be covered with an 85 per cent magnesia covering two or three inches thick. The outer surfaces of all insulating materials should be finished off with a thin coating of hard cement or covered with canvas and painted. If the walls of the setting are constructed with air spaces these spaces should be filled with sand or ashes. The baffles on the interior of the setting should be kept tight so that the gases cannot be by-passed through the heating surfaces. All steam and water leaks around a boiler should be stopped immediately since water coming in contact with heated brick work is likely to cause rapid disintegration of the brick. The setting should be so constructed that the boiler is free to expand without affecting the brick work.

V. INSTALLATION FEATURES AFFECTING DRAFT CONDITIONS

16. *Stacks and Breechings*.—The purpose of a stack is, of course, to supply air to the fuel which is burning on the grates of the boiler furnace and then to remove the flue gases which are formed after they have passed over the boiler surfaces and given up most of their heat to the water and steam in the boiler. The stack may waste coal if the fireman allows it to supply too much or too little air, or if he allows the flue gases to leave the boiler at too high a temperature. Most stacks supply too much air so that a large amount of heat is carried away in the unnecessarily large volume of flue gases formed when this air passes through the fuel bed, where it not only serves to burn the coal but also takes up heat.

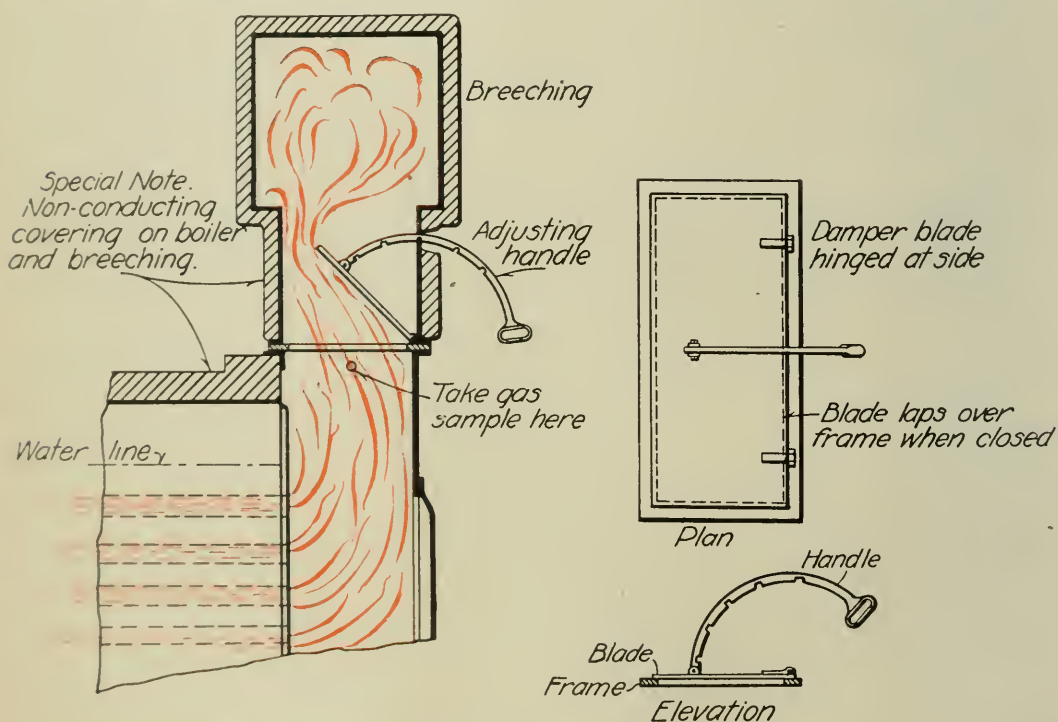


FIG. 11. AN APPROVED FORM OF HINGED DAMPER

The Stack Damper and Its Use

In order to control the amount of air and flue gas passing to a stack, a damper (Figs. 10 and 11) should be installed at the point where the flue gas leaves each boiler. This damper should fit tight

and true and should move easily. An approved form of damper, which is tight fitting, is shown in Fig. 11. It should have a free opening about 25 per cent greater than the area through the tubes. Long narrow dampers are to be avoided wherever possible. For water tube boilers the damper opening should be about 0.25 of the grate area. The damper must be opened only enough to permit the stack to supply the right amount of air to burn completely the fuel fired. When the demand for steam increases, more coal must be burned and the damper must be opened wider in order to supply more air. The air supply should *not* be controlled by opening and closing the ashpit doors, which should stand wide open practically all the time. If the stack is not controlled by the damper, it will always exert its full power on the setting which means that the tendency for air to leak in at any cracks and joints will be as great at half load as at full load, and even with the ashpit doors closed this leakage would be unchanged if the stack damper remained open.

From what has been stated, it is evident that a stack must be capable (at full damper opening) of supplying all the air that the boiler may require when burning coal at the highest rate (pounds per hour) necessary for the maximum load. At all other rates of burning coal the damper must be partly closed, and in many plants, since the stack is too powerful (supplies too much air) even for the highest rate of burning coal, the damper must never be fully opened; otherwise too much air will pass through the grates and fuel will be wasted.

In order that the approximate capacity of a stack may be checked against the load it should carry, Table 3 has been arranged in convenient form for ready reference. Thus, if a certain boiler plant is burning 2,800 pounds of coal an hour at its maximum capacity and the stack is 100 feet high, the diameter should be 60 inches,

Table 3 is a modification of William Kent's stack table and is reliable for the ordinary rates of combustion with bituminous coals. The values in the table give the pounds of coal burned per hour. With coal of fair grade it is necessary to burn about five pounds per boiler horse-power, but with low grade bituminous coal it is necessary to burn from six to eight pounds per boiler horse-power. For example, in the boiler plant already considered, burning 2,800 pounds of coal an hour, a stack 100 feet high and 60 inches in diameter would provide only for 400 boiler horse-power, if a poor grade of middle western coal was used requiring seven pounds per boiler horse-power.

TABLE 3
STACK SIZES BASED ON KENT'S FORMULA

Diameter Inches	Area Square Feet	HEIGHT OF STACK IN FEET										Side of Equivalent Square Stack, In.	Diameter Inches
		50	60	70	80	90	100	110	125	150	175		
		POUNDS OF COAL BURNED PER HOUR											
33	5.94	530	575	625	665	705	745					30	33
36	7.07	645	705	760	815	865	910					32	36
39	8.30	775	845	915	980	1040	1095	1145	1225			35	39
42	9.62	915	1000	1080	1155	1225	1290	1355	1445	1580		38	42
48	12.57	1230	1345	1450	1555	1650	1740	1825	1945	2130	2300	43	48
54	15.90	1590	1740	1880	2010	2135	2245	2360	2515	2755	2975	48	54
60	19.64	2000	2185	2365	2525	2680	2825	2965	3160	3460	3740	54	60
66	23.76	2450	2685	2900	3100	3290	3470	3640	3800	4245	4590	59	66
72	28.27	2955	3230	3490	3735	3960	4175	4380	4670	5115	5525	64	72
78	33.18	3500	3830	4140	4425	4695	4950	5190	5535	6060	6550	70	78
84	38.48	4090	4480	4840	5175	5490	5785	6070	6470	7090	7655	75	84
		HEIGHT OF STACK IN FEET											
		100	110	125	150	175	200	225	250				
		POUNDS OF COAL BURNED PER HOUR											
90	44.18	6690	7015	7480	8195	8850	9465	10040	10580	80	90		
96	50.27	7660	8080	8565	9380	10135	10835	11490	12115	86	96		
102	56.75	8695	9120	9720	10650	11500	12295	13045	13750	91	102		
108	63.62	9795	10270	10950	11960	12960	13850	14695	15490	98	108		
114	70.88	10960	11495	12255	13425	14500	15500	16440	17330	101	114		
120	78.54	12190	12785	13630	14930	16130	17240	18285	19275	107	120		
126	86.59	13485	14145	15080	16515	17840	19070	20230	21325	112	126		
132	95.03	14850	15570	16605	18185	19645	21000	22275	23480	117	132		
144	113.10	17770	18630	19865	21760	23505	25130	26655	28090	128	144		
156	132.73	20950	21965	23420	25655	27710	29625	31425	33120	138	156		
168	153.94	24390	25575	27270	29870	32270	34495	36590	38565	150	168		

“Draft” is in Reality a Pressure

It must be remembered that air enters the ashpit, or rushes through the open fire door, or through any cracks or crevices in the setting or breeching because the surrounding outside air is always

heavier than the hot flue gas in the stack and setting, and hence exerts an inward pressure at all points. The so-called "draft" over the grate as shown by a draft gage (Figs. 3 and 4) is an indication of this difference in pressure or tendency for the outside air to crowd its way into the furnace and boiler setting. In order to understand why this tendency exists and why draft is a true *pressure* and not a suction

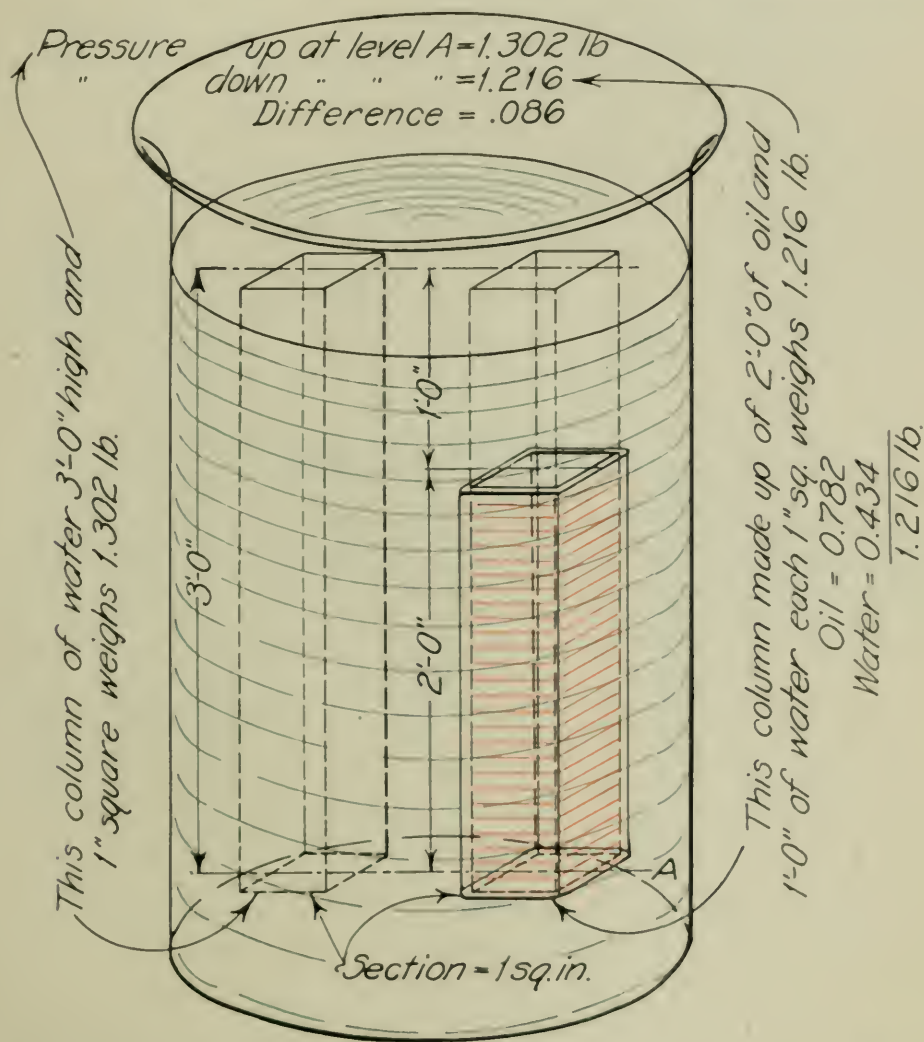


FIG. 12. ISOMETRIC SKETCH ILLUSTRATING THE PRINCIPLE THAT LIGHT FLUIDS OR GASES ARE PUSHED UPWARD WHEN IN CONTACT WITH HEAVIER FLUIDS OR GASES

In this case the lighter fluid, oil (shown in red), is pushed up by the heavier fluid, water (shown in green). The force pushing the oil up the tube is 0.086 pounds per square inch.

refer to Fig. 12. It will be evident that, since the column of water two feet high weighs more per square inch than a similar column of oil, the water will *push* the oil up the tube in the same way that the

cold outside air pushes through the grates of a boiler furnace and forces the hot flue gas up the chimney. If there is a fire burning on the grate the action is continuous, and outside air will continue to

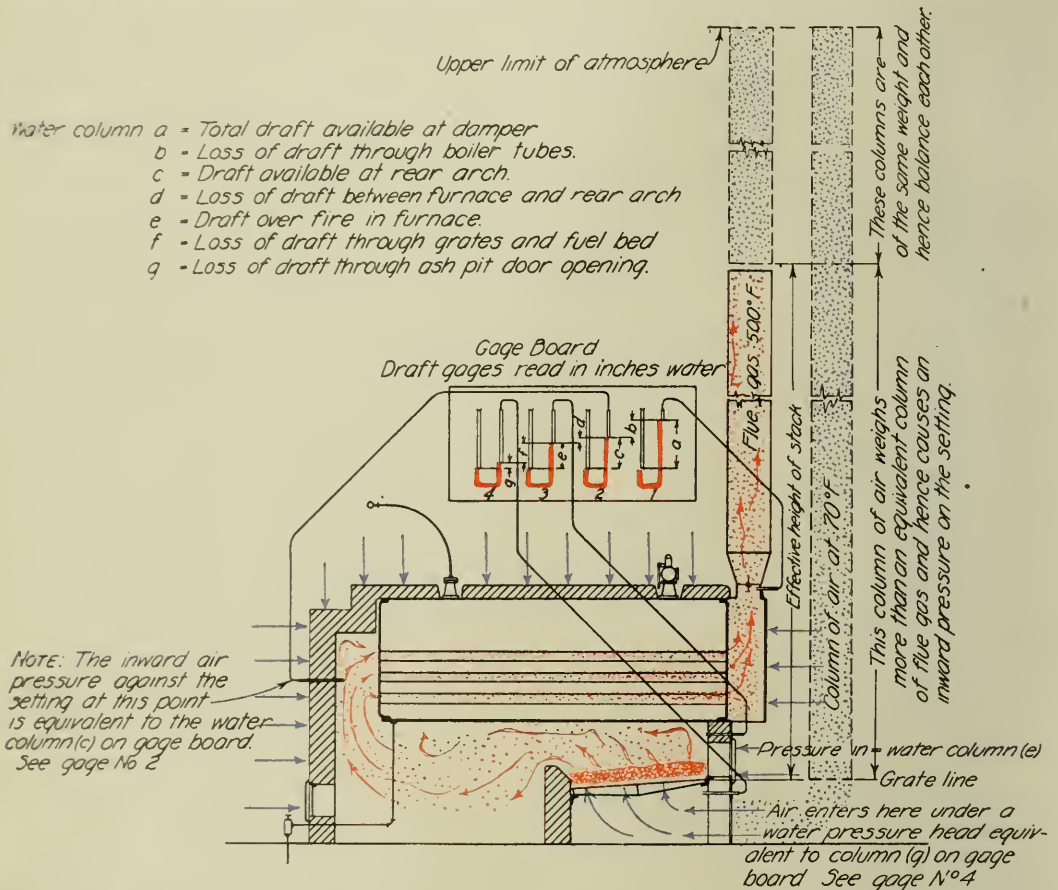


FIG. 13. SKETCH SHOWING VARIATIONS IN DRAFT AT DIFFERENT POINTS AND INDICATING TENDENCY TOWARD AIR LEAKAGE

enter the ashpit (Fig. 13) and push its way through the bed of fuel on the grate, thus promoting the combustion of the fuel.

Air Leaks Affect the Draft and Waste Coal

If cold air leaks into the setting at any point, it will result in two forms of fuel waste. First, it will cool the gases in the setting and in the chimney and will make the draft less. This may make it difficult to burn the fuel properly. Secondly, the air which has leaked in must be warmed up, thus taking heat away from the boiler and wasting it through the chimney, and finally this air will so increase the volume of flue gas that it may be difficult for the chimney to handle the increased volume of flue gas even with the damper wide open.

Breechings for a Battery of Two or More Boilers

Many breechings serving more than one boiler are condemned as being too small because so much unnecessary air is leaking into the various settings that the breeching cannot handle both the leakage and the necessary flue gas. By systematically stopping all the leaks such a breeching will often easily handle the flue gases resulting from the minimum air supply actually required for burning the fuel. Breechings should be at least from 10 to 15 per cent greater in area than the stack to which they connect.

The individual boiler dampers in the throat or uptake connections opening into a breeching should fit accurately and close tightly, otherwise it will be impossible to prevent cold air from entering the main breeching through the damper of a "dead" boiler. This will often seriously affect the operation of all the other boilers by "checking" the draft of the stack which serves the boilers connected to this breeching.

Breechings should be as short and straight as possible, and should have no sharp angles around which the flue gases may swirl and eddy in their passage to the stack. The bad effect of sharp angles is shown in the stack connection marked "B" in Fig. 14. The method of correcting this difficulty is shown at "A" in the same figure. Breechings should be covered with a good heat insulating material or lined with a refractory brick or vitrified material.

All boiler dampers should be "calibrated," that is, should have the operating lever or chain so marked and set that the draft for boiler No. 1, nearest the stack (Fig. 14), will be no greater than for boiler No. 3, farthest from the stack, when both boilers are clean and have the same thickness of fuel bed. To accomplish this result it may be found that the damper on the nearest boiler must be nearly closed most of the time. If all dampers are set at the same angle without regard to their location with reference to the stack, too much air will go through the nearest boiler or the farthest boiler will get too little air and suffer a loss in capacity.

Conditions Under Which a Stack Will Operate Economically

If fuel is to be used economically, the stack must supply the furnace with just enough air to burn the fuel completely. This can only be done when the following conditions are observed:

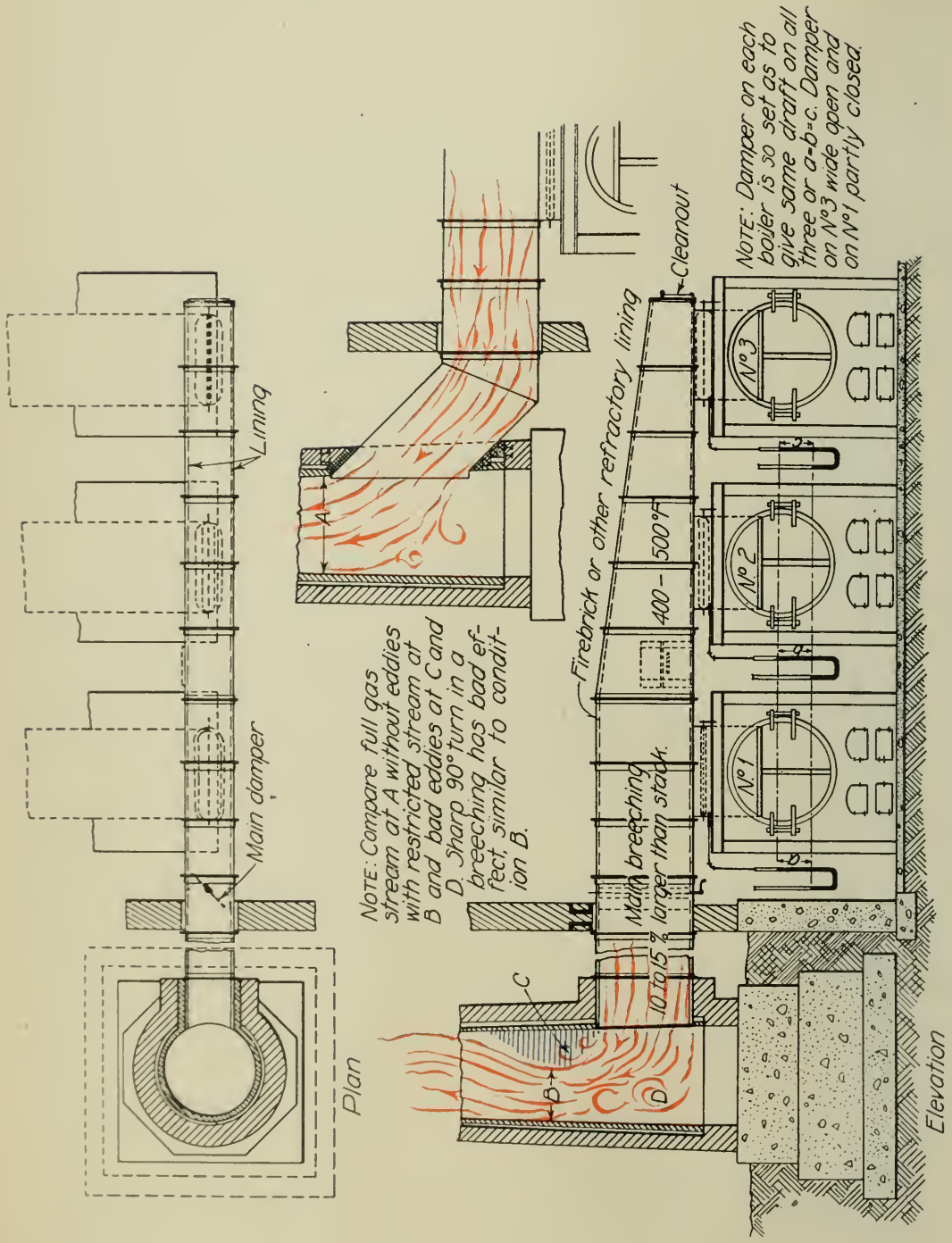


FIG. 14. STACK AND BREECHING CONNECTIONS FOR A BATTERY OF THREE BOILERS

- (1) The setting must be made air tight.*
- (2) All doors and door frames opening into setting and breeching must be made to fit air tight.
- (3) The breeching and stack should likewise be made air tight and should be well insulated to prevent heat loss from the flue gases so that all the heat in the gases may be available for creating draft.
- (4) The air used for burning the fuel should *all* enter through the ashpit, except a limited and carefully regulated air supply which should be admitted through the fire door after firing fresh coal.
- (5) When natural draft is employed, the control of the air should be accomplished by operating the stack or breeching damper according to the rate of burning coal and never by opening and closing the ashpit doors.

* A candle flame is commonly used to detect air leaks into a setting since it will be drawn into any crevice through which air is entering.

VI. FEED WATER HEATING AND PURIFICATION AS FACTORS IN FUEL ECONOMY

17. *Feed Water Purification.*—The majority of waters used for boiler feeding purposes contain more or less impurities which are deposited in the boiler. Such deposits of foreign matter tend to decrease the evaporative capacity of the boiler and, if they are not removed, will frequently cause overheating of tubes and sheets. To overcome these difficulties the impurities in the feed water should be removed before feeding into the boiler.

For convenience of reference, the impurities most often found in feed water and their effects upon the sheets and tubes if permitted

TABLE 4
IMPURITIES IN FEED WATERS, THEIR EFFECTS AND REMEDIES¹

Impurities	Effects	Remedies
1	2	3
Sediment, mud, clay, etc. Bicarbonates of lime, magnesia Sulphates of lime and magnesia	Incrustation and the formation of sludge	Settling tanks, filtration, blowing down. Blow down. Heat feed water. Treat by adding lime. Treat by adding soda. Barium carbonate.
Chloride and sulphate of magnesia Acid Dissolved carbonic acid and oxygen Grease Organic matter	Corrosion	Treat by adding carbonate of soda. Soda or lime. Heat feed water. Keep air from feed water. Add caustic soda or slacked lime. Filter. Iron alum as coagulant. Neutralize with carbonate of soda. Use the best of hydrocarbon oils. Filter. Use coagulant.
Sewage Readily soluble salts in large quantities Carbonate of soda in large quantities ²	Priming	Settling tanks. Filter in connection with coagulant. Blow down. Barium carbonate. New feed supply. If from over treatment, change or modify.

¹ "Steam," published by Babcock & Wilcox Company.

² May cause brittleness in plates. See Bulletin 94, Eng. Exp. Sta. Univ. of Ill., "The Embrittling Action of Sodium Hydroxide on Soft Steel," by S. W. Parr.

to enter the boiler are given in columns 1 and 2 of Table 4. In the last column of this table are given the usual remedies employed to

neutralize or to prevent to a certain degree the effects produced by the various impurities. Some of the impurities found in feed water cause the formation of scale on the sheets and tubes, others cause corrosion of the metal, and still others produce priming, or the carrying over of particles of water with the steam as the latter leaves the boiler.

18. *Treatment of Feed Waters.*—One of the best ways of determining the remedy to be applied in overcoming the injurious effects of the impurities contained in the feed water is to submit a sample of the water to a reliable chemist for analysis and prescription. After such an analysis has been made it is possible to ascertain which one of the following treatments should be applied, chemical treatment, heat treatment, or combined heat and chemical treatment.

Chemical Treatment

The chemical treatment of feed water involves the use of either the lime or the soda process or a combination of these two. The first of these processes in which *slacked lime* is used is well adapted for precipitating the bicarbonate of lime and magnesia contained in the feed water. In the soda process *carbonate of soda* or *caustic soda*, either separately or together, is used for converting the sulphates of lime and magnesia into carbonates or chlorides which may be disposed of by occasional blowing off. The combination of the lime and soda processes, however, is most frequently used. It is satisfactory for treating water containing sulphates of lime and magnesia, carbonic acid, or bicarbonates of lime and magnesia. In this process the sulphates are broken down by the use of sufficient soda, and the necessary lime is added to absorb the carbonic acid not taken up in the soda reaction.

Heat Treatment

The bicarbonates of lime and magnesia so often found in natural waters may be partially precipitated by preheating the feed water in some form of apparatus commonly called a heater. The sulphates of lime and magnesia, however, require high temperatures for complete precipitation and it is impossible to remove these impurities by the simple process of preheating in the ordinary heater using exhaust steam. Instead, live steam heaters and economizers are required for removing them.

Combined Chemical and Heat Treatment

Since preheating of feed water removes the carbonates of lime and magnesia, many water purification systems combine the chemical and heat treatments discussed in the preceding paragraphs, the chemical treatment being used merely for reducing the sulphates.

19. *Boiler Compounds*.—So-called boiler compounds are used rather extensively for treating feed water and no doubt in many cases the results obtained are satisfactory. According to reliable authorities the use of compounds is recommended for the prevention of new scale rather than for the removal of old scale. In general, no compound should be used until the proper advice has been obtained to insure the selection of the right compound for the particular feed water to be treated. In no event should the use of boiler compounds be regarded as a suitable substitute for regular cleaning and inspection.

20. *Feed Water Heaters*.—In any power plant of considerable size cold water should not be fed into the boilers, since all steam boilers are more or less seriously affected by the resulting unequal expansion and contraction. If cold water is forced into the boiler, the tubes of water tube boilers are very likely to become troublesome while in return tubular boilers the seams are liable to develop leaks. Furthermore the feed water cannot be converted into steam until its temperature is raised to the point controlled by the steam pressure, and to do that requires fuel. If the temperature of the feed water can be raised by means of heat which otherwise would be wasted, it is good economy to do so. The preheating of feed water also increases the steaming capacity of the boiler, because of the reduction in the amount of heat to be supplied by the boiler per pound of water evaporated. With certain types of feed water heaters a considerable portion of the scale forming ingredients are precipitated before the water enters the boiler, thus increasing the efficiency and capacity of the boiler as well as effecting a saving in the expense of cleaning out the boiler. In general, it may be stated that one per cent of fuel is saved for every eleven degrees rise in the feed water temperature, provided the heat producing this rise in temperature would otherwise be wasted.

In steam power plants there are two main sources of waste heat, the first being the exhaust steam of the various units, and the second the products of combustion which pass from the boiler to the chimney.

The heat contained in the drips from the high pressure piping system is also a source of loss in many small plants, although its extent is not great. Condensation in the high pressure system, which includes all piping under pressure practically equivalent to boiler pressure, is free from oil and should be returned to the feed water heater by means of pumps or traps. If desired the condensed steam may be returned directly to the boiler. In steam power plants, high pressure steam traps are generally used for automatically draining the condensation from the high pressure lines.

Exhaust Steam Heaters

Heaters using exhaust steam for heating the water may be either of the *open* or *closed type*.

An open heater is one in which the exhaust steam and water mingle, the steam in condensing giving up its heat directly to the water. In general an open heater consists of a shell the upper part of which contains a number of removable trays. The function of these trays is to break up the incoming feed water into thin streams or layers. In passing over the trays, the water mingles with the exhaust steam, and if sufficient steam is supplied temperatures as high as 210 degrees F. may result. It is evident that in an open heater only those scale forming ingredients which will precipitate below 210 degrees F. will be deposited in the heater. Below the trays, the heater is provided with a bed of coke or charcoal through which the water is filtered before the feed pump sends it into the boiler. The function of the coke or charcoal filter is to remove the precipitates and other suspended impurities coming into the heater. Open heaters should always be supplied with a suitable oil separator for removing any oil contained in the exhaust steam.

Closed Heaters

In a closed heater the exhaust steam and feed water do not come into actual contact with each other, the steam giving up its heat to the water by conduction. In one type of closed heater the exhaust steam surrounds tubes through which the water passes. In a second type, the steam passes through tubes which are surrounded by the water. Closed heaters are recommended only for installations where the feed water is free from scale forming ingredients, since there is a tendency for the tube in these heaters to become coated with a deposit of scale, thus materially decreasing the efficiency of the apparatus.

Advantages and Disadvantages of Exhaust Steam Heaters

The advantages of an open heater are as follows:

- (1) The feed water may reach approximately the temperature of the exhaust steam provided sufficient steam is supplied.
- (2) Scale and oil do not effect the transmission of heat.
- (3) The pressure in an open heater is low, practically atmospheric.
- (4) Scale and other impurities precipitated in the heater may easily be removed.
- (5) An open heater is well adapted to heating systems in which it is desired to pipe the returns direct to the heater.
- (6) The initial cost of an open heater is generally less than that of a closed heater.
- (7) With the open heater all the condensed steam is returned to the system.

The disadvantages of an open heater are as follows:

- (1) Some provision must be made for removing oil from the exhaust steam. In modern open heaters this is accomplished by effective oil separators attached directly to and forming a part of the heater.
- (2) "Sticking" or clogging of the back pressure valve may subject the open heater to excessive pressure.
- (3) If the feed water supply is under suction, open heaters may require the use of two pumps, one for hot and one for cold water.

The advantages of a closed heater are as follows:

- (1) The closed heater will safely withstand any ordinary boiler pressure.
- (2) Oil does not come in contact with the feed water.
- (3) It is the only type of heater which may be used in the exhaust main between a prime mover and its condenser.
- (4) Since it is customary to locate a closed heater on the pressure side of the feed pump, only one pump, and that for cold water, is necessary.

The disadvantages of a closed heater are as follows:

- (1) Scale and oil deposits on the tubes lower the heat transmission.
- (2) The temperature of the feed water will always be from four to eight degrees below the temperature of the incoming exhaust steam.
- (3) Scale in the tubes is removed with difficulty.

21. *Economizers*.—An economizer is an arrangement of vertical water tubes arranged in nests, located in the flue between the boiler and the stack. Its purpose is to heat the feed water. The adjacent nests of tubes are connected together by means of expansion bends. To prevent deposits of soot, the tubes are provided with automatic scrapers which are kept moving up and down by means of a suitable mechanism driven by a motor or a small steam engine. Since the temperature of the flue gases is generally about 550 degrees F., it is evident that considerable heat escapes through the chimney. In general, the load factor, the size of the plant, and the cost of fuel are factors which should be considered in determining the advisability of installing an economizer.

22. *Live Steam Heaters*.—Live steam heaters use steam at boiler pressure and, as mentioned in a preceding paragraph, are primarily intended for purifying the feed water. Such heaters are generally not installed unless scale forming impurities are found in the water. At temperatures less than 300 degrees F. the sulphates of lime and magnesia do not entirely precipitate; hence a feed water containing these impurities will not be thoroughly purified by preheating with exhaust steam at atmospheric pressure. Reports of tests tend to show that live steam heaters do not increase boiler efficiency, but merely act as purifiers. Live steam heaters should always be by-passed and so located that the bottom of the shell is at least two feet above the water level in the boiler, thus permitting the purified water to gravitate into the boiler.

23. *Feeding Boilers*.—Water is fed to the boiler by means of an injector or a pump depending upon the size of the plant. The use of an injector does not permit preheating the feed water by means of an open heater; hence relatively cold water is introduced into the boiler, thus decreasing the economy of the plant. It is possible to

install between the injector and the boiler a closed heater, but in such an installation the effectiveness of the heater is decreased because of the heat supplied by the injector. Frequently it is claimed that an injector considered as a combined pump and heater has an efficiency of one hundred per cent since all the heat in the steam used for operating the injector is returned with the water forced into the boiler. Considered as a pump the injector is very inefficient since it requires much more steam to force a given amount of water into the boiler than is required by a pump to do the same amount of work. Furthermore when a pump is used for feeding boilers, the exhaust steam from the pump may be used for preheating the feed water. According to tests, a direct acting steam pump, feeding water through a heater in which the exhaust steam of the feed pump is utilized, shows a much greater saving of fuel than an ejector with or without a heater. In general the feed pump must be located below the water level in the heater, preferably about three feet below, since hot water cannot be lifted by suction.

In order that it may be possible to check the efficiency of the boiler as well as that of the fireman or to determine the most economical fuel, the feed water system should include some form of metering device for measuring the water fed to the boilers. There are a number of metering devices on the market some of which are permanently accurate and some of which must be checked up at frequent intervals. There are now several manufacturers of feed water heaters who are prepared to furnish heaters equipped with water measuring devices.

In addition to measuring the amount of water fed to a boiler, provision should be made for weighing the coal used by each boiler, thus affording a means of determining for each boiler the evaporation per pound of fuel. This will also make it possible to compare the performance of any two boilers in the plant.

For the majority of small boiler plants the feed water should be introduced into the boiler at a constant rate rather than intermittently as is too frequently done.

VII. STEAM PIPING REQUIREMENTS FOR FUEL ECONOMY IN SMALL PLANTS

24. *Possibility of Fuel Loss in the Transmission of Steam.*—In order to promote the economical use of coal, the steam generated should be transmitted to the point of use with as little loss of both heat and steam as practicable. This means that the piping system should be as simple as possible and well insulated. Only short direct runs should be used in connecting boilers, engines, and other apparatus.

All branch lines not in use should have the valves closed so that steam cannot enter them and waste heat by condensing in these dead ends, and the stop valves should be located (Fig. 15) so as to accomplish this purpose. If the engine shown in Fig. 15 is shut down for any length of time the valves at both ends of the engine lead should be closed to prevent steam from condensing in and filling up this lead.

25. *Value of High Pressure Drips as Hot Feed Water.*—Each high pressure header and steam separator should be dripped (Fig. 15) and the hot water returned to the feed-water heater, which should be included in the equipment of every power plant since each 11 degrees F. increase in the feed-water temperature will effect a saving of nearly one per cent in the coal burned. In no case should cold water be fed direct to the boilers, even if live steam has to be used for heating it.

The oily drips from the exhaust steam header (Fig. 15) should be discharged to the sewer or wasted since the oil they carry has an injurious effect upon the boiler tubes and shell, and may do much more harm than the fuel value of the heat contained in these drips is worth.

26. *Leakage Losses at Valves and Fittings.*—The boiler blow-off cock or valve must be perfectly tight to prevent the escape of any hot water to the sewer or other waste channel. If the end of this line cannot be readily inspected to make sure that all blow-off valves are tight, then a tell-tale connection (Fig. 15) with valve should be

placed in the blow-off line beyond the main cock so that by opening it a leak at the blow-off will at once be apparent.

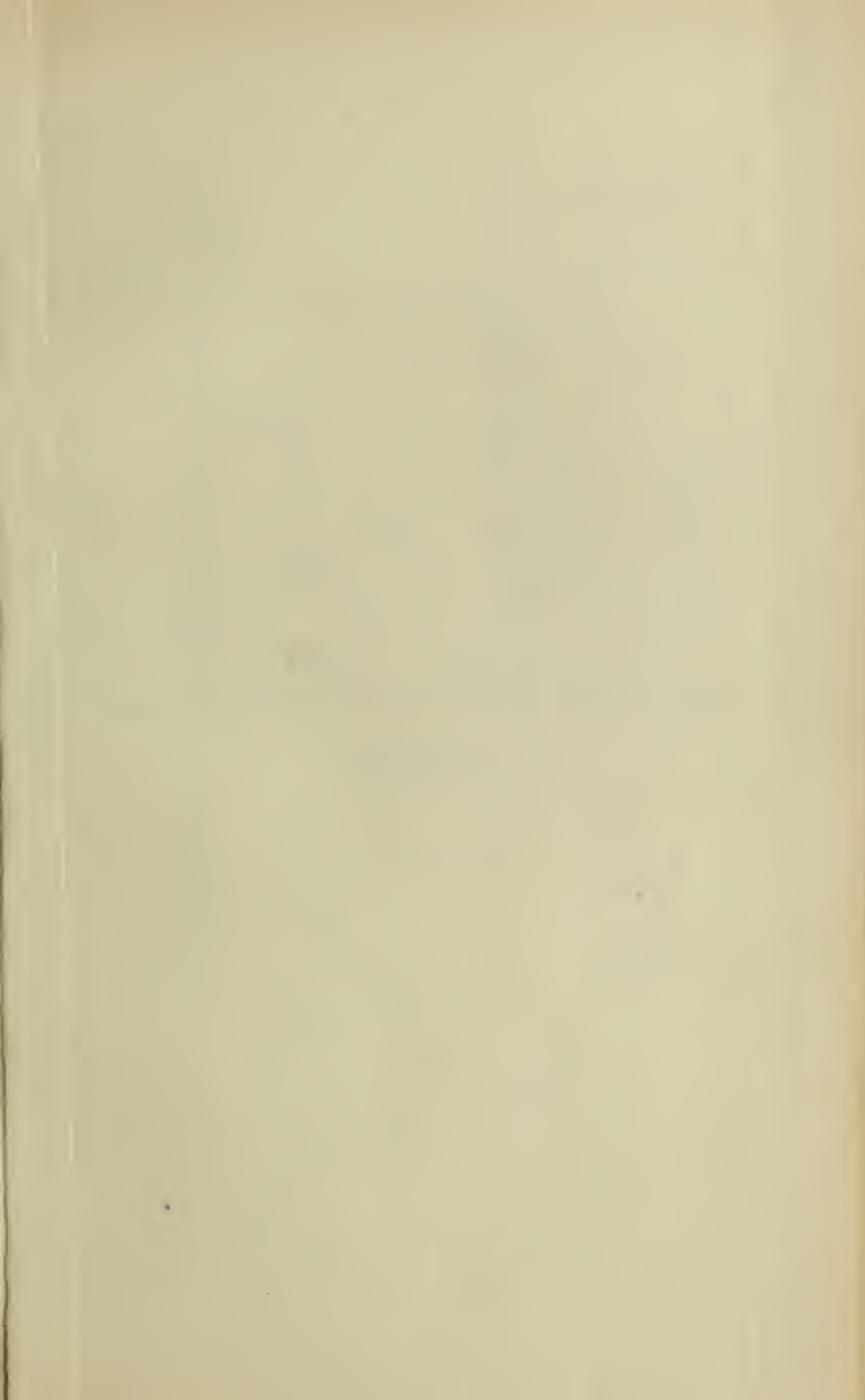
Leaks of hot water or steam which appear in any line either at a valve, flange, or other fitting should be stopped immediately; otherwise a serious waste of fuel may result in making up the heat so lost. An even more serious condition may result if a leak is so located that it discharges water or steam over any part of the boiler or over another pipe since corrosion is very rapid in such a case. In fact, such leaks have started corrosion which later resulted in boiler explosions.

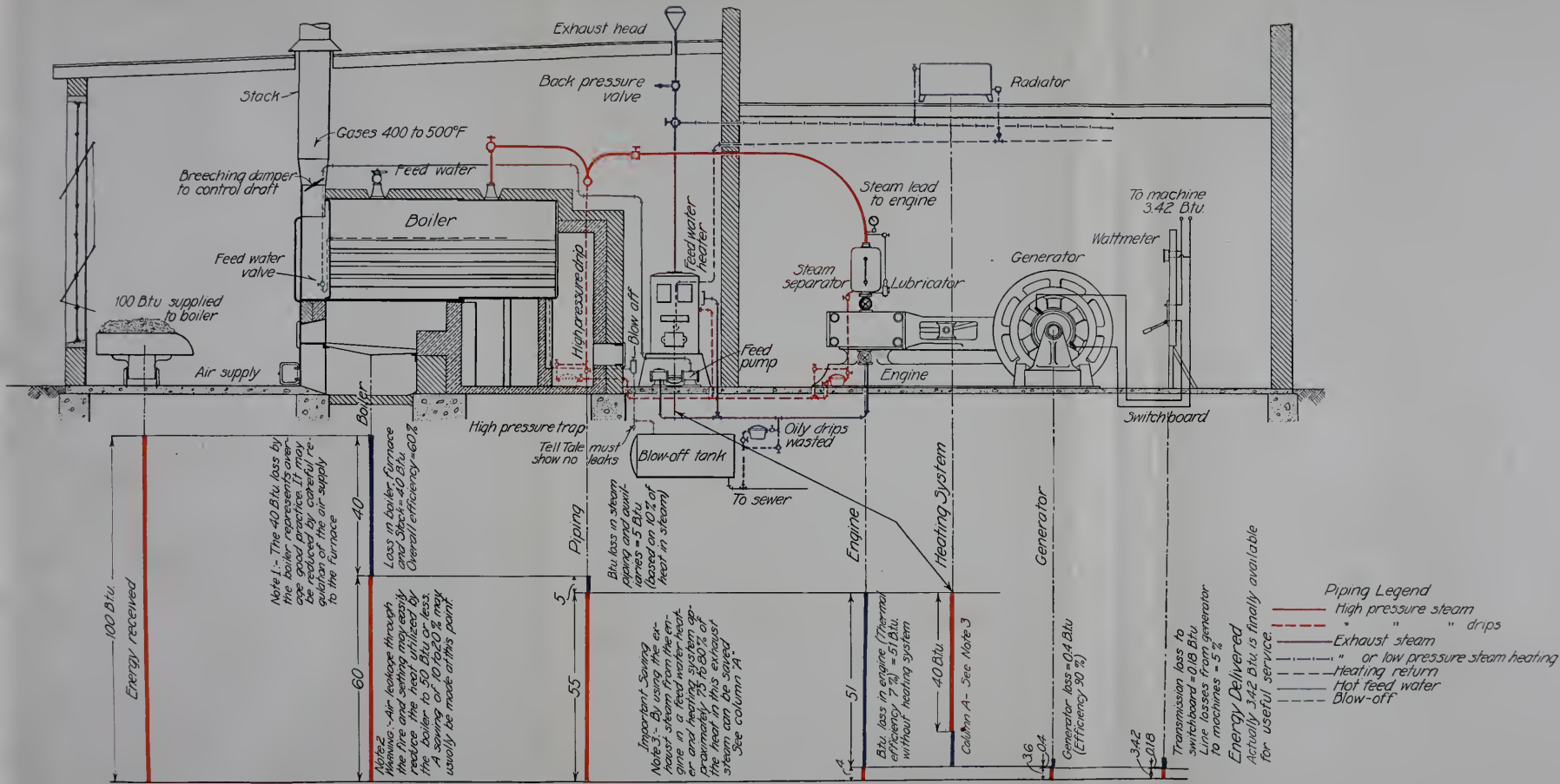
27. *Size of Steam and Exhaust Mains.*—The size and length of steam and exhaust mains may adversely affect the fuel consumption of a plant if these lines are either, (1) too small, or (2) too large for the proper handling of the steam they have to carry. Generally, the lines are too small, but this is not always the case.

If the steam main between boiler and engine is too small or too long, it will be necessary to carry a much higher pressure at the boiler than would otherwise be necessary in order to get the required pressure at the engine. The excessive friction in small steam lines causes this loss in pressure, and if the main is also very long the loss will be still more pronounced. Steam gages at the boiler and at the engine throttle should not show a difference, or a drop in pressure of more than five pounds when the engine is running at full capacity. Should it be necessary to carry a much higher pressure at the boiler than at the engine in order to get a satisfactory operating pressure, some unnecessary coal must be burned since all the heat losses will be slightly increased and leaks will be somewhat more likely to develop.

On the other hand, if this main is too large (which is not so serious) the pressure at the engine throttle will be almost the same as that at the boiler when running at full capacity. The heat losses from oversize pipes and fittings are greater and the first cost of installation is higher than for mains of the proper size.

In the case of the exhaust main, the size is most important since the steam has now expanded and occupies a much greater volume than when it left the boiler. This exhaust steam must be discharged from the engine at the lowest possible back pressure if the engine is to get the most work out of each pound of steam supplied to it and





hence develop the greatest power. With exhaust piping which is too small, a steam gage on the line will show a back pressure of more than two pounds which means that more steam is being taken into the engine to do the work than is necessary. This in turn means coal wasted. Exhaust mains are almost never too large, but in such cases the excess heat loss due to radiation from large exhaust mains is not so serious as in the case of high pressure steam mains since the temperature of exhaust steam is very much less than that of live steam.

28. *Heat Insulating Materials Required on Piping, Boilers, and Breechings.*—The desirability of covering or insulating all high temperature surfaces around a boiler and power plant, when fuel is as expensive and as difficult to obtain as at present, is self evident. It can be shown that by covering all steam and hot-water pipes, fittings, flanges, and valves enough heat can be saved as compared with the loss from bare pipe to pay for the labor and covering material in a very few months. This, of course, applies to the ordinary commercial coverings ranging from one to two inches in thickness. Table 5 shows the saving per year for 100 feet of covered pipe in pounds of coal and in dollars with coal at \$5.00 per ton (2,000 pounds).

The best commercial coverings one inch in thickness will save or prevent the escape of from 75 per cent to 85 per cent of the heat lost from bare pipes. The thickness of the covering should be varied with the temperature of the steam or water in the pipe line, but in general it will pay to use not less than one inch on all lines which are at 200 degrees F. and up to 300 degrees F. Above 300 degrees F. and up to 400 degrees F. it is advisable to use 1½ inch covering and above 400 degrees F., not less than two inches.

The actual heat saving value of commercial pipe coverings now on the market has been the subject of many investigations. The latest work in this field has been done by L. B. McMillan * at the University of Wisconsin and the charts (Figs. 16 and 17) have been made from the results of his tests on bare and covered five-inch steel pipe. These charts show how the heat transmitted by bare pipe compares with the heat transmitted by the same pipe when insulated. The values of only seven of the twenty or more coverings tested are shown. Their efficiency as insulators is easily seen by reading the

* "The Heat Insulating Properties of Commercial Steam Pipe Coverings," Journal, A. S. M. E., Dec., 1915.

TABLE 5
COAL AND STEAM LOSS BASED ON 100 FEET OF UNCOVERED STEEL PIPE

Steam at Gage Pressures of 50, 100, 150, and 200 pounds per square inch. Air Temperature 70 degrees F., feed temperature = 200 degrees F.
Heating value of coal = 12,000 B. t. u. per pound. Boiler and furnace efficiency = 60 per cent.
Coal at \$5.00 per ton. Time = 24 hours per day and 365 days per year.

Note.—A good pipe covering will save 75 per cent to 85 per cent of this loss.

Pipe Size	Surface per 100 Lineal Feet	Pressure = 50 Lb.			Pressure = 100 Lb.			Pressure = 150 Lb.			Pressure = 200 Lb.		
		Steam Con- densed per Hour	Coal per Year	Coal Cost per Year	Steam Con- densed per Hour	Coal per Year	Coal Cost per Year	Steam Con- densed per Hour	Coal per Year	Coal Cost per Year	Steam Con- densed per Hour	Coal per Year	Coal Cost per Year
Inches	Square Feet	Pounds	Tons	Dollars	Pounds	Tons	Dollars	Pounds	Tons	Dollars	Pounds	Tons	Dollars
1	34.5	24.3	16.2	81	32.0	20.6	103	38.6	24.1	121	43.8	26.7	134
1¼	43.5	30.6	20.1	101	40.4	25.8	129	48.8	30.7	154	55.3	33.7	169
1½	49.8	35.1	23.2	116	46.2	29.3	147	55.8	35.0	175	63.3	38.5	193
2	62.2	43.8	28.9	145	57.6	36.8	184	69.7	43.4	217	79.1	48.6	243
2½	75.3	53.0	35.0	175	69.8	44.7	224	84.4	52.6	263	95.7	58.7	294
3	91.6	64.5	42.5	213	84.9	54.3	272	102.7	63.9	320	116.4	71.4	357
3½	104.7	73.7	48.6	243	97.0	61.8	309	117.4	73.1	366	133.1	81.5	408
4	117.9	83.0	54.8	274	109.3	69.6	348	132.2	82.3	412	149.9	91.5	458
4½	131.0	92.2	60.4	302	121.4	77.5	388	146.9	91.5	458	166.5	102.1	511
5	145.6	102.5	67.5	338	134.9	86.3	432	163.2	101.6	508	185.1	113.4	567
6	173.6	122.2	80.6	403	160.9	102.9	515	194.6	121.3	607	220.6	134.9	675
7	199.8	140.6	92.4	462	185.2	118.3	592	224.0	139.7	699	253.9	155.5	778
8	236.0	159.1	104.7	524	209.4	133.6	668	253.4	158.1	791	287.2	175.6	878
9	252.3	177.6	116.9	585	233.8	149.4	747	282.9	176.5	883	320.7	196.2	981
10	281.0	197.8	130.1	651	260.4	166.4	832	315.1	196.7	984	357.2	218.6	1093
11	307.8	216.6	142.4	712	285.3	182.2	911	345.1	215.5	1078	391.2	239.2	1198
12	334.0	235.1	154.6	773	309.5	197.5	988	374.5	233.9	1070	424.5	259.7	1299

Curves taken from "The Heat Insulating Properties of Commercial Steam Pipe Coverings" L.B. McMillan ASME Dec 1915

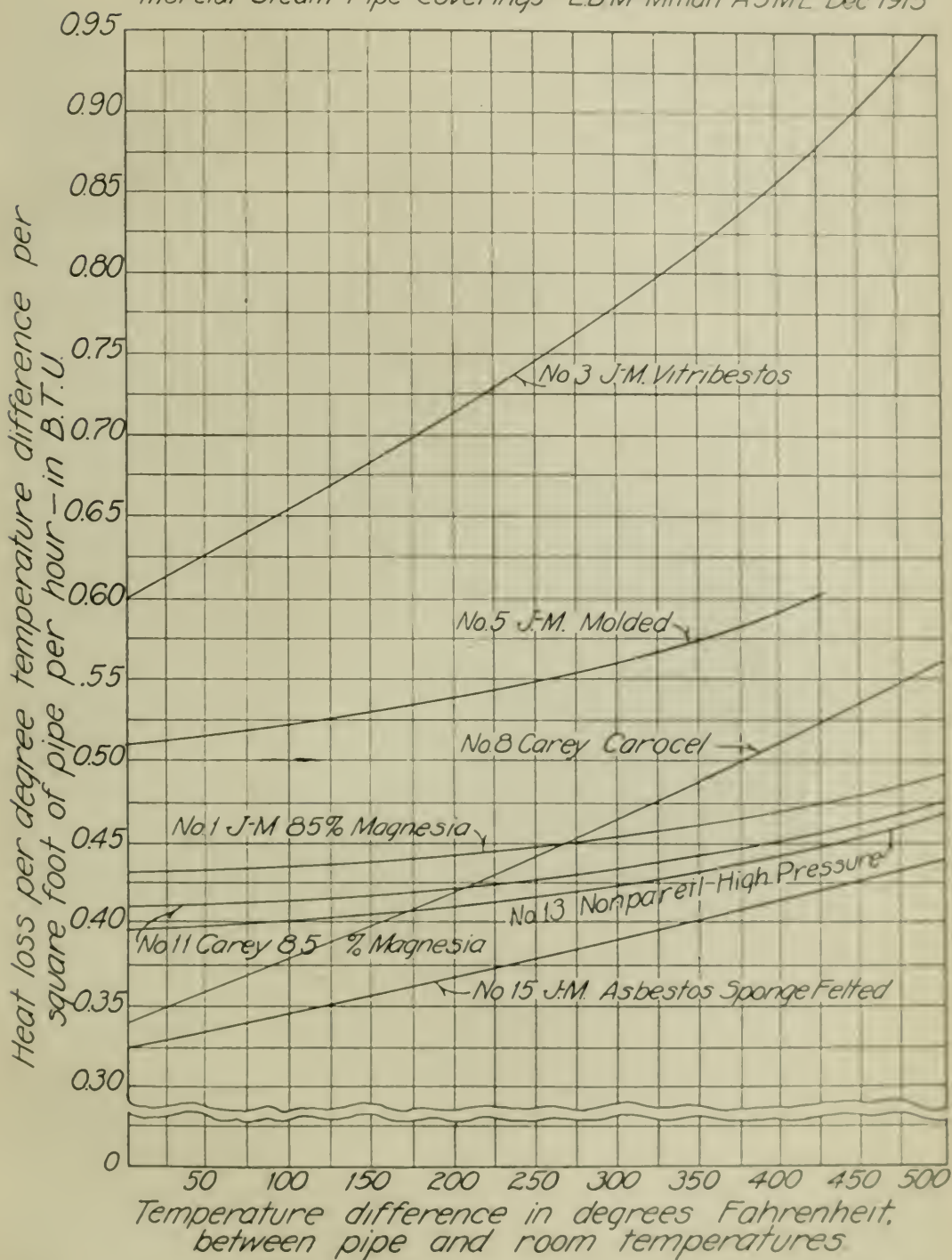


FIG. 16. CHART SHOWING AMOUNT OF HEAT TRANSMITTED BY STEAM PIPES INSULATED WITH COMMERCIAL COVERINGS (See Fig. 17 for Bare Pipe.)

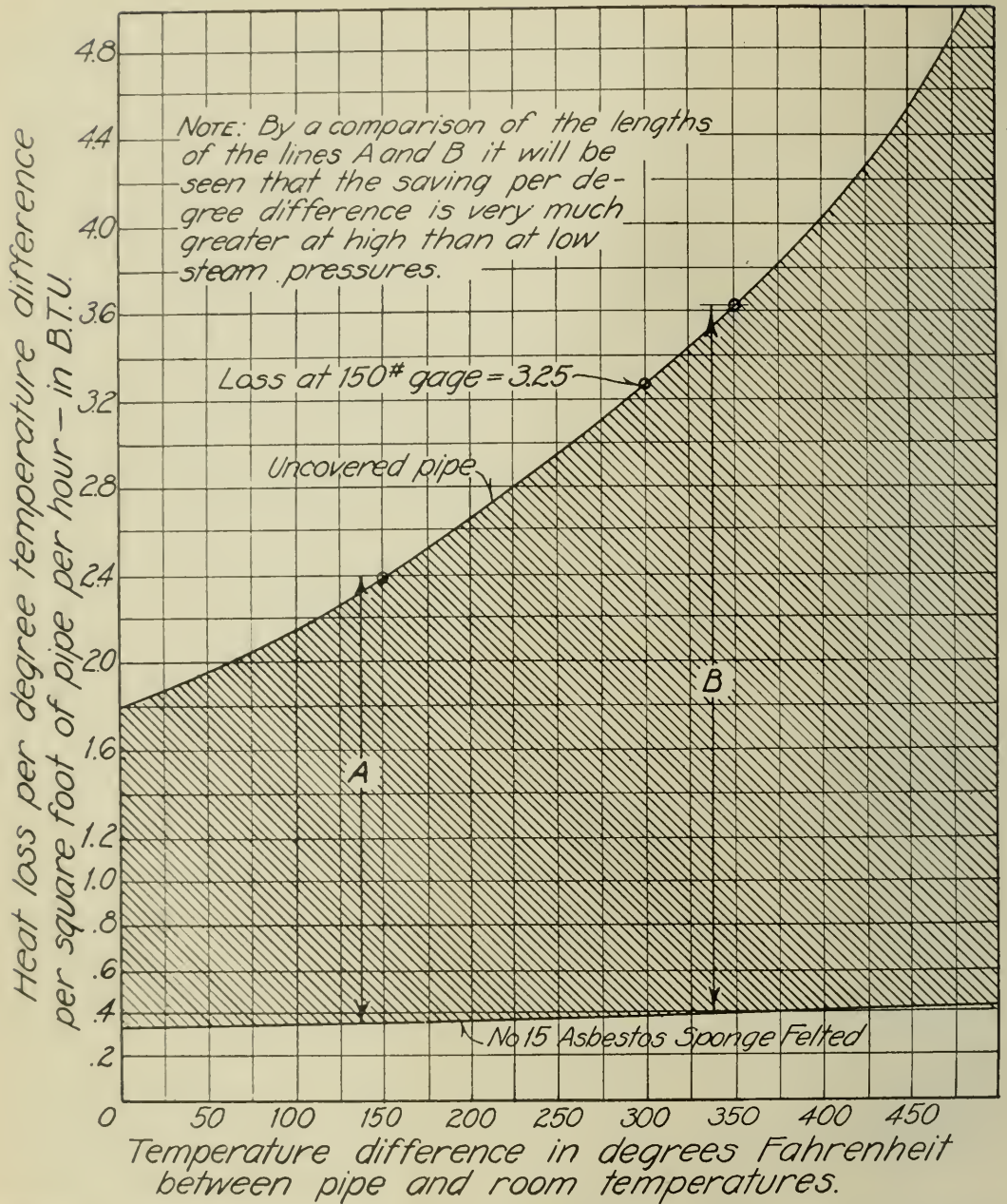


FIG. 17. CHART SHOWING HEAT LOST BY BARE STEAM PIPE AND SAVING WHICH MAY BE SECURED BY USING A GOOD COVERING

scale at the left of the chart, which shows the heat lost or transmitted per hour per square foot of pipe surface per degree difference of temperature between the steam in the pipe and the air outside. The heat loss is expressed in B. t. u.*

* For a definition of B. t. u. see foot-note, p. 17.

The importance of covering steam lines operating at high pressures and temperatures is emphasized by Fig. 17. It will be seen by reference to the upper curve in this diagram that the heat loss per square foot from an *uncovered* five-inch steam line increases very rapidly as the temperature of the steam in the line increases. By covering the line with a good insulator, not only is a large saving effected by reducing the heat loss (see shaded area), but the *increase* in the heat loss per square foot from such a covered line at high temperatures is very much less than for a bare pipe.

In other words, it is most important to cover all high temperature surfaces which are in contact with the air.

The chart (Fig. 17) shows that with steam which is 300 degrees F. above the temperature of the outside air a five-inch diameter standard bare steel pipe transmits about 3.3 B. t. u. per square foot per hour, while the same pipe covered with "Nonpariel High Pressure Covering" transmits (Fig. 16) only 0.425 B. t. u. per hour or thirteen per cent of the heat wasted by the bare pipe. In other words about seven-eighths of the loss has been stopped by the covering.

29. *Requirements for a Good Covering.*—A satisfactory pipe covering must be, (1) unaffected by heat or fire, (2) easily molded and light in weight, (3) impervious to or unaffected by water and steam, (4) non-corrosive in its effect upon metals (steel, iron and brass), (5) structurally fairly strong or self sustaining, and (6) sanitary and not attractive to vermin of any kind. The market affords a great variety of materials at various prices which are used for this purpose. But the purchaser must remember that he is buying heat insulation, not merely covering, and he must assure himself that the material is a practical and effective insulator.

In this connection, it should be noted that the soot which collects on the boiler tubes where no insulation is desired is several times as good an insulator as asbestos, and that the fine ash which also collects on the tubes is almost as good an insulator as soot. In other words, if it is advisable and economical to cover steam pipes with insulation, it is decidedly more economical and necessary to keep boiler tubes absolutely clean and free from the insulating effects of soot and ashes at all times. A similar argument applies to the scale which collects on the water side of the tubes or shell as a result of

infrequent cleaning or failure to employ suitable treatment for the feed water.

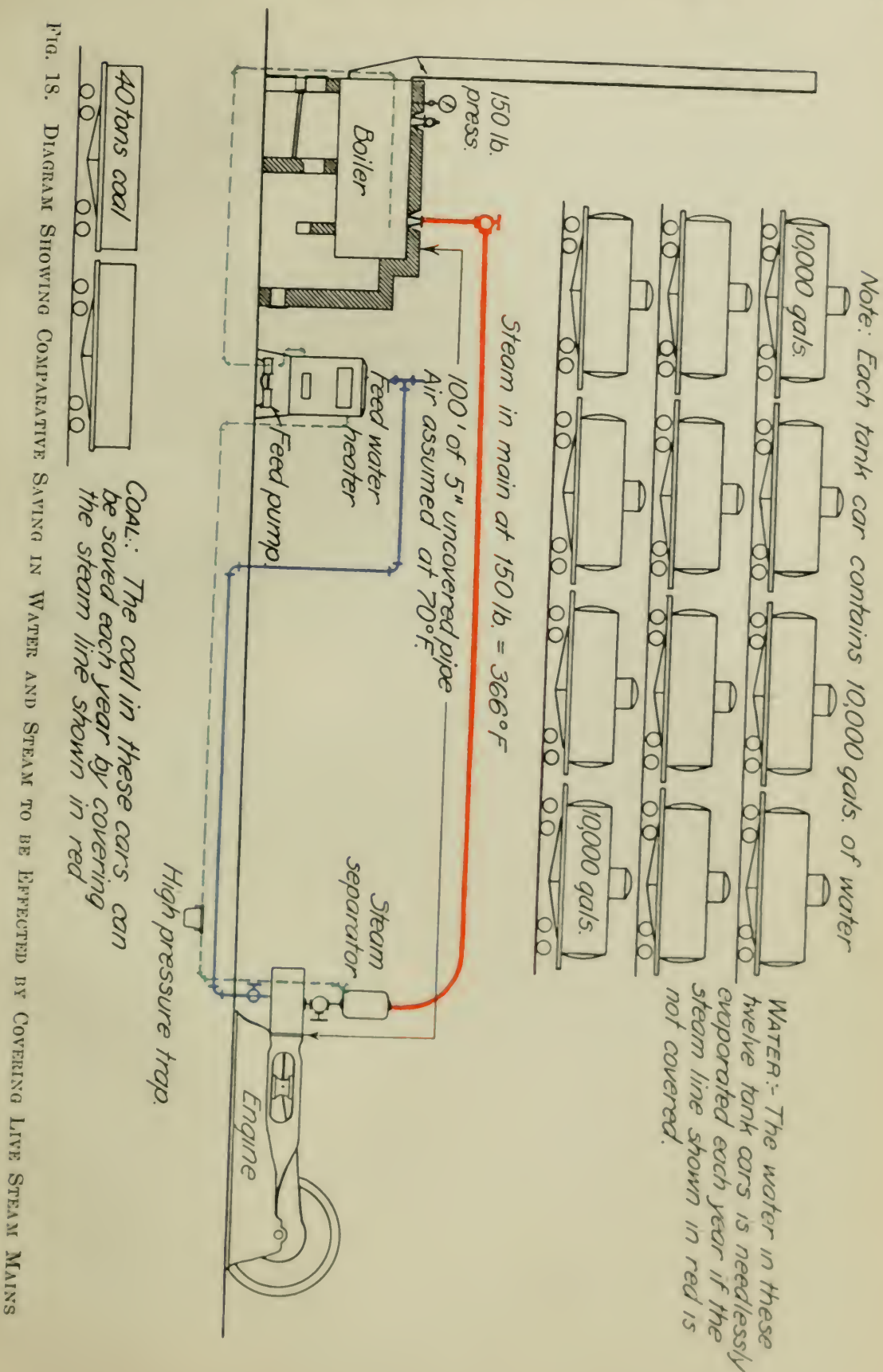
30. *Bad Effects of Water of Condensation in Steam Lines.*—Not only does an uncovered steam line waste heat by transmission to the outside air, but it *greatly increases the condensation*, requiring the boilers to furnish more steam than would be necessary in a covered line. It also adds to the amount of water to be handled by the traps, and results in excessive wear on the valve seats and through the steam ports of the engines. If a trap fails to operate properly, the accumulated water may travel along with the steam at high velocity and develop a “water hammer” which will loosen or break some fitting.

31. *Uncovered Pipes Waste Steam as Well as Coal.*—The owner of a power plant should realize that an uncovered steam main wastes heat and that it should therefore be covered. The loss of heat means a loss of steam by condensation and the generation of steam by the boiler plant which cannot be used.

That this loss is serious and that it requires the boilers, the feed pumps, and the traps to handle much more water than would be necessary if the steam mains were properly covered is shown by Fig. 18. The simple computations necessary to prove this statement for a typical case are given below. The plant shown in Fig. 18 is operating 24 hours per day for 365 days per year. A five-inch steam main, which is uncovered, carries steam at 150 pounds gage pressure to an engine 100 feet away. The air around the main averages 70 degrees F. and the feed water enters the boiler at 200 degrees F.

- (1) Actual tests show that each square foot of pipe loses 3.25 B. t. u. per hour for each degree of difference in temperature between steam inside and air outside (Fig. 17).
- (2) The outside surface of 100 feet of five-inch pipe amounts to 145.6 square feet.
- (3) The total heat loss from the pipe per hour is:
$$3.25 \times 145.6 \times (366-70) = 140,000 \text{ B. t. u.}$$

(The temperature of saturated steam at 150-pound gage is 366 degrees F.)
- (4) This heat is obtained *by condensing* not by cooling some of the steam in the pipe. One pound of steam gives up



858 B. t. u. when it condenses at 150 pounds gage pressure, or the uncovered pipe condenses $\frac{140,000}{858} = 163$ pounds per hour.

- (5) In one year, $163 \times 24 \times 365 = 1,428,000$ pounds of steam wasted; that is, this much steam *never reaches the engine*. At 8 1-3 pounds per gallon, 171,200 gallons of water have been needlessly handled.
- (6) Expressed in another way, this plant has evaporated and sent over 17 tank cars (of 10,000 gallons capacity per car) of water into this line which did not perform useful work. This represents an absolute waste of coal, of steam, and of boiler capacity. At least 75 per cent of this waste could have been prevented by covering the line.
- (7) To evaporate each pound of this water from feed water at 200 degrees F. took $168 + 858 = 1,026$ B. t. u. or a total of $1,026 \times 163 = 167,300$ B. t. u. per hour.
- (8) At 60 per cent efficiency each pound of coal (heat value taken as 12,000 B. t. u. per pound) gives to the boiler 7,200 B. t. u. or the plant is wasting 23.2 pounds of coal per hour to supply the condensation loss.
- (9) The coal required per year is:
 $23.2 \times 24 \times 365 = 203,000$ pounds or 101.5 tons.
- (10) This means that this plant has to burn about $21\frac{1}{2}$ cars of coal (holding 40 tons each), in order to provide for this annual heat loss. A good covering would stop 75 per cent of this waste and save two whole cars or 80 tons of coal a year. Bare steam pipe is a very expensive luxury in any power plant.

The cost of labor and material for covering 100 feet of the five-inch main referred to, including two valves and six fittings, is estimated at about \$160. This is based on present day conditions with the plant located within 150 miles of Chicago, using the best 85 per cent magnesia sectional covering, of $11\frac{1}{2}$ inches in thickness. The use of asbestos sponge felted covering would not add more than four or five per cent to this estimate, and if this work could be executed as part of a large covering job the cost could be reduced about twenty-five or thirty per cent.

VIII. RECORD OF OPERATION

32. *Purpose of the Record.*—The maintenance of such records of operation as may be necessary to enable the superintendent or owner of a plant to determine with reasonable reliability the cost of operation, the relative efficiency of the plant, and the improvement from time to time is essential. For practical purposes, the index to the performance of any steam generating plant lies in the relation between the number of pounds of water evaporated, and the number of pounds of coal fired less the weight of the ash. The record must therefore give the information on the basis of which this relation may be determined at any time, and it should also contain such other data as may be required for detecting and remedying any defects in operation which indicate loss.

33. *Character of the Record.*—It must be recognized that no satisfactory record of operation is possible unless the plant is equipped with certain checking and recording devices which have been recommended in the preceding pages of this discussion. These may be recounted as follows:

- (1) Means of weighing the coal fired for each boiler.
- (2) Means of weighing the ash removed from the pit.
- (3) Some device for weighing or measuring the water fed to the boiler or the steam delivered by the boiler.
- (4) A thermometer for indicating the temperature of the feed water.
- (5) A draft gage connected into the space above the fuel bed and into the ashpit.
- (6) A differential draft gage connected into the space above the fuel bed and into the flue gas passage near the point of discharge from the boiler.
- (7) A CO₂ analyzer.
- (8) A pressure gage at the boiler (pressure gages should also be supplied at the ends of all live steam lines).
- (9) A pyrometer for indicating the temperature of the flue gases leaving the setting.

Some of these devices are already part of the equipment of most

plants and the others may be secured at so slight a cost in proportion to the saving to be effected as easily to warrant their installation.

The weighing of the coal does not present any difficulties. The weighing of the ashes, however, is not so easy owing to the fact that in most plants the ash is wetted down to facilitate handling and to the possibility of a certain amount of unconsumed coal being present in the ash.

The function to be performed by each item of equipment included in the foregoing list has already been explained. The first four items are necessary for arriving at the relationship:

$$\frac{\text{Number of pounds of water evaporated}}{\text{Number of pounds of ash free coal fired}}$$

The last five items of equipment listed are needed to indicate the source of any defects in operation or troubles which may be leading to losses.

The daily record of operation should include the items shown in the form given on page 83.

Items 1, 2, and 3 of this record should cover the entire shift of the firemen for each boiler. Items 4 to 9, inclusive, should contain readings taken for each boiler at regular intervals (the suggested form provides for hourly readings) throughout the shift as frequently as may be practicable. The choice of the individual to be charged with the responsibility of maintaining this record is a matter which will depend largely upon the character and size of the existing organization of each plant. In some cases it may be found satisfactory to entrust the matter to the fireman; in others it may be desirable to assign it to some other employe. It should be recognized in any case that unless the record is maintained with reasonable care and accuracy its value is not great. If carefully kept the record will prove a means of stimulating the interest and coöperation of employes concerned with the operation of the plant as well as the basis for effecting economies the extent of which will in most cases be material.

NAME OF FIRM
DAILY RECORD OF POWER PLANT OPERATION

Date

1st shift M to M.

Name of Fireman.....

2nd shift M to M.

Name of Fireman.....

3rd shift..... M to M.

Boiler No.

Name of Fireman.....

	1st Shift	2nd Shift	3rd Shift
1. No. lb. Coal Fired			
2. No. lb. Ash Removed . . .			
3. No. lb. Water Evaporated			
4. Temp. of Feed Water . . .			
5. Boiler Pressure			
6. Draft Gage Readings . . .			
7. Differential Gage Readings			
8. CO ₂ Readings			
9. Temperature of Flue Gases			

34. *Profit Sharing or Bonus Systems.*—In some plants the practice is followed of permitting firemen and other employes to share in the savings which result from their efforts. Without undertaking either to commend or to condemn the profit sharing system as applied to power plant operation, the suggestion is offered that in any event no such system should be inaugurated until the plant is put in proper condition, i. e., until the setting has been made tight and is well covered, until live steam pipes are covered, and until such other changes have been made and devices installed as have been herein discussed. Having made these mechanical changes and improvements, the importance of which will be reflected in the results of operation, the earnest coöperation of employes concerned with the plant is essential in securing the maximum benefits, and the application of a profit sharing plan may in some cases prove the means of enlisting this coöperation.

IX. SUMMARY OF CONCLUSIONS

35. *Conclusions.*—To enable the plant owner to determine whether or not his installation conforms with the requirements of practice promoting fuel economy and to check up his methods of operation, the essential features discussed in the preceding pages are summarized as follows:

Coal

- (1) Practically the only fuel available for power plant use in Illinois under present conditions is bituminous coal from the central fields of Illinois, Indiana, and western Kentucky.
- (2) The care with which coal is “prepared,” and separated into different sizes is an important factor affecting its value in the power plant.
- (3) The B. t. u. value and the percentage of ash furnish a general guide to the relative values of Illinois coals. Generally the coals having the lowest ash content have the highest B. t. u. value.
- (4) The storage of bituminous coal is both practicable and desirable. Certain precautions, however, must be observed. These are set forth in detail on page 18.*

Principles to be Observed in Firing

- (5) The three fundamental conditions necessary for complete and smokeless combustion of bituminous coal are:
 - (a) A sufficient amount of air must be supplied.
 - (b) The air and fuel must be intimately mixed.
 - (c) The mixture must be brought to the ignition temperature and maintained at this temperature until combustion is complete.
- (6) Since bituminous coal from the central field is practically the only fuel available at present for power plant use in Illinois the boiler setting and the plant in general should be adapted to the economical use of this fuel.

* See also Univ. of Ill. Eng. Exp. Sta. Circular 6, entitled, “The Storage of Bituminous Coal,” by H. H. Stock.

- (7) Every boiler should be equipped with two draft gages, one connected directly in the space over the fire, and one connected both into the space over the fire and into the gas passage below the damper. For every given load, the draft necessary to carry it and the proper thickness of fuel bed with the grade of coal used should be determined.
- (8) In many plants of large and medium size automatic draft control has proved economical, and it is also of advantage in maintaining constant steam pressure.
- (9) Every plant should have some simple type of CO₂ analyzer for obtaining a knowledge of conditions existing within the furnace. (See page 30.)
- (10) Air leakage through the boiler setting should be prevented by properly calking and covering the setting.
- (11) Losses due to the presence of unconsumed coal in the ash should be avoided by seeing that the fire is properly worked and that the grate openings are not too large for the size of fuel fired.
- (12) Sooty deposits on the heating surfaces should be removed frequently. Should the temperature of the gases leaving the boiler exceed 550 degrees F., it probably indicates that the tubes need blowing.
- (13) Scale on the water surfaces of the boiler should not be allowed to accumulate.
- (14) The spreading method of firing in which small quantities of coal are fired at frequent intervals is regarded as a satisfactory method for hand fired plants. (See page 41.)

Features of Boiler Installation

- (15) The foundation for a boiler setting should rest on a firm footing in order to insure a setting which will remain tight and free from any tendency to crack.
- (16) For the complete combustion of bituminous coal boiler settings must provide for the introduction of sufficient air into the furnace, for the proper mixing of this air with the gases given off by the burning coal, and for the maintenance of a high temperature until the process of combustion is complete. This is to be accomplished by means of arches, baffles, and other devices as hereinbefore described. (See page 44.)

- (17) The brick work in setting should be properly pointed up and covered with an insulating material to prevent air leakage. The exposed parts of the shell of horizontal return tubular boilers and of the steam drums of water tube boilers should be covered with a high grade of asbestos insulating material at least two inches thick, or with an 85 per cent magnesia covering two or three inches thick, and the outside finished off with a thin coat of hard cement or covered with canvas and painted. Air spaces in the walls of the setting should be filled with sand or ashes.

Stacks and Breechings

- (18) In order to control the amount of air and flue gas passing to the stack a damper installed at the point where the flue gas leaves the boiler should be regulated so as to permit the stack to supply the right amount of air to burn completely the fuel fired. The air supply should be controlled by this damper and not by opening and closing the ashpit doors, which should stand open practically all the time. Each boiler should have its individual damper.
- (19) The individual boiler dampers should fit accurately and close tight, otherwise it will be impossible to prevent cold air from entering the main breeching through the damper of a "dead" boiler.
- (20) The breeching and stack should be made air tight and should be insulated to prevent heat loss from the flue gases so that all the heat in the gases may be available for creating draft.

Feed Water and Fuel

- (21) If the feed water used causes scale, corrosion, or priming it should be analyzed by a reliable chemist and treated in such manner as he may prescribe.
- (22) It is necessary and economical to heat the feed water. One per cent of fuel is saved for every eleven degrees rise in the feed water temperature.
- (23) For the majority of small boiler plants the feed water should be introduced into the boiler at a constant rate rather than intermittently. The feed water system should include some form of metering device for measuring the water fed to the boiler or the steam delivered by it.

- (24) Means should be provided for weighing the coal fired to each boiler and the ash removed.

Steam Piping Requirements

- (25) The piping system should be as simple as possible and well insulated. Only short direct runs of live steam pipes should be used in connecting boilers, engines, and other steam using apparatus.
- (26) Each high pressure header and steam separator should be provided with drips and the hot water returned to the feed water heater.
- (27) Leakage losses at valves and fittings in all steam and water lines should be stopped at once.
- (28) If a steam main is either too small or too long it will be necessary to carry a higher pressure at the boiler to get the required pressure at the engine. Steam gages at the boiler and at the engine throttle should show a drop in pressure of not more than five pounds when the engine is running at full capacity. If a steam main is too large the pressure at the engine throttle will be almost the same as that at the boiler. The heat losses from oversize pipes and fittings are somewhat greater and the cost of installation is higher than for mains of the proper size. The exhaust piping should be of such size that a gage near the engine will show a pressure of not more than two pounds.
- (29) All steam and hot water piping, fittings, flanges, and valves should be covered with an insulating material. The saving to be affected by such covering is sufficient to repay the cost in the first few months. (See page 76.)

Record of Operation

- (30) A suitable record of operation should be maintained upon the basis of which the superintendent or owner of the plant may determine with reasonable reliability the cost of operation, the relative efficiency of the plant, and the improvement from time to time. From the record of operation it should be possible to determine whether the individual boilers are operating at their rated capacities. In cases in which boilers are operating at less than capacity conditions should be so changed as to require each unit to carry

its full load or an overload. In cases in which it is not possible to balance the load with the combined capacity of units, it is economical to operate as many boilers as possible at capacity and to throw the excess on an extra unit.

- (31) Special emphasis is to be laid upon the problem of the plant owner and upon the part he must play in the program of saving fuel. The economical utilization of fuel in power plants is vital, not so much because it means a cash saving to the owner, but rather because conditions are fast approaching a point at which the owner who does not conserve his fuel may find himself unable to maintain continuous operation.

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1

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

CIRCULAR No. 8

SEPTEMBER, 1918

THE ECONOMICAL USE OF
COAL IN RAILWAY LOCOMOTIVES

PREPARED UNDER THE DIRECTION OF A COMMITTEE CONSISTING OF
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CONTENTS

	PAGE
I. INTRODUCTION	7
1. Scope and Purpose	7
2. Authorship	7
II. THE AMOUNT OF COAL USED BY RAILWAYS	9
3. Fuel Burned in Railway Locomotives	9
III. WHAT BECOMES OF THE COAL AND THE HEAT	11
4. Sources of Loss	11
5. Losses in Transportation and Storage	11
6. Coal Consumed when the Locomotive is Standing	11
7. The Distribution of the Heat Generated in the Firebox	12
The Firebox and Boiler	13
The Locomotive	15
Sheer Wastes	19
8. Summary of Losses	19
IV. HOW TO SAVE COAL	29
9. Means to be Employed	29
10. Selection, Distribution and Storage of Coal	30
11. Roundhouse and Shop Equipment and the Care of Locomotives	30
12. Locomotive Design	31
13. Locomotive Operation	32
Fuel Consumption while not Hauling Trains	32
Theory of Combustion	33

	PAGE
Fuel Consumption while Hauling Trains	35
Firing Instructions	36
The Significance of Smoke	47
V. COAL	49
14. Kind of Fuel Used by Railroads	49
15. Classification of Coal	49
Anthracite	51
Semi-Anthracite	51
Semi-Bituminous	51
Bituminous	52
16. Preparation a Factor Affecting the Value of Coal	55
17. The Selection of Coal	55
18. Purchase of Coal	56
19. Storage of Coal	57
Kinds and Sizes of Coal which may be Safely Stored	58
The Effect of Sulphur on Spontaneous Com- bustion	61
Method of Piling Coal	62
The Effects of Storage upon the Properties of Coal	62
Additional Precautions	63
Expense of Storing Coal	63
VI. SUMMARY	65
20. Ways in which Locomotive Coal may be Saved .	65
In Handling Transportation	65
In Locomotive Design	65

	PAGE
In Maintenance	65
In Handling and Care of Locomotives . .	66
In Locomotive Operation	66
Coöperation	66
21. Estimated Possible Saving to be Effected in the	
Use of Locomotive Coal	66
Executive Officials and Transportation Forces	67
Executive and Motive Power Officials . . .	68
Motive Power Officials and Shop and Round-	
house Forces	68
Roundhouse Forces and Hostlers	69
Engineers and Firemen	70

LIST OF FIGURES

NO.		PAGE
1.	Heat Distribution for the Firebox and Boiler	14
2.	Heat Distribution in the Locomotive	17
3.	Heat and Coal Distribution—Locomotive <i>A</i>	21
4.	Heat and Coal Distribution—Locomotive <i>B</i>	22
5.	Heat and Coal Distribution—Locomotive <i>C</i>	23
6.	Heat and Coal Distribution—Locomotive <i>D</i>	24
7.	The Uniform Fire	37
8.	Banks in the Fire	38
9.	Holes in the Fire	39
10.	The Uniform Fire	40
11.	Banks in the Fire	41
12.	Holes in the Fire	42
13.	Diagram Showing the Chemical Composition and Heat Value of the Several Ranks of Coal	53
14.	A Long Pile of Stored Coal	59
15.	Storage Coal with Unloading Track and Crib	59
16.	A Circular Pile of Stored Coal	60
17.	A Coal Storage Station and Chute	60

LIST OF TABLES

NO.		PAGE
1.	Heat Uses and Losses which Occur on a Locomotive	20
2.	Coal Saving through the Use of the Superheater and the Brick Arch	28
3.	Expense of Storing Coal	64

THE ECONOMICAL USE OF COAL IN RAILWAY LOCOMOTIVES

I. INTRODUCTION

1. *Scope and Purpose.*—Conditions brought about by the war place upon all classes of persons responsibilities which are inescapable. The positive need for the coöperation of the individual and for the exercise of every personal effort in the prosecution of the war is daily becoming more apparent. It is recognized that the greatest service which the individual not actually engaged in military service may render lies in doing what he may to conserve the nation's supplies. One of the greatest opportunities open for service in this respect is in connection with the nation's fuel supply. The successful issue of the war depends upon the economical use of coal both in the home and in the industrial plant and locomotive.

The Engineering Experiment Station of the University of Illinois has undertaken to prepare for users of fuel certain publications containing suggestions and advice relating to fuel conservation. Circulars dealing with the use of coal in heating homes and in generating power have been published as have also bulletins and circulars relating to the extraction of coal in the mines and its storage. The present circular has the purpose of helping railway officials and employes in their efforts to save coal. While it contains little that is new, it presents a simple statement of the facts concerning the choice, distribution, storage, and use of coal, and offers some conservative suggestions concerning the ways in which coal may be burned economically.

This circular is issued with a full understanding and appreciation of the intelligent and wide-spread effort which railway men are making to save coal, and it aims only to supplement these efforts as it may. If it succeeds in stimulating greater interest in the subject and in bringing even a few helpful suggestions to those into whose hands it may fall, it will have served its purpose.

2. *Authorship.*—The facts presented in this circular have been compiled by a committee consisting of J. M. SNODGRASS, Assistant Pro-

fessor of Railway Mechanical Engineering (Chairman), MAJOR E. C. SCHMIDT, U. S. R. Engineer, Conservation Division U. S. Fuel Administration, and Professor of Railway Engineering, University of Illinois (on leave of absence), H. H. STOEK, Professor of Mining Engineering, S. W. PARR, Professor of Applied Chemistry, and C. S. SALE, Assistant Professor of Civil Engineering and Assistant to the Director of the Engineering Experiment Station.

This committee has had the assistance of an advisory committee consisting of E. W. PRATT, Assistant Superintendent of Motive Power, Chicago and Northwestern Railway Company, Chicago, Ill., W. L. ROBINSON, Fuel Supervisor, Baltimore and Ohio Railroad Company, Baltimore, Md., A. N. WILLSIE, Chairman, Fuel Committee, Chicago, Burlington and Quincy Railroad Company, Chicago, Ill., TIMOTHY SHEA, Acting President, Brotherhood of Locomotive Firemen and Enginemen, Cleveland, O., A. B. GARRETSON, President, Order of Railway Conductors of America, Cedar Rapids, Ia., W. S. STONE, Grand Chief, Brotherhood of Locomotive Engineers, Cleveland, O., O. P. HOOD, Chief Mechanical Engineer, Bureau of Mines, Washington, D. C., D. M. MYERS, Advisory Engineer on Fuel Conservation, U. S. Fuel Administration, Washington, D. C., and C. R. RICHARDS, Dean of the College of Engineering and Director of the Engineering Experiment Station, University of Illinois. Each member of this advisory committee has personally reviewed the original manuscript and a meeting was held at Urbana on March 22, 1918, at which the manuscript was examined in detail. The authors gratefully acknowledge the valuable assistance and coöperation of the members of this committee and feel that the value of the publication has been greatly increased as a result of their efforts.

II. THE AMOUNT OF COAL USED BY RAILWAYS

3. *Fuel Burned in Railway Locomotives.*—A considerable portion of the coal bought by railways is used in stationary power plants at shops and roundhouses, and in heating plants at stations and elsewhere. While there are many opportunities for effecting greater economy in the use of stationary plant fuel, this subject is not considered in the present circular; it has already been covered in previous publications* issued by the Station. The use of oil fuel, also, is not discussed, since oil is used on only a few roads and since the extent of its use, under prevailing prices, is decreasing. Many of the suggestions concerning coal are, however, obviously applicable to oil.

During the calendar year of 1917 there were produced in the United States about 545,000,000 tons (2000 pounds) of bituminous or "soft" coal and about 92,000,000 tons of anthracite. The railways of the country used 26 or 27 per cent of the bituminous coal and 6 or 7 per cent of the anthracite, or a total of about 155,000,000 tons. About 140,000,000 tons or about 22 per cent of the total production were burned in locomotive fireboxes. The corresponding amount for the year 1918 is estimated to be about 150,000,000 tons. This fuel is referred to in this circular as locomotive coal. It is not easy to appreciate the significance of such a quantity. It would take 3,000,000 fifty-ton capacity ears to hold it; every coal car in the country would need to be loaded more than three times each year to carry the locomotive coal alone.

Trains carrying it in one load would have a total length of 24,000 miles, or a distance equivalent to the circumference of the earth. During the year 1918 locomotive coal will probably cost the railways more than \$500,000,000; thus the fuel cost will be the largest single item of operating expense except only the wages of trainmen and engine-men.

Obviously the saving of even a small percentage of the large quantity of fuel consumed by locomotives is a result worth striving for. In

*Circular 4, entitled "The Economical Purchase and Use of Coal for Heating Homes with Special Reference to Conditions in Illinois," 1917, and Circular 7, entitled "Fuel Economy in the Operation of Hand Fired Power Plants," 1918, of the Engineering Experiment Station of the University of Illinois.

view of all the earnest effort already being expended to save coal, the possibility of effecting a still further saving is not at once apparent. A study of the problem, however, discloses many means of effecting greater economy which have not as yet been generally employed. The saving on each scoopful of a piece of coal as big as an ordinary egg would amount to one per cent or 1,500,000 tons. Even when firing a modern freight locomotive on a heavy grade, one less scoopful of coal every fifteen minutes, or one less scoopful every three or four miles, would effect a similar saving. A little more personal interest and more care will reduce coal consumption by such a percentage even on railroads where the practice is already excellent, and where fuel conservation is already receiving close attention. On many railroads where efforts at fuel economy are not so well organized much greater savings can undoubtedly be effected.

By a nation-wide attack on this problem involving the concerted interest and action of all concerned, practices relative to fuel conservation may be so improved as to save at least 9,000,000 tons of coal per year. This amount is more than enough to supply the needs for an entire year of such a system as the Chicago and Northwestern Railway, or the Atchison, Topeka and Santa Fe, each of which operates about two thousand locomotives. This amount of coal would be sufficient to coal a fleet of transports, United States to France, which would be capable of carrying an army of three to five millions of soldiers with supplies and equipment.

III. WHAT BECOMES OF THE COAL AND THE HEAT

4. *Sources of Loss.*—Any attempt to economize in the use of fuel must be based upon a clear understanding of what happens to the coal from the time it is loaded at the mines until it reaches the locomotive firebox, and what becomes of the heat liberated in the firebox. Each operation from the mining of the coal to its combustion in the firebox presents possibilities of loss or waste.

In the following sections the several possible sources of waste are discussed and the ways in which coal may be used without doing any useful work whatever are pointed out. It is not the purpose to consider at this point the ways in which savings may be effected, but simply to indicate the points to which attention and effort should be directed.

5. *Losses in Transportation and Storage.*—In transit from the mines to the chutes there is some direct loss of coal from the cars due to carelessness in loading. Probably this loss is not large, but it can be and should be entirely eliminated. At the chutes little coal, as such, is lost but its fuel value is sometimes considerably diminished by improper handling which causes undue breakage and which ultimately results in a direct loss on the locomotive by increasing the cinder discharge through the stack because of the excess of fine material in the coal.

At certain seasons and under certain market conditions it is essential for the railways to store large quantities of coal, generally in outdoor piles. These piles frequently take fire with the result that the fuel value of the coal so stored is destroyed or impaired. Except in rare instances, this is a preventable loss resulting from improper methods of storage. These losses can generally be avoided by exercising precautions which are discussed in a later chapter of this circular.

6. *Coal Consumed when the Locomotive is Standing.*—About one-fifth of all locomotive coal is consumed at periods when the locomotive is not doing useful work, in firing up, in waiting for trains, in standing on sidings and passing tracks, in delays at terminals, and in waiting at ashpits. During the year 1918 nearly 30,000,000 tons of coal will be used during these periods.

While this consumption is not directly productive, it is, of course, not all to be regarded as waste. Locomotives must be fired up and their fires must be cleaned; a certain amount of time must also inevitably be spent in yards, terminals, and sidings. All these are essential activities incident to operation, but since so large an amount of coal is involved it is clearly necessary that under present conditions these periods should be reduced to a minimum. The saving of fuel in this connection reduces itself to a question of saving time in these operations.

This is a part of the fuel conservation problem over which trainmen have little or no control; roundhouse men can help in one or two respects, but the chief responsibility lies with operating officials and dispatchers. The results to be obtained will come chiefly from forethought and attention to the details of operation. The best results have undoubtedly not been reached even on the best managed and best equipped roads. The magnitude of the amount of coal involved should provide a strong incentive to renewed effort and care.

It is proper to point out, however, that on some railroads any very radical decrease in this part of the coal consumption involves expensive and time-consuming physical improvements such as the redesign of terminals and the enlargement or increase in the number of sidings. Such work requires large expenditures of capital and cannot always be done hurriedly even when the money is available. The responsibility for determining whether such improvements may be made lies, of course, with executive officers. Where such improvements looking toward extensive savings are impossible, that fact should not diminish the enthusiasm for attempting to effect the smaller savings which are possible in any situation.

7. *The Distribution of the Heat Generated in the Firebox.*—There remain four-fifths of the locomotive coal which is burned while locomotives are hauling trains. For the year 1918 this will amount to about 120,000,000 tons. Naturally, the campaign for economy has always been most vigorously waged with respect to this fuel and the greatest effort in the future must continue to be directed at the possibility of effecting greater savings in the use of it.

In this section it is the purpose to show what becomes of the heat generated in burning this coal—to show, that is, where the heat goes while the locomotive is at work on the road, always with the view of

disclosing the avenues of loss and waste as well as the points at which efforts at conservation are likely to prove most effective.

Although many avenues of waste occur in connection with the actual burning of the coal and the heat generated in the firebox, it should be recognized that enginemen have no control over some of the losses which occur. Many kinds of fuel waste can be prevented only by better locomotive maintenance, and by better roundhouse and shop practice. Motive power officials and shop and roundhouse employes must assume the responsibility for heat losses which result from poor locomotive maintenance.

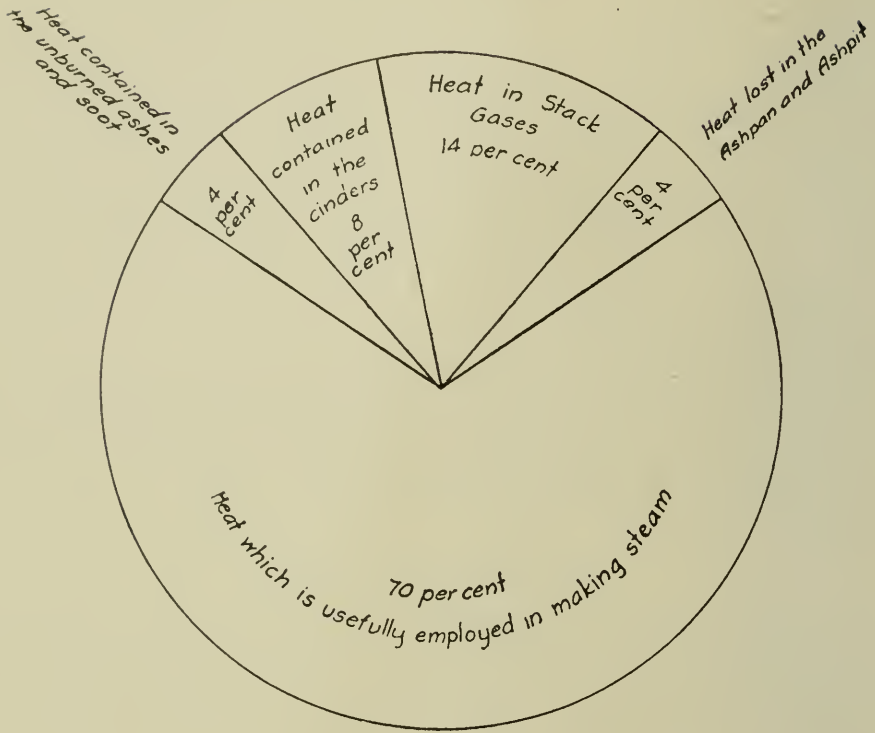
Certain maintenance improvements and certain matters of design such as the installation of washout apparatus, superheaters, or brick arches require such expenditures of capital and time that they cannot be considered matters for emergency treatment.

The Firebox and Boiler

Fig. 1 shows graphically what may become of the heat contained in the coal which is shoveled into the firebox in so far as the firebox and boiler are concerned. The proportions are intended to be representative of a modern locomotive, burning from 70 to 80 pounds of western bituminous coal per square foot of grate surface per hour.

The largest area represents that portion of the heat of the coal which is usefully employed in making steam. The four smaller areas represent avenues of loss and the size of these areas with the percentage figures which accompany them indicates the possible magnitude of these losses under normal operation. The losses may be much greater when locomotives are in poor condition. In all cases, however, they may vary to a considerable extent depending upon the degree of excellence with which the locomotive is fired and handled.

The ashpan and ashpit losses, the losses represented by the unburned gases and cinders, and those due to the heat in the escaping gases indicate the directions in which efforts to effect savings should be made. The largest of these losses, that due to heat in the gases escaping from the stack, is in large part unavoidable, since these gases must always escape at a temperature near to or somewhat above the temperature of the steam in the boiler. Under service conditions also the loss due to unburned fuel discharged into the ashpan or to cinders and combustible gases discharged through the stack will always take place to some extent. In such losses there is, however, opportunity



100% coal burned in locomotives annually	(1918 est) 150,000,000 tons
70% is applied in making steam	105,000,000 tons
14% goes out of the stack as heat in gases	20,800,000 tons
8% goes out of the stack as cinders	12,000,000 tons
4% goes out of the stack as unburned gases	6,000,000 tons
4% goes into the ashpan and ashpit	6,000,000 tons

(These figures are based upon good performance, which is much better than the average performance of all the locomotives of the country.)

FIG. 1. HEAT DISTRIBUTION FOR THE FIREBOX AND BOILER

for considerable savings. While it is impossible to eliminate entirely any of these losses, each may, through skilful and careful firing and engine operation, be reduced. The total loss from the four sources amounts to 30 per cent of the heat value of the coal fired, and the difference between good and poor practice with a given locomotive represents at least an appreciable percentage of the total coal used. What it means to save one per cent of the total coal used in locomotives has already been pointed out.

The Locomotive

Fig. 2 presents graphically and in somewhat different form the same information which is presented by Fig. 1 together with some additional facts relating to the avenues of use and loss as the steam passes through the engines and the front end of the locomotive. This illustration shows the possible distribution of the heat when coal is being burned at the rate of 70 or 80 pounds per square foot of grate surface per hour. As in Fig. 1 there is shown a 4 per cent ashpan loss, a 14 per cent loss to heat in the stack gases, a 4 per cent loss to unburned gases, and an 8 per cent loss to cinders. The remaining 70 per cent of the heat of the coal is stored in the steam. Of this stored heat, 5 per cent is shown as lost by radiation, 6 per cent goes to the operation of the air pumps, blower, etc., and 59 per cent goes to the cylinders. Of the heat going to the cylinders, a large part, 52 per cent of the original heat of the coal, is exhausted and passes out through the stack. Of the remaining 7 per cent, 1 per cent is used up in overcoming the friction of the locomotive, and 6 per cent is obtained as work at the drawbar.

The losses shown by Fig. 2 as occurring after the heat is stored in the steam are those due to radiation, to steam using devices other than the main cylinders, to the heat of the exhaust steam, and to the heat required to overcome the friction of the locomotive itself.

The radiation losses probably cannot be greatly reduced, and whatever reduction is possible is a matter of design and maintenance in connection with the lagging.

That portion of the heat going into the air pumps and other steam using devices is a loss only in so far as more steam is used in this way than is necessary. The operation of bell ringers, reverse gear, and grate shakers as well as the lighting and heating require the consumption of steam and coal. The estimate that 6 per cent of the heat is

given to air pumps and other steam using devices is a comparatively low average value. Air leaks, particularly in freight service, may greatly increase this loss. The air pumps ordinarily use the greatest amount of steam for auxiliary work and the amount of coal and steam which they use depends very largely upon how well the air brake equipment is maintained. The opportunity for saving here lies in the proper maintenance of all auxiliary apparatus and its careful operation by the enginemen.

The largest heat loss is that of 52 per cent in the steam which is exhausted from the cylinders and passes out through the stack. This exhaust steam, however, does a large amount of useful work in forcing air through the grates and in forcing the gases of combustion through the firebox and boiler. The possibility of reducing this loss is mainly a matter of design, although maintenance and the care with which the throttle and reverse lever are handled are important factors in the problem.

About one per cent of the total heat of the coal or one-seventh of the energy developed in the cylinders is consumed in operating the locomotive itself, or overcoming locomotive friction. The percentage lost as locomotive friction is a serious one since it affects directly the effective work done at the drawbar. Assuming that, in the case given, the locomotive friction loss could be reduced from one per cent to three-fourths of one per cent, then the effective work at the drawbar would be increased from 6 per cent to $6\frac{1}{4}$ per cent; that is, the effective work would be increased about 4 per cent. This would mean that the locomotive could do the same work at the drawbar with a saving of about 4 per cent of its coal. Savings in connection with locomotive friction are largely matters of proper maintenance of the locomotive running parts and of careful lubrication.

The heat losses so far considered are those which to a greater or less extent exist under normal conditions, or with the locomotive in what may be called good operating condition. Most of these losses such as the radiation loss, the loss of heat out of the stack, or the heat which goes to the performance of work necessary to the operation of air pumps or draft apparatus are in part unavoidable. With the locomotive in poor condition, these losses will be larger. In like manner if the locomotive is not skilfully fired and operated, most of these losses will be increased. The opportunity for saving comes in reducing these losses to the lowest possible amounts through better design, better maintenance, and skilful engine operation and firing.

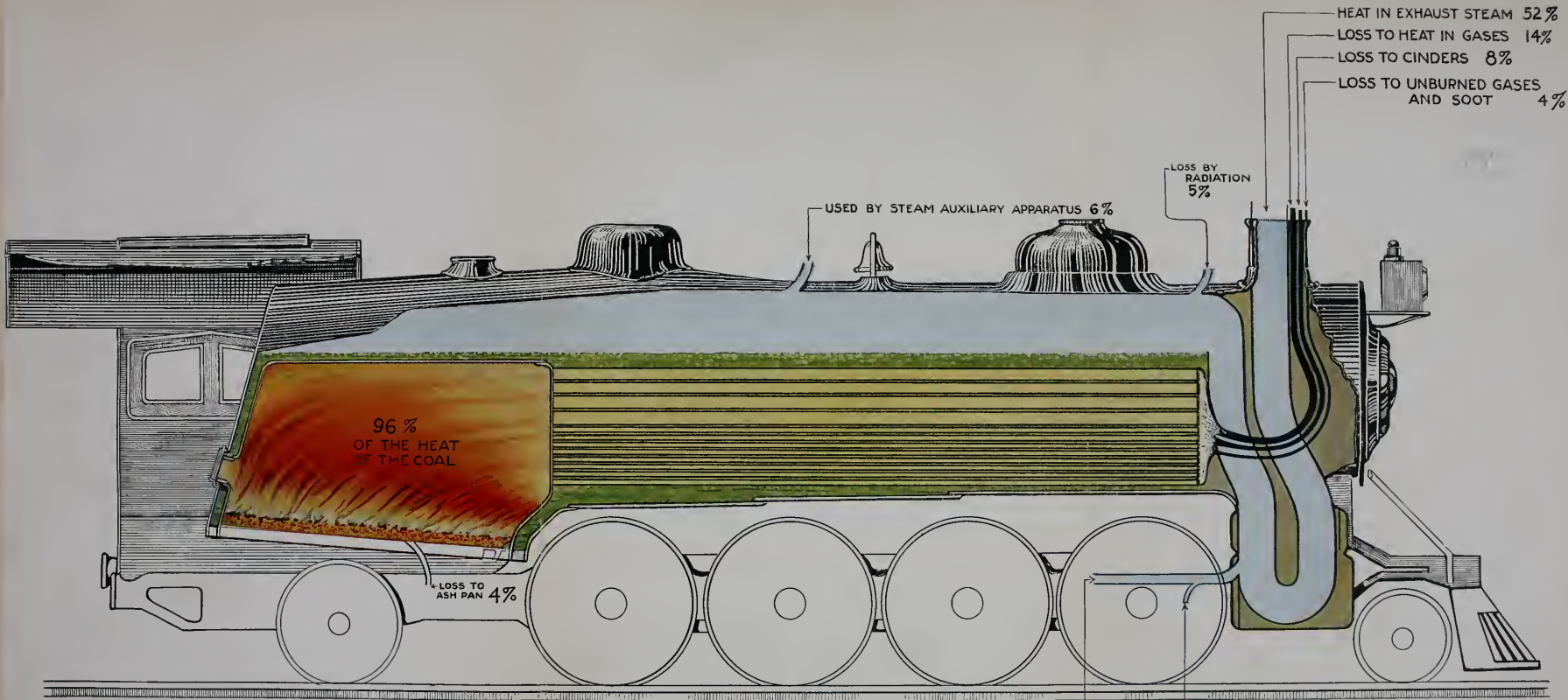


FIG. 2. HEAT DISTRIBUTION IN THE LOCOMOTIVE

Combustion in the firebox liberates heat from the coal.

Combustion is not complete, and all the heat of the coal is not liberated. Cinders and combustible gases escape unburned from the firebox and pass out through the stack.

Some of the heat liberated from the coal is carried out of the stack by the stack gases. Air and coal at a temperature of say 70 degrees F. go into the firebox; the gases which they form come out of the stack at 500 or 600 degrees F. For each ton of coal which goes into the firebox about 12 tons of air also go in and about 13 tons of gases pass out of the stack. It takes some of the heat of the ton of coal to heat these 13 tons of gases from 70 degrees to 600 degrees.

The heat liberated in the firebox, except that which is used to heat the stack gases, passes through the firebox sheets, the boiler tubes, and the superheater units to the water and steam.

Some heat is lost by radiation. This comes from all parts of the locomotive which are warmer than the surrounding air. The most of the heat lost by radiation comes out of the steam.

The heat in the steam is used for auxiliary purposes, is lost as leaks, or goes to the cylinders to do work.

Some coal is lost to the ashpan and ashpit.

Of the heat going to the cylinders a large portion passes out the stack in the exhaust steam. This exhaust steam produces the draft necessary to force into the firebox and through the firebox, boiler, and front end the 12 tons of air and the 13 tons of gases which are moved for each ton of coal fired.

That portion of the heat converted into work in the cylinders moves the locomotive and its train. A portion moves the locomotive and its parts and the remainder becomes work usefully applied in hauling tonnage.

Fig. 2 is shown without a brick arch. Firebox conditions similar to those indicated might be more readily maintained in fireboxes with arches.

Fig. 2 and the succeeding colored figures are somewhat conventional representations of locomotive firebox conditions. The intense, almost white, heat which often exists in practice cannot be represented satisfactorily with colors. The coloring and definition are somewhat exaggerated in the desire to emphasize the particular features being illustrated or under discussion.

Sheer Wastes

In addition to the losses already considered, there are certain losses which may be called complete or sheer wastes. Such wastes give no return for the heat lost and are for the most part avoidable. Leaks of steam or hot water as at the mudring or of steam at the piston rod or valve stem packing mean the escape of heat and its complete loss. Steam leaks into the firebox at the superheater header or from the steam pipes mean direct loss of heat and probably a greater loss due to the poor conditions which such leaks bring about in connection with engine performance. Front end air leaks mean a coal loss, and blows past the engine pistons and valves greatly increase the amount of steam required and consequently the amount of coal which must be burned. Considerable steam may be wasted through safety valves. Forty or fifty pounds of coal per minute may be required to furnish steam to discharging safety valves. Frequent and long continued blowing of safety valves means the burning of more coal than is necessary.

In the preceding discussion the percentage of the available energy which is obtained at the drawbar as work has been assumed to be usefully employed. Improper or poor maintenance of cars and car equipment may, however, cause a waste of the power delivered at the drawbar, and therefore may require the consumption of a larger amount of coal than would otherwise be necessary. Sticking brakes, poorly maintained or poorly lubricated journals, or other defects which unnecessarily increase train resistance or cause delays or stops result in a waste of coal. The importance of the loss which results from air line leaks has already been pointed out. Losses of considerable importance may likewise take place in connection with train heating; such losses could be reduced and the comfort of passengers increased by the more general use of thermometers or even of thermostatic control apparatus in passenger coaches. Fuel wastes which may be eliminated through better maintenance of cars and car equipment should not be overlooked by those responsible for this part of the maintenance work.

8. *Summary of Losses.*—A graphical summary of the coal or heat uses and losses which have been discussed is presented as Figs. 3, 4, 5, and 6, each illustration representing the distribution of losses for a different locomotive or for a different set of operating conditions. These performance conditions may be described as ranging from very

good to poor. In all four cases the locomotive is regarded as delivering the same amount of work at the drawbar.

Table 1 lists the ten avenues of use or loss for the heat from the coal fired to the locomotive.

TABLE 1
HEAT USES AND LOSSES WHICH OCCUR ON A LOCOMOTIVE

(a)	1. Heat primarily useful or that which is obtained as work at the drawbar
(b)	Heat used for the purpose of operating the locomotive: 2. By the air pumps and other steam using apparatus 3. As exhaust steam and for the production of draft 4. In overcoming locomotive friction
(c)	Heat lost or wasted due to: 5. Steam leaks to the outside air or from the boiler to the firebox or into the front end. Steam from safety valves, etc. 6. Unburned fuel to the ashpan and ashpit 7. Unburned fuel as cinders 8. Escaping combustible gases 9. Heat in the escaping gases 10. Radiation

All the items listed in Table 1 except that of useful work indicate avenues of possible loss and present opportunities for effecting economies. The percentage of the coal going to the different items will in practice vary widely depending upon the skill and care with which the firing is done and with which the locomotive is handled, upon the kind and condition of the coal, upon the design of the locomotive including its equipment of fuel saving devices, and upon the condition of the locomotive as to maintenance and roundhouse care.

Fig. 3 shows the percentage of heat uses and losses which have already been given in connection with Figs. 1 and 2. The values in the column headed "Estimated Possible Savings" indicate the points at which the greatest opportunities for saving lie, and also indicate roughly the amount of saving which it should be possible to make through improvements, principally in locomotive firing and operation, and through better locomotive maintenance. These estimated possible savings amount to 6 per cent of the total coal fired, and are to be made in connection with six of the different losses which exist.

The coal losses which occur on a given locomotive and the coal savings which can be made are so closely related that a change effecting any one loss generally effects to a greater or less extent most of the other losses. A saving of one per cent of the coal value through the better operation and better maintenance of air pumps and air brake

equipment means that less coal will have to be burned. Less coal burned means less steam necessary to produce draft, and therefore a saving in the exhaust steam is effected. The burning of less coal also means a reduction in the quantity of coal lost in the form of cinders, unburned gases, and heat in the escaping gases. Any improvement, whether in operation or maintenance, which brings about better fire-box conditions will ordinarily effect savings not only in connection

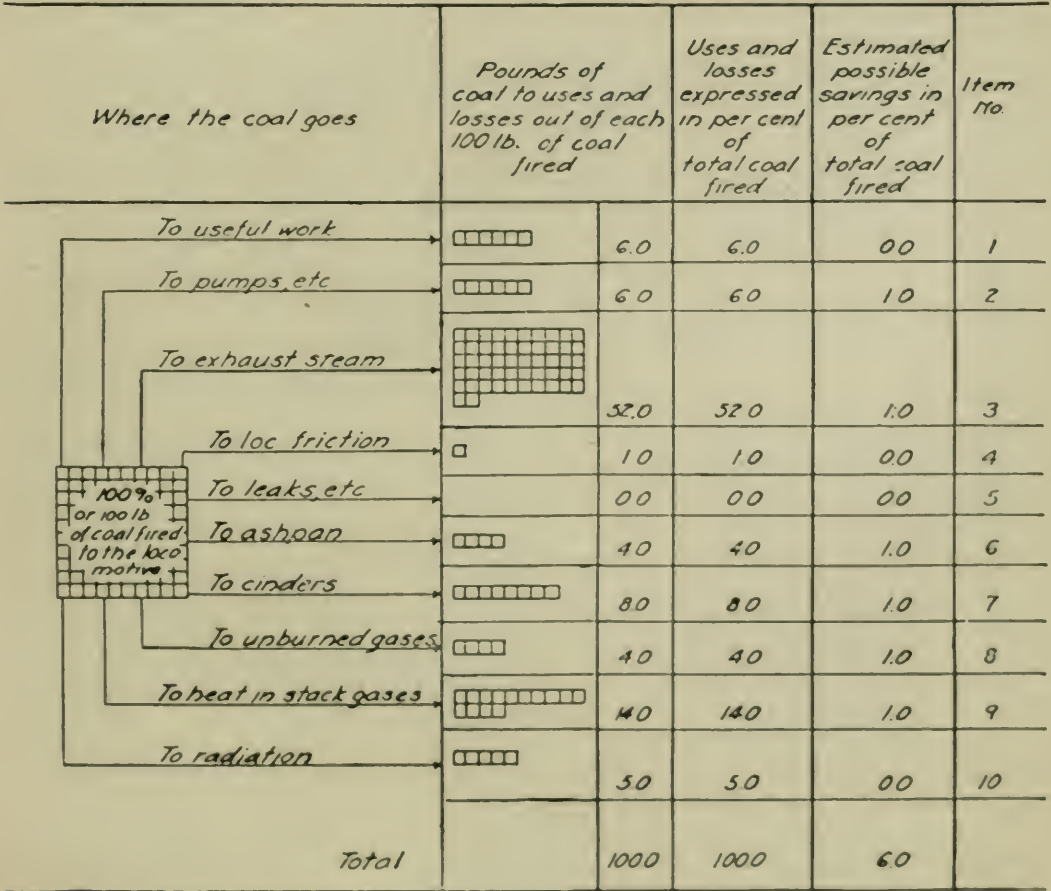


FIG. 3. HEAT AND COAL DISTRIBUTION—LOCOMOTIVE A

Locomotive A is assumed to be a modern locomotive in good average condition, using western bituminous coal and burning from 70 to 80 pounds of coal per square foot of grate surface per hour.

Sixty-five per cent of the heat has been stored in the steam, 5 per cent having been lost by radiation. The efficiency of the boiler is 65 per cent.

Work equivalent to the heat of 6 pounds of coal, or 6 per cent of the total coal, is obtained for each 100 pounds of coal fired. The overall locomotive efficiency is 6 per cent.

Coal savings amounting to 6 per cent of the coal fired should be made by reducing coal uses and losses other than that obtained as useful work at the drawbar.

Each small square in the above diagram represents one pound of coal.

with one loss, but generally in connection with four losses, those to ashpan, to cinders, to unconsumed gases, and to heat in the stack gases.

The percentages given in Fig. 3 refer to average performance on a locomotive in good condition, and the savings here estimated as possible might not be readily obtainable on some locomotives which are in the very best of condition and are being well fired and handled. On

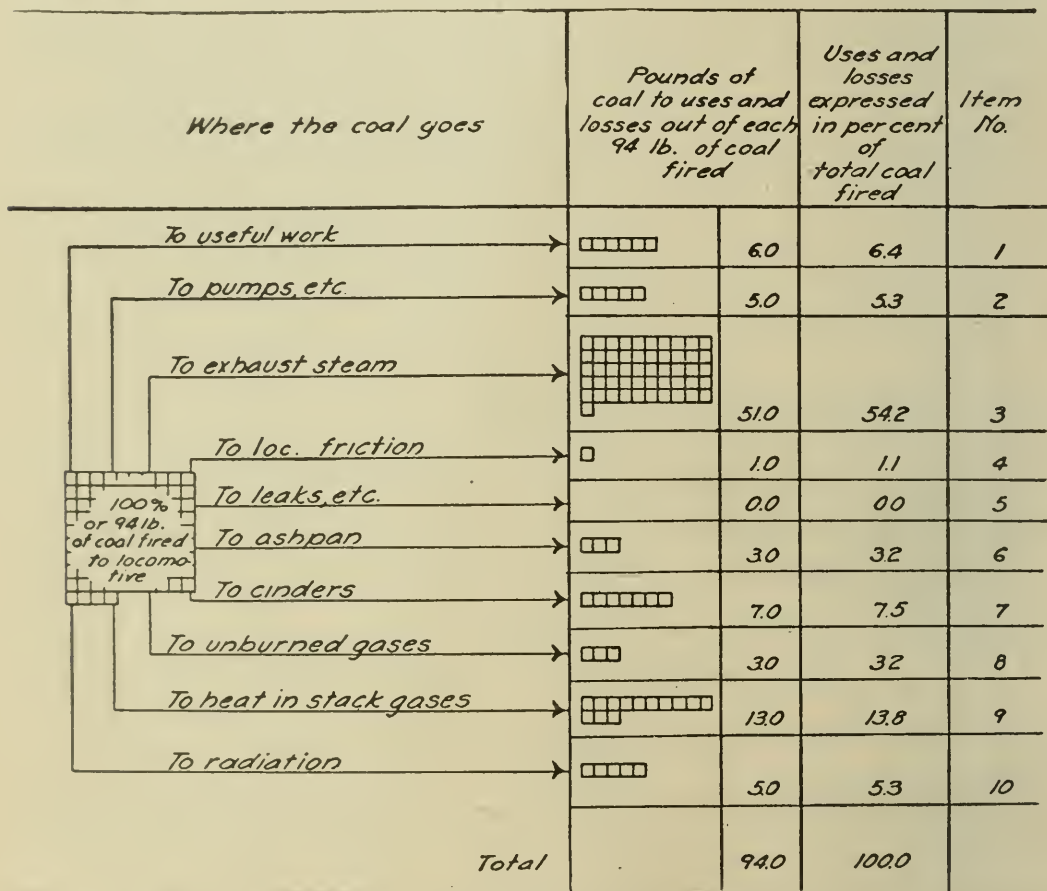


FIG. 4. HEAT AND COAL DISTRIBUTION—LOCOMOTIVE B

Locomotive B is assumed to be in somewhat better condition than locomotive A due to better maintenance and care and is assumed to have been more carefully fired and operated in order to save coal.

Six pounds of coal per 100 pounds fired have been saved as compared with the performance of locomotive A.

Sixty-seven per cent of the heat has been stored in the steam, 5 per cent having been lost by radiation. The boiler efficiency is 67 per cent.

Work equivalent to the heat of 6 pounds of coal, or 6.4 per cent of the total coal, is obtained for each 94 pounds of coal fired. The overall locomotive efficiency is 6.4 per cent.

The diagram shows this saving to have been brought about principally through improvement of combustion conditions and in part through saving steam at pumps, etc., and at the exhaust.

Each small square in the above diagram represents one pound of coal.

the other hand, many locomotives are not in all respects in the best condition and are not fired and handled with the greatest possible care. With many of these locomotives much greater savings would be possible than those indicated by Fig. 3. Fig. 4 indicates the distribution of the coal or heat assuming that the 6 per cent saving indicated as possible by Fig. 3 has been made.

Fuel losses are greatly affected by steam leaks. Direct steam

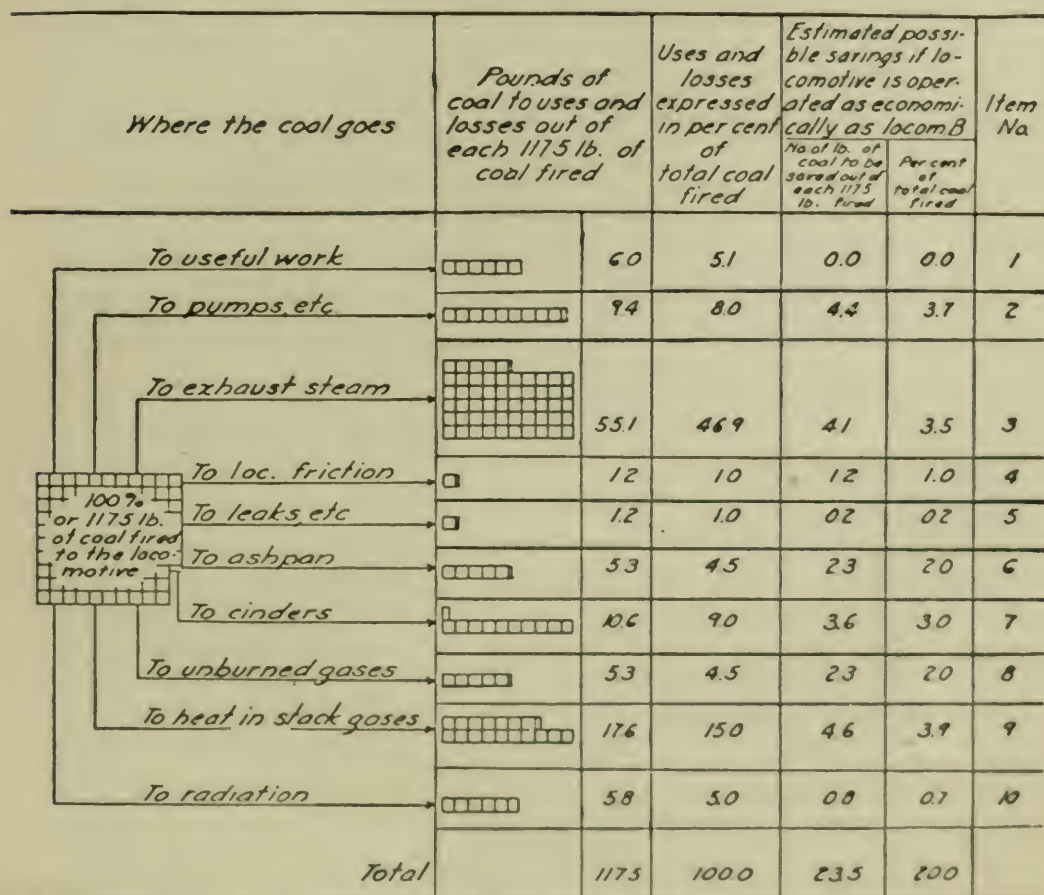


FIG. 5. HEAT AND COAL DISTRIBUTION — LOCOMOTIVE C

Steam leaks, poorer general condition of the locomotive, and less skilful firing and operation are assumed to have caused much less satisfactory performance for locomotive C.

Sixty-two per cent of the heat has been stored in the steam, 5 per cent having been lost by radiation. The boiler efficiency is 62 per cent.

Work equivalent to the heat of 6 pounds of coal, or 5.1 per cent of the total coal, is obtained for each 117.5 pounds of coal fired. The overall locomotive efficiency is 5.1 per cent.

Coal savings ranging from 0.2 to 3.9 per cent of the coal fired are estimated as possible in connection with the uses and losses shown on the diagram. Twenty per cent of the coal of locomotive C would be saved if it were operated as well as locomotive B.

Each small square in the above diagram represents one pound of coal.

leaks to the outside air whether from the boiler or from the cylinders and valve chambers mean a direct loss of coal. Steam leaks into the firebox aside from being a direct loss may occasion a still greater loss through increasing the ashpan, cinder, unburned gas, and heat in gas losses through the resulting poor firebox conditions. Steam leaks past cylinder and valve packings greatly increase the heat loss in the exhaust steam, and by requiring more coal also increase the losses which

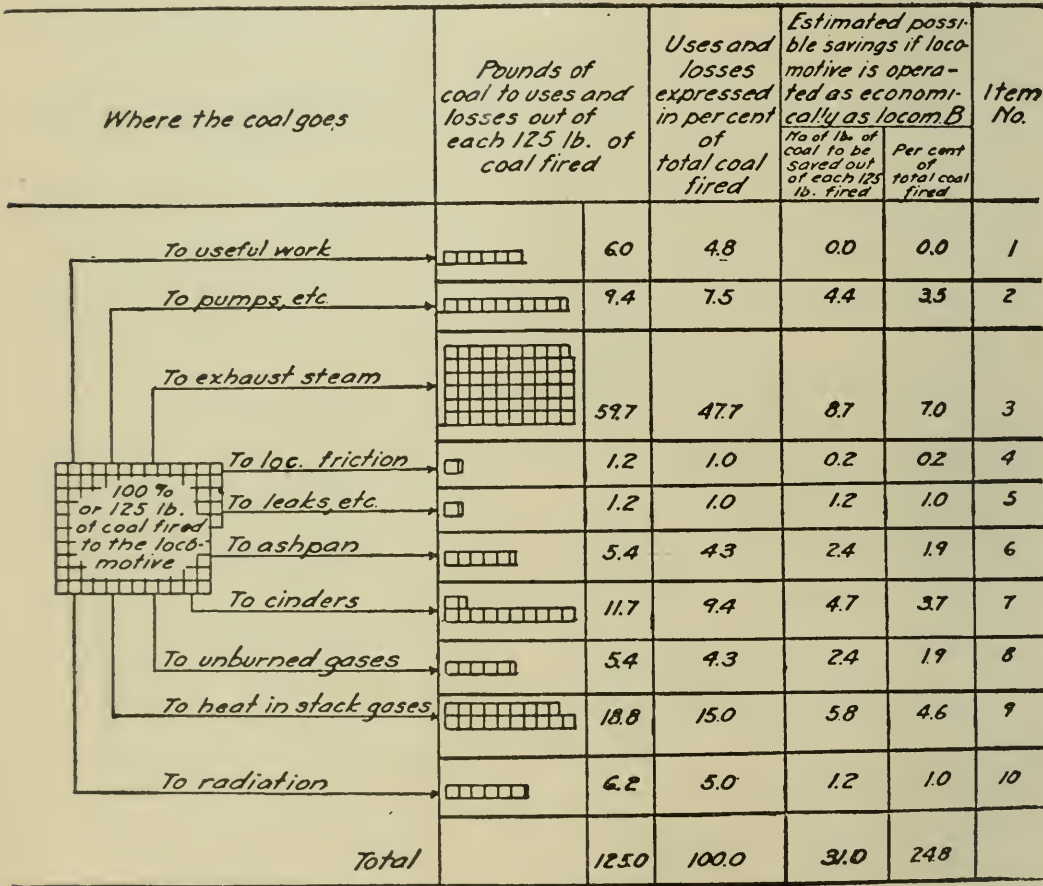


FIG. 6. HEAT AND COAL DISTRIBUTION — LOCOMOTIVE D

A comparatively large loss in the exhaust steam, in leaks, and in poor combustion conditions causes a large coal loss for locomotive D.

Sixty-two per cent of the heat has been stored in the steam, 5 per cent having been lost by radiation. The boiler efficiency is 62 per cent.

Work equivalent to the heat of 6 pounds of coal, or 4.8 per cent of the total coal, is obtained for each 125 pounds of coal fired. The overall locomotive efficiency is 4.8 per cent.

Coal savings ranging from 0.2 to 7.0 per cent of the coal fired are estimated as possible in connection with the uses and losses shown on the diagram. Twenty-four and eight-tenths per cent of the coal of locomotive D will be saved if it is operated as efficiently as locomotive B.

Each small square in the above diagram represents one pound of coal.

take place in connection with the combustion process. Steam leaks or air leaks into the front end also necessitate the use of more steam to produce the draft required, thus materially increasing the exhaust steam loss and so also effecting other losses. The losses just mentioned including direct steam leaks are made evident through the increase in the losses which are listed in Table 1; thus an air leak into the front end or a blow past the cylinder packing increases the exhaust steam loss and in general increases the losses under items 6, 7, 8, and 9.

A steam leak into the front end makes a direct loss, item 5, and in general increases the losses under items 3, 6, 7, 8, and 9. It is well known that leaks to the firebox or cylinder blows may produce losses so great that an engine failure results. In such cases the amount of fuel wasted is, of course, very large.

Fig. 5 indicates the heat distribution where the losses are greater than those shown by Figs. 3 and 4. The pumps and auxiliary apparatus use 8 per cent, and the direct steam losses account for one per cent of the heat of the coal, while the ashpan, cinder, unburned gas, and stack heat losses are all greater. More steam is used in the cylinders under the conditions of Fig. 5 so that the loss through the exhaust is also greater. The direct leak loss and losses arising from it are due to improper maintenance. The increased loss at the pumps and other auxiliary steam using apparatus may be due largely to increased train line air leaks and to poor operation or maintenance of the pumps and other steam using devices. The increased losses in the exhaust steam, the ashpan, the cinders, the unburned gas and the stack heat are likewise probably due in part to less skilful firing and engine operation and in part to poor maintenance of the locomotive. Fig. 5 shows that locomotive C has used 17.5 pounds more coal than locomotive A, and 23.5 pounds more coal than locomotive B in producing the same amount of useful work. If locomotive C were operated with the same economy as locomotive B, a saving of 20 per cent of the fuel of locomotive C would result.

Fig. 6 indicates the heat distribution when there is a somewhat heavier loss through the exhaust steam such as might be caused by bad leaks past the cylinder packing. This illustration shows locomotive D as using 31 pounds of coal more than locomotive B for work equivalent to 6 pounds of coal delivered at the drawbar. The operation of locomotive D with the same economy as locomotive B would mean a saving of practically 25 per cent of the fuel of locomotive D.

As a locomotive approaches its "shopping" time, it probably carries more scale, leaks are more common, packings do not fit so closely, and its general condition is such that coal losses are greater than would be the case if the locomotive were kept in better condition throughout the period between shoppings.

Important coal savings which can be considered as immediately possible are to be made for the most part in two ways, by improving the mechanical condition of locomotives through better care and maintenance and by exercising greater care in the firing and operation of the locomotive while in service. For a given locomotive the total possible saving can be effected only when attention is given to both these matters, that is, when the engine is both better cared for and better handled. With locomotives in good condition, the total possible savings are comparatively small, but more careful engine operation and more skilful firing will even in this case yield results worth while. With locomotives in less satisfactory condition, the total possible savings become greater and in general the greater part of such savings can be effected through better maintenance and care.

Table 1 and Figs. 3, 4, 5, and 6 show nine ways in which coal may be used or wasted other than in producing useful work at the drawbar. The careful consideration of these avenues of waste makes it evident that somewhat better maintenance and roundhouse care and somewhat better firing and operation should result in an average coal saving on all the railroads of the country which can be conservatively estimated as at least 6 per cent of the total coal fired.

Certain improvements which may be considered matters of design rather than of maintenance may often be adopted to aid in reducing the amount of coal or steam used. Front end arrangements, grates or other firebox parts, and cylinder or other packings indicate points at which through better design the coal uses and losses which have been discussed may be decreased.

The values for "Estimated Possible Savings" shown in the preceding diagrams are not intended to indicate the possible savings through the installation of some of the more important fuel saving devices such as the superheater. It has already been pointed out that since the installation of such devices usually requires executive action, they probably cannot be generally adopted and installed as emergency matters. The brick arch can, however, often be added to existing equipment with comparatively little loss of time and without great

expense. On account of the great reduction in fuel losses which can often be made through the use of the more important fuel saving devices, their more extended use under present conditions is to be urged.

Since the possibility for economy through the employment of these means is great, it is proper to point out the extent of the fuel saving which may be expected upon locomotives equipped with such devices as compared with similar locomotives without these devices. The report of the American Railway Master Mechanics' Association Committee on Superheater Locomotives for the year 1916* contains the following statement concerning economies:

"In view of the many complete reports which have already been rendered on tests covering the economies effected through the application of superheater, and superheater and brick arch, your committee does not believe it necessary to publish further data in this report. However, on a conservative basis, it is felt that an economy of fifteen to twenty-five per cent in fuel and twenty to thirty per cent in water consumption can be expected in every-day operation through use of the superheater and brick arch. Numerous tests have shown greater economies.

"On a number of railroads the application of superheaters has reduced the time of freight trains on the road ten to fifteen per cent and eliminated one stop for coal and two stops for water over one freight engine division."

Test data from various sources covering both road and laboratory tests indicate that locomotives equipped with superheaters burn from 6 per cent to 30 per cent less coal, for the same work, than similar locomotives not equipped with superheaters. In like manner data relating to the brick arch indicate possible coal savings ranging from 5 per cent to 12 per cent.

Table 2 presents information concerning fuel savings which have been effected through the use of the superheater and the brick arch.

While it may not always be possible under the varying conditions of road service to obtain as great savings as those indicated in Table 2, a very great opportunity for effecting fuel economy is presented through the more general use of the important fuel saving devices. The total saving of locomotive fuel which may be accomplished in this way is, of course, dependent upon the extent to which old locomotives may be supplied with such devices and the extent to which new locomotives fitted with such devices may be employed.

*Proceedings American Railway Master Mechanics' Association, Vol. 42, p. 131, 1916.

TABLE 2
COAL SAVING THROUGH THE USE OF THE SUPERHEATER
AND THE BRICK ARCH

Coal Saving Due to the Superheater Per Cent	Source of Information	Coal Saving Due to the Brick Arch Per Cent	Source of Information
6 to 17	Publication 126 Carnegie Institution of Washington	5 to 8	Report of Coatsville Tests, W. F. M. Goss
20 to 30	A. R. M. M. Ass'n. Proceedings, 1911	12	Report of Coatsville Tests, W. F. M. Goss
12.3 to 29.8	Penn. R. R. Co. Test Dept., Bulletin No. 21	6 to 8	Penn. R. R. Co. Test Dept., Bulletin No. 30
26.8	A. R. M. M. Ass'n. Proceedings, 1914	12 to 13	Penn. R. R. Co. Test Dept., Bulletin No. 6

IV. HOW TO SAVE COAL

9. *Means to be Employed.*—The character and extent of the losses which may occur in transporting, storing, and handling coal for locomotive use, the losses which may occur through terminal, round-house, and road delays, when firing, cleaning and turning locomotives, and the losses which occur in connection with the burning of the fuel and the use of the steam have been summarized and discussed. The effort has also been made to show in what connection it is possible to effect fuel economy and to indicate approximately the magnitude of the possible saving. It is the purpose of the present chapter to point out how these savings can be made and to indicate the officials or employes who would assume the responsibility for making them.

Coal can be saved in connection with :

- (1) The selection, distribution, and storage of coal
- (2) Roundhouse and shop equipment and the care of locomotives
- (3) Locomotive design
- (4) Locomotive operation

It should further be emphasized that only through coöperation upon the part of all who are in any way concerned with locomotive fuel can the highest degree of economy be attained.

In addition to the four avenues of economy mentioned there are certain matters of organization or operation which affect fuel economy in a broad sense or which under present conditions should be adopted in the effort to save coal. A properly organized fuel department should do much to prevent coal wastes. This department may have charge of the purchase, inspection, and distribution of fuel, exercise supervision looking to its economical use, prepare and make use of records relating to fuel consumption and standards of performance, and have charge of such educational work as is to be carried on for the purpose of promoting fuel economy. The establishment of proper standards of performance with regard to fuel consumption together with thorough going supervision by road foremen of engines or fuel supervisors are of the greatest importance in the matter of coal savings. Managing officials may find it possible to save coal through the

rearrangement of schedules or through a reduction in the weight or speed of some of the heavier and faster trains. Careful consideration may show that certain locomotives in freight service are not operating economically because of overloading. Heavy loading of locomotives is being relied upon at the present time to move large amounts of traffic. The traffic gain may be of much greater importance than the amount of fuel wasted. Where such wastes occur, however, they should be recognized and prevented if possible. On the other hand, the elimination of some fast freight runs and the handling of greater tonnage per train at slower speed may be a possible means of saving coal.

10. *Selection, Distribution and Storage of Coal.*—Certain coal savings can be effected through the exercise of care in selecting the kind or size of coal best fitted for the service or for the locomotives in which it is to be used. Coal can be saved by preventing the actual loss of coal during its distribution as from cars, tenders and coal chutes, and by preventing fire loss or deterioration in storage coal. The selection, distribution, and storage of coal and their relation to losses and possible savings are discussed in this circular in the chapter on coal.

11. *Roundhouse and Shop Equipment and the Care of Locomotives.*—Coal can be saved by improving shop and roundhouse facilities for the maintenance and care of locomotives. While it is difficult to estimate the exact extent of the coal savings to be effected through improved roundhouse and shop facilities, it is known that a locomotive in poor condition mechanically or one in which scale, soot and leaks have become serious may use 10 to 40 per cent more coal than a locomotive in good condition. Modern facilities properly operated should help maintain all locomotives in better average condition. Good coal chutes, ashpits, inspection pits, hot water washing and filling systems, and good general roundhouse equipment and organization make it possible to care for locomotives properly within a reasonable time, prevent the turning of engines without sufficient cleaning or maintenance work, and thus prevent the waste of coal which results from dirty, leaky, and poorly maintained locomotives.

Good shop facilities are also important in the matter of saving coal through their relation to locomotive maintenance. Every piece

of poor or careless work whether in connection with the boiler parts, the engine parts, or the auxiliary apparatus means a loss of coal later through less satisfactory locomotive performance. Shop men and shop officials should recognize the responsibility they bear in matters of this kind. Leaks mean losses which are often repeated day after day. Ill-fitting parts mean loss of power and of coal. Good workmanship always means fuel economy.

Whatever may be the shop and roundhouse facilities available it should be remembered by the roundhouse forces that locomotive fuel economy is very largely dependent upon how well and how thoroughly locomotives are cleaned and cared for between trips. The following up of enginemen's reports and thorough inspection make it possible for work upon the locomotive to be most effective. Careful repairs of leaks, of grates, of arches, and of locomotive running parts mean fuel economy. Boilers should be properly washed and kept free from heavy scale. Leaks, which are the cause of many of the most serious fuel losses, are often due to scale and mud. Tubes, tube sheets and superheater flues and units should be cleaned of soot and ashes. Soot and scale often cause very large fuel losses; their removal means fuel saved. To send out a locomotive in poor condition means a serious waste of coal.

12. *Locomotive Design*.—Executive and motive power officials can save coal through attention to matters of locomotive design and equipment. The fact that difficulties concerning expenditures of time and money are involved ought not to prevent the careful consideration of such matters. The saving of coal which can be effected along these lines is most important.

It may be possible to save coal:

By using grates and grate bars better suited to the kind and size of coal.

By providing ash pans with greater openings for the admission of air.

By making the front end arrangement more economical in the use of exhaust steam or in utilizing the heat generated in the firebox.

By making packing and lubrication devices more effective in reducing friction.

The installation of superheaters and brick arches upon existing equipment should receive careful consideration and installations should be made as rapidly as possible where warranted. Under present conditions the more general installation of important fuel saving devices of this kind is advisable even though the financial savings may be slight.

While engineers and firemen are not primarily responsible for matters of locomotive maintenance and design, they can aid materially by making careful and specific reports about all matters requiring attention. Many defects such as leaks can be discovered only by engineers and firemen while the locomotive is at work on the road, and their coöperation in reporting trouble is essential to good maintenance. Under the common practice of pooling engines, the incentive to be scrupulous in this regard is not so great as under the old plan, but the responsibility nevertheless remains.

Fuel Consumption while not Hauling Trains

13. *Locomotive Operation.*—It has already been pointed out that about one-fifth of all locomotive fuel is burned during the time when the locomotive is not hauling trains.

Yardmasters can save coal by delivering engines quickly to the roundhouse and thereby cutting down the time during which fires must be kept burning.

Fires kept up during delays at ashpits mean a waste of coal, and the elimination of such delays may be the means of important fuel savings.

Coal can be saved through economical methods of kindling fires and through the use of economical kindling material. If considerable coal falls through the grates when building fires, it should be saved and not allowed to become mixed with the ash.

When fires are maintained in the house, coal can be saved by covering the stacks to hold the fire and retain the heat. A light metal cover having an adjustable slide which may be opened sufficiently to permit the emission of condensed vapors and gases created when the locomotive is standing with banked fires can be made very cheaply.

Coal can be saved through the coöperation of roundhouse and transportation forces in the matter of calling engines, thus preventing their standing under steam longer than is necessary.

Yardmasters, dispatchers, and trainmen can save coal through

assisting in the elimination of yard and other terminal delays. Locomotives in readiness to handle trains are burning coal rapidly no matter how carefully the enginemen may handle them.

While engines are standing at meeting points or on sidings, fires must be maintained. A large amount of fuel is burned under such circumstances. Transportation forces can effect a substantial coal saving by eliminating as large a part of these delays as possible.

Every stop on the road means the consumption of considerable coal. Every unnecessary stop means a waste of coal. Transportation men, trainmen, and enginemen should all assist in preventing unnecessary stops.

Transportation employes should appreciate that their branch of the service offers important opportunities for the saving of coal. Through action on their part that portion of the fuel now being burned on locomotives not actually used in hauling trains may be materially diminished, and a coal saving of great importance accomplished.

Theory of Combustion

There are three factors involved in the process of combustion:—

- (1) There must be fuel
- (2) There must be oxygen
- (3) There must be an ignition temperature

Thus stated it would seem to be a simple matter, but effective combustion, under the conditions which exist in the locomotive firebox, presents problems which are more complex and difficult than those found in the stationary plant.

The essential constituents of coal, that is, the combustible or heat-producing substances, are carbon and hydrogen. The mineral matter and the moisture constitute a non-burnable portion and comprise a very considerable part, approximately 25 per cent, of the average bituminous coal.

In the process of combustion, every particle of the actual combustible material must be supplied with an exact and definite quantity of oxygen; otherwise the burning process does not take place or the process is incomplete.

The carrier of oxygen is air, but air is not all oxygen, four-fifths of its volume being nitrogen. Nitrogen is an inert gas which in no way contributes to the process of combustion but rather retards it through

its diluting effect which makes the process of complete and quick mixing of oxygen with the fuel more difficult.

Because the locomotive firebox is small the rapidity of combustion is of vital importance. The rapidity of combustion depends upon two factors: the completeness of mixture of air with combustible gases, and the temperature. If the fuel mixture is as intimate as is found in a gasoline engine cylinder, the combustion proceeds with explosive rapidity, even though the gases are surrounded by cold surfaces. The high temperature of gases alone without good mixing of air will not of itself produce rapid combustion; the more necessary factor is intimate mixture of gases at the ignition temperature. To increase temperatures above the ignition point tends to accelerate the process. Combustion attains the greatest rapidity with perfect mixtures at the highest temperature. Under ordinary conditions in the locomotive firebox the temperature is high enough to produce rapid combustion, provided the mixing is complete.

The average bituminous coal contains approximately 75 per cent of actual combustible material, about half of which must burn as volatile matter in the combustion space. The other half, or possibly about 40 per cent, is in the form of fixed carbon, which burns nearly completely on the grate. The volatile part of the combustible, which is driven from the coal and must be burned in the combustion space, consists of complex gases which are hard to burn. On the other hand, the fixed carbon remaining on the grate, when burning, produces gas which is simple and is easy to burn. The rate at which air is forced through the fuel bed controls the rate of combustion of the coal. The completeness of combustion of the gases above the fuel bed depends upon the air furnished above the fuel bed, either through the door, staybolt openings, or holes in the fire. In locomotive practice a large part of this supplementary air must enter through the holes in the fire, the intense pulsating draft requiring relatively small holes to supply this air. It is essential to keep an even firebed without visible holes or black spots in order to prevent an excess of air being drawn through the holes in the fuel bed. When burning coal at 60 pounds per square foot of grate per hour, free openings for air from two to four per cent of the grate area will supply ample air for combustion of the gases above the fuel bed. The most skilful firing is none too good to maintain an even bed which will restrict the free air openings below the figures given.

With regard to the volatile portion of the fuel, most of it is given off in a few seconds after the coal reaches the fuel bed. The heavier tar-like constituents are all driven off by the time the coal has attained a dull red heat. The volatile matter from the coal is a mixture of complex gases requiring a very large amount of air for complete combustion. Some of the gases take ten times their own volume of air and other gases as much as one hundred times to produce complete combustion. The composition of these gases depends upon the conditions under which combustion proceeds. If there is a lack of oxygen, these complex gases will be rapidly decomposed by the high temperature, with the ultimate product of carbon in the form of soot, hydrogen, and carbon monoxide. Probably 95 per cent of this carbon formation takes place within the first foot above the fuel bed. In the absence of oxygen, extremely high temperatures accelerate the breaking down of the heavy hydrocarbons into soot near the fuel bed. One reason why locomotive firing requires skill is that the conditions giving high economy are not definitely controllable by the fireman. Immediately after firing green coal on a uniformly level firebed the small holes are made smaller just at the time when more air is needed, and these holes grow larger and deliver an increasing amount of air just when the fire requires a decreasing supply of air. The only way to approximate a satisfactory condition is to make the charges and firing intervals very small so that extreme conditions do not depart materially from average conditions of air supply.

For efficient combustion the following conditions must be satisfied:

- (1) Fuel and air must be admitted in as nearly a uniform manner as possible.
- (2) A proper distribution and thorough mixing of the air with the combustible gases in the firebox must take place.
- (3) A sufficiently high and uniform firebox temperature must be maintained.

Fuel Consumption while Hauling Trains

Four-fifths of all locomotive fuel are used while the locomotive is in operation actually hauling trains. For the year 1918 this will amount to about 120,000,000 tons of coal. The large amount of coal involved adds emphasis to the desirability of employing any means

which may be available to effect greater economy in the use of fuel in locomotives during the time they are performing useful work. Since locomotives are directly under the charge of the engineer and fireman at this time, the responsibility lies largely with them.

Firing instructions both printed and verbal are commonly based upon the assumption that the locomotive is in good condition. While losses due to defects in design and improper maintenance may be great, engineers and firemen should recognize that, with any locomotive, whatever its general condition, the difference between careful and skilful operation and firing, and poor and indifferent operation may easily account for five or ten per cent of the coal required to perform a given amount of work.

Firing Instructions

The principal purpose of firing instructions is to help to reduce some or all the fuel losses which have been considered in the section on "The Distribution of the Heat Generated in the Firebox" and which are listed in Table 1, page 20. In connection with the process of combustion the firing instructions aim to prevent loss by showing how the three conditions necessary to satisfactory combustion may be brought about (see "Theory of Combustion," page 35).

Instructions regarding level firing and the avoidance of banks, holes and clinkers, and regarding door control, blower operation, and grate operation relate to the supplying of the proper amount of air and the thorough admixture of this air with the burning fuel. Instructions relating to firing at a uniform rate, to spreading the coal, and to closing the firedoor are largely aimed at securing a sufficiently high and uniform firebox temperature.

A level, bright fire should be maintained. In general also the fire should be as light as is consistent with the work which the locomotive is doing and with the character of the fuel being used. Certain precautions are to be observed in maintaining a light, bright, and level fire. Large lumps of coal should not be fired; they tend to make the fire uneven. All large lumps should be broken into pieces not larger than three or four inches.

Apply fresh coal to the parts of the fire which are the brightest and thinnest, that is, where the coal is needed most to keep the fire level and uniformly bright.

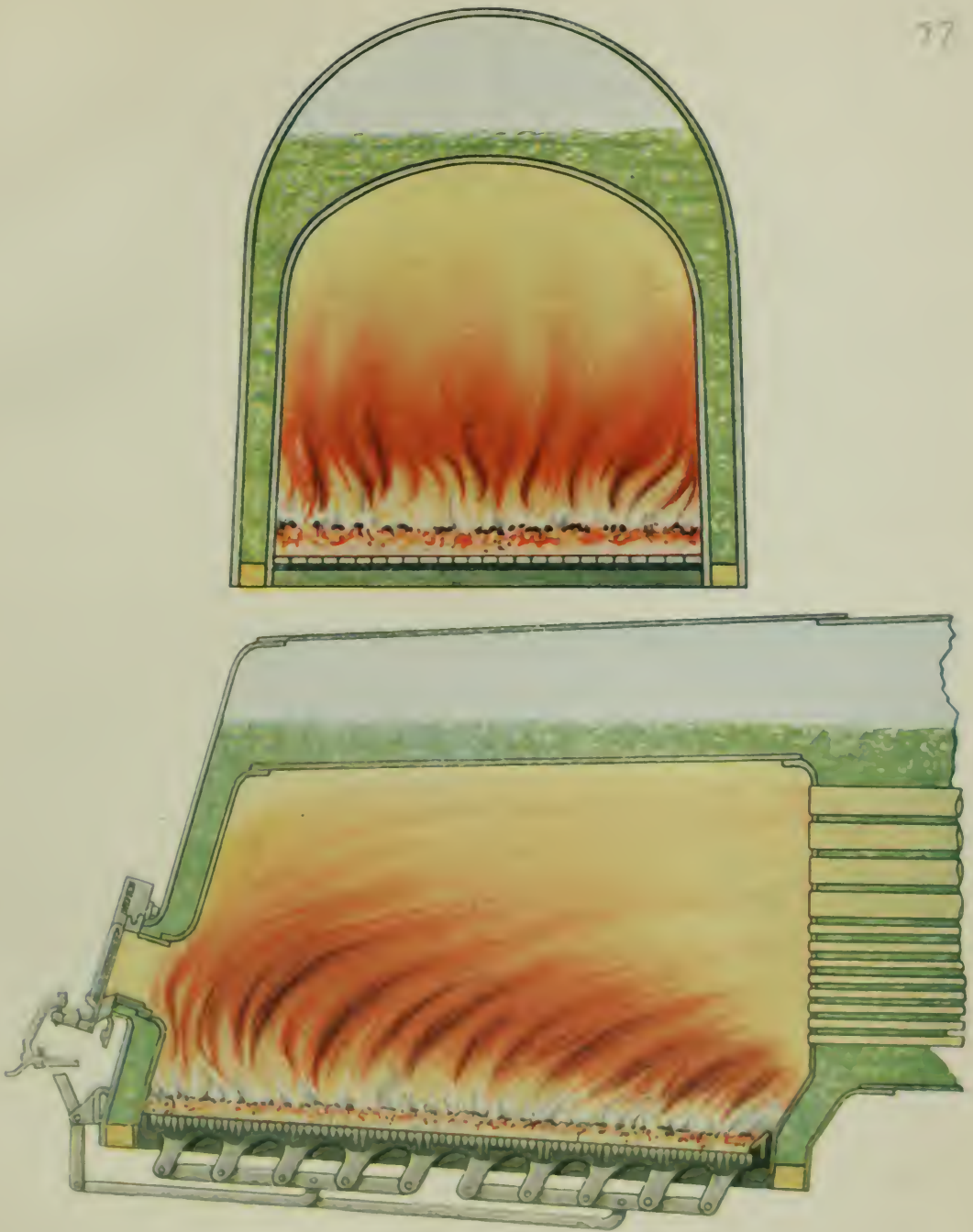


FIG. 7. THE UNIFORM FIRE

The condition here represented shows the admission of enough and not too much air. With the grates and draft apparatus in proper condition, a level and uniform firebed permits air to enter without undue obstruction over the entire grate surface and at the same time prevents cold air from entering through holes or thin spots.

With a light, level and bright fire most of the air supply enters the firebox through a uniform firebed. The firebed being uniform, without holes, clinkers, or lumps, the air enters in many small streams equally distributed over the entire grate surface. Every part of the fuel bed and of the gases just above the fuel bed has its share of the uniformly distributed and finely divided air streams which are entering through the grate.

High and uniform firebox temperatures result from a proper air supply and a good mixture and most of the coal and gases burn before they can escape from the firebox. High firebox temperatures insure the burning of combustible gases which with lower firebox temperatures escape without giving up their heat.

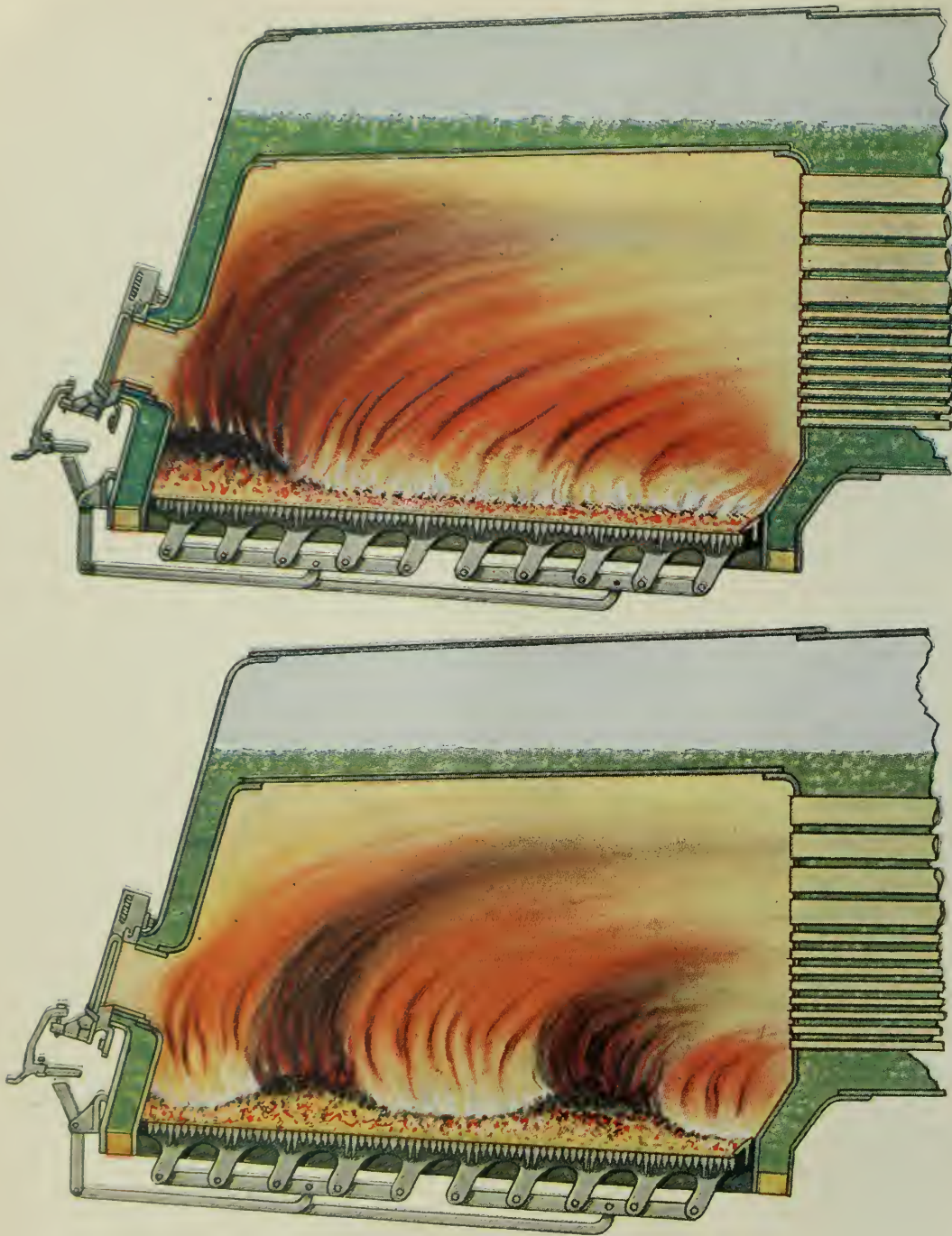


FIG. 8. BANKS IN THE FIRE

The condition here represented shows the results of not enough air or of too much air. Banks may restrict the air supply to such an extent that there is not enough air to insure combustion. Even though large quantities of air may enter through the thinner parts of the fire or through the fire door, the combustible gases arising from the banks may not receive a sufficient supply of air before they escape from the firebox. The firebox may be receiving too much air and still not burn the coal because of the poor mixing.

Above the thinner parts of the fire there may be more air than is needed to burn the fuel with which it comes in contact. This surplus air does not assist in the combustion process, but reduces the firebox temperature and carries off heat through the stack.

Where the coal and gases are heaviest the incoming air supply is least, and neither the incoming air nor the gases arising from the fuel are sufficiently divided into numerous well distributed small streams to permit easy and rapid mixing. Gases arising from the fuel bed reach the flues in a very short time and mixing must be rapid.

The banks of green coal and the smoky gas arising from them tend to cool the firebox as a whole. Some parts of the fire burn much more brightly than others and the temperature throughout the firebox is not uniform. Smoke and unburned gases result from this condition.

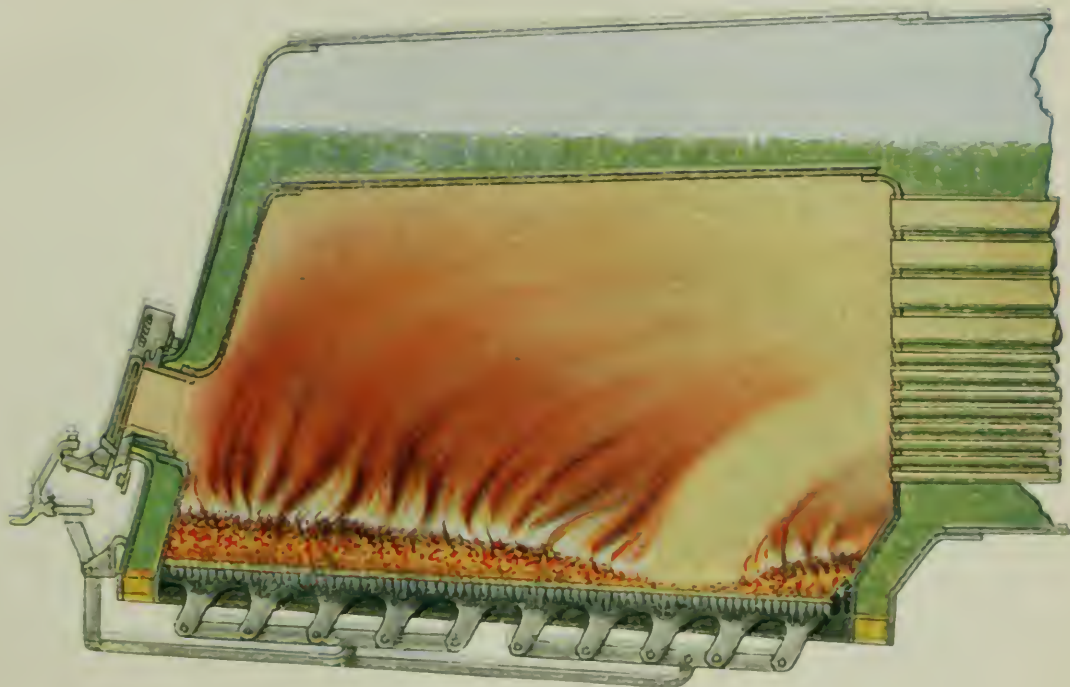


FIG. 9. HOLES IN THE FIRE

When holes occur in the firebed the result is too much air in parts of the combustion chamber and not enough air in others.

Holes permit too much air to enter the firebox for the amount of coal which is being burned where the holes exist. The air coming through the hole is in a comparatively solid stream, and where this air comes through there is little fuel to burn. At points where there is fuel on the grate not enough air is coming through, and the opportunity for good mixing is lost.

The large volume of cold air coming through the hole cools that part of the firebox and combustion in other parts of the firebox is retarded.

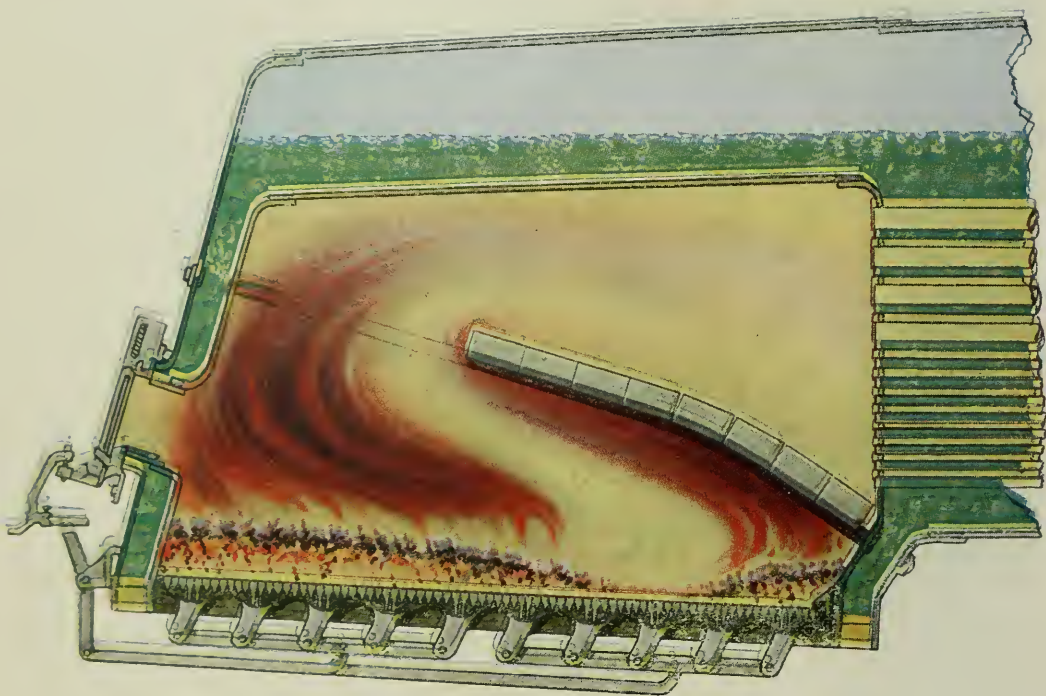


FIG. 12. HOLES IN THE FIRE

This figure is similar to Fig. 9 but shows a firebox equipped with the brick arch. The combustion conditions shown are intended to be practically the same as in Fig. 9 with combustion somewhat better as the gases approach the flues above the arch. The intention is to show similar combustion conditions rather than the effect of the arch.

Fire the coal in small amounts and at regular intervals. One or two shovelful at a firing should ordinarily give the best results. With large fireboxes and high rates of combustion it is often found necessary to apply from three to five shovelful. In any case each shovelful should be spread to that part of the fire where it is most needed to keep the firebed level and to keep the fire as a whole as bright as possible.

Large amounts of coal applied at one firing give off so large a volume of gases that they cannot all be burned before escaping from the firebox; consequently firebox temperatures are lowered, and the air supply through the grate is restricted or broken up into uneven streams. The result is poor combustion and coal loss. Proper mixture of air and fuel, and sufficiently high firebox temperatures produce good firebox conditions. Uniformity in the firing of coal, that is, the firing of small amounts at regular intervals, promotes these conditions.

Firing instructions concerning the spreading of the coal, cross firing, and firing slightly heavier next the firebox sheets to prevent excessive air entering along the sides and ends are intended, first, to help in securing a sufficient but not too great air supply through the fuel bed, and secondly, to insure the mixing of the air thoroughly with the burning fuel. The fuel bed being uniform and of equal thickness throughout, the air will come through it in a uniform manner and so be more or less thoroughly mixed both with the fuel burning on the grate and with the burning gases arising from the fuel bed.

Fig. 7 illustrates a level uniform fire in which a uniform supply of air is passing through the fuel bed, good admixture of air, fuel and gases is taking place, and a high firebox temperature is being maintained. These three conditions result in practically complete combustion and in the greatest possible fuel economy.

Fig. 8 shows firebeds in which banks have been permitted to form either because of heavy firing under the firedoor or because of uneven firing on other parts of the grate. Fig. 9 shows a hole or spot in the fire. Firebeds in which bad clinkers have formed constitute another example of poor firing conditions. Banks, holes, and clinkers all tend to interfere with the uniform flow of air through the fuel bed. Banks and clinkers restrict the flow of air through parts of the bed and accelerate it through other parts. Holes permit large amounts of cold air to flow through them and retard the flow of air through other parts of the fuel bed. Undue lack of uniformity in the fuel bed interferes

with a uniform supply of air, prevents the proper mixing of air and fuel, and tends to lower the temperature of the firebox.

Figs. 10, 11 and 12 present illustrations similar to Figs. 8, 9 and 10, except for the addition of the brick arch. In general, it may be said that it should be easier to maintain good firebox conditions where there is a brick arch and that the bad effects due to banks or holes will be reduced through the action of the arch. The arch produces a much longer average path for the gas to travel from the surface of the firebed to the flues, thus permitting and assisting in a more intimate mixture of the air with the burning gases. The hot arch brick also assists in maintaining a uniform temperature in the firebox. In all the figures the attempt has been made to illustrate certain combustion conditions as related to air supply, air mixture and firebox temperature, and to emphasize the importance of maintaining proper conditions in these respects irrespective of any particular firebox or type of locomotive.

Care should be exercised in shaking grates, the purpose of which operation should be to remove such ash and clinker as may be necessary in order to admit sufficient air but to avoid disturbing the fuel bed so much that holes or additional clinker may be formed. Preferably the grates should be shaken when the locomotive is standing, since holes are likely to result from shaking the grates when running. Many western bituminous coals such as those from Illinois, Indiana and western Kentucky cannot be satisfactorily burned without frequent shaking of the grates. Some eastern coals may be burned with practically no shaking of the grates.

The firedoor should, in general, be closed immediately after each shovelful of coal is fired. This practice prevents an improper air supply, which cannot properly mix with the burning gases and helps maintain a uniform firebox temperature. When the firedoor is wide open a large volume of cold air rushes into the firebox. This cold air lowers the firebox temperature and, being in one large stream, does not readily become mixed with the gases arising from the fuel bed. Too much air may be supplied through the firedoor under almost all firing conditions.

Immediately after a shovelful of coal is thrown on the fire, the gases arising from that coal require more air above the fuel bed than is otherwise needed. Leaving the firedoor on the latch is sometimes advantageous at this time since it allows a certain amount of air to

enter. In general, however, the ashpan openings, the grate openings, and the condition of the firebed are such that the greater part of the air required for combustion is drawn through the grates and the opening or partial opening of the firedoor to supply air rendered unnecessary. In all cases the instructions regarding the firedoor are aimed at the proper air supply, the proper mixing, and the maintenance of a high firebox temperature.

The blower should not be so used as to draw an unnecessarily large amount of air through the firebox. The throttle should not be so operated as to disturb the firebed and the slipping of drivers should be avoided since the firebed is disturbed through the violent action resulting from the exhaust steam.

The waste of steam through the safety valve should be avoided at all times. Careful attention to the handling of the fire at the time of a stop or when standing or drifting will prevent blowing off and wasting fuel.

Careful attention to the handling of the injectors will save coal in several ways. Waste of steam through safety valves may be prevented through the use of the injector. The water level in the boiler should never be so high as to cause water or very moist steam to be carried over to the superheater or to the cylinders. A very serious fuel loss may occur from this cause. In general the injectors should feed the boiler at a fairly uniform rate. It is often possible to make firebox conditions more nearly uniform and more satisfactory through the operation or regulation of the injectors. When the demand upon the locomotive is light, the operation of the injectors makes the demand upon the firebox somewhat heavier; and when the demand upon the locomotive is heavy, a reduction in the amount of water supplied makes the demand upon the firebox lighter than would otherwise be the case. The injectors should feed the water required by the boiler in such manner as to assist in maintaining uniform and satisfactory firebox conditions and they should not be so handled as to occasion direct loss of steam through the safety valves, or to cause priming.

The reverse lever and throttle should always be so handled as to use the minimum amount of steam. The following extracts concerning throttle opening and reverse lever control are taken from the instructions concerning fuel economy which have been adopted as recommended practice by the American Railway Master Mechanics' Association.*

* Proceedings of the American Railway Master Mechanics' Association, Vol. 49, pp. 753, 755, 1916.

“The locomotive should be operated with a full throttle opening (except when starting or drifting) when the cut-off is 25 per cent of the stroke, or greater; but if 25 per cent cut-off with full throttle gives more power or speed than is needed, the reverse lever should be left at 25 per cent cut-off and the throttle partly closed as necessary. With locomotives using superheated steam it is well to use 15 per cent cut-off instead of 25 per cent, as mentioned.

“Superheater locomotives should be operated with a full throttle opening and reverse-lever control, as far as service conditions will permit, the exceptions being: When starting a train, when using a very small quantity of steam, and when drifting.”

The careful handling of the throttle and reverse lever saves steam at the cylinders and also makes it possible for the fireman to do his work more easily and with a greater saving of fuel in the firebox.

Certain other firing precautions relating to injector operation or firing in anticipation of heavy demands for steam are for the purpose of avoiding unusual “peaks” in the demand made on the firebox and the uneconomical forcing of the fire. Very heavy firing is wasteful as compared with moderate rates of firing. Where demands upon the locomotive, such as may be caused by a heavy grade, can be anticipated a somewhat gradual building up of the fire will help to equalize the demands made upon the firebox. This practice will to a certain extent prevent high rates of combustion and so will be economical of fuel. The careful operation of the injectors before and during such “peaks” and the careful operation of the locomotive as a whole, together with the handling of the fire to the best advantage, will do much to prevent the necessity for very high rates of combustion and the loss of fuel which attends such periods of forced firing.

Still other precautions are for the purpose of eliminating or reducing coal losses which occur more or less independently of the efficiency of the combustion process. Coal when placed on tenders should be trimmed so that there shall be no loss through falling off on the road bed. Coal should not be allowed to fall or blow away from the engine deck. The sprinkler hose should be used enough to keep down dust and to prevent fine coal from blowing about.

Unnecessary stops for coal or water or other unnecessary stops which may be within the control of the engine crew should not be made. Each stop and start requires the burning of additional coal. Prevention of delays of this kind in so far as they may come within the control of enginemen will save coal.

The direct loss of coal or partially burned coal through the grates to the ashpan should, of course, be prevented in so far as possible. The careful handling of the grates and the exercise of care when starting, cleaning, or banking fires will save coal.

Locomotives should not be brought into terminals with a fire heavier than is necessary properly to handle the train and the locomotive. A large fire which must burn out or be dumped when the fire is cleaned means a direct waste of fuel.

Every pound of steam which can be saved in the operation of the air pumps or in the use of other steam using devices and every steam leak which can be prevented or stopped means a corresponding saving of fuel. In like manner the economical use of air means coal economy. Air leaks and wastes often mean much larger coal losses than is commonly realized.

It takes extra power to operate valves and other parts of the locomotive mechanism which are not properly lubricated and this waste of power means a waste of coal.

The economical use of steam, other than in the main cylinders, the economical use of air, the prevention of leaks whether of coal, water, steam, air, or heat, and proper lubrication all mean economy and coal saving.

The Significance of Smoke

The emission of visible smoke from the stack of a locomotive is evidence of the presence of unconsumed volatile matter and soot, or the heavy distillates of coal, and indicates that conditions in the firebox are not such as to promote fuel economy. The direct loss represented by the smoke itself is not serious, but the conditions of which smoke is an indication may often result in waste.

The prevention of smoke depends to a large extent upon an adequate air supply, its proper mixture with the combustible gases, and the maintenance in the combustion chamber or firebox of a temperature sufficiently high to insure the ignition and combustion of the distillates arising from the coal. These distillates must be heated quickly and kept at a high temperature until the process of combustion is completed. The brick arch is an aid in promoting better conditions in the firebox and consequently in reducing smoke, since it helps to mix the air and gases and to maintain a uniformly high firebox temperature. The careful use of steam jets is also an aid in reducing smoke.

In general, it should be recognized that the emission of smoke is an indication of unsatisfactory conditions in the firebox which should be corrected as promptly as possible.

The discharge from locomotive smoke stacks contains not only the unconsumed distillates of coal but an amount of cinders and unburned fuel particles which have a unit heating value equivalent to about three-fourths that of the original fuel. This discharge of cinders varies according to the rate of combustion from a fraction of one per cent to as much as twenty per cent of the coal fired. The amount discharged at high rates of combustion is proportionately much greater than at low rates. For this reason, as for others which are discussed elsewhere, firemen should endeavor to maintain as low and uniform a rate of combustion as will meet the demands made upon the locomotive.

V. COAL

14. *Kind of Fuel Used by Railroads.*—The fuel used by railroads is normally that which is nearest at hand, or that which is most easily and most cheaply obtainable. In the early days wood was extensively used in locomotives, but it has now practically disappeared as a fuel. In a few districts where oil is available it is used as locomotive fuel, but coal constitutes the fuel now almost universally used in locomotives.

The location of many railroads has been determined by the location of coal fields because, as noted elsewhere, not only do the railroads consume in their operation about 25 per cent of the coal output of the country, but about one-third of the freight hauled by railroads consists of coal. It is important, therefore, for railroads to have access whenever possible to coal fields, not only to insure their own supply of fuel, but also to insure a certain amount of freight traffic. Owing to these conditions practically every variety of coal available in the United States is used to a greater or less extent by the railroads.

15. *Classification of Coal.*—Some coals may be classified upon a strictly chemical basis, but in general it is necessary to consider both the chemical composition and the physical characteristics. The United States Geological Survey has proposed the term “rank” for the different kinds of coal as determined by the chemical and physical characteristics, but this term has not yet come into general use. Upon the basis of chemical and physical properties coals may be classified or ranked as *anthracite*, *semi-anthracite*, *semi-bituminous*, *bituminous*, *sub-bituminous*, and *lignite*.

All these coals are composed of the following materials in varying proportions:

Solid or fixed carbon which burns with a glow and with little or no flame.

Gases or volatile matter which escape from the coal when it is heated and which burn with a flame.

Gases or volatile matter and water which escape from the coal when it is heated and which do not burn.

Ash or mineral matter which will not burn and which remains as ashes after the carbon and gases are burned.

The relative proportions of the materials in different coals determine their value for particular purposes. Fuels having a large amount of fixed carbon and a relatively small amount of volatile matter burn with a short flame and the whole process of combustion takes place at or near the surface of the fuel bed. Such fuels can be burned without developing visible smoke. On the other hand, coals containing a relatively large amount of volatile matter and a lower amount of fixed carbon than the high carbon coals burn with a longer flame because the volume of combustible gases distilled from them is greater, and the combustion of these requires an appreciable period of time. Because of the difficulty of completely burning this greater amount of gas which escapes from the coal, the amount of visible smoke is usually greater, unless special devices are used for furnishing an adequate supply of air to burn this gas before it escapes.

The water and non-combustible gases which are present in all coals and which are detected only by chemical analysis escape up the stack unnoticed and without being of any benefit in the process of combustion. In fact, these constituents represent a definite loss because they do not furnish heat by combustion and because they absorb and carry off heat which would otherwise be available for useful work. The term *moisture* in coal does not refer to the water adhering to the surface of the lumps, but to that contained within the pores of the coal. Moisture in coal is not evident to the eye and a coal containing a high percentage of moisture by analysis may appear to be perfectly dry.

The ash content of different coals varies widely. It not only has no heating value, and, therefore, represents a portion of the coal for which no return is received, but it may hinder the free burning of the combustible components of the coal and, if it contains certain mineral substances, may by clinkering, greatly interfere with the process of combustion and with the cleaning of grates.

It should be clearly understood that ash will *not* burn and that no treatment with chemicals, secret processes, or other devices will cause it to burn. Ash is objectionable not only because it may interfere with the burning of the coal, but because it means an increased cost for handling.

The ash in coal may be classified under two divisions: (a) That which is a definite part of the coal itself and is invisible to the eye and which cannot, therefore, be separated by hand, and (b) That

which is due to rock, shale and slate, which becomes mixed with the coal in mining and which can be removed in great part either in the mine or in the tipple.

Anthracite

Anthracite, commonly known as "hard coal," contains a high percentage of fixed carbon, very little volatile, does not coke, and is smokeless. It was formerly extensively used for locomotive purposes by the so-called "anthracite" roads, or roads traversing the anthracite district of northeastern Pennsylvania. Owing, however, to the increasing value of and demand for anthracite for domestic purposes, it is now used on locomotives mainly for passenger service or where smokeless burning is required.

Semi-Anthracite

Semi-anthracite is a hard coal and has a high percentage of fixed carbon but has neither of these properties to so great an extent as has anthracite. It may be classed as an intermediate variety between true anthracite and true bituminous. The amount of semi-anthracite in the United States is small, and it is employed for railroad use only to a very limited extent.

Semi-Bituminous

The term "semi-bituminous" is unfortunate, since it implies a lower grade of coal than bituminous. Actually semi-bituminous coals are better than bituminous. Semi-bituminous coals have a relatively high percentage of fixed carbon which makes them nearly smokeless. Consequently most of these coals are known as "smokeless coals." Semi-bituminous coals, in general, have the highest heating value of any of the coals and are usually low in ash and moisture. They are also usually very friable, breaking easily into small pieces and dust. They are particularly adapted for purposes where smokeless combustion is essential, as with the navy. Their high value precludes their general use as locomotive fuel except upon the roads running through the districts in which the coal is mined.*

Semi-bituminous and bituminous coals may be either coking or non-coking. A true coking coal when fired swells, becomes pasty, and

*Semi-bituminous coals are mined in certain districts in West Virginia, Virginia, eastern Kentucky, western Maryland, and in some parts of Pennsylvania. Semi-bituminous coal is also mined in Arkansas and eastern Oklahoma and a small amount is found further west.

fuses into a mass of more or less porous coke. Such coke will burn without flame and will hold fire a considerable period. This fusing or coking takes place without respect to the size of the piece of coal. A non-coking coal does not swell and become pasty, but burns away gradually to ash, the pieces becoming smaller and smaller as combustion proceeds. There is a gradual gradation from true coking to true non-coking coals, and many coals cannot be distinctly placed in either class. Coal which will not coke on a furnace grate may, however, give good coke in by-product coke ovens, particularly when mixed with other more easily coking coals. This is the case with many Illinois coals.

Bituminous

The term "bituminous" coal as generally understood applies to a group of coals which have relatively high percentages of volatile matter. Generally the volatile matter and the fixed carbon are nearly equal, but this fact alone is not distinguishing, since the same statement may be made with regard to sub-bituminous coal and lignite. The principal characteristic which distinguishes bituminous coals from the lower ranks of coals, such as sub-bituminous and lignite, is its weathering property. Most of the bituminous coals can be exposed to the air for considerable periods, without material physical or chemical deterioration. Sub-bituminous and lignite coals, on the other hand, break up easily when exposed to the weather. Bituminous coals vary widely in their physical properties and in their heat producing qualities and many attempts have been made to subdivide them, but so far no scheme has met with general approval. The bituminous coals include some of the best coking coals, most of the gas coals, and the cannel coals. In general the bituminous coals decrease in value from the East toward the West, the highest rank of bituminous coal being found in the East and the lowest in the West. There are, however, some striking exceptions to this general rule.

The eastern bituminous coals contain from 5 to 10 per cent of ash, from 25 to 35 per cent of combustible gases, from 2 to 5 per cent of moisture and non-combustible gases, and from 55 to 65 per cent of fixed carbon. These coals are more generally of the coking variety than are the Middle West coals. In general, they are higher in heating value and lower in ash, but break up more easily and are not so well suited for transportation and for-repeated handling as are many of the Middle West coals.

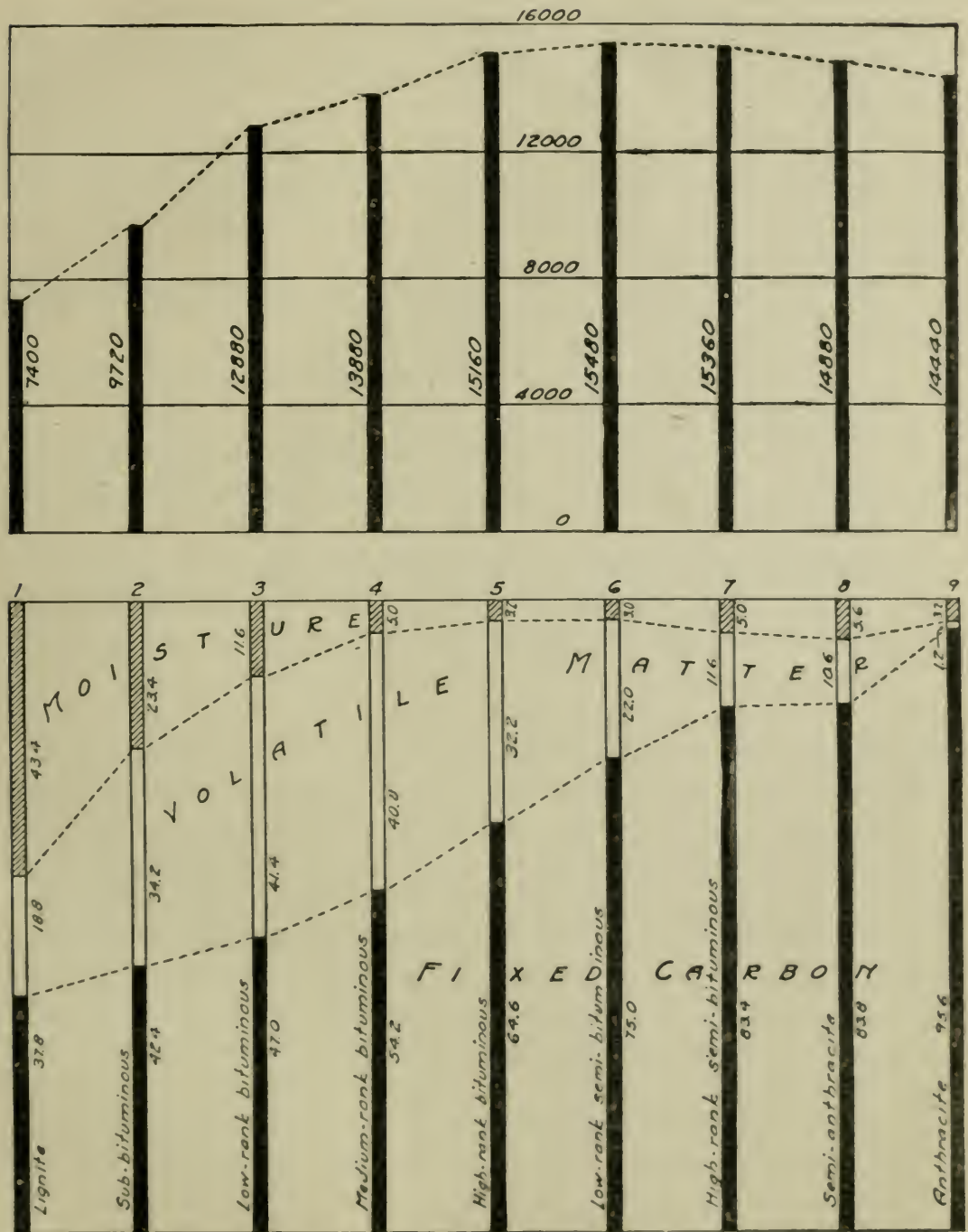


FIG. 13. DIAGRAM SHOWING THE CHEMICAL COMPOSITION AND HEAT VALUE OF THE SEVERAL RANKS OF COAL

The upper diagram shows the heat value of the samples of coal represented in the lower diagram, computed on the ash-free basis. The lower diagram shows the variations in the fixed carbon, the volatile matter, and the moisture of coals of different ranks, from lignite to anthracite, computed on samples as received, on the ash-free basis.

Bituminous coals are more extensively used in locomotives than other kinds. This fact is probably due to the wide geographical distribution of bituminous coal and to the many varieties which are available.

Fig. 13 shows graphically the proximate chemical composition of the various coals referred to and their heat producing values. The analyses have been recalculated to the "ash free" basis so as to make them strictly comparative.*

The lower diagram of Fig. 13 shows the following general differences between the different ranks of coal:

- (1) The proportion of fixed carbon increases quite regularly from lignite to anthracite.

- (2) The amount of moisture in the higher ranks of coals is small—it is about the same for semi-anthracite and high rank semi-bituminous—and increases rather rapidly for the lower ranks, lignite showing a very high percentage.

- (3) The greatest amount of volatile matter is found in the medium and low rank bituminous coals.

The upper diagram of Fig. 13 shows the heat values of the same coals on a similar "ash free" basis. It is sometimes wrongly inferred that the heat value of a coal depends upon its content of fixed carbon. This is not true, for the heat value of pure carbon is only 14,600 B. t. u., whereas coal No. 6 has a heat value of 15,480 B. t. u. The heat value of a coal depends largely upon the content of carbon and hydrogen. The carbon has a heat value of 14,600 B. t. u. and the hydrogen a value of 62,000 B. t. u. The greater heating value of coals containing less fixed carbon but more volatile matter than anthracite, for instance, is due to the fact that they contain hydrogen, which when burned produces greater heat than the same weight of carbon. Comprehensive tables of analyses of coal from all parts of the United States are contained in Bulletin 29 of the State Geological Survey, Urbana, Illinois, entitled "Purchase and Sale of Illinois Coal under Specifications," by S. W. PARR, and in Bulletin 3 of the Illinois Coal Mining Investigations, Urbana, Ill., entitled "Chemical Study of Illinois Coal," by S. W. PARR and in the publications of the U. S. Bureau of Mines and U. S. Geological Survey, the latest of these being Professional Paper 100A previously mentioned.

*These values are given in U. S. Geological Survey Professional Paper 100A, "The Coal Fields of the United States," by M. R. Campbell.

16. *Preparation a Factor Affecting the Value of Coal.*—Coal occurs in the earth in beds or seams, and usually in a solid mass as a rock. In mining it is blasted with powder, shoveled into cars, and conveyed to the surface. In the process of mining and handling it becomes broken up into pieces of all sizes. It may have some rock or dirt from the floor and roof of the mine mixed with it or there may be bands of layers of shaly matter in the coal seam itself. Coal as it comes from the mine is, therefore, not usually in condition for direct delivery to the consumer, but should first be “prepared” in order to remove these impurities and to separate it into the proper sizes for various markets. The impurities are removed, in the large sizes of coal by picking them out by hand, and in the smaller sizes by treating the coal in cleaning machinery. Separation into different sizes is accomplished by sending the coal over screens having holes of the proper sizes.

As a general rule, railroads have used for locomotives run of mine coal or various sizes of lump and egg. Screenings have recently been used on stoker fired engines which are being employed more and more extensively.

17. *The Selection of Coal.*—The beginnings of fuel conservation may be made in the purchase of the coal. While the purchasing agent is frequently limited in his choice by the presence of mines on the railway line, by the necessity of taking sizes of coal for which there is little other market, and by the fact that not all sizes are available in sufficient quantities for railway use, there are frequently occasions when these limitations are largely removed. Under these circumstances the size of coal should be chosen not only with respect to its price but with respect to its steam producing capacity.

There is considerable difference between the steam producing capacity in locomotives of various sizes of coal. No one expects, for example, to get as much steam from a ton of screenings burned in a locomotive as from a ton of mine run coal. Tests* recently made on a Mikado locomotive in the locomotive laboratory of the University of Illinois show the relative value in locomotive service of six different sizes of coal. All six sizes were from the same mine in Franklin County, Illinois, and while the results of the tests are strictly appli-

*“Comparative Tests of Six Sizes of Illinois Coal on a Mikado Locomotive,” Univ. of Ill. Eng. Exp. Sta., Bul. 101, 1917.

cable only to this coal, they probably apply in some measure to the great bulk of soft coal used on locomotives.

The tests were run at two rates of combustion; a medium rate at which about 4,000 pounds of coal were burned per hour, and a high rate at which about 7,500 pounds were burned per hour. Comparisons between the sizes were made upon the basis of their steam producing ability, that is, upon the basis of the evaporation per pound of coal which took place with each size.

At the medium rate the coals stand in the following order: 3-inch by 6-inch egg, mine run, 2-inch lump, 2-inch by 3-inch nut, 2-inch screenings, and 1 $\frac{1}{4}$ -inch screenings. Assuming mine run coal to produce 100 per cent of steam, the egg coal produced 2 per cent more and the lump and nut coals respectively 1 per cent and 2 per cent less. The 2-inch screenings produced 7 per cent less steam, and the 1 $\frac{1}{4}$ -inch screenings 15 per cent less.

The relation is somewhat different for the higher rate of combustion. Again assuming the mine run in the high rate test to produce 100 per cent of steam, the nut coal produced 6 per cent more, the egg 5 per cent more and the lump 3 per cent less. The 2-inch and 1 $\frac{1}{4}$ -inch screenings produced respectively 13 per cent and 18 per cent less.

These facts should enable the railway purchasing agent to form some idea of the sizes of coal with the greatest steam producing capacity although the fact that the coal used in these tests came from a single mine should not be overlooked. Obviously the choice of coal can seldom be made solely on the basis of the steam producing qualities of the fuel, since the market price must also be taken into consideration. While no general statement can be made regarding the relative prices warranted for the different sizes of fuel, the question is somewhat more fully discussed in the bulletin mentioned.

In the present emergency all purchasers of fuel are glad to get coal of any kind and they are allowed little opportunity to choose. The problem is to get coal enough of any sort for the purposes in hand, but these temporary conditions ought not to prevent the recognition of the fact that a considerable saving in coal and money may be effected by a proper choice of fuel.

18. *Purchase of Coal.*—The custom of purchasing coal on the B. t. u. basis is increasing and if the specifications for such purchase are properly drawn and understood it is the logical way to buy coal.

It is equivalent to buying so many heat units instead of so many tons of coal and is an effort to get the greatest amount of heat for a dollar. Upon this basis a purchaser should be able to determine whether a low priced coal which gives less efficient boiler service and requires more labor and expense in handling ashes is really cheaper than a higher priced coal.

With reference to the selection of most locomotive coals, the B. t. u. value and the percentage of ash furnish a general guide to their relative merits. The ash content and character directly affect the B. t. u. value, so that the relative value may be expressed with fair accuracy by the B. t. u. value alone, although the value of any coal drops off more rapidly than its B. t. u. value when the ash content exceeds 10 or 15 per cent.

It should be understood that the purchase of coal on the B. t. u. basis does not insure maximum efficiency from the fuel, because a high grade coal carelessly fired or having a bad clinkering ash may give poorer results than a low grade coal carefully fired. In other words, the high B. t. u. value of a coal establishes a presumption in favor of that coal which must be tested by actual trial under good operating conditions. The person who furnishes coal of a high B. t. u. value cannot be held responsible for poor results obtained through improper use. The great disadvantage with regard to the purchase of coal on the B. t. u. basis lies in the difficulty of obtaining samples for analysis which are truly representative of the coal received by the consumer. Too much emphasis can hardly be placed upon the necessity for exercising great care in the taking of samples in order that they may truly represent the coal for which they stand.

19. *Storage of Coal.*—Coal is stored by railroads to guard against a shortage which may result from failure to receive coal from the mines on account of strikes, explosions, interruptions in traffic, or other causes, and to use coal loading equipment in company service when it is not in demand for commercial service. There is an advantage also in hauling large quantities of coal in the summer months when the cost of haulage is lower than at any other season.

Railroad storage has been very fully discussed in the Proceedings of the International Railway Fuel Association and very full details will be found in Circular 6 of the Engineering Experiment Station of the University of Illinois. The Fuel Station Committee of the Inter-

national Railway Fuel Association suggests that all new mechanical coaling plants be so designed as to provide a storage adjunct, so that coal may be unloaded into a receiving hopper from which it may be distributed either to the coaling station direct or to a storage pile. The plant should be designed so as to permit the recovery of the coal from the storage pile without the use of railroad cars.

The usual method of storing railroad coal, as illustrated in Fig. 14, is to start a pile on the ground and gradually to raise the track on top of the coal pile until a height of 10 to 20 feet is reached, the end of the track being supported on a crib as shown in Fig. 15. The coal is then reclaimed with a locomotive crane or steam shovel, or by hand shoveling into cars.

Although this is a simple and cheap way to store coal, the conditions of storage are favorable to spontaneous combustion. Especially is this true since run of mine coal is generally stored and the weight of the locomotive and loaded cars on top of the coal break it up and produce fine material in the center of the pile while the lumps roll to the bottom of the outside slope and afford passages through which the air may reach the fine material. If stored in piles as shown in Fig. 14 the pile should not be continuous, but passageways should be left through the pile at regular intervals in order to facilitate the removal of the coal in the case of heating.

Fig. 16 shows a circular storage pile used in connection with a coaling station, and Fig. 17 a large coaling station. Many of the railroads issue instructions with regard to the method of storing coal, but these instructions are too frequently not carried out.

The following general conclusions regarding storage are based upon answers to a questionnaire sent to several hundred firms and individuals storing coal under different conditions. It is believed that the statements made represent very closely the present consensus of opinion on the subject.

Kinds and Sizes of Coal which may be Safely Stored

Although it is undoubtedly true that some coals may be stored with greater safety than others, the danger from spontaneous combustion is due more to improper preparation and piling of coal than to the kind of coal stored.

Most varieties of bituminous coal may be safely stored if of proper size and if free from fine coal and dust. The coal must be handled in



FIG. 14. A LONG PILE OF STORED COAL



FIG. 15. STORAGE COAL WITH UNLOADING TRACK AND CRIB

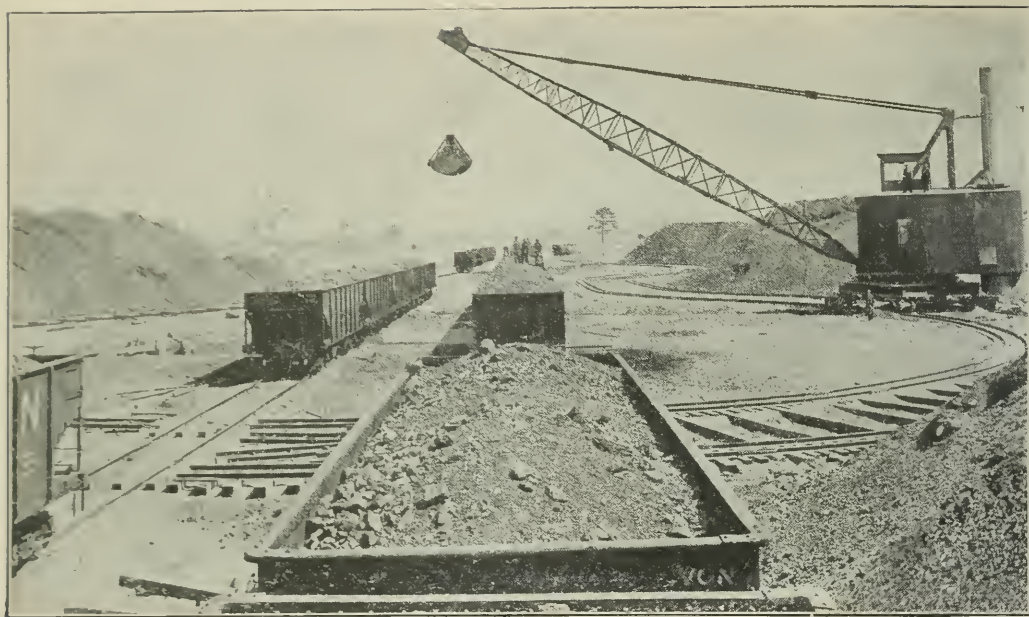


FIG. 16. A CIRCULAR PILE OF STORED COAL

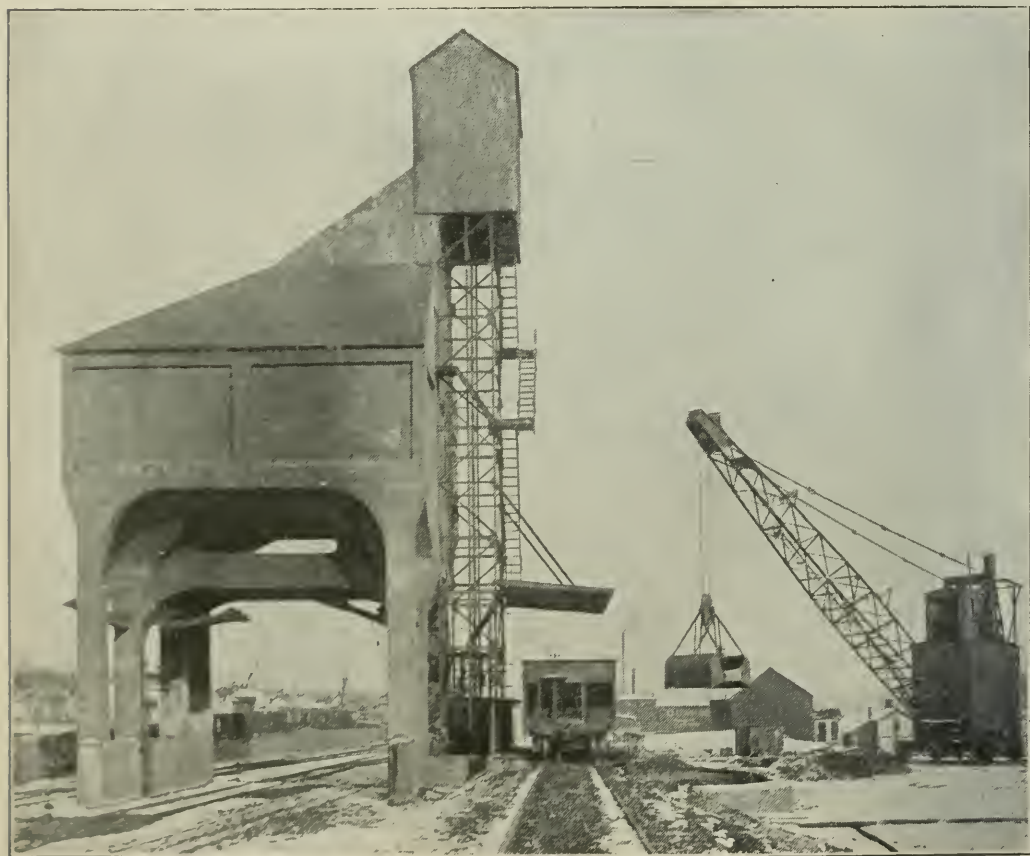


FIG. 17. A COAL STORAGE STATION AND CHUTE

such manner as to prevent excessive breakage and the consequent production of much fine coal and dust when being placed in storage, because spontaneous combustion is due mainly to the oxidation of the coal surface. The danger of spontaneous combustion in storing coal is very greatly reduced, if not entirely eliminated, by storing only lump coal from which the dust and fine coal have been removed. Of two coals, the one which is less friable should be chosen for storage purposes because less dust and fine coal will be produced in its handling. All varieties of bituminous coal can be stored under water, since water excludes the air and prevents spontaneous combustion.

Fine coal or slack has sometimes been safely stored in cases in which air has been excluded from the interior of the pile. Exclusion of air from the interior may be accomplished (1) by a closely sealed wall built around the pile or (2) by close packing of the fine coal. Any pile of slack requires careful watching to detect evidences of heating and means should be provided for moving the coal promptly if heat develops. The only absolutely safe way to store slack or fine coal is under water.

Many varieties of mine run coal cannot be stored safely because of the presence of fine coal and dust mixed with the lumps.

Coal exposed to the air for some time may become "seasoned" and thus may be less liable to spontaneous combustion because of the oxidation of the surfaces of the lumps of coal, but opinions are by no means unanimous on this point.

It is believed by many that damp coal or coal stored on a damp base is peculiarly liable to spontaneous combustion, but the evidence on this point is by no means conclusive. It is safer not to dampen the coal when or after it is placed in storage.

The Effect of Sulphur on Spontaneous Combustion

It has been shown by experimentation that the sulphur contained in coal in the form of pyrites is not the chief source of spontaneous combustion, as was formerly supposed, but the oxidation of the sulphur in the coal may assist in breaking up the lumps of coal and thus may increase the amount of fine coal which is particularly liable to rapid oxidation. Even this opinion is not unanimously endorsed. In spite of experimental data showing that sulphur is not the determining element in spontaneous combustion, the opinion is wide-spread that,

if possible, it is well for storage purposes to choose a coal with a low sulphur content.

Method of Piling Coal

To prevent spontaneous combustion, coal should be so piled that air may circulate freely through it and thus may carry off the heat due to oxidation of the carbon, or it should be so closely packed that air cannot enter the pile and oxidize the fine coal.

Stratification or segregation of fine and lump coal should be avoided, since an open stratum of coarse lumps of coal provides a passage through which air may enter and come in contact with the fine coal, thus oxidizing it and starting combustion.

If space permits, low piles are preferable since in low piles the coal is more fully exposed to the air and is better cooled than in high piles and in case of heating it can be more readily and quickly moved. A disadvantage of high piles lies in the greater difficulty of moving the coal quickly, if necessary. The idea that a high pile causes heating at the bottom is erroneous, since as many fires take place near the top as near the bottom, and as many near the outside as near the interior of the pile. If possible, the coal should be divided by alleyways so as to facilitate rapid loading out the coal in case of necessity, and so that an entire coal pile may not be endangered by a local fire.

Much of the attempted ventilation of coal piles has been inadequately done through the use of only an occasional ventilation pipe, which has been not much more than a place in which to insert a thermometer. The practice of placing ventilating pipes closely together has been used in Canada and is reported to be effective.

Water is an effective agent in quenching fire in a coal pile only if it can be applied in sufficient quantities to extinguish the fire and to cool the mass. A small quantity of water is not effective. Unless there is an ample supply of water thoroughly to quench the fire and to cool the pile, it is very dangerous to add any water to a coal pile.

Coals of different varieties should if possible not be mixed in storage, for the coal possessing the greatest tendency toward spontaneous combustion may jeopardize the safety of other varieties not so liable to spontaneous combustion.

The Effects of Storage upon the Properties of Coal

The heating value of coal as expressed in B. t. u. is decreased very little by storage, but the opinion is wide-spread that storage coal burns

less freely than fresh coal. Storage does, however, have the physical effect of producing an amount of fine coal and dust which, when fired in locomotives, tends to increase the percentage of unburned coal and cinders discharged through the stack. Dirt, cinders or other foreign matter are sometimes picked up when stored coal is reclaimed and the burning qualities of the fuel thereby seriously impaired.

The coking properties of most coals seem to decrease as a result of storage, but coals vary greatly in this respect.

The deterioration of coal stored under water is negligible, and such coal absorbs very little extra moisture. If only part of a coal pile is submerged, the part exposed to the air is still liable to spontaneous combustion.

Additional Precautions

The best preventive of loss in coal storage is regularly to inspect the pile. If the temperature reaches 150 degrees F., the pile should be carefully watched and if the temperature rises to 175 or 180 degrees F. the coal should be removed as promptly as possible. The coal should be thoroughly cooled before being replaced in storage.

Storage appliances and arrangements should be so designed as to make it possible to remove the coal quickly if necessary, and coal should not be stored in large piles unless provision is made for loading it out quickly.

Pieces of wood, greasy waste, or other easily combustible material mixed in a coal pile may form a starting point for a fire, and every effort should be made to keep such material from the coal as it is being placed in storage.

It is important that coal in storage should not be subjected to such external sources of heat as steam pipes, because the susceptibility of coal to spontaneous combustion increases rapidly as the temperature rises.

Expense of Storing Coal

The following statements or estimates of the expense of storing coal are given by the railroads noted:

TABLE 3 *
EXPENSE OF STORING COAL

Source of Information	Cost in Dollars per Ton		
	Storing	Reclaiming	Total
Clinchfield Fuel Co., Dante, Va., Crane & Trestle.....	.0671	.0655	.1326
Estimate of C. G. Hall, former Secretary Int. Ry. Fuel Association.....			.2628
Missouri, Kansas & Texas Railroad, including cost of track	.035	.035	.07
Chicago, Lake Shore & South Bend Railroad.....			.12
Grand Trunk Pacific Ry.....	.08	.062	.142
Atlantic Coast Line.....			.04-.05
Central of Georgia Railroad.....	.0258	.0209	.0467

* These estimates vary widely according to the conditions under which the storing was done and for this reason are not directly comparable. The conditions of storage in each of the cases covered in this table are set forth in detail in Circular 6, entitled "The Storage of Bituminous Coal," by H. H. Stock, Univ. of Ill. Eng. Exp. Sta., March, 1918.

VI. SUMMARY

20. *Ways in which Locomotive Coal may be Saved.*—Large coal savings may be made by slightly modifying the methods of handling transportation, by the use of carefully designed equipment and auxiliary apparatus, by better maintenance of equipment, by better handling of locomotives when not hauling trains, and by more careful attention to the firing and operation of the locomotive when hauling trains. The importance of coöperation in the matter of fuel economy should not be overlooked.

In Handling Transportation

Transportation should be so handled, train schedules should be so arranged, and the calling of locomotives should be so carefully watched that waste of coal through unnecessary terminal delays and delays on sidings may be eliminated. Thirty million tons of coal per year are burned on locomotives when they are not hauling trains. Transportation officials have an opportunity to save some of this coal.

In Locomotive Design

The responsibility for installing equipment of the best design and for the use of at least some of the most approved fuel saving devices, such as the superheater and brick arch, rests upon railway executive officials. The fact that changes along such lines take time and require capital expenditure may possibly remove them from the class of emergency measures, but this fact does not lessen the possibility of effecting large coal savings through these means. A more extended use of fuel saving devices of proved merit should be made.

In Maintenance

Proper maintenance of equipment, particularly of locomotives, is one of the principal means through which large fuel savings may be made. The importance of this matter is often overlooked or is not fully appreciated. While a large number of persons is concerned with maintenance matters, the responsibility rests principally upon officials of the mechanical department and upon the shop and round-house forces.

In Handling and Care of Locomotives

In addition to matters of maintenance, roundhouse men are in position to effect substantial savings in the direct use of coal when cleaning, building, and maintaining locomotive fires, and they can effect savings of still greater importance through proper boiler washing and cleaning of flues. The care which a locomotive receives and the work which is done upon it between trips is of the greatest importance in relation to fuel economy. Adequate roundhouse and other terminal facilities should be provided as rapidly as possible.

In Locomotive Operation

Engine crews are in direct charge, hauling trains, while 80 per cent of the locomotive fuel is being burned. While the extent of the saving to be effected in the burning of this fuel often depends in part upon the design, upon the maintenance and care of the equipment, and upon activities of transportation officials, there still remain with the engineers and firemen opportunities for fuel saving of the greatest importance.

Coöperation

The highest degree of economy in the use of locomotive fuel cannot be attained without coöperation. Executive officials must furnish funds for the equipment and labor necessary to promote fuel saving work. There must be close coöperation between the transportation and mechanical departments in order that terminal and other delays may be reduced and that power and fuel may be handled and used to the best advantage. Shop men and officials must appreciate the importance of proper maintenance of locomotives, cars, and auxiliary equipment. Roundhouse forces and engine crews must work together if locomotives are to be economical in the use of coal both while on the road and while being cared for between trips. Engineers must render carefully prepared and specific work reports, and the roundhouse forces must act in a thorough going way upon the matters reported and return locomotives to the roadmen in the best possible condition.

21. *Estimated Possible Saving to be Effected in the Use of Locomotive Coal.*—The railways used in locomotives during the year 1917 about 140,000,000 tons of coal. The corresponding amount for 1918

will be about 150,000,000 tons. A saving of six per cent of this fuel is readily possible through the coöperative action of railroad officials and employes. Such a saving would amount to 9,000,000 tons. A much larger saving is ultimately possible through a general improvement in equipment and the adoption of approved fuel saving devices.

Six per cent of the locomotive coal is sufficient to operate 2500 additional modern locomotives; it is equivalent to the fuel supply of some of our largest railway systems; it would coal sufficient transports to take an army of many millions of men to France.

Every person connected with the railroad service should ask himself:

- (1) How much coal can be saved by the group of which I am a part?
- (2) What are the ways in which I can assist in saving coal?
- (3) Am I doing my part as an individual to save coal?
- (4) Am I coöperating with others so as to assist in securing the greatest combined saving?

Executive Officials and Transportation Forces

Executive officials, transportation officials and employes, particularly superintendents, train masters, dispatchers, fuel agents, and inspectors can effect coal savings in many ways. They should be able materially to reduce stand-by losses and to effect economies in connection with the loading of locomotives and in the selection, distribution, and handling of locomotive coal.

Operating officials should thoroughly acquaint themselves with the fuel problem and should take such action as will impress all officials and employes under them with the importance and possibilities of fuel savings. Too much emphasis can hardly be placed upon the necessity of operating officials taking an active interest in the matter of fuel economy and upon their giving personal and financial support to fuel saving measures.

Engines should, for fuel economy, neither be operated with very light loads nor with loads so heavy that they cause poor combustion conditions, or occasion undue delays through stalling or other accidents.

If possible reduce speed in passenger work or eliminate some passenger runs. Coal savings can be made in this way.

Arrange freight train schedules and the methods of calling engines so as to prevent locomotives standing and wasting coal through delays at terminals, on sidings, at meeting points, and at ashpits.

Assist in furnishing the most suitable kind of coal for the various types of locomotives or classes of service.

Prevent loss of coal in transit, at coal chutes, and at storage piles.

See that coal is so handled that it is not unnecessarily broken up, thereby impairing its value as fuel.

Executive and Motive Power Officials

Executives including motive power officials can effect important fuel savings through matters of design, improved equipment, and better shop and roundhouse facilities. They should consider the purchase of and should apply, so far as may be possible, improved motive power and equipment. They should install as rapidly as possible such fuel saving devices as the superheater and the brick arch; and they should improve locomotive terminal facilities and improve or extend existing shop and roundhouse equipment to the extent necessary to promote better maintenance and care of locomotives.

Motive Power Officials and Shop and Roundhouse Forces

Mechanical department employes and officials concerned with the maintenance of equipment have opportunity to stop some of the largest coal losses which take place in locomotive operation. Every feature of shop and roundhouse management, equipment, or operation which will make it possible to maintain locomotives in the best possible condition means a saving of fuel. A locomotive in very poor condition may use from 20 to 40 per cent more fuel than when in thoroughly good condition. Those responsible for the proper maintenance of locomotives are responsible for a large part of the coal saving that it is possible to make at the present time. Engineers and firemen, no matter how skilful or careful, cannot use coal economically on poorly maintained equipment. It is also more difficult for engineers and firemen to know whether or not they are handling their locomotive to the best advantage when it is not in proper condition, and the incentive to meet all the requirements of economical operation is lacking when poorly maintained equipment is being handled.

Locomotives should be maintained to prevent serious steam, air and water leaks.

Steam pipes and superheaters should be tested and kept free from leaks.

Leaks past the cylinder and valve packing and past rod packings should be prevented.

Leaks from the boiler through leaky mudring, flues, or at other points should be prevented.

Air leaks to the front end should be stopped.

Air pumps and all auxiliary steam using apparatus should be kept in the best possible condition. Locomotive friction should be kept as low as possible by having the mechanism well maintained.

Radiation losses should be kept as low as possible through properly maintained lagging.

Exhaust nozzle opening should be as large as is consistent with the proper steaming of the locomotive.

The locomotive should be equipped with injectors of the proper size.

All auxiliary apparatus such as power reverse mechanism and automatic firedoors should be in condition to be used to the best advantage.

Firemen cannot meet the requirements for maintaining proper firebox conditions unless firedoors, arches, grates, ashpans, and dampers are in good condition, and unless the flues are kept clean. Shop and roundhouse men must meet the requirements of proper maintenance.

Roundhouse Forces and Hostlers

Roundhouse men and officers concerned with the handling and care of the locomotive between trips are immediately concerned with keeping the locomotive in the best possible condition. It has already been pointed out that one of the best means of effecting a saving of coal is by keeping locomotives in somewhat better general condition. This is to be done largely through proper cleaning of flues, washing of boilers, and attention to light repairs. Savings are to be made in connection with the coal which is used on the locomotive while it is being turned, particularly in connection with coal or live fire lost at the ashpit.

Flues should be cleaned after every trip, and special attention should be given to superheater flues. Boilers should be washed frequently.

When locomotives are cleaned the firebox should be thoroughly inspected in order that all necessary cleaning and repairs relating to grates, arches, and flues may be attended to.

In cleaning, building, banking and keeping up fires, every effort should be made to save fuel.

Engineers and Firemen

Engineers and firemen who are in charge while the greater part of the coal is used must accept a considerable part of the responsibility for effecting savings. Engineers through their methods of handling the locomotive can increase or decrease the amount of coal burned to a very marked degree. Firemen by maintaining the best possible combustion conditions can reduce some of the most important losses which occur, such as those to the ashpit and to the stack.

Firemen should keep constantly in mind the three conditions which are necessary for complete combustion; first, that enough and not too much air be admitted to the firebox; secondly, that the air be thoroughly mixed with the coal burning on the grate and with the combustible gases arising from the fuel bed; thirdly, that a sufficiently high and uniform firebox temperature be maintained.

Determine how these conditions can be produced and endeavor to obtain them. Maintain a level, bright fire.

All lumps of coal should be broken into pieces not larger than three or four inches in diameter. Ordinarily, coal should be broken to the proper size before being placed on the tender. In so far as is possible, the fireman should do his part to avoid losses which are caused by firing large lumps of coal.

Do not let banks or holes form in the fire.

Prevent clinker formation so far as possible.

Fire the coal in small amounts; from one to four shovelful at a time.

Spread the coal over the brightest and thinnest parts of the fire.

Close the door after each shovelful of coal is fired.

Shake the grates only when necessary, and then only lightly. Avoid disturbing the fuel bed to such an extent that holes and additional clinker form and that live fuel is lost to the ashpan. Preferably, the grates should not be shaken when running, particularly when the locomotive is working very hard. Air for combustion comes through the grates and firebed. When coming into the firebox in this way it

is more nearly uniformly distributed than if it comes through the door and so will be better mixed with the burning fuel and gases.

Do not rake the fire unless it seems absolutely necessary in order to fill a hole or break up a bank.

Anticipate heavy demands which may be made upon the locomotive and have the fire built up as much as may be possible to meet them.

Anticipate periods when very light demands may be made upon the locomotive and avoid "popping off" or opening the firedoor.

Do not have more fire than is necessary when the terminal is reached.

Do not let coal fall or blow away from the engine deck. Keep the engine deck clean. Use the sprinkler hose enough to keep down dust and prevent fine coal from blowing away.

Locomotive engineers should handle the throttle and the reverse lever so as to be economical of steam and should assist the fireman by using as little steam as possible and by operating the locomotive in such manner as to disturb as little as possible uniform conditions in the firebox.

In general a full throttle opening should be used, exceptions being when starting or drifting, or when working with short cut-off.

Injectors should feed the boiler at a fairly uniform rate, and should be so handled as to assist the fireman in maintaining uniform and satisfactory firebox conditions. The boiler should not be filled to a point likely to cause priming.

Waste of steam through safety valves should be avoided. All auxiliary apparatus should be handled so as to be as economical of steam and air as is possible.

Unnecessary stops and delays should be avoided so far as possible.

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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

BULLETIN No. 105

MAY, 1918

HYDRAULIC EXPERIMENTS WITH VALVES,
ORIFICES, HOSE, NOZZLES, AND
ORIFICE BUCKETS

PART I

LOSS OF HYDRAULIC HEAD IN SMALL VALVES

BY ARTHUR N. TALBOT

PROFESSOR OF MUNICIPAL AND SANITARY ENGINEERING
IN CHARGE OF THEORETICAL AND APPLIED MECHANICS

AND

FRED B SEELY

ASSISTANT PROFESSOR OF THEORETICAL AND APPLIED MECHANICS

PART II

THE FLOW OF WATER THROUGH SUBMERGED ORIFICES

BY FRED B SEELY

ASSISTANT PROFESSOR OF THEORETICAL AND APPLIED MECHANICS

PART III

FIRE STREAMS FROM SMALL HOSE AND NOZZLES

BY VIRGIL R FLEMING

ASSISTANT PROFESSOR OF APPLIED MECHANICS

PART IV

THE ORIFICE BUCKET FOR MEASURING WATER

BY MELVIN L. ENGER

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ENGINEERING EXPERIMENT STATION

PUBLISHED BY THE UNIVERSITY OF ILLINOIS, URBANA

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PREFACE

AS a part of the experimental work conducted in the Hydraulic Laboratory of the University of Illinois a number of problems has been investigated which has not been large enough in scope to warrant publication as separate bulletins. It seems well, however, to put on record the results of such experiments, and this bulletin presents a record of four of these problems. It is believed that the four papers will be found to be of use in various aspects of engineering practice even though the experiments are not exhaustive investigations.

The investigations for the most part have been the outgrowth of experimental work begun by students, largely as thesis work, and carried on over a period of several years.

The variety of conditions under which the flow of water takes place, the possibility of large changes in the state of the flow due apparently to small changes in the form of the passages through which the water flows, and the necessity of persistent effort in subjecting assumptions and analytical deductions to experimental verification, make it desirable to report all hydraulic experimental results which are believed to be reliable.

A part of the experimental results herein reported has appeared in the publication of a technical society. The material, however, has been expanded in this bulletin and will be found in a more convenient form for use.

ARTHUR N. TALBOT

FRED B SEELY

Editors

PART I

LOSS OF HYDRAULIC HEAD IN SMALL VALVES

By ARTHUR N. TALBOT

PROFESSOR OF MUNICIPAL AND SANITARY ENGINEERING
IN CHARGE OF THEORETICAL AND APPLIED MECHANICS

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CONTENTS

PART I

LOSS OF HYDRAULIC HEAD IN SMALL VALVES

	PAGE
I. INTRODUCTION	7
1. Preliminary	7
2. Acknowledgment	7
II. APPARATUS AND METHOD OF EXPERIMENTING	8
3. Valves	8
4. Method of Experimenting	8
III. EXPERIMENTAL RESULTS AND DISCUSSION	10
5. Loss of Head	10
6. Earlier Experiments on Gate Valves	18
7. Coefficients of Discharge for Gate Valves	20
8. Summary	21

LIST OF FIGURES

NO.		PAGE
1.	Longitudinal Sections of Valves Tested	9
2.	Arrangement of Apparatus	11
3.	Curves Showing the Relation between the Velocity and Head Lost in 1-inch Gate Valve	13
4.	Curves Showing the Relation between the Velocity and Head Lost in 2-inch Gate Valve	13
5.	Curves Showing the Relation between the Velocity and Head Lost in 1-inch Globe Valve	14
6.	Curves Showing the Relation between the Velocity and Head Lost in 2-inch Globe Valve	14
7.	Curves Showing the Relation between the Velocity and Head Lost in 1-inch Angle Valve	15
8.	Curves Showing the Relation between the Velocity and Head Lost in 2-inch Angle Valve	15
9.	Curves Showing the Relation between Coefficients of Loss and Valve Openings	16
10.	Curves Showing the Effect of Gradual Change of Section through 2-inch Globe Valve	17

LIST OF TABLES

1.	Experimental Values of Coefficients of Loss	16
2.	Values of the Coefficient of Loss for Gate Valves of Various Diameters Due to Partial Closure Only	19
3.	Experimental Values of the Coefficients of Discharge for Gate Valves .	20

HYDRAULIC EXPERIMENTS

WITH

VALVES, ORIFICES, HOSE, NOZZLES, AND ORIFICE BUCKETS

PART I

LOSS OF HYDRAULIC HEAD IN SMALL VALVES

I. INTRODUCTION

1. *Preliminary.*—Part I of this bulletin presents the results of experiments on the flow of water through 1-in. and 2-in. gate valves, 1-in. and 2-in. globe valves, and 1-in. and 2-in. angle valves. The loss of head caused by each valve, expressed in terms of the velocity head in the pipe, is given for four different ratios of the height of the valve opening to the diameter of the full valve orifice, namely, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1. The coefficients of discharge are also given for the gate valves for each of the four valve openings.

In a long pipe line the total amount of lost head is caused chiefly by pipe friction, the resistance due to a valve being comparatively small except for very small valve openings.

In a variety of cases, however, where valves are used on comparatively short pipe lines as, for example, in hydraulic elevator service, in office buildings, and in special apparatus it is important to know the lost head caused by small valves of different kinds when set at various positions. Very few experimental results have been published on this subject, particularly for globe and angle valves. Any experimental work, furthermore, which helps to indicate the laws governing the flow of water should prove of value. With these facts in mind the results herein recorded have been prepared.

2. *Acknowledgment.*—The experiments herein considered were performed as student thesis work in the Hydraulic Laboratory of the University of Illinois by M. E. THOMAS, class of 1906, under the direction of PROFESSOR ARTHUR N. TALBOT. Unusual care in the experimenting is reflected in the congruity of the data presented in Mr. Thomas' thesis.

II. APPARATUS AND METHOD OF EXPERIMENTING

3. *Valves.*—The valves used were bought in the open market and tested just as received. The passages through the 2-in. globe valve were then modified by the use of plaster of paris to give a more gradual change of section (see Fig. 10), and this valve was tested again. The types or forms of the interiors of the valves and the dimensions of some of the passageways through the valves are shown in Fig. 1. The 1-in. globe valve and the 1-in. angle valve were made by the Western Tube Company. All the other valves were made by the Crane Company.

4. *Method of Experimenting.*—The arrangement of the apparatus is shown in Fig. 2. The test valve was placed in a horizontal pipe to which water was supplied from a standpipe under a static head of about 50 feet. The quantity of water discharged through the valve was regulated by another valve downstream from the test valve. The volume discharged in a certain time was measured in a calibrated pit and the time taken with an ordinary watch from which the rate of discharge was calculated. Three-way gage connections for obtaining the pressure head in the pipe were made at a section one foot upstream and one foot downstream from the valve. Care was taken to avoid having these connections project into the interior of the pipe. It was found by experiment that when any two of the three pressure connections at either section were closed, the same difference of head was registered as when all three connections at either section were open. The three-way connections were used, however, in all the experiments. The difference in the pressure heads at the two sections was measured by a differential mercury gage. A Crosby pressure gage was also attached at each section to serve as a rough check on the differential gage. The lost head due to the pipe friction for the two feet of pipe between the two sections was assumed to be as given in Weston's Tables of Friction of Water in Pipes. This amount of lost head was subtracted from the reading of the differential mercury gage in determining the loss of head caused by the valve.

The loss of head and the corresponding rate of discharge and velocity in the pipe were determined for each of four valve openings for each of the six valves tested. The valve openings used were such that the heights of the openings were one-fourth, one-half, three-fourths, and one times the diameter of the full valve orifice.

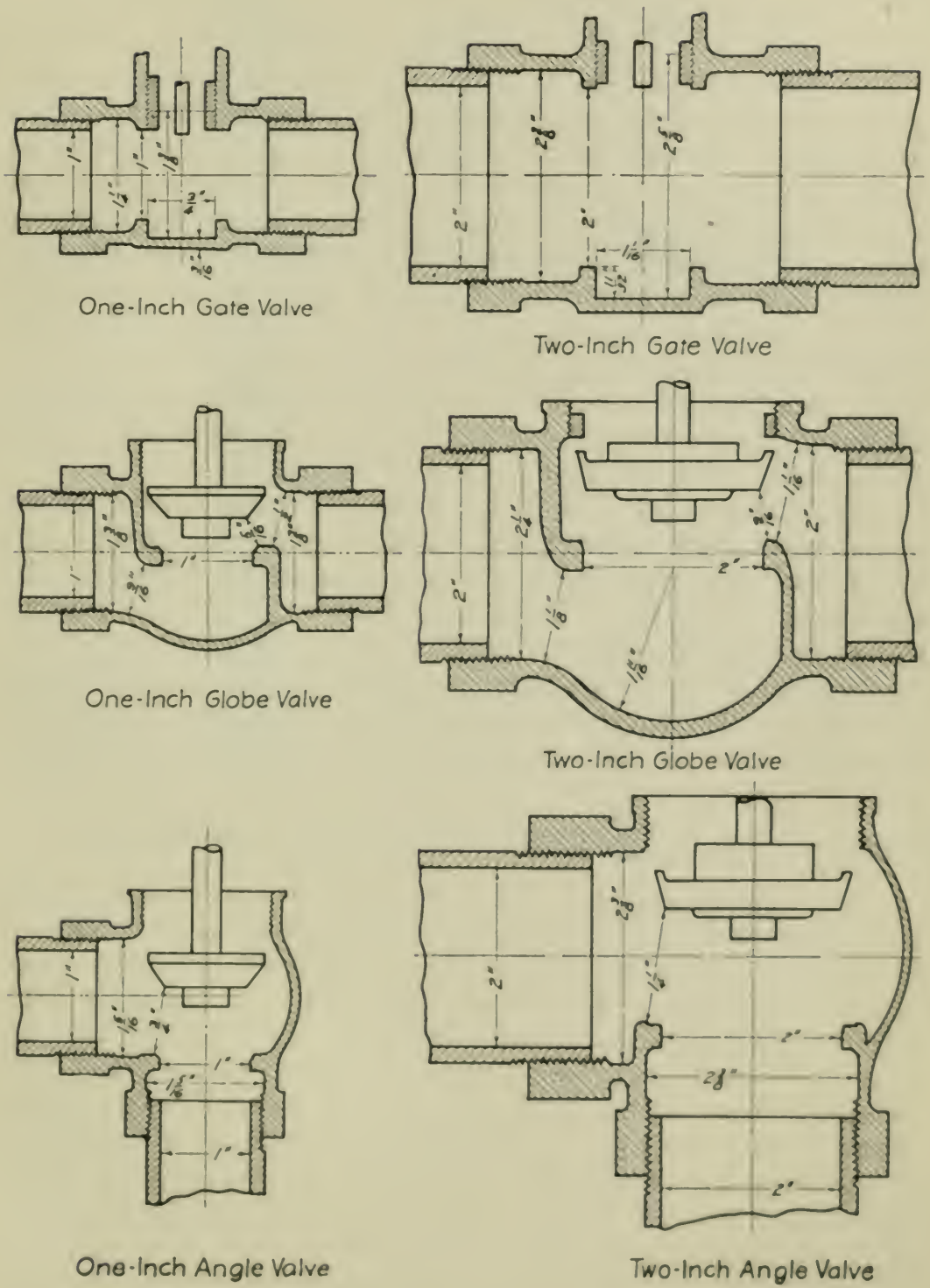


FIG. 1. LONGITUDINAL SECTIONS OF VALVES TESTED

III. EXPERIMENTAL RESULTS AND DISCUSSION

5. *Loss of Head.*—In Fig. 3 to 8 values of the lost head caused by the valve are plotted as abscissas and the mean velocity in the pipe as ordinates. The assumed value of the friction head for the two feet of pipe between pressure connections is subtracted from the differential mercury gage reading in plotting the abscissas. There is, of course, some doubt concerning the correct allowance to be made for this pipe friction. The loss of head due to this cause, however, will be relatively small except for the larger valve openings. It will be noted from the curves in Fig. 3 to 8 that the range of velocity in the pipe varied of course with the kind of valve and with the amount of valve opening. The smallest mean velocity in any case was about $\frac{1}{4}$ ft. per sec., while the maximum mean velocity was about 40 ft. per sec.

The curves in Fig. 3 to 8 give values of the loss of head caused by the valves which vary as the square of the velocity in the pipe, that is, the lost head due to the valve may be expressed in terms of the velocity head in the pipe. This fact is shown very clearly by plotting the values from the curves in Fig. 3 to 8 on logarithmic paper; the curves showing the relation between the lost head, h , and the velocity in the pipe, v , become parallel straight lines with a slope varying but little from two, the slope indicating the exponent in the equation $h = kv^n$. That is, $h = kv^2$ or, when expressed in terms of the velocity head in the pipe, $h = \frac{mv^2}{2g}$ in which m is called the coefficient of loss.

Values of the coefficients of loss for the valves with the various valve openings as obtained from the curves in Fig. 3 to 8 are given in Table 1. These values have been plotted in Fig. 9 as abscissas against the valve openings as ordinates. From these curves and also from Table 1 the resistance to flow caused by the three kinds of valves may be compared at various valve openings. It will be noted that the loss of head varies in a quite different manner with the amount of valve opening for these three kinds of valves, for instance, a comparison of the results for the valves when completely opened shows that a globe valve causes more than twice as much loss of head as the corresponding size of angle valve, while a gate valve causes markedly less loss than either a globe or an angle valve, the velocity in the pipe being the same in the three cases. As the valve is gradually closed, the resistance to flow of the angle valves increases the least (decreasing at first) while the resistance

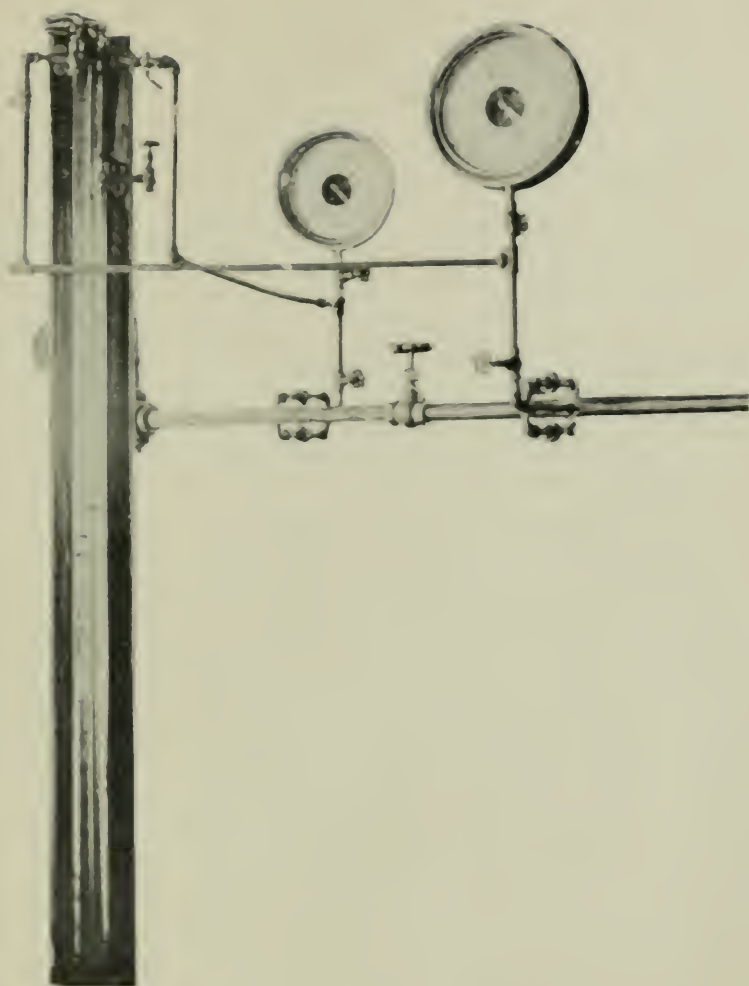


FIG. 2. ARRANGEMENT OF APPARATUS

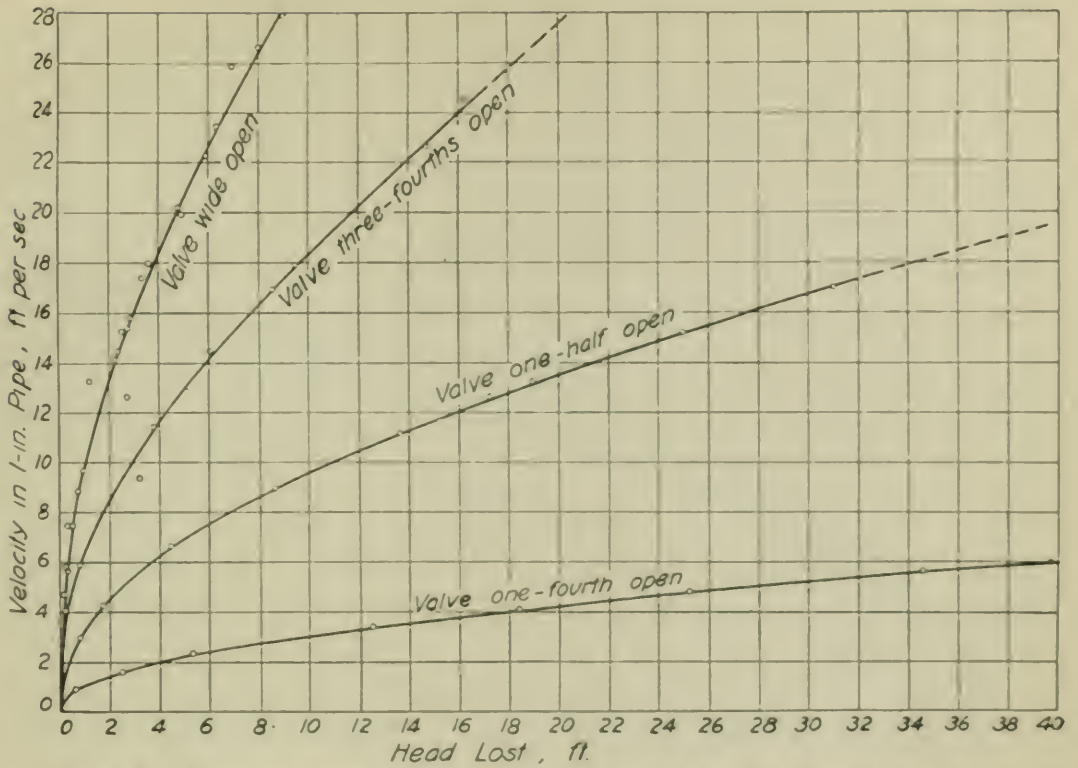


FIG. 3. CURVES SHOWING THE RELATION BETWEEN THE VELOCITY AND HEAD LOST IN 1-INCH GATE VALVE

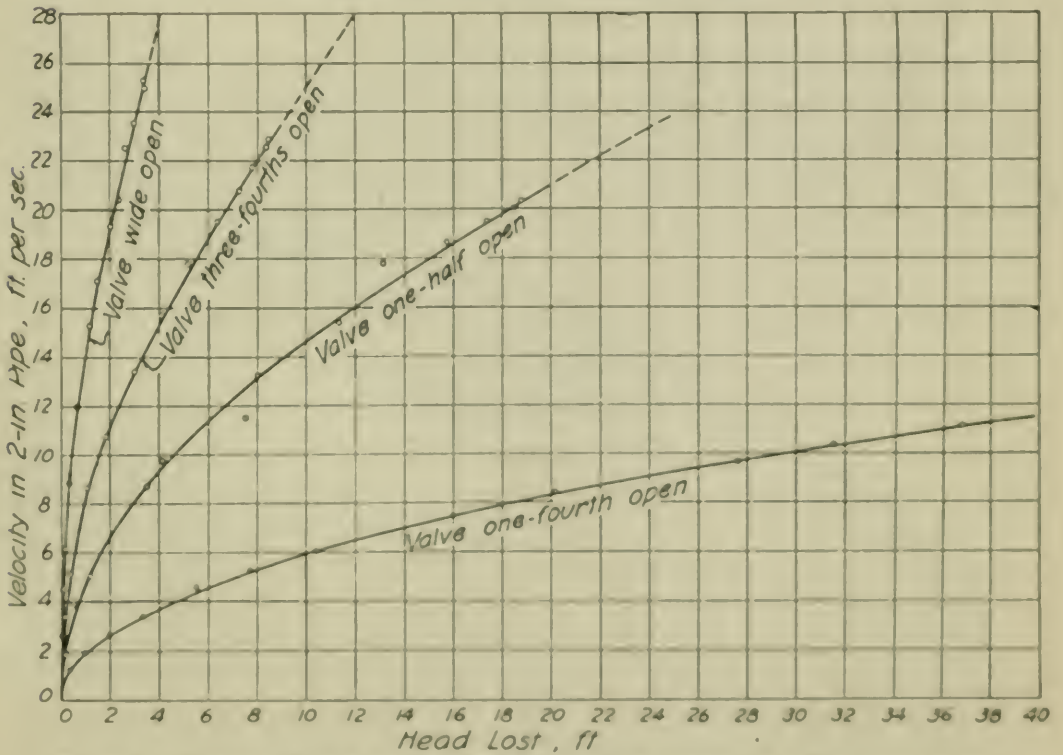


FIG. 4. CURVES SHOWING THE RELATION BETWEEN THE VELOCITY AND HEAD LOST IN 2-INCH GATE VALVE

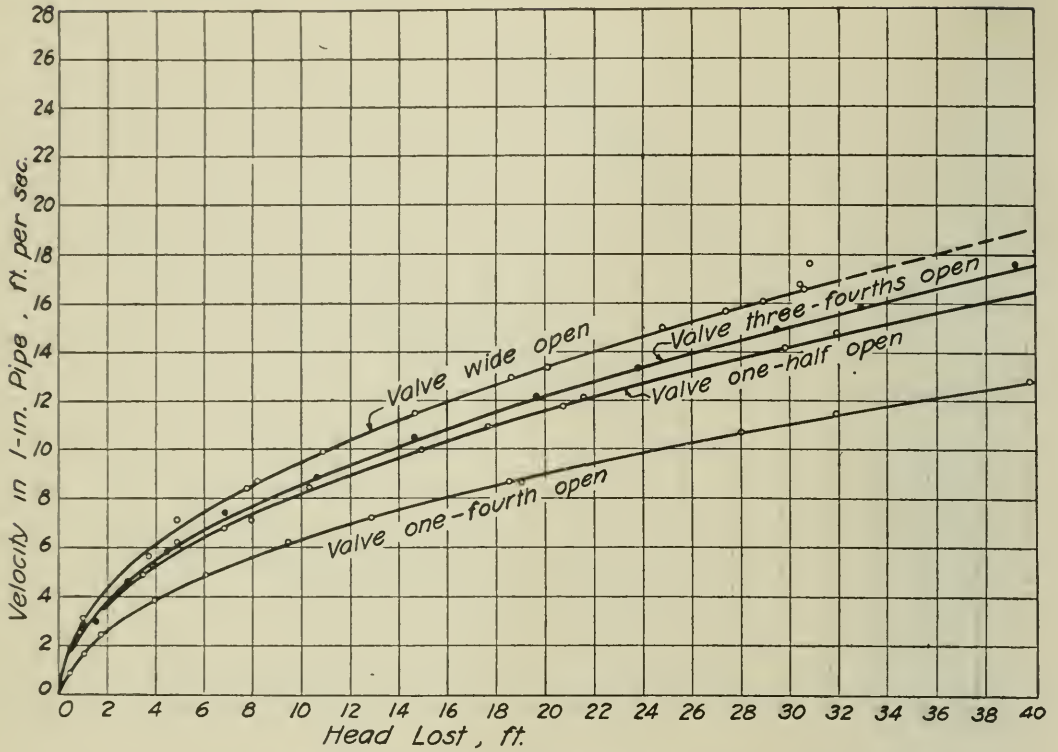


FIG. 5. CURVES SHOWING THE RELATION BETWEEN THE VELOCITY AND HEAD LOST IN 1-INCH GLOBE VALVE

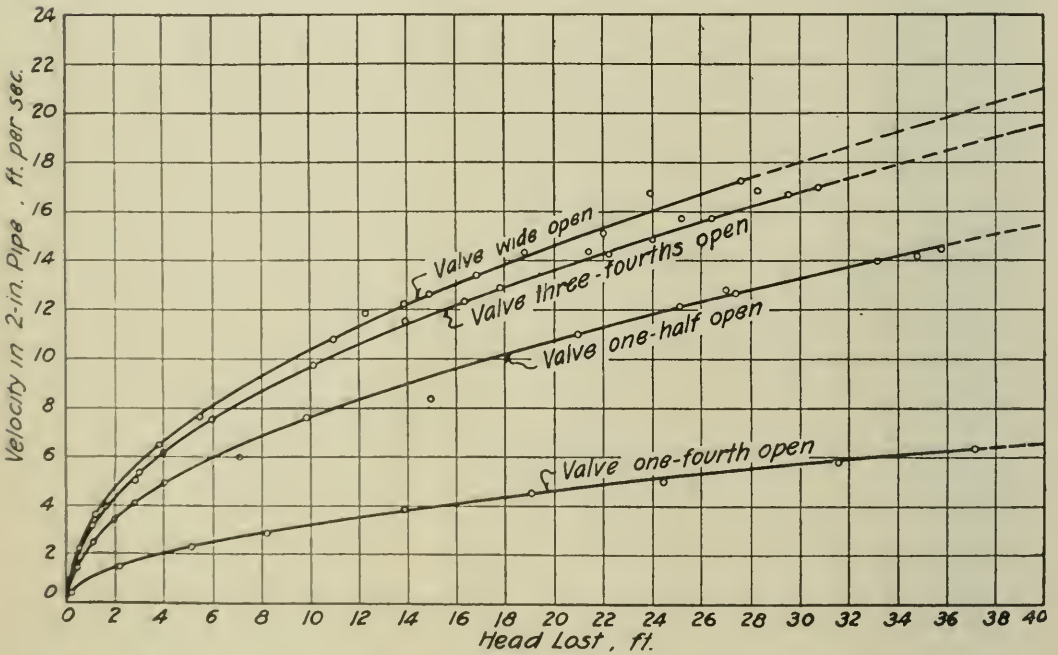


FIG. 6. CURVES SHOWING THE RELATION BETWEEN THE VELOCITY AND HEAD LOST IN 2-INCH GLOBE VALVE

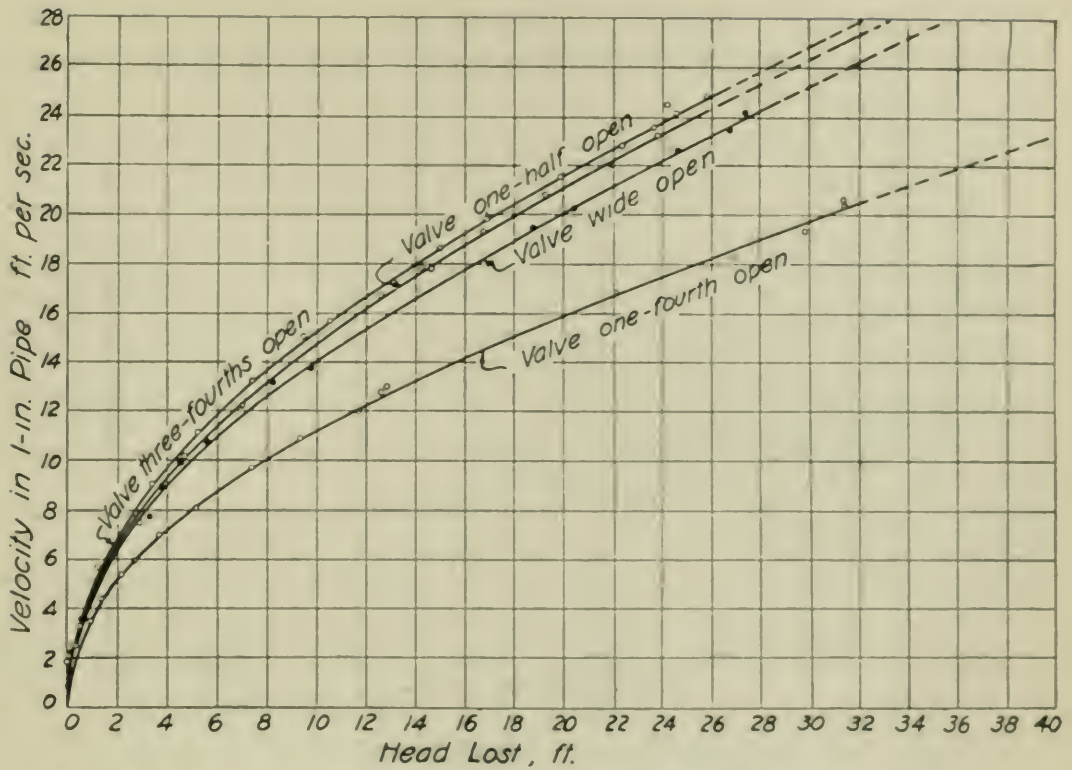


FIG. 7. CURVES SHOWING THE RELATION BETWEEN THE VELOCITY AND HEAD LOST IN 1-INCH ANGLE VALVE

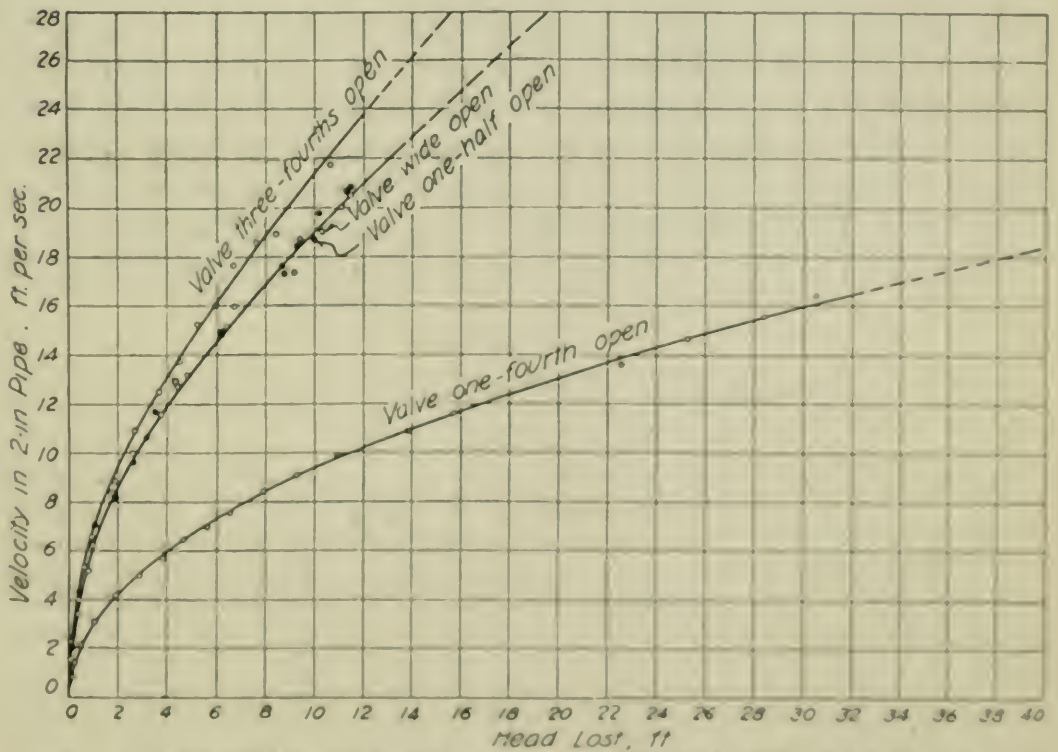


FIG. 8. CURVES SHOWING THE RELATION BETWEEN THE VELOCITY AND HEAD LOST IN 2-INCH ANGLE VALVE

of the gate valves increases the most rapidly, although the rate of increase in any case is comparatively small until the valve is at least one-half closed.

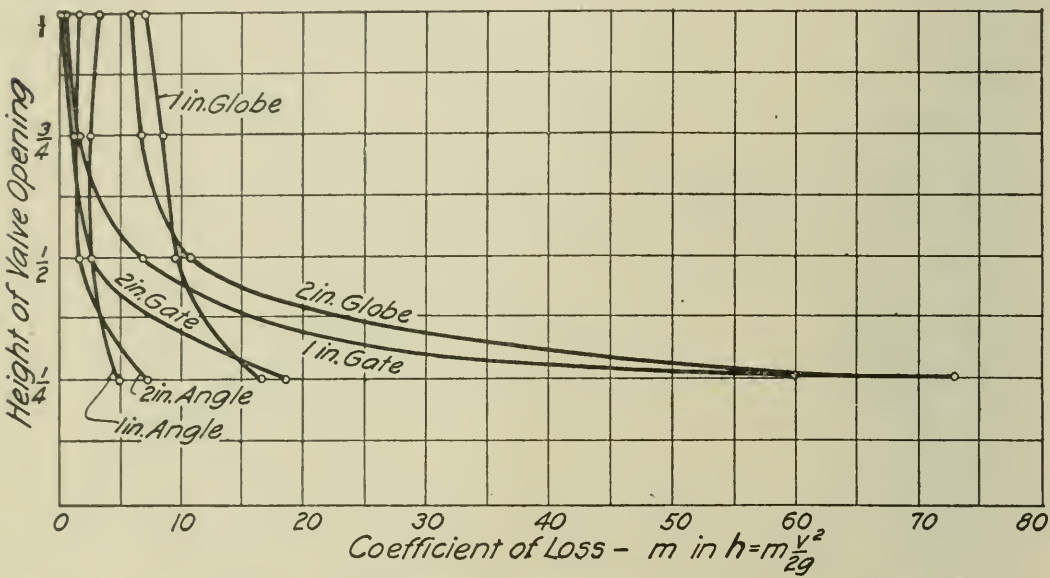


FIG. 9. CURVES SHOWING THE RELATION BETWEEN COEFFICIENTS OF LOSS AND VALVE OPENINGS

Fig. 9 also indicates that the proportions or form or shape of the passageways of the valve of a given type or kind is a very important factor in causing loss of head. This fact is shown by a comparison of the results for the 1-in. globe valve with those for the 2-in. globe valve and also by a comparison of the results of the 1-in. angle valve with those of the 2-in. angle valve. Each of these 1-in. valves was of a somewhat different form from that of the corresponding 2-in. valve as may be seen in Fig. 1. It will be noted from Fig. 9 and Table 1 that

TABLE 1
EXPERIMENTAL VALUES OF COEFFICIENTS OF LOSS

Values of m in $h = \frac{mv^2}{2g}$

Ratio of Height of Valve-Opening to Diameter of Full Valve Opening	Gate Valves		Globe Valves		Angle Valves	
	1-inch Diameter	2-inch Diameter	1-inch Diameter	2-inch Diameter	1-inch Diameter	2-inch Diameter
$\frac{1}{4}$	73.0	18.8	16.6	60.0	5.00	7.3
$\frac{1}{2}$	7.0	2.94	9.62	10.9	2.90	1.70
$\frac{3}{4}$	1.84	1.06	8.75	6.84	2.72	1.44
1	0.74	0.35	7.12	6.0	3.23	1.70

for the smaller valve openings the 1-in. globe valve and the 1-in. angle valve cause less resistance to flow than the corresponding 2-in. valves. The difference is especially large in the case of the globe valves. This unexpected result seems to be due chiefly to the better shaped discharge passages (more gradual expansion) as the water makes its exit from the 1-in. globe valve.

Experiments were made on the 2-in. globe valve to see if a more gradual change in sections through the valve would cause less loss of head. This gradual change was made by filling in part of the passageway with plaster of paris, as shown in Fig. 10. This modified valve was then tested with the valve one-half open and wide open, the results for which are shown in Fig. 10. It will be seen that this modification had no

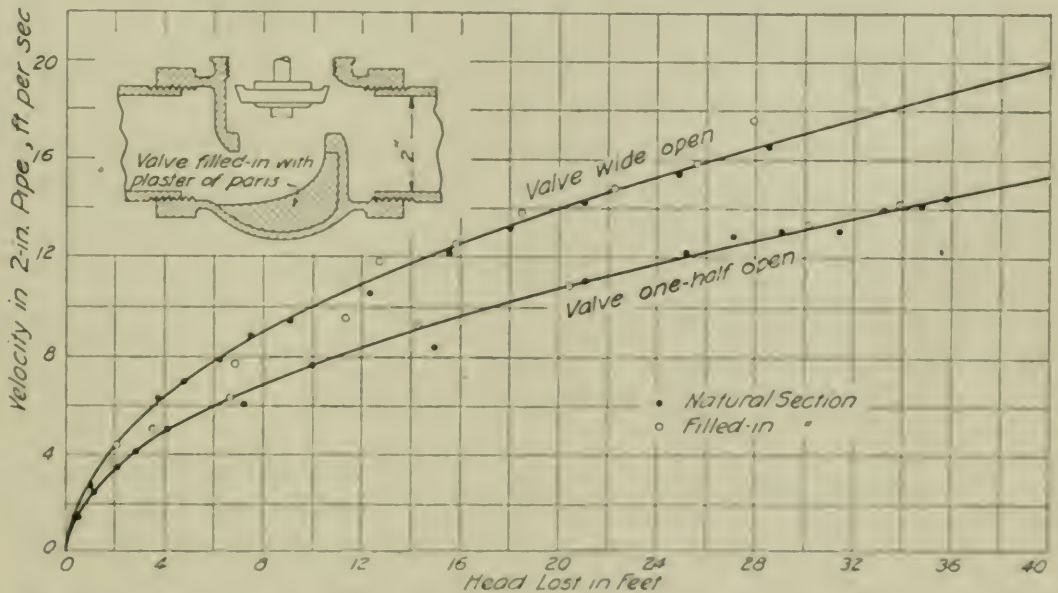


FIG. 10. CURVES SHOWING THE EFFECT OF GRADUAL CHANGE OF SECTION THROUGH 2-INCH GLOBE VALVE

effect on the amount of head lost. This suggests that the lost head in a small globe valve is caused more by the form or shape of the passageway at exit from the valve than by the form of the passages through the valve. Other valve openings and other modifications of the passageways, however, may give better results.

In the case of angle valves the loss of head is not a minimum for the greatest valve opening as is shown in Table 1 and in Fig. 9. For the 2-in. angle valve the lost head is the same when the valve is only one-half open as it is when the valve is wide open, the velocity in the pipe for the two valve openings being the same, that is, the coefficient

of loss is the same for these two valve openings. When this valve is three-fourths open, however, the coefficient of loss is about 20 per cent less than when the valve is one-half open or wide open. The 1-in. globe valve caused a smaller amount of lost head when it was one-half and three-fourths open than it did when wide open, the velocity in the pipe being the same for each of the valve settings. The difference, however, between the coefficients of loss for these three valve openings is not large. The reason that the minimum loss of head in the angle valves occurs when the valve is about three-fourths open is probably because at this opening the water can flow through comparatively large openings all around the valve disc meeting with less abrupt changes of directions than when the valve is wide open. In the latter case there is much turbulent action due to the impact of the water against the bottom of the valve. As the valve opening decreases from the three-fourths open position, the greater resistance due to the narrowing passages causes the lost head to increase again.

The assumption is sometimes made that for comparatively small valves of like type or kind the loss of head varies inversely with the diameter of the valve. For the larger valve openings this assumption is probably approximately true, but from the foregoing results and discussion it would seem that at least for globe and angle valves the form or shape of the passages of the valve is a determining factor in the amount of head lost at the smaller valve openings.

6. *Earlier Experiments on Gate Valves.*—Among the first reliable published results on valves were those by Weisbach.* The largest gate valve used by Weisbach was a little less than two inches. Globe and angle valves, at least of modern construction, were not tested. Other experiments on gate valves have been reported by Magruder† on $\frac{3}{8}$ -in., $\frac{1}{2}$ -in., $\frac{3}{4}$ -in., 1-in., and 1½-in. gate valves, by Folwell‡ on a 4-in. gate valve, by Kuichling¶ on a 24-in. gate valve, and by J. Waldo Smith§ on a 30-in. gate valve. In Smith's experiments the 30-in. valve was located in a 42-in. pipe with increaser-shaped or Venturi-shaped approaches, and in Kuichling's experiments the valve was placed in one branch of a Y only a few feet from the section where the Y started to branch. The methods of determining the lost head in the

* *Mechanics of Engineering* (Coxe's translation).

† *Engineering Record*, Vol. XL, p. 78, 1899.

‡ *Engineering News*, Vol. XLVII, p. 302, 1902.

¶ *Trans. Am. Soc. Civ. Eng.* Vol. XXVI, p. 439, and Vol. XXXIV.

§ *Trans. Am. Soc. Civ. Eng.* Vol. XXXIV, p. 235 (p. 243), 1895.

various experiments were also different. For these reasons it is obvious that the results of these experiments are not directly comparable.

TABLE 2

VALUES OF THE COEFFICIENT OF LOSS FOR GATE VALVES OF VARIOUS DIAMETERS DUE TO PARTIAL CLOSURE ONLY

Ratio of Height of Opening to Diameter of Full Valve Orifice	Weisbach	Kuichling	Smith	Folwell		This Bulletin	
	2½-inch Diameter	24-inch Diameter	30-inch Diameter	4-inch Diameter		2-inch and 1-inch	
	Parallel Sides	Parallel Sides	Parallel Sides Venturi- Shaped Ap- proaches	Parallel Sides	Wedge Shaped	Parallel Sides	
						2-inch Diameter	1-inch Diameter
0			...				
3/100	..	.	950.0	.	.	.	.
1/10	..	.	128.0	.	.	..	...
¼	98.0	..	90.0	72.3	101	..	
¼	17.0	22.7	17.0	16.8	20.5	18.45	72.3
⅓	5.5	8.63	7.5	6.19	8.0	7.0 ¹	16.0 ¹
½	2.1	3.27 ₁	3.5	2.58	2.72	2.59	6.26
¾	0.81	1.09	1.5	1.22	1.5	1.2 ¹	2.5 ¹
¾	0.26	0.25	0.50	0.55	0.66	0.71	1.10
¾	0.07	0.019	0.19	0.20	0.16	0.15 ¹	0.70 ¹
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00

¹ Interpolated from curve.

In Table 2 are given the values of the coefficients of loss as obtained by the various experimenters mentioned previously, as well as the values obtained in the experiments herein reported. These values of the coefficients of loss are those due to partial closure of the valves only, that is, in excess of the loss of head caused by the valve when wide open. Smith's experiments are the only ones in which valve openings less than one-eighth were used. There is considerable chance for error in the results obtained for the very small valve openings, due chiefly to the uncertainty in securing the valve setting desired. Table 2 indicates a rather close agreement in the coefficient of loss for all the gate valves having diameters of 2 in. or greater, and for valve

openings of $\frac{1}{4}$ or perhaps $\frac{1}{8}$ and greater. The values for the 1-in. valve show a considerable increase in the lost head over that for valves of 2-in. diameter and greater. It is probable also that there is considerable variation in the smaller valves of any one type and size.

7. *Coefficients of Discharge for Gate Valves.*—In order to determine the rate of discharge through a pipe a partially closed valve has sometimes been used. This requires the values of the coefficients of discharge of the valve for various valve openings since the rate of discharge, q , is found from the expression, $cA\sqrt{2gh}$, in which c is the coefficient of discharge, A the area of the valve opening, and h the difference in pressure heads on the two sides of the valve (lost head), velocity of approach being neglected. The average values of the coefficients of discharge for the 1-in. and 2-in. gate valves as found in the experiments reported in this bulletin are given in Table 3. The value of the

TABLE 3
EXPERIMENTAL VALUES OF THE COEFFICIENTS OF DISCHARGE FOR
GATE VALVES

Values of $c = \frac{q}{A\sqrt{2gh}}$

Ratio of Height of Valve-Opening to Diameter of Full Valve-Opening	Coefficient of Discharge		Area of Valve-Opening Square Inch	
	1-inch Valve	2-inch Valve	1-inch Valve	2-inch Valve
$\frac{1}{4}$	.48	.88	.195	.826
$\frac{1}{2}$	.67	1.00	.450	1.80
$\frac{3}{4}$	.88	1.12	.660	2.67
1	1.16	1.70	.785	3.14

coefficient varied somewhat with the velocity for any given valve opening. Because of the uncertainty of obtaining the exact valve setting desired and the corresponding uncertainty in the area of the valve opening, the values of the coefficients of discharge given in Table 3 cannot be considered as refined determinations.

It will be noted that the coefficient of discharge increases directly with the valve opening for each of the gate valves for a range of valve

openings of $\frac{1}{4}$ to $\frac{3}{4}$ or perhaps greater. The more the valve is opened the greater is the velocity of approach toward the valve and since the velocity of approach is not considered in the calculation of the coefficient of discharge the value of the coefficient increases with the valve opening. The coefficient of discharge for the valves used by Kuichling and Smith decreased slightly until the valve was about one-fourth open and then increased rapidly for further openings of the valve. Gibson with a $2\frac{1}{2}$ -in. flat disc stop valve found nearly a constant coefficient of discharge of 0.80. These variations in the coefficients of discharge are not surprising considering the wide range of conditions covered by the experiments. They suggest, however, that if gate valves are to be used for determining the rate of discharge in pipes with reasonable accuracy much more experimenting is required, or better, where it is possible, experiments should be performed under service conditions to calibrate the particular valve to be used.

8. *Summary.*—The following brief summary is given as applying to 1-in. and 2-in. valves of the three kinds tested (gate valves, globe valves, and angle valves) with valve settings ranging from one-fourth open to wide open.

(1) The loss of head caused by small valves varies as the square of the velocity in the pipe for all the valve openings; hence the lost head may be expressed as a constant times the velocity head in the pipe, $\left(h = \frac{mv^2}{2g}\right)$.

(2) When wide open a globe valve causes more than twice as much loss of head as an angle valve of the same size, while a gate valve causes much less loss of head than either a globe or an angle valve, the velocity in the pipe being the same in the three cases.

(3) The loss of head for an angle valve is somewhat less when about three-fourths open than when wide open, the velocity in the pipe being the same in each case.

(4) The loss of head for each valve, as the valve is closed from a wide open position, varies comparatively little with the valve opening until the valve is at least one-half closed. As further closure takes place the loss of head of the globe valves and gate valves increases rapidly and is considerably larger than that of the angle valves.

(5) The form or shape of the passageways through a globe or angle valve has a large influence on the loss of head for the small

valve openings. The portion of the passageways in which the form seems of greatest importance is in the exit from the valve rather than in the passageways leading to the valve disc or seat. On account of the influence of the form or shape of the valve no law giving the relation of the lost head to the diameter of the valve can be stated for valve settings less than five-eighths open. For larger valve openings than this, the lost head seems to vary approximately inversely as the diameter.

(6) The use of the lost head through a partially closed valve as a means of determining the flow can be only a very rough method of measurement unless the particular valve to be used is calibrated under service conditions. Even then the difficulty in obtaining the desired valve setting may introduce considerable uncertainty in the results.

PART II

THE FLOW OF WATER THROUGH SUBMERGED
ORIFICES

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CONTENTS

PART II

THE FLOW OF WATER THROUGH SUBMERGED ORIFICES

	PAGE
IV. INTRODUCTION	27
9. Preliminary	27
10. Acknowledgment	28
V. APPARATUS AND METHOD OF EXPERIMENTING	29
11. Orifices	29
12. Tank Used and Method of Experimenting	30
13. Method of Calculating the Coefficient of Discharge	33
VI. EXPERIMENTAL RESULTS AND DISCUSSION	34
14. Coefficients of Discharge	34
15. Results Obtained by Earlier Experimenters	37
16. Comparison with Discharge into Air	39
17. Summary	40

LIST OF FIGURES

NO.		PAGE
11.	Tank Used and Arrangement of Apparatus	31
12.	Diagrams Showing Values of Coefficients of Discharge of Circular Submerged Orifices for Various Velocities	34
13.	Diagrams Showing Values of Coefficients of Discharge of Square Submerged Orifices for Various Velocities	34
14.	Diagrams Showing Values of Coefficients of Discharge of Rectangular Submerged Orifices for Various Velocities	35
15.	Curve Showing the Relation between Coefficient of Discharge for Circular Orifice and Diameter of Orifice	36
16.	Curve Showing the Relation between Coefficient of Discharge for Square Orifice and Side of Orifice	36
17.	Curve Showing the Relation between Coefficient of Discharge of Rectangular Orifice and Short Side of Orifice (other Side being Six Inches, in Each Case)	36
18.	Curves Showing Relation between Coefficient of Discharge and Area of Orifices	37

LIST OF TABLES

4.	List of Orifices Used	29
5.	Values of Coefficient of Discharge for Submerged Orifices for Velocities from One-half to Five Feet per Second	35
6.	Results Obtained by Earlier Experimenters on Submerged Sharp-edged Orifices	38

PART II

THE FLOW OF WATER THROUGH SUBMERGED ORIFICES

IV. INTRODUCTION

9. *Preliminary.*—Part II of this bulletin presents the results of experiments on submerged sharp-edged orifices of various shapes and sizes discharging under moderately low and under very low heads. The orifices used were of three shapes, circular orifices with diameters from 1 in. to 6 in., square orifices with sides from $\frac{1}{2}$ in. to $5\frac{1}{2}$ in., and rectangular orifices having one side range from $\frac{1}{2}$ in. to 2 in., the other side being 6 in. in each case. The coefficient of discharge is given for each orifice for a velocity range of approximately $\frac{1}{2}$ ft. per sec. to 4 ft. per sec. This range corresponds roughly to a range of head on the orifice of 0.006 ft. to 0.08 ft.

Considerable experimenting has been done on orifices discharging into air, particularly on sharp-edged circular orifices of rather small size although the results are somewhat discordant. Comparatively little experimental work, however, has been carried out on submerged orifices. While the orifice has lost some of its importance as a water measuring device due to the development of other methods, it is, nevertheless, of importance to determine how the rate of discharge is affected by the shape and the size of the orifice and also by the head on the orifice, particularly the effect of very low heads which the submerged orifice makes possible.

The submerged orifice may be of particular importance in cases which require the measurement of water with as small a loss of head as possible as, for example, in determining the discharge from a water turbine when operating under a low head. The decrease in the available head on the turbine made necessary by the proper setting of a weir may be an important factor in the installation.

There is a feeling among some engineers that the importance of the so-called standard orifice (sharp edges, complete contraction without velocity of approach, etc.) has been over-emphasized and that beveled-edged orifices are better adapted at least to conditions where the orifice may be obstructed and the edge soon worn off, as, for example, in measuring the water supplied to water wheels through flume or bulk-

head openings. There exist, no doubt, some grounds for this feeling. A sharp edged orifice (an opening in a thin plate), however, is subject to less variation in its construction than a beveled-edged orifice. This fact is of considerable importance where accuracy is essential. It is felt that the submerged orifice, both beveled-edged and sharp-edged, is worthy of more attention than has been accorded it.

10. *Acknowledgment.*—The experimenting was done in the Hydraulic Laboratory of the University of Illinois. Some of the results herein presented have been taken from the theses of W. R. ROBINSON of the class of 1906 and G. D. PHILLIPS of the class of 1907, and some of the results also, particularly at the low heads, have been taken from a second thesis presented by Mr. Robinson in 1909. All the thesis work was conducted under the direction of PROFESSOR ARTHUR N. TALBOT. The careful way in which this preliminary experimenting was done has made the results of the theses of much value. During 1914 and 1915 the writer spent considerable time in checking the results of the theses work and extending certain parts of the investigation.

V. APPARATUS AND METHOD OF EXPERIMENTING

11. *Orifices.*—The orifices used were of three different shapes. Four of the orifices were circular with diameters of 1 in., 2 in., 4 in., and 6 in. Five were square with sides of $\frac{1}{2}$ in., 1 in., 2 in., 4 in., and $5\frac{1}{2}$ in. Three were rectangular with dimensions of $\frac{1}{2}$ in. by 6 in., 1 in. by 6 in., and 2 in. by 6 in. In each case the orifice was formed in a cast iron plate $\frac{1}{2}$ in. thick and $10\frac{1}{2}$ in. in diameter, a sharp edge being formed by beveling at 45 degrees. Except for a few small nicks the edges were sharp and the areas closely true to shape. The dimensions of the orifices were carefully determined (except for the 1-in. circular orifice) by an inside micrometer for dimensions greater than 1 in. and inside screw calipers for dimensions less than 1 in. A list of the orifices used and the areas as determined from the measured dimensions are given in Table 4. The 1-in. circular orifice was broken before

TABLE 4
LIST OF ORIFICES USED

Form of Orifice	Nominal Size	Measured Area square feet
Circular	1 in. diam.	not measured
	2 in. diam.	0.0219
	4 in. diam.	0.0883
	6 in. diam.	0.1967
Square	$\frac{1}{2}$ in. by $\frac{1}{2}$ in.	0.001735
	1 in. by 1 in.	0.00698
	2 in. by 2 in.	0.0279
	4 in. by 4 in.	0.1109
	$5\frac{1}{2}$ in. by $5\frac{1}{2}$ in.	0.2105
Rectangular	$\frac{1}{2}$ in. by 6 in.	0.0206
	1 in. by 6 in.	0.0418
	2 in. by 6 in.	0.0838

its dimensions were taken so that the nominal diameter (1 in.) has been used in the calculations. There may be some error, therefore, in the results for this orifice.

12. *Tank Used and Method of Experimenting.*—The same tank was used in all the experiments, the dimensions and general arrangement of which is shown in Fig. 11.* The tank was divided into two compartments by a vertical partition in which the orifice was placed, holding the orifice in a vertical plane.

The water coming from the laboratory standpipe was supplied to the tank through a 6-in. supply pipe and also through a $\frac{3}{4}$ -in. pipe, the latter making possible a finer adjustment in maintaining a constant head. After passing through baffle boards the water flowed through the orifice and finally left the downstream compartment by passing out through small openings in the end of the tank, the flow through which was regulated by placing stoppers in some of the holes. These holes were arranged in two narrow portions in the end of the tank, one near each side of the tank, and the stoppers were arranged so as to give nearly a uniform distribution from each of the two sets of openings. This arrangement, it was found, helped to maintain steady conditions.

The quantity of water discharged was determined by weighing for the small discharges and by measuring in a pit for the larger discharges. The pit was about 6 ft. deep, and 7.995 ft. in diameter. The value for the diameter is the average of a large number of readings of a micrometer attached to a rigid stick. The rise in the pit was determined by a vertical graduated rod which could be read directly to 0.02 ft. and to 0.004 ft. by estimating. A float was attached to the bottom of the rod and a still basin was provided. The water was wasted into another pit through a movable spout until the surface of the water in the measuring pit became fairly still so that an accurate reading of the rod could be taken. A hook gage was used to test the accuracy of the float and rod. At the end of the experiment the water was again wasted in the same manner. A calibrated stop watch gave the time corresponding to the rise in the pit.

The head causing flow through the orifice is the difference in the levels of the water surfaces in the two compartments of the tank. This head was measured in nearly all the experiments by means of hook gages. These gages were read directly to 0.001 ft. and to 0.0005 ft. by estimating. Vertical 2-in. pipes attached toward the bottom of the tank served as still basins for the hook gages. The level of the water in the upstream compartment was determined by the use of one

*A view of the tank is shown in Fig. 5 of Bulletin No. 96 of the Engineering Experiment Station of the University of Illinois.

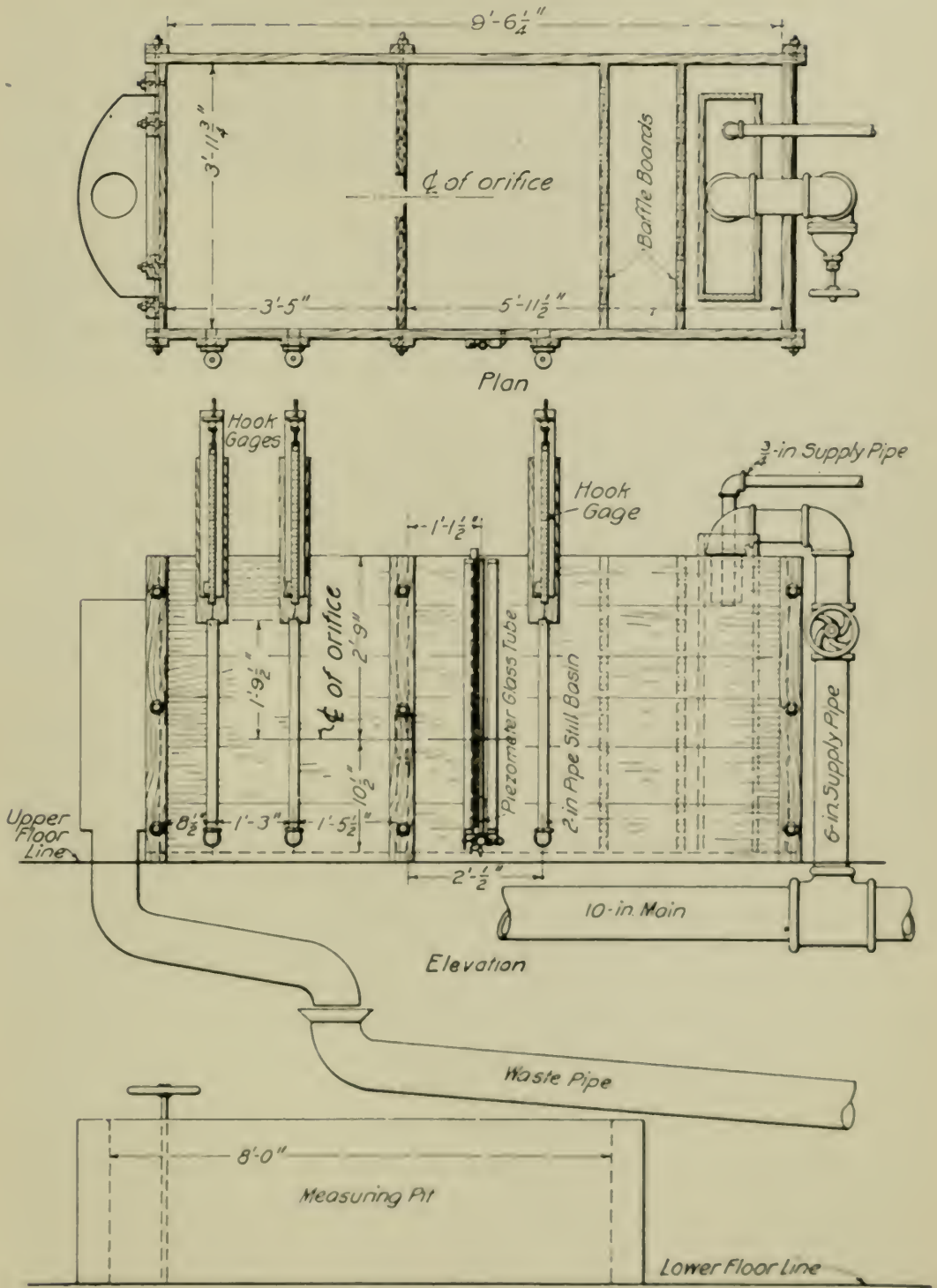


FIG. 11. TANK USED AND ARRANGEMENT OF APPARATUS

hook gage only, while two gages were used on the downstream compartment in the earlier experiments. It was found, however, that for the lower heads the two gages gave practically the same result, while for the higher heads the gage nearer the partition gave less fluctuation. For these reasons and because of less difficulty in getting simultaneous readings of only two gages, it was decided to take readings with one gage only on each compartment.

Zero readings of the hook gages were obtained by reading the gages when the tank was nearly full and when no water was allowed to escape, the levels of the water surfaces in the two compartments then being the same. Zero readings were taken frequently during the experimenting.

For most of the heads above 0.3 ft., the head was measured by two vertical peizometer glasses, one attached near the bottom of each compartment, the difference in readings of which (corrected for zero reading) gave the head to 0.001 ft. These two methods overlapped somewhat so that certain heads were measured by both methods.

Leakage from the tank and from the measuring pit was determined several times during the progress of the experimenting and was found to be negligible.

An experiment or run consisted of the following: A sufficient number of stoppers was removed from the end of the tank to give the desired discharge and the inflow through the 6-in. and $\frac{3}{4}$ -in. pipes was then adjusted until the difference in levels of the water surfaces in the two compartments of the tank became constant. The $\frac{3}{4}$ -in. supply pipe was used to make the final adjustment of the head and to hold the head constant throughout the experiment. After obtaining a constant head, the waste pipe shown in Fig. 11 was pulled from beneath the discharge pipe, thus allowing the water to discharge into the measuring pit until the rise in the pit was sufficient to allow its measurement without appreciable error and also to allow time for an accurate measurement of the head. The head was taken as an average of from two to ten readings of the hook gages, the larger number being necessary with the higher velocities on account of the greater fluctuations of the water levels due to the more turbulent conditions of the water, especially in the downstream compartment. Each experiment was repeated, as a rule, three times, although in some cases as many as eight or ten runs were made.

13. *Method of Calculating the Coefficient of Discharge.*—The head, h , causing flow through the orifice is the difference in the levels of the water surfaces in the two compartments of the tank. The ideal rate of discharge is $A \sqrt{2gh}$ in which A is the area of the orifice in square feet and g is the acceleration due to gravity in feet per second per second; hence the coefficient of discharge, c , is found from,

$$c = \frac{q}{A \sqrt{2gh}}$$

where q is the measured rate of discharge in cubic feet per second, as determined from the measured weight or volume discharged and the corresponding time.

VI. EXPERIMENTAL RESULTS AND DISCUSSION

14. *Coefficients of Discharge.*—Fig. 12, 13, and 14 show the experimental values of the coefficients of discharge for the various orifices tested. Each plotted point represents the average of from two to ten experiments at practically the same head. It will be noted that

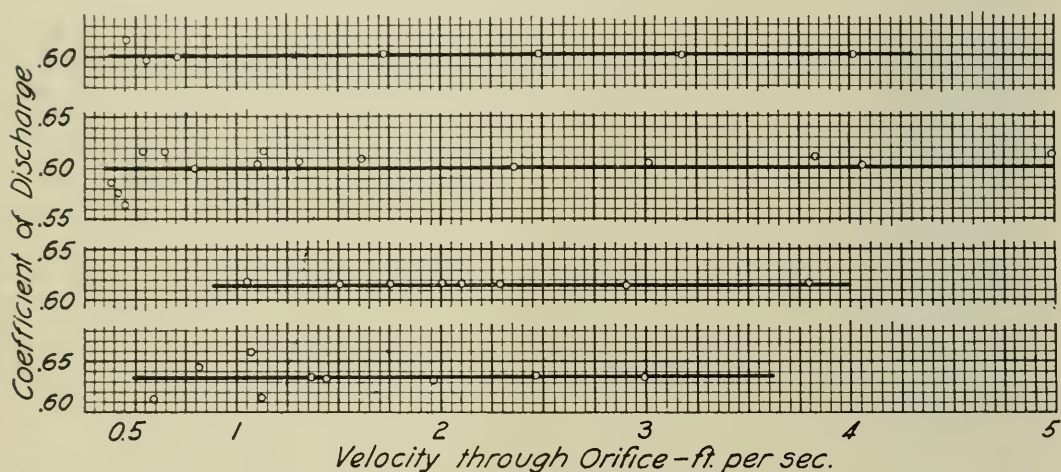


FIG. 12. DIAGRAMS SHOWING VALUES OF COEFFICIENTS OF DISCHARGE OF CIRCULAR SUBMERGED ORIFICES FOR VARIOUS VELOCITIES

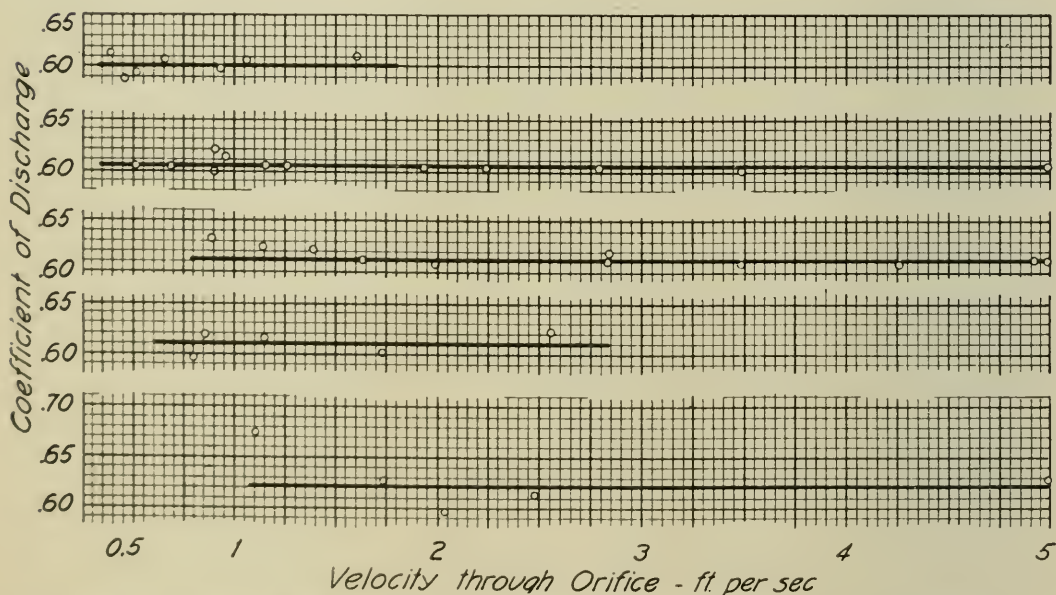


FIG. 13. DIAGRAMS SHOWING VALUES OF COEFFICIENTS OF DISCHARGE OF SQUARE SUBMERGED ORIFICES FOR VARIOUS VELOCITIES

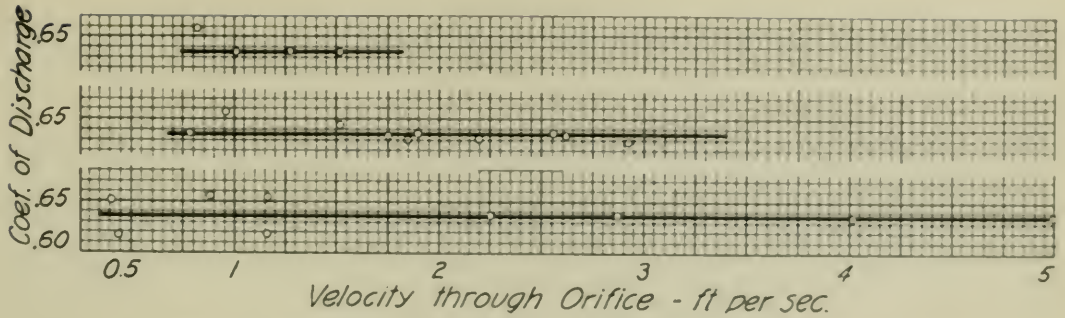


FIG. 14. DIAGRAMS SHOWING VALUES OF COEFFICIENTS OF DISCHARGE OF RECTANGULAR SUBMERGED ORIFICES FOR VARIOUS VELOCITIES

for any given orifice the coefficient is constant for the whole range of velocity used in these experiments which in most of the cases is about $\frac{1}{2}$ ft. per sec. to 4 or 5 ft. per sec. This velocity range corresponds roughly to a range in head of 0.006 to 0.08 ft. and as may be expected the values of the coefficient show the greatest variation at the very low heads.

TABLE 5

VALUES OF COEFFICIENT OF DISCHARGE FOR SUBMERGED ORIFICES FOR VELOCITIES FROM ONE-HALF TO FIVE FEET PER SECOND

Kind of Orifice	Nominal Size	Coefficient of Discharge
Circular	1 in. diameter	0.6351
	2 in. diameter	0.615
	4 in. diameter	0.600
	6 in. diameter	0.600
Square	$\frac{1}{2}$ in. by $\frac{1}{2}$ in.	0.620
	1 in. by 1 in.	0.610
	2 in. by 2 in.	0.610
	4 in. by 4 in.	0.605
	$5\frac{1}{2}$ in. by $5\frac{1}{2}$ in.	0.600
Rectangular	$\frac{1}{2}$ in. by 6 in.	0.635
	1 in. by 6 in.	0.635
	2 in. by 6 in.	0.635

¹ Probably somewhat in error since diameter was not measured; nominal diameter used in calculations.

Table 5 and Fig. 15, 16, and 17 show how the coefficient of discharge for the orifices of any given shape varies with the diameter or

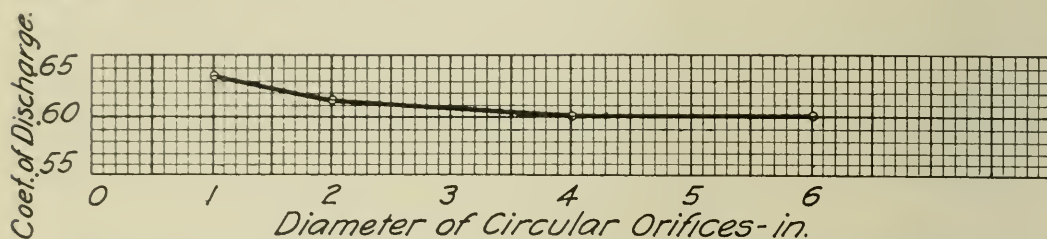


FIG. 15. CURVE SHOWING THE RELATION BETWEEN COEFFICIENT OF DISCHARGE FOR CIRCULAR ORIFICE AND DIAMETER OF ORIFICE

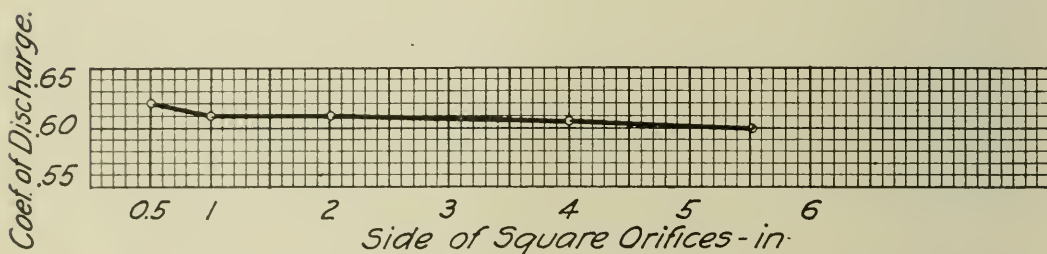


FIG. 16. CURVE SHOWING THE RELATION BETWEEN COEFFICIENT OF DISCHARGE FOR SQUARE ORIFICE AND SIDE OF ORIFICE

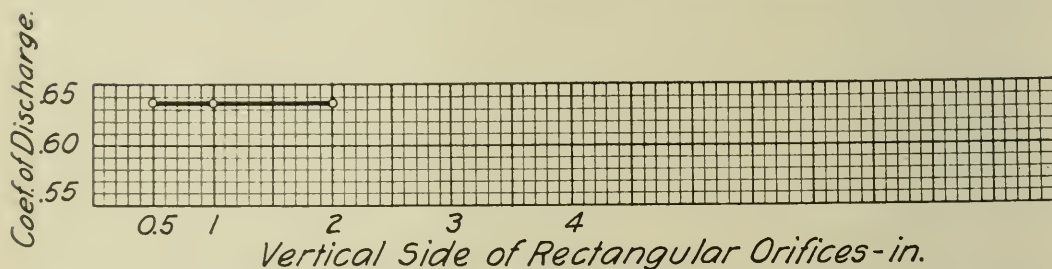


FIG. 17. CURVE SHOWING THE RELATION BETWEEN COEFFICIENT OF DISCHARGE OF RECTANGULAR ORIFICE AND SHORT SIDE OF ORIFICE
(OTHER SIDE BEING SIX INCHES IN EACH CASE)

side of the orifice, while from Fig. 18 a comparison may be made between the coefficients of discharge for the different shaped orifices on the basis of their areas. These figures show that the coefficient of discharge for circular and square orifices decrease as the size increases until an area of 8 or 10 square inches is reached after which the coefficient has a constant value of not far from 0.60. This indicates that complete contraction does not take place with the smaller orifices. Because of the uncertainty of the exact diameter there is some doubt, however, concerning the correct value for the 1-in. circular orifice. It will be noted also that the coefficient of discharge for the rectangular orifices

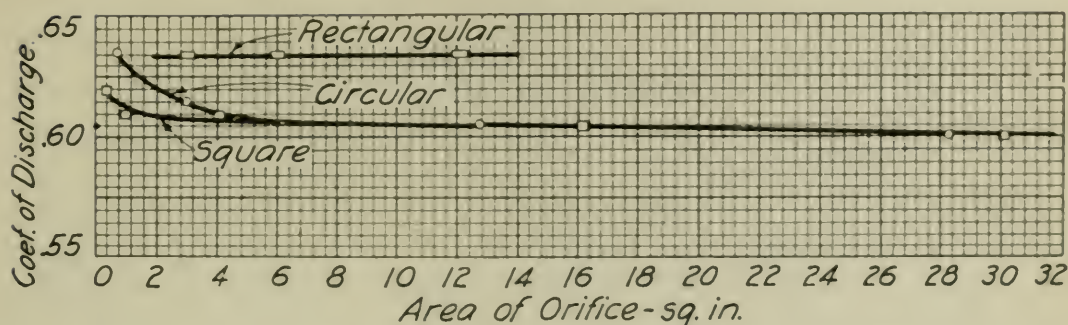


FIG. 18. CURVES SHOWING RELATION BETWEEN COEFFICIENT OF DISCHARGE AND AREA OF ORIFICES

remain constant for the range of areas used in these experiments and that its value is larger than that for circular and square orifices of the same area. Fig. 18 indicates furthermore that as the area of the orifices decreases below 8 sq. in., the coefficient of discharge for circular orifices increases faster than that for square orifices. These observations suggest that the longer side of the rectangular orifices has a controlling influence in determining the rate of discharge for a given head and that the corners of a small square orifice are inefficient in discharging water as compared with the form of a circular orifice of the same area.

15. *Results Obtained by Earlier Experimenters.*—In order to compare the results given in this bulletin with those of earlier investigations and to extend the study to include higher heads and velocities, the results given in Table 6 have been condensed from available published data. It will be noted that the results are not entirely concordant, but considering the different arrangements and methods of measuring the head and the rate of discharge, the results show a very good agreement. The low value of the coefficient of discharge found by Francis is due no doubt to the fact that the rate of discharge was measured over a weir on which the head was rather small. From Table 6 it will be seen that in some of the earlier investigations the coefficient of discharge increased slightly with the head while in others the coefficient decreased, and in still others it showed no systematic change. In all cases the value of the coefficient of discharge is not far from 0.60. The small square orifice (1.2 in. by 1.2 in.) used by Hamilton Smith gave a slightly larger coefficient than the circular orifice with a diameter of 1.2 inches. This result is the reverse of that found in the experiments described in this bulletin. The values also of the coefficient of discharge for circular and square orifices as found by

TABLE 6
RESULTS OBTAINED BY EARLIER EXPERIMENTERS ON SUBMERGED
SHARP-EDGED ORIFICES

Circular Orifices				Square Orifices			
Source	Diam-eter inches <i>d</i>	Head feet <i>h</i>	Coeffi- cient of Dis- charge <i>c</i>	Source	Dimen- sions inches	Head feet <i>h</i>	Coeffi- cient of Dis- charge <i>c</i>
Francis	1.22	1.024	.592	Hamilton Smith, Jr.	0.6 by 0.6	0.35	.6201
		1.324	.592			2.21	.6092
		1.490	.592			4.06	.6068
		1.499	.593				
		1.514	.591				
Hamilton Smith, Jr.	0.6	0.437	.6183	Hamilton Smith, Jr.	1.2 by 1.2	0.207	.6117
		2.16	.6041			0.410	.6091
		4.08	.6016			0.771	.6053
						1.52	.6055
Hamilton Smith, Jr.	1.2					2.32	.6040
						3.11	.6052
						3.95	.6043
		0.250	.6048			Ellis	12 by 12
		0.648	.6027	3.92	.602		
		0.985	.6025	7.99	.606		
		1.51	.6006	11.58	.605		
		2.00	.6006	14.31	.611		
		2.58	.5997	16.22	.606		
		2.99	.5989	18.45	.606		
		3.57	.5987				
		3.97	.5992				
Ellis	12.0	2.60	.607	Balch	12 by 12	0.363	.5940
		4.71	.590			0.750	.5940
		6.41	.606			0.771	.5932
		8.10	.599			0.826	.5982
		8.80	.600			0.905	.5950
		12.09	.600			1.134	.5960
		14.25	.601			1.371	.5970
		16.29	.602			2.097	.6056
		18.66	.599			2.636	.6105
Balch	12.0					3.220	.6095
						3.975	.6148
		0.145	.5909	Stewart	48 by 48 (3.72 in. thick)	.05	.626
		0.469	.5902			.10	.608
		0.851	.5912			.15	.605
		1.254	.5993			.20	.605
		1.612	.5921			.25	.606
		2.012	.5924			.30	.610
		2.421	.5954	Rectangular Orifices			
		2.949	.5967	Hamilton Smith, Jr.	0.6 by 3.6	0.614	.6219
		3.410	.6006			1.63	.6207
		4.015	.6054			2.77	.6188

Hamilton Smith are slightly less than those herein reported in Table 5. Omitting the values as given by Francis it will be observed that there is very little difference between the coefficients for the small and the large orifices, the value of the coefficient varying only slightly from 0.60. From the results obtained in the present investigation as given in Table 5 and in Fig. 12, 13, and 14, it will be seen that the coefficient varies more with the size of the orifice than is shown by the results of the earlier experiments, as given in Table 6.

It will be observed also that the coefficient of discharge for the rectangular orifice used by Hamilton Smith is somewhat smaller than that herein reported. It may seem that the diverging sides of the orifices used in the experiments reported in this bulletin (orifice plate $\frac{1}{2}$ -in. thick) would form a diverging mouthpiece, particularly in the case of the smaller orifices, but experiments* on diverging mouthpieces have shown that a mouthpiece having a total angle of divergence of 90 degrees has very little, if any, effect on the rate of discharge.

16. *Comparison with Discharge into Air.*—The experiments on sharp-edged orifices with discharge into air are more numerous than for submerged discharge. The experiments of Bilton and to a less degree those by Judd and King, and those by Mair and by Ellis indicate that there is a critical head for each circular orifice above which the coefficient remains constant. Bilton concludes that "circular orifices of $2\frac{1}{2}$ -in. diameter, and over, under heads of 17 in., and over, have a common coefficient of discharge lying between 0.59 and 0.60 but which is probably about 0.598 (subject to the head being not less than 2 or 3 diameters)." The results of the experiments of Hamilton Smith, as is well known, indicate that the coefficient of discharge gradually decreases as the size of the orifice increases, and also decreases as the head increases until at a head of 100 ft. all orifices, regardless of the size or the shape, have a common coefficient of discharge.

The results of the experiments on submerged orifices herein reported seem to indicate, as previously noted, that orifices having diameters greater than about $2\frac{1}{2}$ in. (or sides, if square) have a common coefficient of discharge which is very close to 0.60. There seems, however, to be no evidence of a critical head since the coefficient remains constant for the whole range of head used, nor is there evidence of a critical head in the results obtained by earlier experimenters on submerged orifices as given in Table 6.

*"The Effect of Mouthpieces on the Flow of Water Through a Submerged Short Pipe." Univ. of Ill. Eng. Exp. Sta., Bul. 96, 1917.

From a study of the experimental results on orifices with discharge into air it is believed that the coefficient of discharge for submerged orifices are the same as those for discharge into air for the same heads and sizes and shapes (except for very small heads). It is doubtful if the statement sometimes made, namely, that the coefficient of discharge for submerged orifices is about one per cent less than that for free discharge, is justified.

17. *Summary.*—The following brief summary is given as applying to submerged sharp-edged orifices for velocities from $\frac{1}{2}$ to 5 ft. per sec.

(1) The coefficient of discharge for a circular, a square, or a rectangular submerged orifice does not vary with the velocity.

(2) Circular and square submerged orifices having areas greater than about 10 sq. in. have a common coefficient of discharge varying but little from 0.60.

(3) Rectangular submerged orifices having one side from 3 to 12 times the other side have a constant coefficient of discharge which is larger than that for circular and square orifices of the same size, particularly for the larger areas, at least up to a size of 12 sq. in.

(4) The flow of water through submerged sharp-edged orifices is very nearly the same as that for the same kind of orifices with discharge into air, provided the head is not less than 2 or 3 diameters when the discharge is into air.

PART III

FIRE STREAMS FROM SMALL HOSE AND NOZZLES

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CONTENTS

PART III

FIRE STREAMS FROM SMALL HOSE AND NOZZLES

	PAGE
VII. INTRODUCTION	45
18. Scope of Experiments	45
19. Acknowledgment	46
VIII. APPARATUS AND METHOD OF EXPERIMENTING	47
20. Hose and Nozzles	47
21. Method of Experimenting	48
IX. EXPERIMENTAL RESULTS AND DISCUSSION	49
22. Results from Freeman's Experiments	49
23. Experimental Data	49
24. Friction Factors	53
25. Coefficients of Discharge	54
26. Height and Horizontal Distance of Jets	55
27. Effect of Cylindrical Tip	56
28. Requirements for Temporary Fire Protection for the Interior of Buildings	57
29. Summary	59

LIST OF FIGURES

NO.		PAGE
19.	Longitudinal Sections of Nozzles Tested	47
20.	Cross-section of Piezometer Coupling	48
21.	Diagram Showing Friction Factors in Rubber-lined and Unlined Hose .	53
22.	Diagram Showing Coefficients of Discharge of Nozzles	54

LIST OF TABLES

7.	Freeman's Results for 1-in., $1\frac{1}{8}$ -in. and $1\frac{1}{4}$ -in. Nozzles Attached to 2 $\frac{1}{2}$ -in. Hose	50
8.	Results of Experiments at University of Illinois with $\frac{5}{16}$ -in., $\frac{7}{16}$ -in., and $\frac{1}{2}$ -in. Nozzles Attached to $1\frac{1}{2}$ -in. Hose	51

PART III

FIRE STREAMS FROM SMALL HOSE AND NOZZLES*

VII. INTRODUCTION

18. *Scope of Experiments.*—Part III presents the results of experiments on 1½-in. hose and nozzles. Both rubber-lined hose and unlined linen hose were used. Three sizes of conical nozzles were tested, the diameters of the nozzle openings being $\frac{5}{16}$ in., $\frac{7}{16}$ in., and $\frac{1}{2}$ in.

The loss of head in the hose due to friction and the corresponding friction factor are given for each hose for a range in velocity from about 4 to 8 ft. per sec. The coefficient of discharge for each nozzle is recorded for a range in pressure at the base of the nozzle from about 10 to 85 lb. per sq. in. The height and the horizontal distance which the jets reached are also recorded. The influence of a cylindrical tip on a nozzle is brought out and some discussion is given concerning the quantity of water required for temporary fire protection for the interior of buildings.

The importance of adequate fire protection has become so well recognized that most buildings, even those of moderate size, are equipped with some sort of fire apparatus for immediate service in case of fire in the interior of the building and until the city fire department arrives. The ordinary water buckets and portable chemical fire extinguishers have in a large measure been supplemented with small fire hose. Few data are available concerning the hydraulics of small fire streams. Many inquiries concerning the discharge from small nozzles and the loss of head in small hose led to the tests which are herein described. The tests were undertaken with the object of acquiring data and putting the results into so workable a form that it would be easy to compute the quantity of water delivered by a nozzle of the size ordinarily used in the fire protection of the interior of buildings or to compute the pressure necessary in the mains to give an effective fire stream from such nozzles, and also to throw some light upon the quantity of water which would be considered sufficient for temporary protection.

* The experiments used in Part III of this Bulletin were reported in the Proceedings of the Fifth Meeting of the Illinois Water Supply Association, p. 170, 1913.

19. *Acknowledgment.*—The experiments here used were conducted at the University of Illinois under the direction of the writer by E. O. KORSMO and A. B. NEININGER of the class of 1911 as thesis work. Much credit is due them for the care and thought given the problem and the thoroughness with which they did their work. The water for the experiments was drawn from the University mains. The experiments for determining the height and the horizontal distance the jets would reach were conducted out of doors. The other experiments were carried on in the Hydraulics Laboratory.

VIII. APPARATUS AND METHOD OF EXPERIMENTING

20. *Hose and Nozzles.*—Rubber-lined cotton hose and unlined linen hose having a nominal diameter of $1\frac{1}{2}$ in. were used, the length of the test section for determining the lost head being 50 ft. in each case. The hose taken was from the racks in the University buildings and is representative of hose of this size commonly in use.

Three $1\frac{1}{2}$ -in. conical nozzles having different sizes of openings, as shown in Fig. 19, were tested. The first nozzle had a diameter of

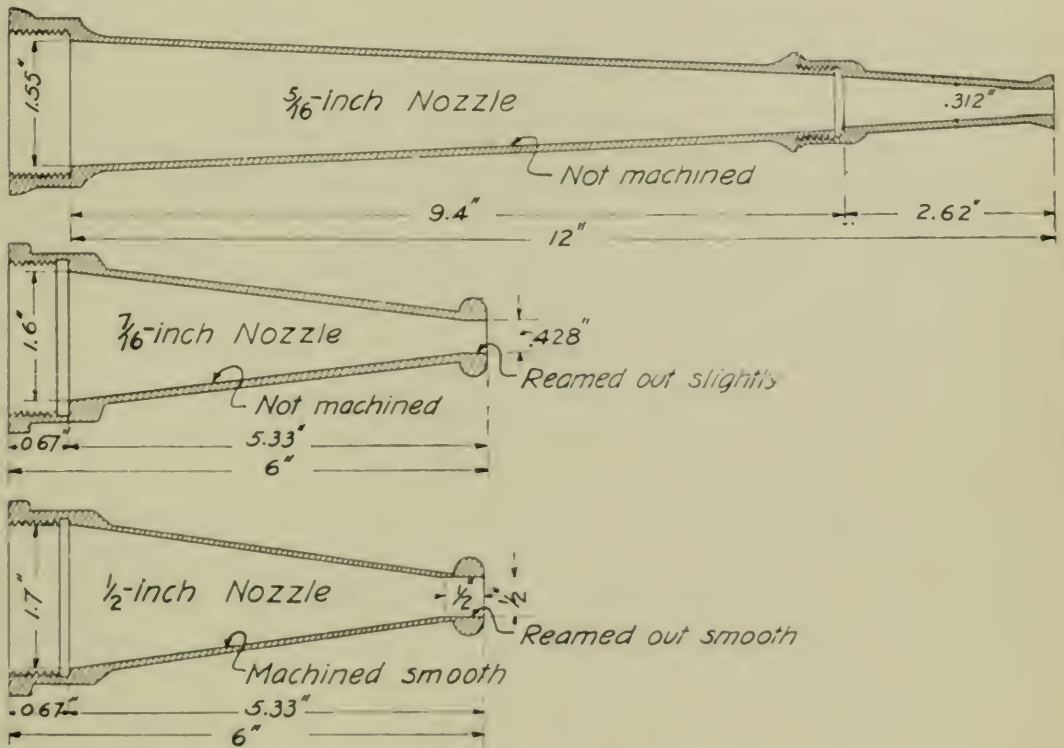


FIG. 19. LONGITUDINAL SECTIONS OF NOZZLES TESTED

$\frac{5}{16}$ in. The second nozzle had a diameter of 0.428 in., which is approximately $\frac{7}{16}$ in., and in compiling the tables, corrections were made so as to apply to a $\frac{7}{16}$ -in. nozzle. The third nozzle had a diameter of $\frac{1}{2}$ in. The $\frac{5}{16}$ -in. nozzle was 12 in. long while the other two were only 6 in. long (see Fig. 19). The $\frac{5}{16}$ -in. and the $\frac{7}{16}$ -in. nozzles were rough on the interior surfaces, having been left just as they came from the molds, the prints of the sand core being plainly visible. The tips had been smoothed slightly by running a drill through the

opening, but the cylindrical portion made by the drill was very short in both cases. The $\frac{1}{2}$ -in. nozzle was made from a $\frac{7}{16}$ -in. nozzle. The entire inner surface was machined smooth and a $\frac{1}{2}$ -in. reamer was run through the opening making a cylindrical portion $\frac{1}{2}$ in. long.

21. *Method of Experimenting.*—The loss of head was measured over a length of fifty feet of the hose by means of a differential mercury gage. The average pressure at a section of the hose was obtained with a piezometer connection or coupling of the Freeman type. A cross-section of one of these couplings is shown in Fig. 20. The discharge

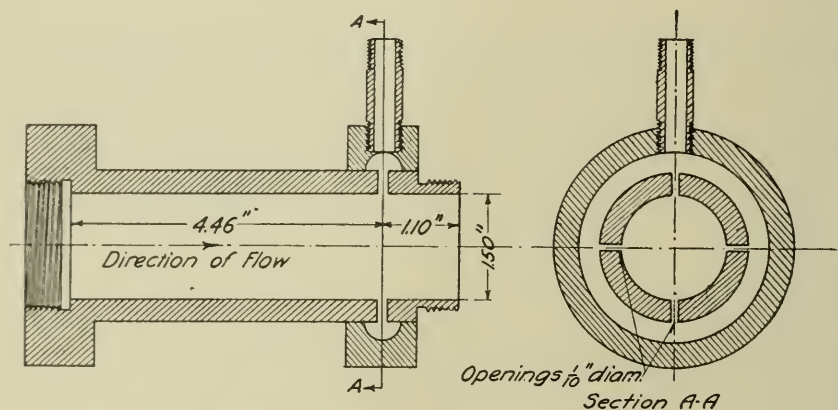


FIG. 20. CROSS-SECTION OF PIEZOMETER COUPLING

through the hose when determining the lost head in the hose was measured with a calibrated nozzle. When determining the coefficient of discharge for the nozzles the discharge was measured by weighing. The pressure at the base of the nozzle was measured with a calibrated pressure gage.

The vertical heights attained by the streams were determined by means of a transit and the horizontal distances reached were found by measuring with a tape from stakes which were driven in the ground at frequent intervals and at known distances from the nozzle.

IX. EXPERIMENTAL RESULTS AND DISCUSSION

22. *Results from Freeman's Experiments.*—In 1888 John R. Freeman conducted an extensive series of tests upon 2½-in. fire hose and nozzles.* In general, Freeman arrived at the following conclusions: Smooth conical nozzles give coefficients of discharge as high as any other form of nozzle, the jets reach farther and the streams remain solid for greater distances than for any other form of nozzle of the same size of opening and with the same pressure at the base of the nozzle. For smooth conical nozzles 1⅛ or 1¼ in. in diameter, a coefficient of discharge of 0.977 may be taken with great confidence that it will not be more than one-half of 1 per cent in error. The coefficient will be slightly larger for smaller nozzles. The nozzle makes a very convenient method of measuring water. The friction is but slightly more in smooth rubber-lined hose than in clean iron pipe of the same diameter. The friction in unlined linen hose is about two and one-third times as much as in smooth rubber-lined hose. A hose elongates from 2 per cent to 5 per cent with a pressure of 50 lb. per sq. in. This elongation produces a sinuosity which increases the loss of head about 6 per cent. Care should be exercised that there is no abrupt change of section in the hose couplings and that no washers or gaskets are so left as to impede the flow of water.

It is frequently recommended that a 250 gal. per min. fire stream be used in business districts, while a 175 or a 200 gal. per min. stream may be used in a residential district. These discharges correspond to a nozzle pressure of 40 to 50 lb. per sq. in., and a hydrant pressure of 80 to 110 lb. per sq. in. These values refer to outside service. Table 7 gives data for 2½-in. hose and nozzles for three different sizes of nozzle openings taken from Freeman's results. This table is convenient for making calculations for outside fire protection.

23. *Experimental Data.*—Table 8 gives the more important data of the experiments with hose and nozzles herein reported. Values are given for the pressures at the base of the nozzles, the discharges, the loss of head in the hose, and the vertical and horizontal distances reached by the jets. Other results discussed have been calculated from the data in this table.

* "Experiments Relating to the Hydraulics of Fire Streams." Trans. Am. Soc. Civ. Eng., Vol. XXI, p. 304, 1889.

TABLE 7

FREEMAN'S RESULTS FOR 1-IN. 1⅛-IN. AND 1¼-IN. NOZZLES ATTACHED TO 2½-IN. HOSE

1-INCH NOZZLE

Pressure Base of Nozzle	Discharge	Loss of Head in 100 Feet of Hose		Vertical Height of Jet for Good Fire Stream	Horizontal Distance	
		Rubber Lined	Unlined Linen		Jet for Good Fire Stream	Extreme Drops at Level of Nozzle
Lb. per sq. in.	Gallons per minute	Lb. per sq. in.	Lb. per sq. in.	Feet	Feet	Feet
20	132	5	10	35	37	77
30	161	7	15	51	47	109
40	186	10	20	64	55	133
50	208	12	25	73	61	152
60	228	15	30	79	67	167
70	246	17	35	85	72	179

1⅛-INCH NOZZLE

Pressure Base of Nozzle	Discharge	Loss of Head in 100 Feet of Hose		Vertical Height of Jet for Good Fire Stream	Horizontal Distance	
		Rubber Lined	Unlined Linen		Jet for Good Fire Stream	Extreme Drops at Level of Nozzle
Lb. per sq. in.	Gallons per minute	Lb. per sq. in.	Lb. per sq. in.	Feet	Feet	Feet
20	168	8	16	36	38	80
30	206	12	25	52	50	115
40	238	16	33	65	59	142
50	266	20	41	75	66	162
60	291	24	49	83	72	178]
70	311	28	57	88	77	191

1¼-INCH NOZZLE

Pressure Base of Nozzle	Discharge	Loss of Head in 100 Feet of Hose		Vertical Height of Jet for Good Fire Stream	Horizontal Distance	
		Rubber Lined	Unlined Linen		Jet for Good Fire Stream	Extreme Drops at Level of Nozzle
Lb. per sq. in.	Gallons per minute	Lb. per sq. in.	Lb. per sq. in.	Feet	Feet	Feet
20	209	12	25	37	40	83
30	256	19	38	53	54	119
40	296	25	51	67	63	148
50	331	31	63	77	70	169
60	363	37	76	85	76	186
70	392	43	88	91	81	200

TABLE 8

RESULTS OF EXPERIMENTS AT UNIVERSITY OF ILLINOIS WITH 5/16-IN., 7/16-IN.
AND 1/2-IN. NOZZLES ATTACHED TO 1½-IN. HOSE

5/16-INCH NOZZLE

Pressure Base of Nozzle	Discharge	Loss of Head in 100 Feet of Hose		Vertical Height of Jet for Good Fire Stream	Horizontal Distance	
		Rubber Lined	Unlined Linen		Jet for Good Fire Stream	Extreme Drops at Level of Nozzle
Lb. per sq. in.	Gallons per minute	Lb. per sq. in.	Lb. per sq. in.	Feet	Feet	Feet
20	12	.7	1.3	28	15	53
30	15	1.1	1.9	32	18	63
40	17	1.5	2.6	34	21	71
50	19	1.8	3.2	35	23	78
60	21	2.2	3.9	36	26	84
70	23	2.6	4.5	37	28	90
80	24	2.9	5.2	38	29	96
90	26	3.3	5.9	39	30	102
100	28	3.7	6.5	40	31	107

7/16-INCH NOZZLE

Pressure Base of Nozzle	Discharge	Loss of Head in 100 Feet of Hose		Vertical Height of Jet for Good Fire Stream	Horizontal Distance	
		Rubber Lined	Unlined Linen		Jet for Good Fire Stream	Extreme Drops at Level of Nozzle
Lb. per sq. in.	Gallons per minute	Lb. per sq. in.	Lb. per sq. in.	Feet	Feet	Feet
20	25	2.8	5.1	23	10	45
30	30	4.2	7.7	27	13	54
40	35	5.6	10.2	30	16	63
50	39	7.0	12.8	32	18	70
60	43	8.5	15.3	33	20	77
70	47	9.8	17.8	34	21	84
80	50	11.1	20.3	35	23	94
90	53	12.7	22.9	36	24	99
100	56	14.1	25.5	37	25	106

1/2-INCH NOZZLE

Pressure Base of Nozzle	Discharge	Loss of Head in 100 Feet of Hose		Vertical Height of Jet for Good Fire Stream	Horizontal Distance	
		Rubber Lined	Unlined Linen		Jet for Good Fire Stream	Extreme Drops at Level of Nozzle
Lb. per sq. in.	Gallons per minute	Lb. per sq. in.	Lb. per sq. in.	Feet	Feet	Feet
20	33	5.2	9.5	34	15	63
30	40	7.7	14.4	37	20	79
40	46	10.2	18.8	38	25	91
50	52	12.8	23.8	39	30	102
60	57	15.4	28.5	40	33	111
70	61	18.0	32.7	41	37	120
80	65	20.5	38.4	42	40	127
90	69	23.0	42.0	43	43	134
100	73	25.6	47.0	44	46	140

24. *Friction Factors.*—The curves of Fig. 21 show the friction factors for each kind of hose used and for velocities in the hose ranging

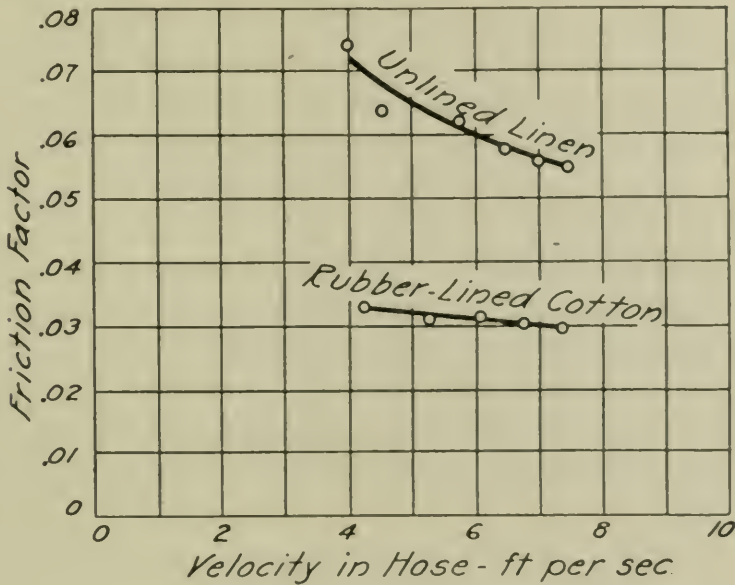


FIG. 21. DIAGRAM SHOWING FRICTION FACTORS IN RUBBER-LINED AND UNLINED HOSE

from 4 to 8 ft. per sec. These curves cover the range of velocities which would be met in ordinary use. The friction factor f is computed from the formula

$$h = \frac{fl v^2}{d 2g}$$

h = head lost in feet of water

l = length of hose in feet

d = diameter of hose in feet

v = velocity of the water in the hose in feet per second

g = acceleration due to gravity in feet per second per second

The loss of head in the rubber-lined hose varies almost directly as the square of the velocity and is about the same as the loss of head in clean iron pipe of the same diameter. The friction factor for the unlined linen hose decreases as the velocity increases, or in other words the loss of head does not vary directly as the square of the velocity, the ratio of the lost head to the square of the velocity being larger for the lower velocities. The reason that the friction factor for unlined linen hose decreased more rapidly with the velocity than does that for rubber-lined cotton hose may be that the diameter of the unlined hose

is increased more than that of the rubber-lined cotton hose by the increasing pressures which accompany the increasing velocities. This would make the value of d larger and the value of v smaller in the equation for f than was actually used. It is probable, furthermore, that the increasing pressure decreases the roughness of the unlined linen hose more than it does for rubber-lined hose. In general the lost head in the unlined linen hose is about twice as great as in the rubber-lined cotton hose. If an average value of the friction factor (0.06) is used for the unlined linen hose, no great error will enter into the results under ordinary circumstances. The length of hose will ordinarily not be more than 100 feet and for this length about 10 lb. per sq. in. will be the maximum loss of head in the unlined linen hose under working conditions with nozzles giving streams up to $\frac{1}{2}$ in. in diameter. An error as large as 10 per cent in the calculation of the loss of head in the hose would affect the nozzle pressure not more than one pound per square inch.

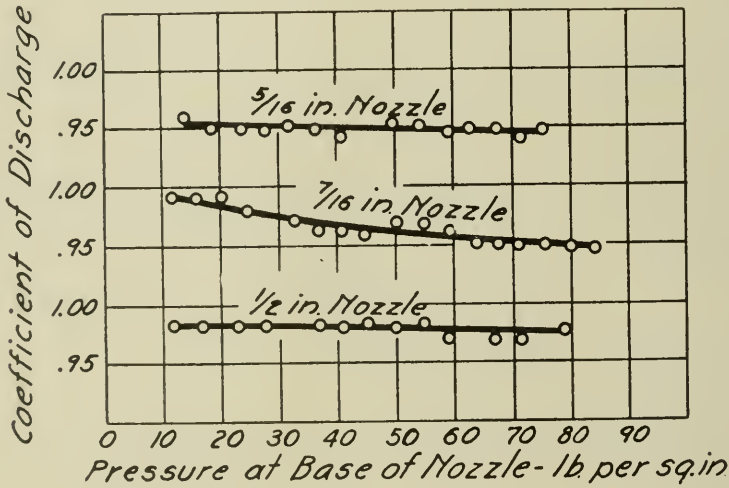


FIG. 22. DIAGRAM SHOWING COEFFICIENTS OF DISCHARGE OF NOZZLES

While the loss of head in the unlined linen hose is about twice as great as the loss of head in the rubber-lined hose, the linen hose has several advantages. It is much lighter to handle, folds up in less space on the wall racks, costs only about 50 to 60 per cent of the cost of rubber-lined hose and, in an ordinary building, its life is much longer.

25. *Coefficients of Discharge.*—The coefficients of discharge for each of the three sizes of nozzles are given in Fig. 22 for pressures

at the base of the nozzle ranging from about 10 to 85 lb. per sq. in. This range of pressures corresponds to a range in the velocity of the issuing jet from a minimum of about 35 ft. per sec. with the $\frac{1}{2}$ -in. nozzle to a maximum of about 185 ft. per sec. with the $\frac{5}{16}$ -in. nozzle. The coefficient of discharge is the ratio of the measured discharge to the ideal discharge. The measured discharge was weighed and the volume computed from the weights. The ideal discharge was computed from the formula

$$q = A \sqrt{2gh}$$

q = discharge in cubic feet per second. A = area of the opening of the nozzle in square feet. g = acceleration due to gravity in feet per second per second. h = pressure at the base of the nozzle in feet of water. The velocity of approach to the nozzle was negligible and was, therefore, not considered in the equation for the ideal discharge. The pressure at the base of the nozzles was measured with calibrated pressure gages.

The coefficient of discharge for the $\frac{7}{16}$ -in. and the $\frac{1}{2}$ -in. nozzles is nearly constant for all pressures and averages 0.98. The coefficient is slightly lower than 0.98 for the $\frac{7}{16}$ -in. nozzle at the higher pressures. The $\frac{5}{16}$ -in. nozzle gives a coefficient of 0.95. The $\frac{5}{16}$ -in. nozzle is 12 inches long while the other two are only 6 inches long, and this greater length adds somewhat to the friction and lowers the coefficient of discharge for the $\frac{5}{16}$ -in. nozzle.

The $\frac{5}{16}$ -in. and the $\frac{7}{16}$ -in. nozzles were rough on the interior surfaces, having been left just as they came from the molds. The tips had been smoothed slightly by running a drill through the opening, but the cylindrical portion made by the drills was very short in each case and the nozzles gave streams which sprayed badly a short distance away. The $\frac{1}{2}$ -in. nozzle was made from a $\frac{7}{16}$ -in. nozzle. The entire inner surface was first machined out in hopes that it would prevent the spraying of the jet, but the nozzle gave a stream which appeared no better than before machining. Then a $1\frac{5}{32}$ -in. reamer and finally a $\frac{1}{2}$ -in. reamer were run through the opening, each reducing the spraying. The $\frac{1}{2}$ -in. reamer made the cylindrical portion of the opening $\frac{1}{2}$ -in. long, and the resulting nozzle gave a very good stream. An opening larger than $\frac{1}{2}$ -in. could not be made in the nozzle because of the thinness of the walls.

26. *Height and Horizontal Distance of Jets.*—The heights and the horizontal distances reached by the jets from each of the three nozzles

used are given in Table 8. As stated, the vertical heights were measured by means of a transit and the horizontal distances were measured with a tape from stakes which were driven in the ground at frequent known space intervals. The observations were made when a moderate wind was blowing which interfered with the streams considerably. A stream was considered good for the distance in which practically all the water would pass through a circle whose diameter was 18 inches. The value was an arbitrary selection and the streams might be considered by some as effective for greater distances than those given in Table 8. The streams, however, beyond the sections chosen, diverged rapidly and the selection of a circle larger than 18 inches would have added but a few feet to the distances given in Table 8 in any case.

27. *Effect of Cylindrical Tip.*—The tests show clearly the importance of a smooth cylindrical opening at the tip of the nozzle. A comparison of the results of the tests on the $\frac{1}{16}$ -in. and the $\frac{1}{2}$ -in. nozzles for vertical heights and horizontal distances of the jets will show this difference. In the case of the $\frac{1}{16}$ -in. nozzle with a pressure of 30 lb. per sq. in. at the base of the nozzle the vertical height of the jet was 27 ft. as compared with 37 ft. for the $\frac{1}{2}$ -in. nozzle for the same pressure. Likewise the horizontal distance reached with the $\frac{1}{16}$ -in. nozzle was 13 ft. as compared with 20 ft. with the $\frac{1}{2}$ -in. nozzle. Similar comparisons may be made for other pressures at the base of the nozzle. The appearance of the jets showed a much greater difference than the data would indicate. It must be remembered that the two nozzles were alike and gave streams which appeared to be the same before one was reamed out to a larger size.

It will be noted also that in the case of the $\frac{5}{16}$ -in. nozzle for a pressure of 30 lb. per sq. in. at the base of the nozzle, the vertical and horizontal distances reached by the stream were respectively 32 and 18 ft., which indicate that the improvement in the carrying capacity of the $\frac{1}{2}$ -in. nozzle over that of the $\frac{1}{16}$ -in. nozzle was not due to the smoother condition of the interior surface of the $\frac{1}{2}$ -in. nozzle, but rather to the effect of the cylindrical tip. The condition of the interior surface of the nozzle to within one-half inch of the end does not seem to affect appreciably either the quantity of discharge or the quality of the stream.

It seems important, therefore, that the tip of the nozzle should be reamed out for a distance of at least $\frac{1}{2}$ in. in order to obtain a good fire stream. It is probably true also that for nozzles somewhat larger

than those used in these experiments the length of the cylindrical portion should be more than $\frac{1}{2}$ in., perhaps equal to the diameter of the issuing stream.

28. *Requirements for Temporary Fire Protection for the Interior of Buildings.*—Small fire hose and nozzles should be used as a temporary protection and brought into play until greater relief is at hand. They must necessarily operate under ordinary working pressures in the mains more often than under fire pressures. With 40 lb. per sq. in. as an average pressure in the mains, there should be, after deducting for losses in the hose and connecting pipes, about 30 lb. per sq. in. at the nozzle. This pressure, of course, would be still further reduced if the nozzle used was at a higher elevation than the main. With a nozzle pressure of 30 lb. per sq. in. the $\frac{1}{2}$ -in. nozzle will discharge 40 gal. per min., the $\frac{7}{16}$ -in. and the $\frac{5}{16}$ -in. nozzles will discharge 30 and 15 gal. per min., respectively. It is felt that the discharge from the two smaller nozzles is not great enough for effective work. It is true that the pressure at the nozzle for the smaller sizes with a given pressure in the main will be somewhat greater than for the $\frac{1}{2}$ -in. nozzle, because of the decreased velocity in the hose which will give a smaller loss of head, but this difference in pressure will not be enough to increase the discharge materially for an ordinary length of hose. The discharge from the $\frac{5}{16}$ -in. nozzle is too small to be very effective even at higher pressures. The discharge for a pressure of 100 lb. per sq. in. is but 28 gal. per min. It is recommended that $\frac{1}{2}$ -in. nozzles be used with $1\frac{1}{2}$ -in. hose. For nozzles larger than $\frac{1}{2}$ in., the discharge would become greater and increase the loss of head in the hose to such an extent that there would not be enough nozzle pressure left to produce a stream which would carry a sufficient distance.

With the aid of the tables the discharge for any of the nozzles may be readily computed for any pressure in the mains. If the nozzle is at a higher elevation than the main, subtract from the pressure in the main an amount equal to 0.434 times the difference in elevation in feet between the nozzle and the main. Take a discharge from the table for any pressure at the base of the nozzle for the size of nozzle used, then take the corresponding value of the head lost in the kind of hose used, multiply this value by the length of hose in feet used and divide by 100. The result gives the total loss in the hose for the assumed discharge. If there is any connecting pipe, the loss in it will be the same as the loss in a corresponding length of rubber-lined hose.

Add the losses in the pipe and hose to the pressure at the base of the nozzle for the assumed discharge to obtain the pressure in the main (corrected for the difference in elevation) necessary to produce this discharge. The discharge will vary as the square root of this pressure. Letting q' = the assumed discharge, P' = the pressure in the main (corrected for the difference in elevation) which will produce this discharge, q = the discharge to be determined, P = the actual pressure in the mains and H = difference in elevation between the nozzle and the main in feet gives the relation

$$q = q' \sqrt{\frac{P - 0.434H}{P'}}$$

which gives the required discharge.

To illustrate the use of the formula the following assumptions are made. Pressure in mains, $P = 60$ lb. per sq. in., 80 ft. of linen hose, 50 ft. of 1½-in. connecting pipe, elevation of nozzle above main 30 ft. and ½-in. nozzle used.

Assume a discharge of 46 gal. per min. and from the table the following values are obtained:

$$\text{Nozzle pressure} = 40$$

$$\text{Loss in hose} = \frac{80 \times 18.8}{100} = 15.0$$

$$\text{Loss in pipe} = \frac{50 \times 10.2}{100} = 5.1$$

$$\text{Total} = P' = 60.1$$

Substituting in the formula

$$q = 40.7 \text{ gal. per min.}$$

The following method may be used to determine the discharge for any size of nozzle for any pressure in the mains. Assume any pressure at the base of the nozzle, h' , in feet of water. The discharge for this pressure may be determined by the formula

$$q' = cA \sqrt{2gh'}$$

q' = discharge in cu. ft. per sec.

c = coefficient of discharge and may be taken as 0.98

A = area of opening of nozzle in sq. ft.

$2g = 64.4$ ft. per sec. per sec.

Determine the velocity in the hose for this discharge from the formula

$$v = \frac{q'}{a}$$

v = velocity in hose in ft. per sec.

a = area of hose in sq. ft.

Determine the loss in the hose from the formula

$$h_2 = \frac{fl v^2}{d 2g}$$

h_2 = head lost, in feet

f = friction factor which may be taken as 0.03 for rubber lined hose or 0.06 for unlined linen hose

l = length of hose in feet

d = diameter of hose in feet

v = velocity in hose in ft. per sec.

$2g$ = 64.4 ft. per sec. per sec.

If there is any pipe connecting the hose to the main, the loss for it may be computed by the same formula as for the hose, using 0.03 for the friction factor for 1½-in. pipe. Call this lost head h_3 .

The pressure in the main to give the assumed nozzle pressure is

$$H' = h' + h_2 + h_3$$

This pressure will be in feet of water. Then using the relation

$$q = q' \sqrt{\frac{H}{H'}}$$

gives the required discharge. If the main is below the nozzle, subtract the difference in elevation in feet from H in the formula.

It is recognized that this method is not strictly accurate since the head does not vary exactly as the square of the discharge, but the results obtained will be close enough for practical use.

29. *Summary.*—The following brief summary is given as applying to small hose and nozzles with velocities in the hose ranging from about 4 to 8 ft. per sec. and with pressures at the base of the nozzle ranging from about 10 to 85 lb. per sq. in.

(1) The friction factor (f in the equation for the lost head, $h = f \frac{l v^2}{d 2g}$) for rubber-lined hose varies but little with the velocity in the hose and is nearly the same as for clean iron pipe of the same diameter.

(2) The friction factor for unlined linen hose decreases as the velocity increases. In general the loss of head in unlined linen hose is about twice as great as in rubber-lined hose of the same diameter and for the same velocity.

(3) The nozzle should have a smooth cylindrical tip at least one-half inch long to keep the jet from spraying. A cylindrical tip is a much more important factor in securing a good fire stream than a smooth surface in the interior of the nozzle.

(4) Nozzle openings commonly in use to supply fire streams in the interior of buildings seem too small for adequate temporary fire protection. It is recommended that a nozzle with a $\frac{1}{2}$ -in. opening be used with a $1\frac{1}{2}$ -in. hose in order to secure a sufficient quantity of water for an effective fire stream.

(5) The coefficient of discharge of a small conical nozzle varies but little with the velocity and is close to 0.98. The value of 0.95 obtained with the $\frac{5}{16}$ -in. nozzle, which was 12 in. long as compared with 6 in. for the other nozzles tested, indicates, however, that the nozzle should be short to obtain the value of 0.98. A cylindrical tip on the nozzle seems to have little influence on the coefficient of discharge.

PART IV

THE ORIFICE BUCKET FOR MEASURING WATER

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CONTENTS

PART IV

THE ORIFICE BUCKET FOR MEASURING WATER

	PAGE
X. INTRODUCTION	65
30. Purpose	65
31. Acknowledgment	66
XI. APPARATUS AND METHOD OF CALIBRATING	67
32. Orifice Bucket	67
33. Method of Calibrating Orifice Bucket	71
XII. EXPERIMENTAL DATA AND DISCUSSION	72
34. Fifteen-inch Orifice Bucket having Fifty-six Orifices	72
35. Fifteen-inch Orifice Bucket having Only Three Orifices	73
36. Twenty-inch Orifice Bucket having Nineteen Orifices	73
37. Conclusions	76

LIST OF FIGURES

NO.		PAGE
23.	Fifteen-inch Orifice Bucket having Fifty-six Orifices	67
24.	View Showing Orifice Bucket in Use	69
25.	Twenty-inch Orifice Bucket having Nineteen Orifices	68
26.	Calibration Curves for 15-inch Orifice Bucket having Fifty-six Orifices .	72
27.	Calibration Curves for 15-inch Orifice Bucket having Only Three Orifices	74
28.	Calibration Curves for 20-inch Orifice Bucket having Nineteen Orifices .	75

PART IV

THE ORIFICE BUCKET FOR MEASURING WATER

X. INTRODUCTION

30. *Purpose.*—The purpose of Part IV is to describe a method of measuring water by means of a simple, portable, and inexpensive device, here called an orifice bucket, and to present experimental data applying thereto for a range of conditions sufficient to indicate that the device is reliable for use in engineering practice. An orifice bucket is a cylindrical vessel into which water to be measured falls vertically and passes out through a number of holes or orifices in the bottom. A vertical glass tube placed just outside the bucket is connected to the sides of the bucket near the bottom, and the height of the water in the tube indicates the head on the orifice.

The orifice bucket was devised for the purpose of measuring the discharge of several artesian wells pumped by means of air lift, the water from each of which discharged into a separate cistern or small reservoir through a vertical pipe. In each case the water left the pipe with considerable blast and momentum. Several possible methods for the measurement of the discharge were considered but were thought to be impracticable for various reasons or inapplicable for the particular case. After some preliminary laboratory experimenting an orifice bucket was devised which served very satisfactorily in determining the discharge from each of the wells. It was at first feared that the water would enter the bucket with such a blast that entrained air would enter the vertical glass tube and cause trouble in determining the height of water in the bucket. There was, however, no trouble from this cause and the fluctuations of the water level in the glass tube offered no serious difficulties.

The orifice bucket has also given satisfaction in tests made to determine yields of well pumps of the reciprocating type. It should give satisfactory results in the field where simplicity of construction and portability are desirable and where extreme accuracy is not of great importance.

31. *Acknowledgment.*—The orifice bucket was developed by the writer through experimental work in the Hydraulic Laboratory of the University of Illinois during 1910 and 1911.* Considerable improvement has been made in the arrangement of certain parts of the bucket by I. W. FISK, P. S. BIEGLER, and P. J. NILSEN, of the department of electrical engineering, in connection with tests on electric motor-driven deep-well pumps. Some of the experimental data herein presented were obtained by them, to whom acknowledgment is made.

* A part of the results here presented was published in the Proceedings of the Third Meeting of the Illinois Water Supply Association, p. 87, 1911.

XI. APPARATUS AND METHOD OF CALIBRATING

32. *Orifice Bucket*.—Fig. 23 shows the construction and dimensions of one of the first orifice buckets used in the experiments, and Fig. 24 shows the bucket in use. This bucket weighed 23 lb.

As previously stated an orifice bucket is a cylindrical vessel having holes or orifices in its bottom and into which water to be measured

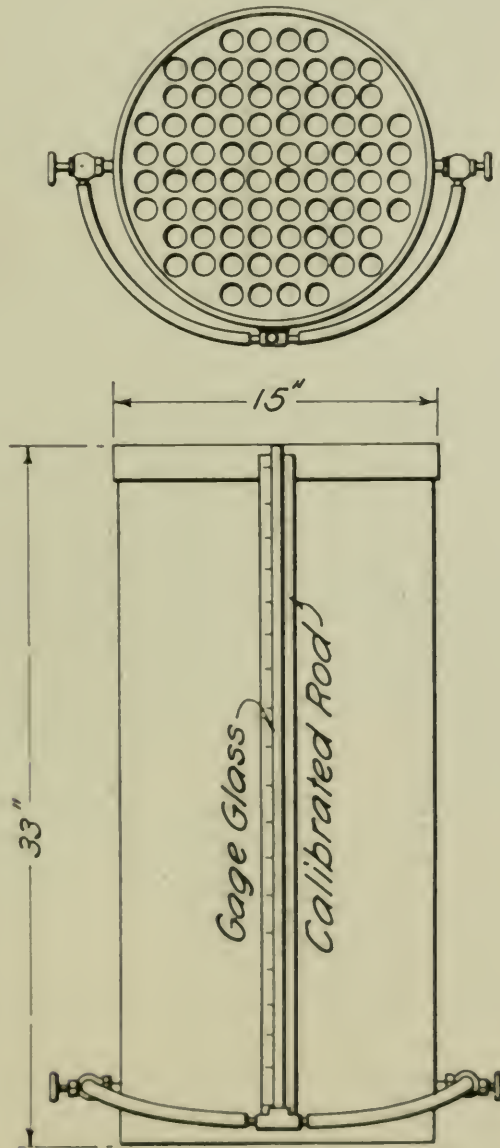


FIG. 23. FIFTEEN-INCH ORIFICE BUCKET HAVING FIFTY-SIX ORIFICES

falls vertically, the head of water on the orifices being indicated by the height of the water in a vertical glass piezometer tube attached near the bottom of the bucket.

Fig. 25 shows the construction of the most elaborate orifice bucket which has been used. It is provided with a short tube checker-work to smooth out the flow of the water on its way to the orifices in the

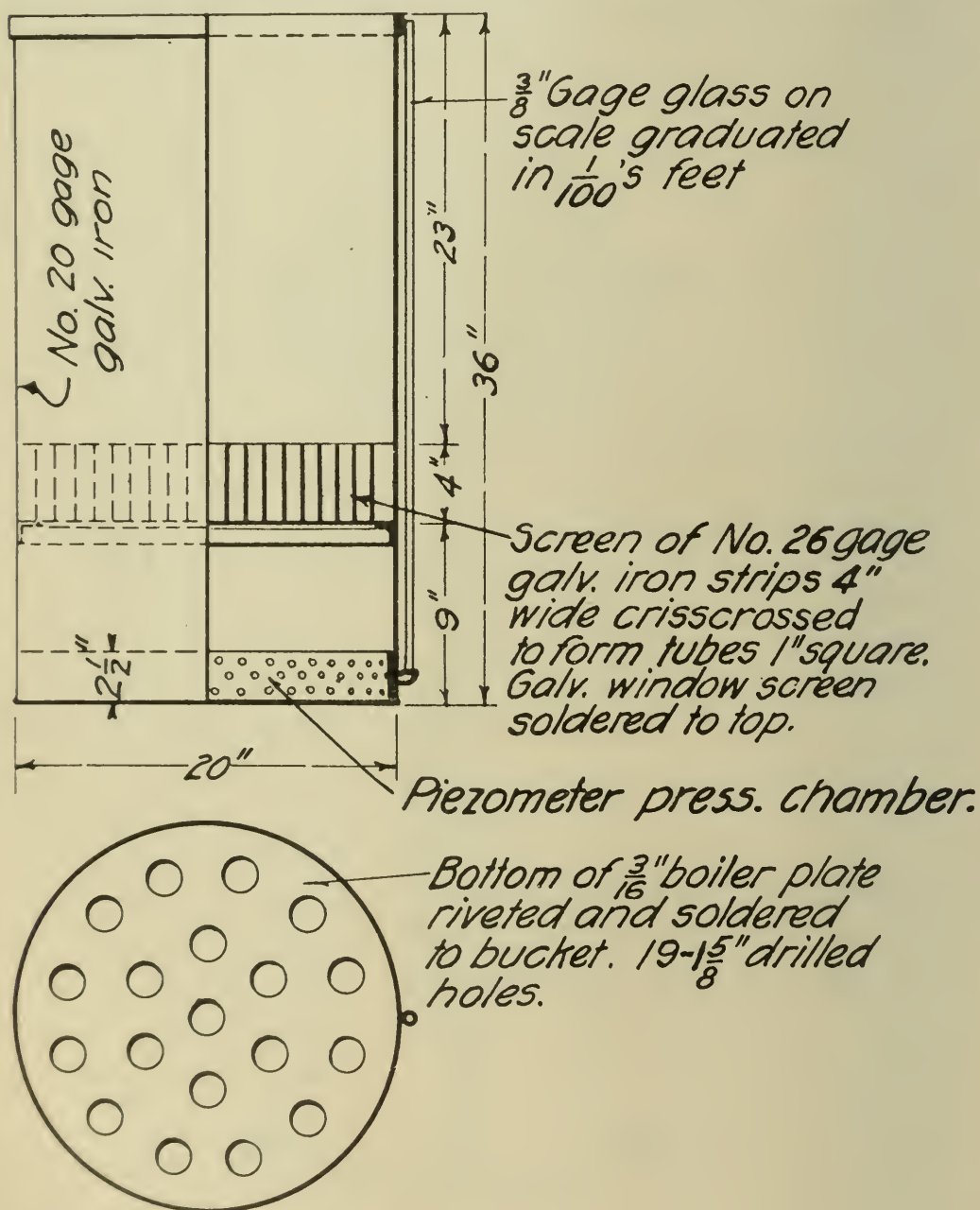


FIG. 25. TWENTY-INC ORIFICE BUCKET HAVING NINETEEN ORIFICES

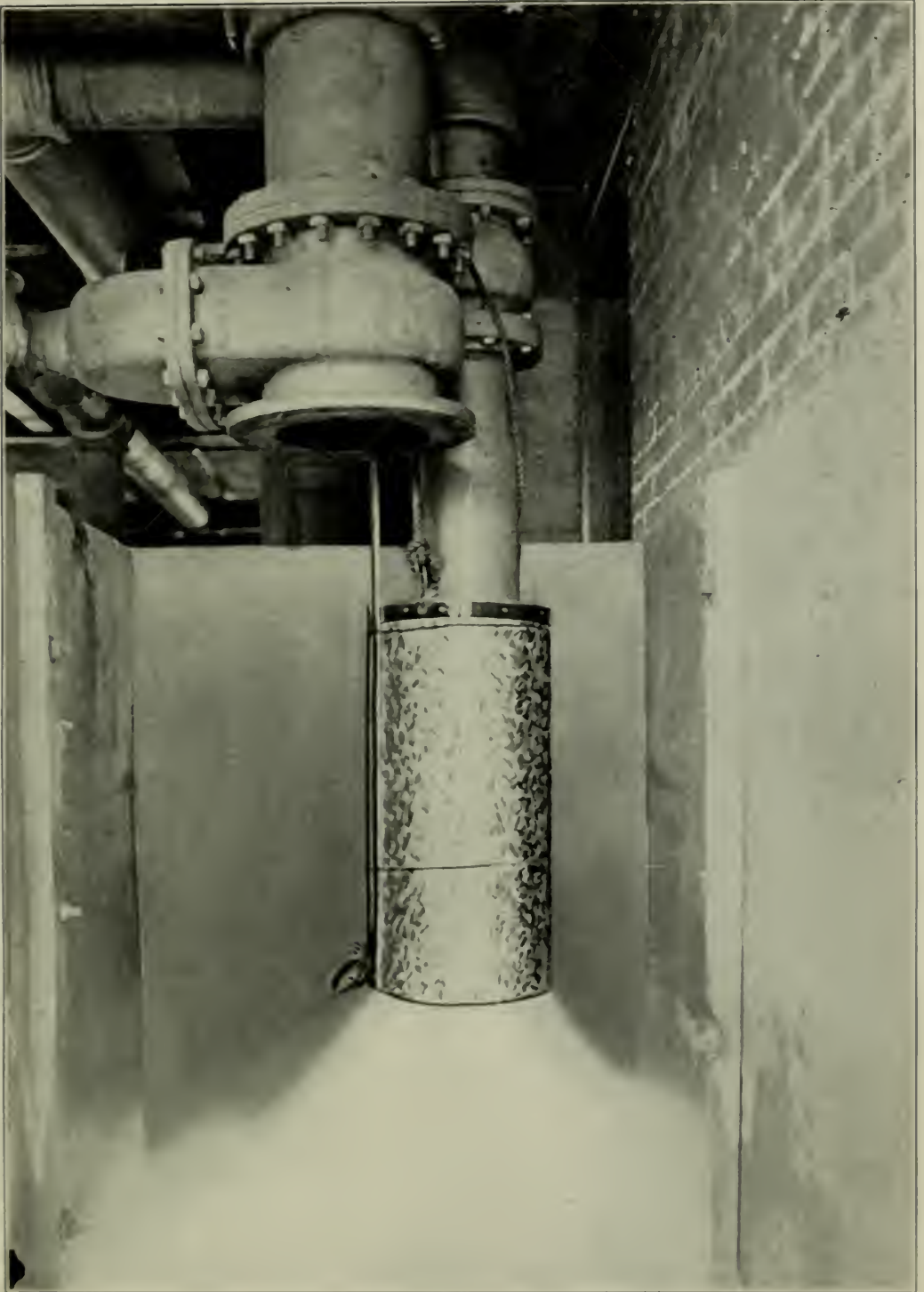


FIG 24. VIEW SHOWING ORIFICE BUCKET IN USE

bottom of the bucket. The vertical glass tube is connected to a piezometer chamber or ring around the base of the bucket, pressure being transmitted to the piezometer ring through a large number of small holes.

The orifice bucket may be adapted for the measurement of water for a considerable range in the discharge by varying the head on the orifices and also by varying the number of holes which are stopped or plugged with corks or wooden stoppers. The range in the capacities of the orifice buckets which have been used is from about 40 to 1000 gal. per min.

33. *Method of Calibrating Orifice Bucket.*—In calibrating the orifice bucket it was hung underneath a vertical pipe as shown in Fig. 24. The quantity of water discharged was measured with a 6-in. Venturi meter in most of the calibration tests although a calibrated measuring pit was used in some of the tests to determine the volume discharged in a given time.

With a given number of holes open, the flow in the orifice bucket was regulated by means of a valve between the Venturi meter and the bucket until the height in the bucket remained constant. The Venturi meter reading and the head on the orifices were then taken. This procedure was repeated for several different heads and for different numbers of orifices open.

The effect of varying the conditions of flow was investigated somewhat. The height of the free fall of the water from the inflow pipe to the orifice bucket was varied; likewise different sizes of pipe were used giving different velocities to the stream entering the bucket. The stream was also allowed to enter near to one side of the bucket instead of at the center. Different groupings of the open orifices, furthermore, were tried, and different methods were employed in attempting to spread or distribute the inflowing stream.

In using the orifice bucket it is necessary to estimate the average head shown in the glass tube because there is some fluctuation. The amount of the fluctuation may be reduced by throttling the valve in the connection of the glass tube to the orifice bucket. If the proper conditions are observed, there should be little trouble from this source. It should be remembered that the rate of discharge is proportional to the square root of the head and that the effect of the error which might occur in the head reading itself is thus reduced in determining the discharge.

XII. EXPERIMENTAL DATA AND DISCUSSION

34. *Fifteen-inch Orifice Bucket Having Fifty-six Orifices.*—Fig. 26 shows the calibration curves for the 15-inch orifice bucket shown in Fig. 23 and 24. There were fifty-six 1-in. holes in the bottom of the bucket giving a maximum capacity of about 1000 gal. per min. With all the orifices open the rate of discharge was varied from about 600 to 1000 gal. per min. by varying the head from about $\frac{3}{4}$ ft. to 2 ft. With thirty-two orifices open the discharge had a range of about 300 to 600 gal. per min. by varying the head from about $\frac{1}{2}$ ft. to 2.5 ft. In closing the twenty-four orifices, corks were used of such size that they projected but little into the bucket. It was found that in filling the orifices a symmetrical arrangement gave somewhat steadier action, particularly when the orifices near the circumference were the ones filled. The inflowing stream was discharged from an 8-in. pipe. A

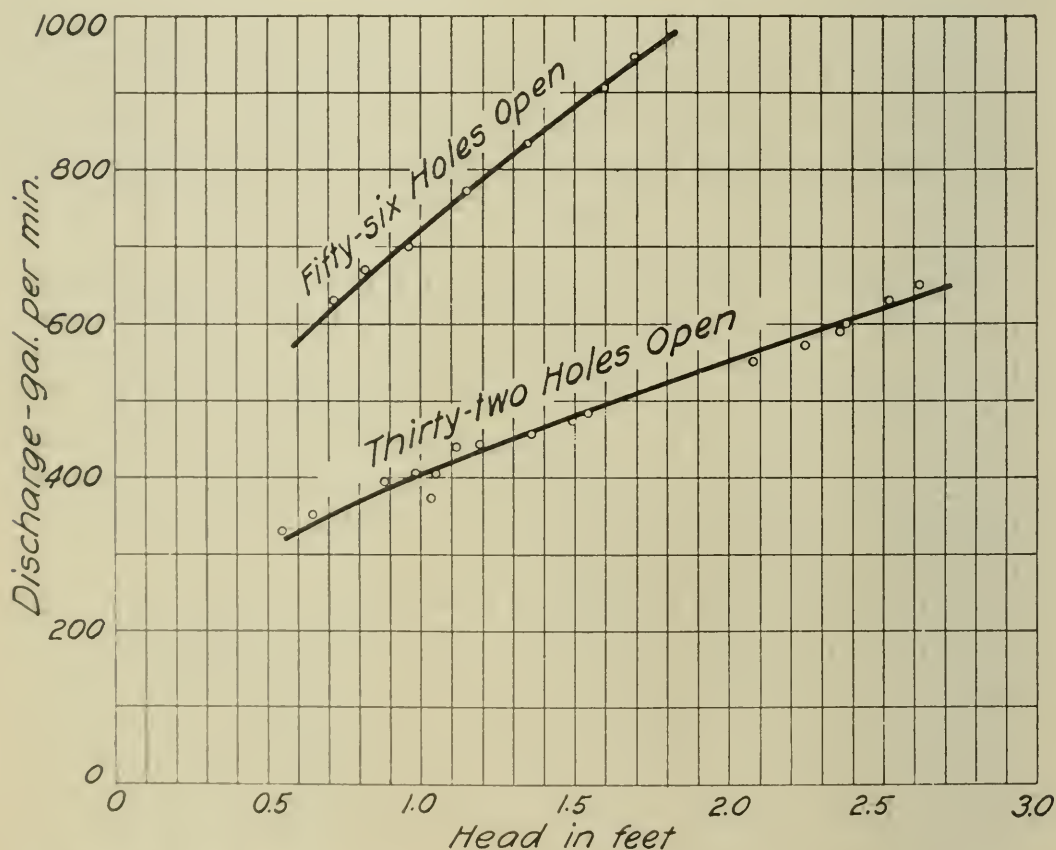


FIG. 26. CALIBRATION CURVES FOR 15-INCH ORIFICE BUCKET HAVING FIFTY-SIX ORIFICES

3-in. pipe was also tried but did not give satisfactory results, on account of the high velocity which produced an extremely agitated condition of the water in the bucket. This condition may be overcome, however, by use of a deflector or distributor, such as an open bag or sack attached to the end of the discharge pipe.

The rate of discharge for any other number of open orifices for this bucket may be obtained from the equation

$$q = 12.8 n \sqrt{h}$$

which represents fairly well the relation between the quantity, q , in gal. per min., the number of orifices open, n , and the head in the bucket, h , in ft. The experiments give an average coefficient of discharge for the 1-in. orifices of this bucket of about 0.63.

35. *Fifteen-inch Orifice Bucket Having Only Three Orifices.*—Fig. 27 shows an orifice bucket of the same dimensions as the one just described but with three iron tubes about 1 in. long inserted in a 1-in. wooden bottom. It was provided with two screens through which the water passed on its way to the orifices in the bottom of the bucket.

Fig. 27 also shows the calibration curves for this orifice bucket. It will be noted that the discharge ranges from about 35 to 115 gal. per min. This orifice was constructed and calibrated for immediate use and not for experimental purposes. The calibration curves are of value in indicating the reliability of the orifice bucket under a rather wide range in the details of its construction.

36. *Twenty-inch Orifice Bucket Having Nineteen Orifices.*—An illustration of the most elaborate orifice bucket used in the experiments is shown in Fig. 25, the capacity of which is about 1000 gal. per min. It contains a checkerwork of vertical tubes through which the water flows in passing to the orifices. The gage glass which indicates the head on the orifices is connected to a piezometer ring or chamber around the base of the bucket. The pressure of the water in the bucket is transmitted to the piezometer chamber through a large number of $\frac{1}{8}$ -in. holes. The bottom of the bucket consists of $\frac{3}{16}$ -in. boiler plate in which nineteen $1\frac{1}{8}$ -in. circular holes are drilled.

The calibration curves for this orifice bucket are shown in Fig. 28, for all holes open and for ten holes open. The discharge for any other number of orifices open may be found with a fair degree of accuracy from the equation

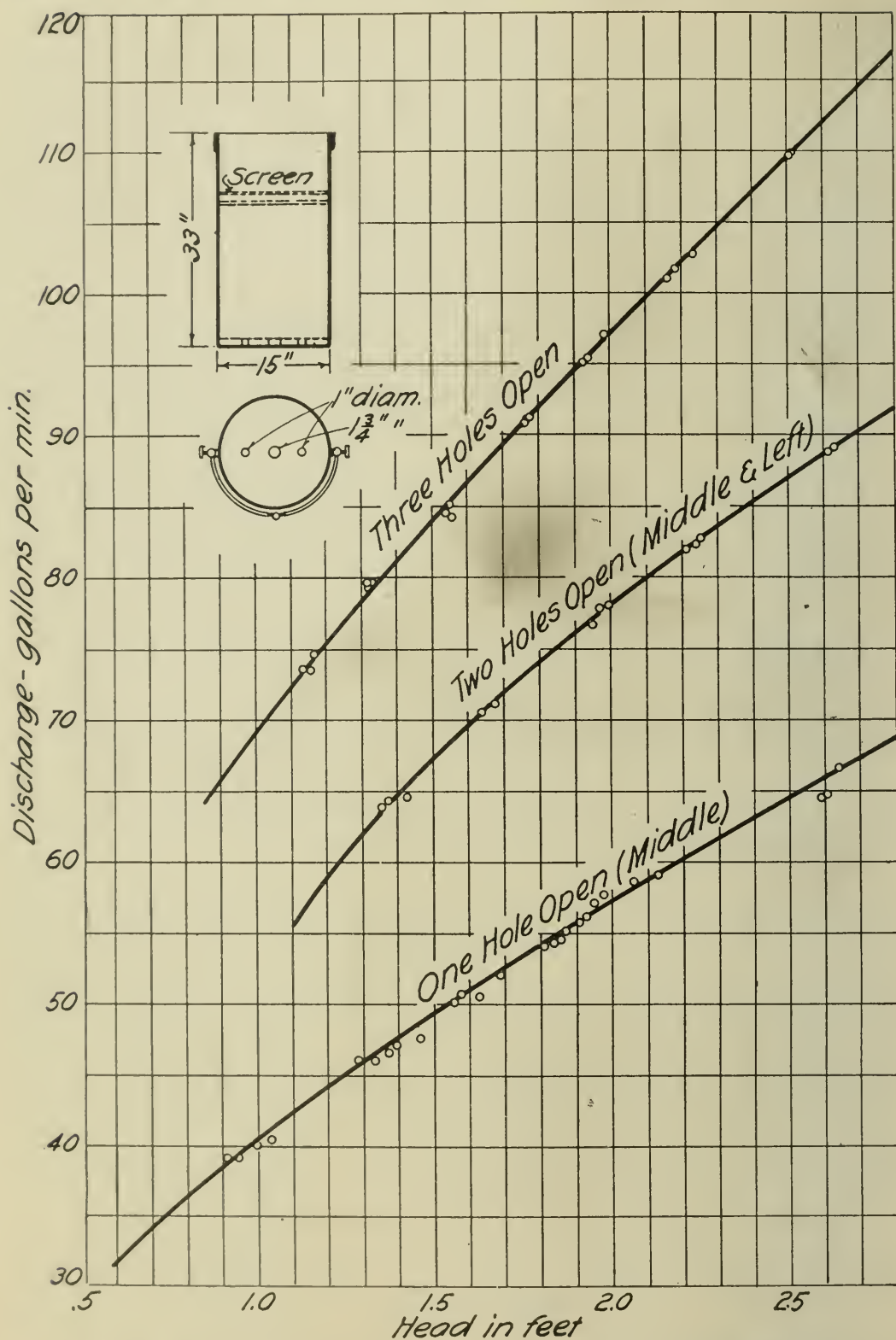


FIG. 27. CALIBRATION CURVES FOR 15-INCH ORIFICE BUCKET HAVING ONLY THREE ORIFICES

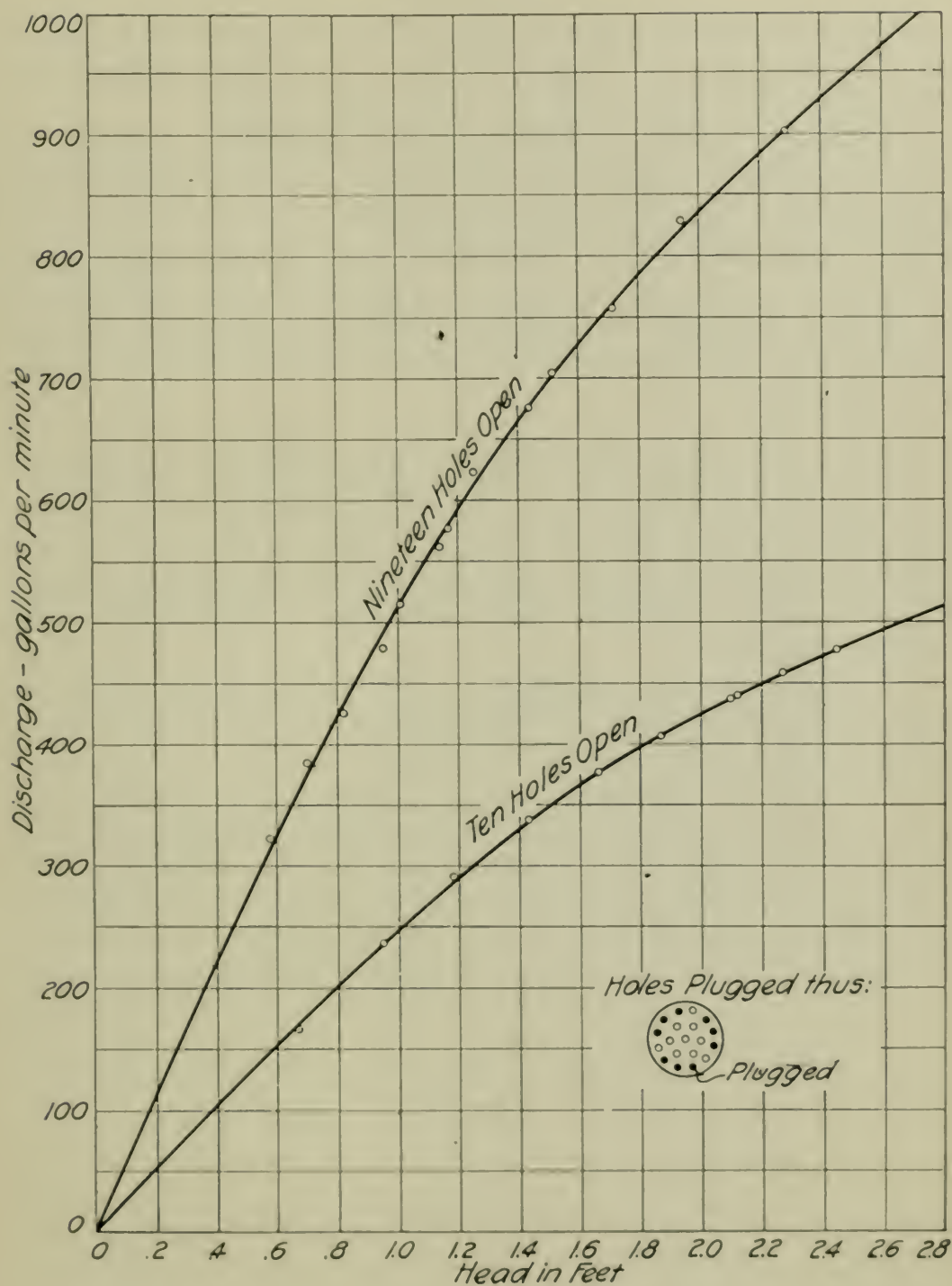


FIG. 28. CALIBRATION CURVES FOR 20-INCH ORIFICE BUCKET HAVING NINETEEN ORIFICES

$$q = 32.7 n \sqrt{h}$$

in which q is expressed in gal. per min., n is the number of holes open, and h is the head on the orifices in feet. The average coefficient of discharge for the 1½-in. orifices of this bucket is 0.61.

The curves in Fig. 28 were obtained when the bucket was supported firmly in an upright position with the stream to be measured discharging vertically in the center of the bucket and with the free fall into the bucket small. The velocity of the inflowing stream, furthermore, was not high (2 or 3 ft. per sec.), thereby causing but little agitation of the water in the bucket. Experiments, however, in which more or less variation from these conditions were allowed indicated that no serious errors resulted.

37. *Conclusions.*—The conditions under which the discharge of water has to be measured are so varied and the purpose or aim in determining the discharge differs so much in different problems that nearly any one of the many common methods of measuring water has a rather restricted field of usefulness, while some methods apply only to very special conditions.

The orifice bucket is designed to meet rather special conditions. It is peculiarly adapted for the measurement of water where a device which is portable (light weight and small size), simple in construction, and low in cost are essential features. The measuring capacity, moreover, covers a considerable range. The orifice bucket is particularly fitted for the measurement of water when the water discharges with considerable blast and momentum from the end of a vertical pipe, in such a manner that the spray covers the entire surface of the water in the bucket, as in the case of air lift pumping. When so used the orifice bucket gives results which should be correct within 5 per cent if the proper precautions are observed in its use, as is shown by the calibration curves, and correct within 10 per cent for the more unfavorable conditions to be met in the field. The highest accuracy is obtained when the orifice bucket is supported rigidly in an upright position with the center of the discharging stream vertically over the center of the bucket. The free fall of the water should be as small as possible and the velocity of the water as it enters the bucket should not be large, unless the stream is distributed, so as to avoid high local velocities in the bucket. The orifice bucket, however, gives very satisfactory results even when there are considerable deviations from these desirable conditions and renders a service for which other measuring devices may not be adapted.

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- Bulletin No. 9.* An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Revised Edition 1912. *Fifty cents.*
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- Bulletin No. 16.* A Study of Roof Trusses, by N. Clifford Ricker. 1908. *None available.*
- Bulletin No. 17.* The Weathering of Coal, by S. W. Parr, N. D. Hamilton, and W. F. Wheeler. 1908. *None available.*
- Bulletin No. 18.* The Strength of Chain Links, by G. A. Goodenough and L. E. Moore. 1908. *Forty cents.*
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UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

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MAY, 1918

TEST OF A FLAT SLAB FLOOR OF THE
WESTERN NEWSPAPER UNION BUILDING

BY

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CONTENTS

	PAGE
1. Preliminary	7
2. Acknowledgment	7
3. The Building	8
4. The Test	13
5. Deflection of Slab	19
6. Cracks in Slab	19
7. Load-strain Diagrams	21
8. Stresses in Reinforcing Bars in Upper Side of Slab	25
9. Stresses in Reinforcing Bars in Lower Side of Slab	27
10. Strains in Concrete at Upper Surface of Slab	27
11. Strains in Concrete at Lower Surface of Slab	27
12. Influence of Position of Bar	29
13. Resisting Moment Accounted for by Stresses in Reinforcing Bars	31
14. Bending of Columns	39
15. Time Effect of Load and Recovery upon Removal of Load .	40
16. General Comments	41
17. Wrecking of the Floor	43
18. Summary	43

LIST OF FIGURES

NO.	PAGE
1. View of Western Newspaper Union Building at Time of Test	9
2. View Showing Full Load on Floor	9
3. Thickness of Floor in Test Area	11
4. Arrangement of Reinforcement in Test Area	12
5. Location of Test Panels	15
6. Location of Gage Lines on Upper Side of Slab	16
7. Location of Gage Lines on Lower Side of Slab	17
8. Load-deflection Diagrams and Location of Deflection Points	18
9. Location of Main Cracks in Upper Side of Slab	20
10. Location of Main Cracks in Lower Side of Slab	21
11. Load-strain Diagrams for Gage Lines on Reinforcing Bars on Upper Side of Slab	22
12. Load-strain Diagrams for Gage Lines on Reinforcing Bars on Upper and Lower Sides of Slab	23
13. Load-strain Diagrams for Gage Lines in Concrete on Upper and Lower Sides of Slab	24
14. Stresses in Reinforcing Bars in Pounds per Square Inch at Load of 913 Pounds per Square Foot	26
15. Unit-strains in Concrete at Load of 913 Pounds per Square Foot . . .	28
16. Position of Neutral Axis for the Several Layers of Bars at Column 22 .	30
17. Sections of Positive Moment and Negative Moment Considered in the Calculations	32
18. Sections of Positive and Negative Moment Considered in the Calculation of the Resisting Moments Accounted for by Stresses in the Rein- forcing Bars	35
19. Load-strain Diagrams for Gage Lines Located on Columns	39
20. View Showing Arrangement of Reinforcement at Column 22	45
21. View of Slab after Reinforcing Bars were Exposed	45

LIST OF TABLES

NO.	PAGE
1. Compression Tests of Concrete Prisms	13
2. Tension Tests of Reinforcing Bars	14
3. Resisting Moments Accounted for by Stresses Observed in Reinforcing Bars	36

TEST OF A FLAT SLAB FLOOR OF THE WESTERN NEWS- PAPER UNION BUILDING

1. *Preliminary.*—This bulletin gives the results of the test made on a four-way reinforced concrete flat slab floor of the Western Newspaper Union Building in Chicago in August and September, 1917. A load of 913 lb. per sq. ft. was applied over four panels. The building, which was nine years old at the time of the test, was to be torn down to clear the site for the new Union Passenger Station; the opportunity was utilized to apply a test load much greater in proportion to the design load than had been used in previous tests of buildings. The test was carried far enough to give stresses in the reinforcing bars and concrete markedly higher than have been obtained in other building tests. The information on the action of the slab in its various parts given by the strain measurements has an important bearing on the design of the flat slab structure.

2. *Acknowledgment.*—The test was made as investigative work of the Engineering Experiment Station. The testing work was done under the direct supervision of Mr. GONNERMAN. He and Mr. N. E. ENSIGN, Associate in Theoretical and Applied Mechanics, acted as observers. The results have been reduced and prepared for publication as a bulletin of the Engineering Experiment Station.

Acknowledgment of valuable aid received in carrying out the test is made. The PORTLAND CEMENT ASSOCIATION furnished the labor for preparing for the test and for hauling the loading material and loading and unloading the floor. The UNIVERSAL PORTLAND CEMENT COMPANY assisted in making arrangements and gave assistance on the test. The pig iron used for loading material was lent by the ILLINOIS STEEL COMPANY. The freight on the pig iron was borne jointly by the PENNSYLVANIA RAILROAD and the PORTLAND CEMENT ASSOCIATION. Opportunity to use the building for the purpose of the test was given by the CHICAGO UNION STATION COMPANY; the test was made at the suggestion of A. J. HAMMOND, Principal Assistant Engineer. The CONDRON COMPANY provided an assistant for tracing and checking the transfer of the loading material.

3. *The Building.*—The Western Newspaper Union Building was an eight-story reinforced concrete structure located at Clinton and Adams Streets, Chicago. The building was erected in the spring of 1909 by the George Hincheliff Company, contractors, according to plans furnished by S. N. Crowen, architect, and Ritter and Mott, engineers. It had been in use by a printing company until 1916. The floor tested had been occupied by printing presses. Fig. 1 is a view of the building at the time of the test; the wrecking of the building had begun.

Two types of floor construction were used in the building. The first five floors were slab and girder type; the sixth, seventh, and eighth floors were Turner mushroom flat slab type (four-way reinforcement). The floors of the building were divided into panels 17 ft. 5½ in. by 19 ft. 4½ in. The test was made on the sixth floor. This floor was designed for a live load of 250 lb. per sq. ft. and was nominally 8½ in. thick. A considerable variation in thickness was found, the measured thickness over the test area ranging from 7.5 to 9.8 in. Fig. 3 gives the thickness of the floor at a number of places as determined by readings with an engineer's level. In general, the thickness was greater away from the columns than in the vicinity of the columns. The interior columns were octagonal in form, 24 in. in short diameter below the floor tested and 21 in. in short diameter above it. The inside diameters of the hooping of the columns on the fifth and sixth floors are given on the plans as 21 in. and 18 in., respectively. The column capitals were pyramidal; the short diameter at the top of the capital was 54 in. The building plans called for 15 5⁄8-in. round bars in each of the four bands of reinforcement in the floor slab and indicated that over most of the columns in the test area there were laps in certain bands. After the test was made, the floor was broken into and the location and extent of all laps and the position of reinforcing bars with respect to the surfaces of the slab were found. Fig. 4 shows the arrangement of the reinforcement found over the test area, including the position of the laps. In several places the arrangement of reinforcement differs from that given in the building plans. In three places in rectangular bands the reinforcement for positive moment was double that given on the plans (30 bars instead of 15). The lapping of bars at columns was generally greater than that indicated on the plans. At column 15 three bands were lapped; at columns 14, 16, 21, 26, 27, and 28, two bands; and at columns 22 and 23, one band. In most cases, the length of lap and its position were such that the extra



FIG. 1. VIEW OF WESTERN NEWSPAPER UNION BUILDING AT TIME OF TEST



FIG. 2. VIEW SHOWING FULL LOAD ON FLOOR

metal was effective in regions of greatest moment. In some cases the laps were poorly arranged, as at columns 15, 16, and 27. No reason is apparent for the lapping of bars between columns. There was no reinforcement for negative moment in the region midway between columns. The eight $1\frac{1}{4}$ -in. column rods were bent out into the slab, and two circumferential ring rods (circles of 5 ft. 6 in. and 8 ft. 6 in. diameters) rested on these and supported the lower layers of reinforcing bars. The measurement of position of bars with respect to

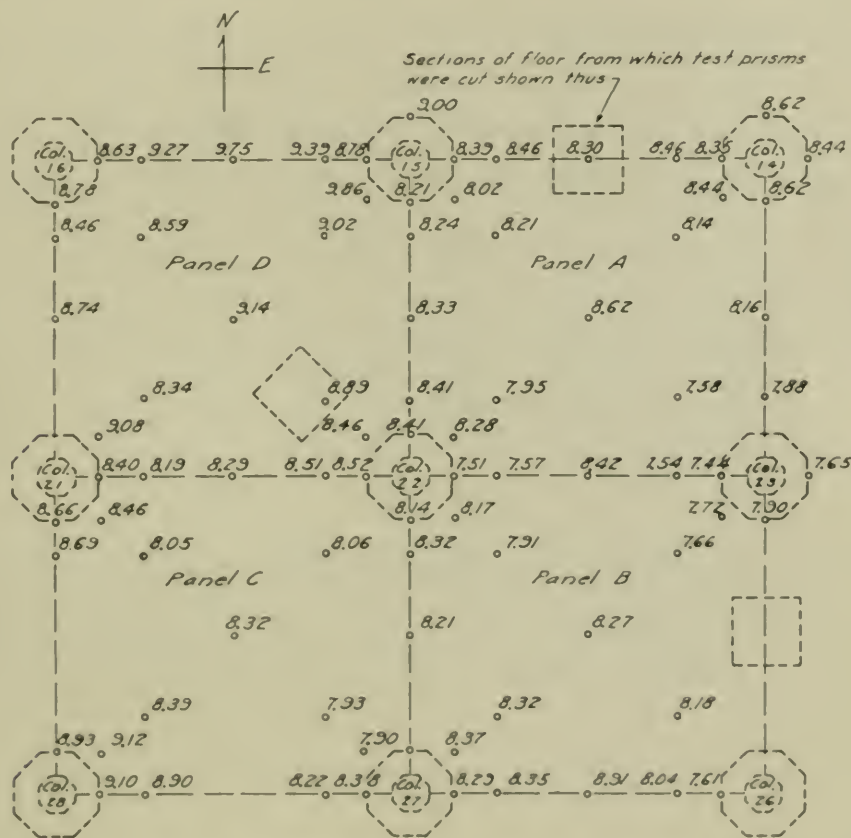


FIG. 3. THICKNESS OF FLOOR IN TEST AREA

the surface of the slab showed considerable variation at the several columns. The method of lapping was not always the same; in some cases the laps of a given band were not at the same level—at columns 15 and 22, for example, there were five layers of $\frac{5}{8}$ -in. bars besides the circumferential and bent-out column bars. At the columns the distance of the centers of the bars of the top layer from the upper surface of the slab varied from 0.90 in. to 2.00 in., and that of the lower layer from 3.60 to 4.00 in. At points between columns the

centers of the bars of the rectangular bands were from 2.30 in. in one case to 4.20 in. above the lower surface of the slab. In the region of the center of the panel the centers of the lower layer of diagonal bars were from 1.20 to 2.20 in. above the lower surface of the slab.

The variation in amount of reinforcement available at the different sections of the slab due to the diversity in amount and position

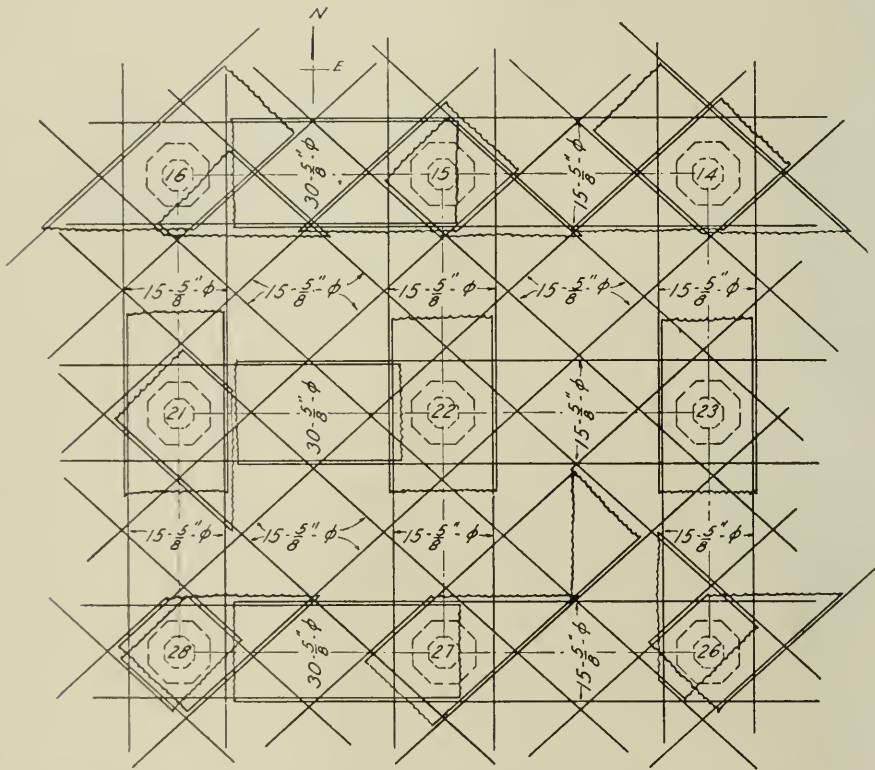


FIG. 4. ARRANGEMENT OF REINFORCEMENT IN TEST AREA

of laps, and the variation in depth of reinforcing bar and thickness of slab, as well as variations in the quality and stiffness of the concrete in different parts of the loaded area, may be expected to cause some lack of uniformity in the stresses and deflections at points similarly located on the test area.

The concrete in the slab was 1-2-4 mix; in the columns 1-1-2 mix was used. The coarse aggregate was gravel. At the time of the test the building was eight years old. Pieces of the concrete were cut out from the floor and sawed into test prisms approximately 5 in. square and 16 in. long; one face of the prism was coincident with the upper

surface of the slab. The concrete was taken from parts of the slab (indicated in Fig. 3) which had not been highly stressed and which was relatively free from reinforcing bars and cracks. The results of the tests are given in Table 1. The strength of these test prisms

TABLE 1
COMPRESSION TESTS OF CONCRETE PRISMS

Specimen	From Panel	Length inches	Section inches	Loaded Area sq. in.	Maximum Applied Load pounds	Unit Compressive Strength lb. per sq. in.	Modulus of Elasticity lb. per sq. in.
A1	A	16.9	4.6 by 4.8	22.1	71 000	3210	4 500 000
A2	A	16.8	4.7 by 4.7	22.1	119 300	5400	5 100 000
A3	A	16.2	4.9 by 3.9	19.1	104 400	5460	4 800 000
					Average	4690	4 800 000
B1	B	18.0	6.5 by 4.5	29.3	93 400	3190	4 200 000
B2	B	15.0	4.7 by 4.7	22.1	79 100	3580	3 500 000
					Average	3385	3 850 000
D1	D	16.0	4.5 by 3.0	13.5	48 600	3600	4 600 000
D2	D	16.5	5.1 by 2.5	12.8	42 800	3340	4 500 000
D3	D	10.0	4.9 by 3.1	15.2	60 600	3990	
D4	D	9.5	5.0 by 3.1	15.5	59 700	3850	
D5	D	8.0	4.5 by 3.2	14.4	48 200	3350	
					Average	3626	4 550 000

ranged from 3190 lb. per sq. in. in panel B to 5460 lb. per sq. in. in panel A, and the initial modulus of elasticity from 3 500 000 to 5 100 000 lb. per sq. in. When the floor was broken up after the test, a noticeable difference was found in the quality of the concrete in the four test panels. The concrete in panels A and D appeared much stronger and harder than that in panels B and C. That in panel D was very hard.

Steel coupons were cut from reinforcing bars at different places in the tested floor. The results of the tension tests of these bars are given in Table 2. The bars gave an average yield point by drop of beam of 63 600 lb. per sq. in. and an average ultimate strength of 101 300 lb. per sq. in.

4. *The Test.*—The method of testing was similar to that used in previous buildings tests, as described in Bulletin No. 64 of the University of Illinois Engineering Experiment Station, "Tests of Reinforced Concrete Buildings Under Load." The loading material was pig

TABLE 2
TENSION TESTS OF REINFORCING BARS

Specimen No.	Average Diameter inches	Yield Point lb. per sq. in.	Ultimate Strength lb. per sq. in.	Elongation in 8 Inches per cent	Reduction of Area per cent
1	.602	66 800	103 500	17.2	37.6
2	.603	59 900	96 200	17.8	34.0
3	.619	69 100	115 600	17.8	33.4
4	.625	55 400	86 700	21.5	48.0
5	.626	64 000	100 000	16.2	39.4
6	.625	66 500	115 000	14.6	45.3
7	.625	64 900	101 600	15.1	45.3
8	.615	60 000	99 000	18.6	45.3
9	.621	65 700	102 700	16.9	41.7
Average		63 600	101 300	17.2	41.1

iron. The pig iron was hauled from the freight yards to the building in auto trucks. The net weight of each truck load of iron was obtained by weighing on a certified scale before it was hauled to the building. At the building the pig iron was loaded on hand trucks, hoisted to the test floor by means of an electric freight elevator, and then placed on the test area by hand. A record was kept of the number of truck loads placed on each panel and the total weight on each panel was obtained from the truck weights.

The load was applied over the four interior panels of the sixth floor. Fig. 5 gives the location of the panels tested. The load on each panel was divided into quarters by means of aisles 6 to 8 in. wide extending at right angles to each other along the center lines of the panels and along the boundaries between panels. The space occupied by the aisles and by the boxes built on the floor to afford access to the gage lines amounted to 17 per cent of the panel areas. The final load on the slab was 913 lb. per sq. ft., a total load of 308 400 lb. per panel. Fig. 2 shows the full load in place.

Gage lines were prepared in advance of the test—103 on the reinforcing bars and 75 on the concrete. Fig. 6 and 7 show the location of the gage lines on the upper and lower sides of the slab. To insure reliability of initial readings, three sets of strain gage readings (and more on many of the gage lines) were taken before the load was applied. Strain gage readings were taken at loads of 234, 425, 637, 855, and 913 lb. per sq. ft. of panel area. In each case except for the first load two complete sets of strain readings were taken on the reinforcing bars and the concrete, and sometimes more. The deflection of the slab was measured at 20 points. The location of the deflection points are

shown in Fig. 8. The appearance of cracks was also noted. Readings of deformation and deflection at the more important points were taken from time to time as the load was being placed on the floor.

Readings were also taken three days after the maximum load had

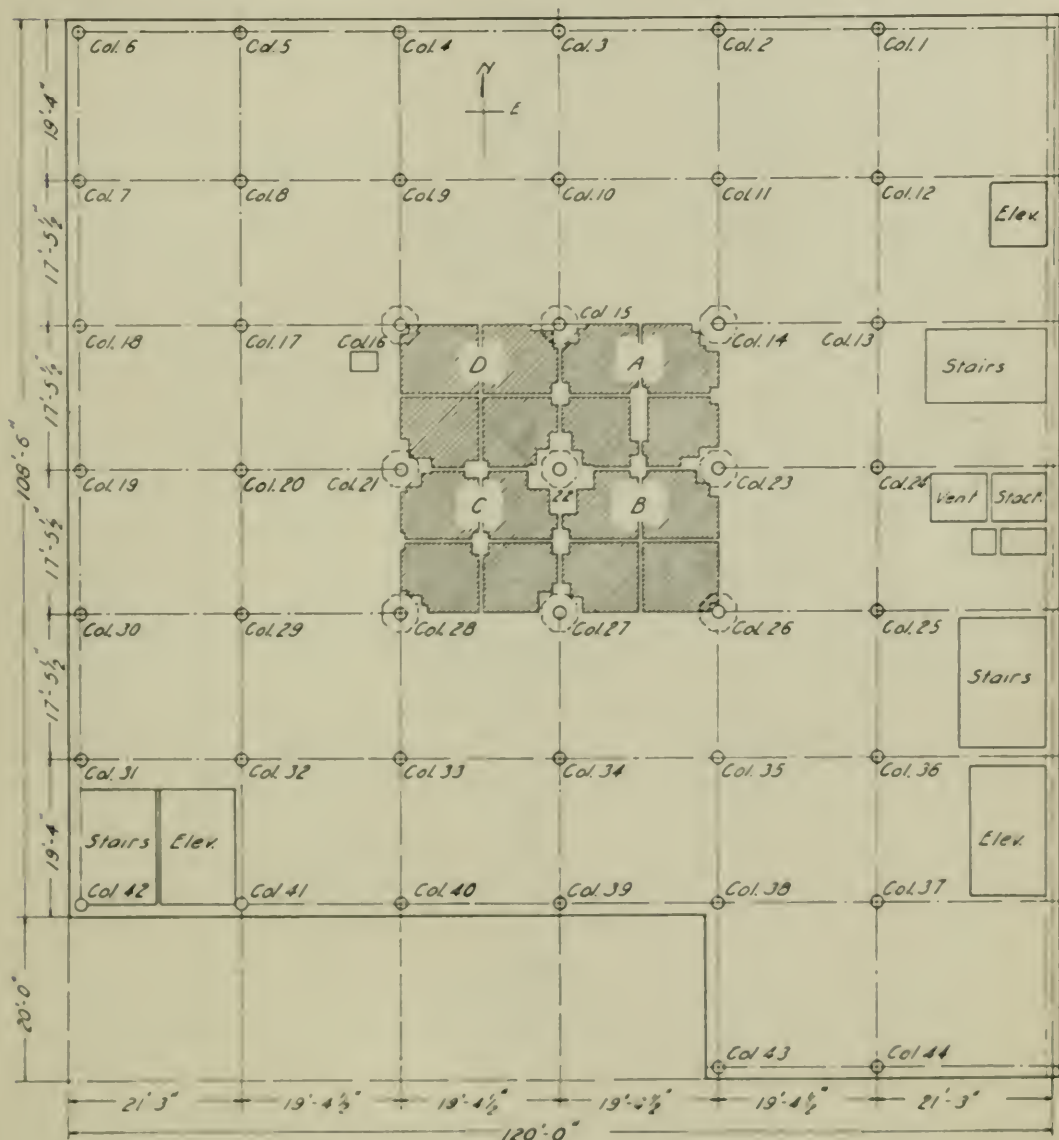


FIG. 5. LOCATION OF TEST PANELS

been placed on the floor in order to get information on the time effect of the load upon deformations and deflections. After the removal of the load readings were taken to find the amount of recovery in the floor.

The test area was chosen where there would be the least effect of floor openings and where the building plans showed laps in only the rectangular bands over the column in the center of four panels. In a building test the considerable time required to reduce the data of the

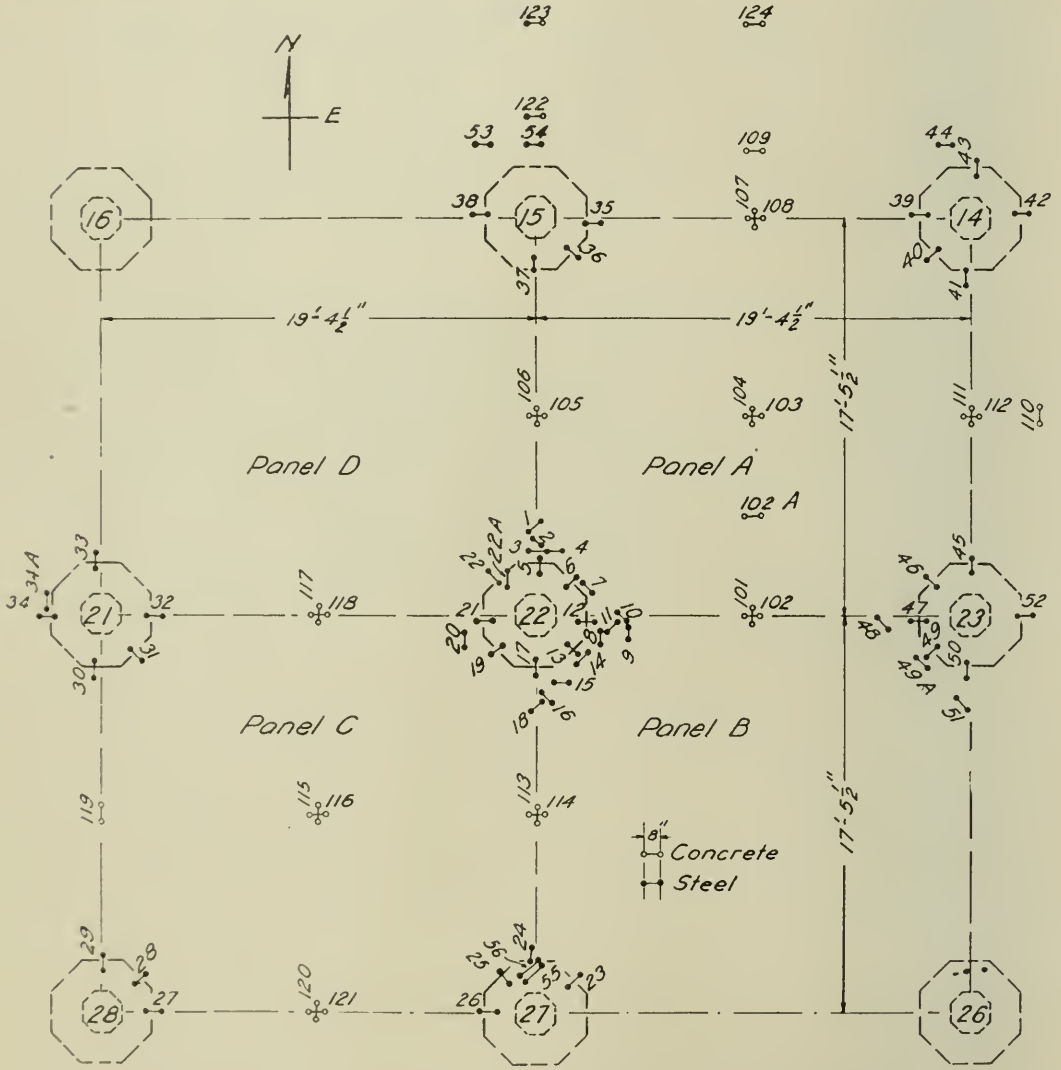


FIG. 6. LOCATION OF GAGE LINES ON UPPER SIDE OF SLAB

readings into form for analysis renders it necessary to restrict the number of gage lines and so their distribution over the test area becomes a matter of importance. The gage lines in this test were placed with a view of getting some information on (1) the amount and distribution of the stresses in the reinforcement along sections through

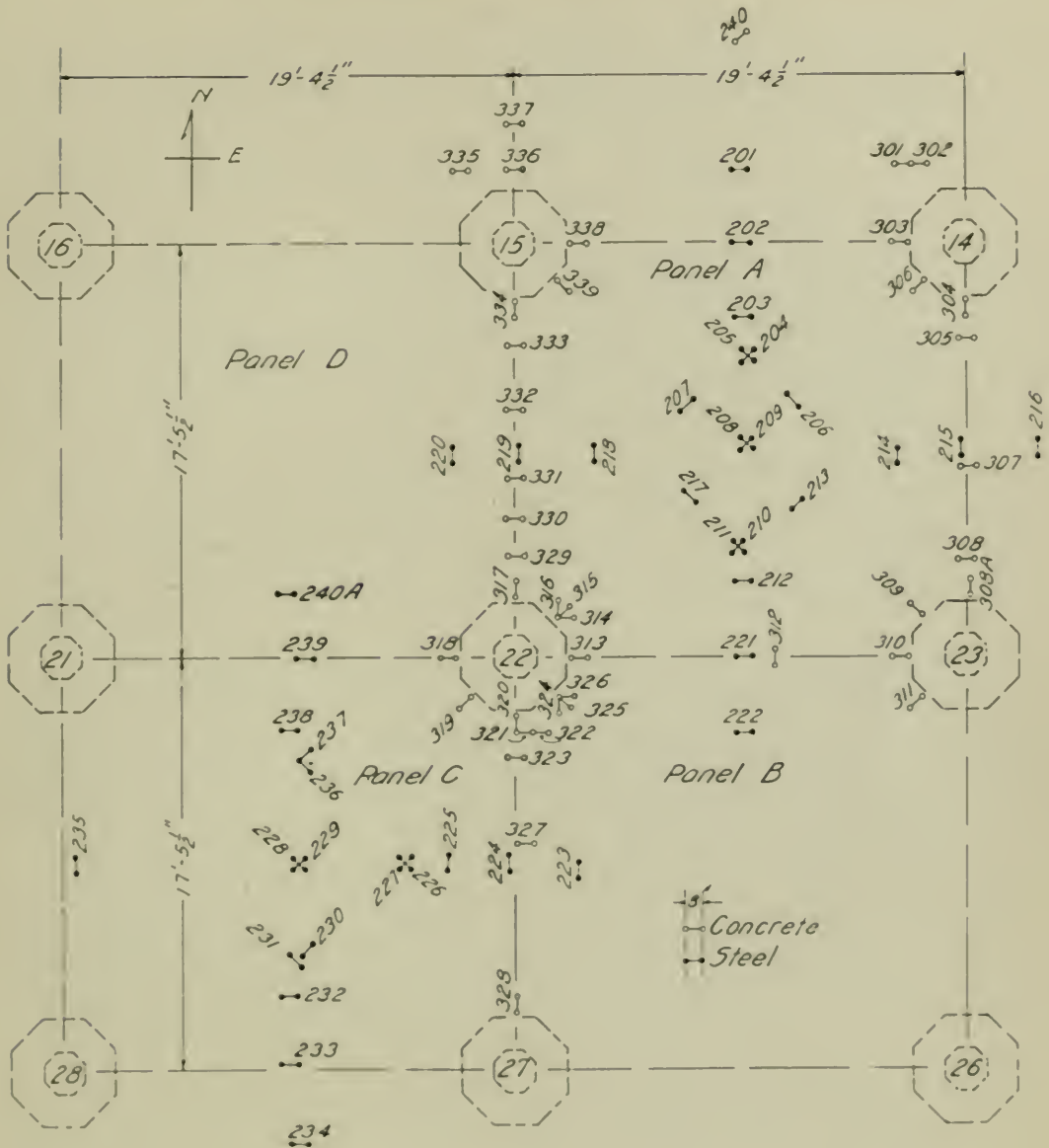


FIG. 7. LOCATION OF GAGE LINES ON LOWER SIDE OF SLAB

the panel centers and through the panel edges, and at other points, (2) the strain in the concrete at the more important points in the slab, (3) the moment of resistance accounted for by the stresses in the reinforcing bars which cross sections through the panel centers and through the panel edges, and (4) something on the stresses in columns at the edges of the loaded area due to bending under the applied load.

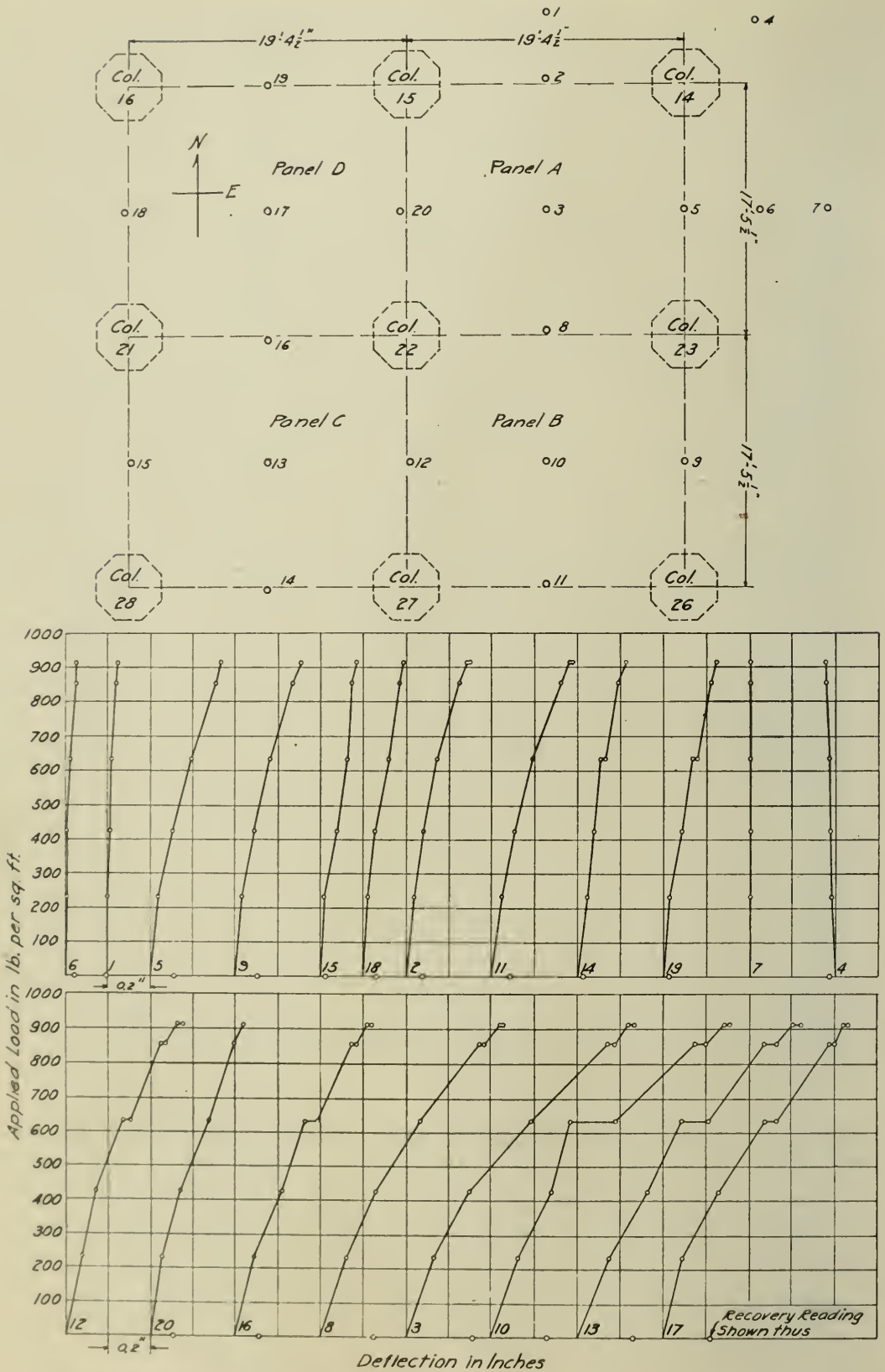


FIG. 8. LOAD-DEFLECTION DIAGRAMS AND LOCATION OF DEFLECTION POINTS

The gage lines in panels A and C with few exceptions were laid out in duplicate in order that readings obtained on the gage lines in the one panel might serve as a check on readings obtained at corresponding gage lines in the other, and also to find out whether there was similarity of action in the two panels.

5. *Deflection of Slab.*—The deflections of the slab at the several deflection points are plotted in Fig. 8, the diagrams for points similarly located being grouped together. The recovery of deflection one day after the load was removed is indicated by the points plotted at the bottom of the diagram. The second point plotted for the load of 637 lb. per sq. ft. is the deflection 16 hours after the last of the load was applied; the second point for the load of 855 lb. per sq. ft., 37 hours after; the second point for the load of 913 lb. per sq. ft., 66 hours after. Where no second point is plotted, the change in deflection was negligible.

The deflections at the centers of panels A, B, C, and D under the load of 913 lb. per sq. ft. were 1.06, 1.12, 1.04, and 0.87 in., respectively. It may be noted in this connection that panels C and D had a greater amount of reinforcement than A and B and that panel D was thicker. The concrete of panels A and D was of unusually good quality.

Of the deflections at the middle of the inner edges of the loaded panels, that at point 8 (Fig. 8) was considerably greater than that at point 16, and that at point 12 was more than at point 20. The differences are explainable by differences in quality of concrete and in amount and arrangement of reinforcement.

At the outer edges of the loaded area, the deflections at points 5 and 9 were considerably greater than at points 15 and 18; and the deflections at points 2 and 11 were greater than at points 14 and 19—the same circumstances explain these differences in deflections.

It may be noted that points 1 and 6 (Fig. 8) distant one-quarter of the panel lengths from the panel edges gave a measurable deflection. Point 7 (center of adjacent panel) remained stationary and point 4 showed an upward movement.

6. *Cracks in Slab.*—It was noted before the load was applied that there were numerous checks in the upper surface of the slab in the regions around the column and along the panel edges. Most of these were evidently surface checks; others were tension cracks formed under previous loads. The latter opened upon the application of load.

Fig. 9 gives the location of the more important cracks on the upper side of the slab which either opened or formed under the test load. These cracks were all open cracks—much more marked than hair cracks. Under the load of 913 lb. per sq. ft. they ranged in width from 0.02 to 0.06 in. These cracks show the regions of high tensile stress in the top of the slab. The main cracks at the columns were generally at or near the edges of the column capital; they branched out to join the cracks extending along the panel edges between the columns. The cracks at the capitals of the columns bordering the loaded area were

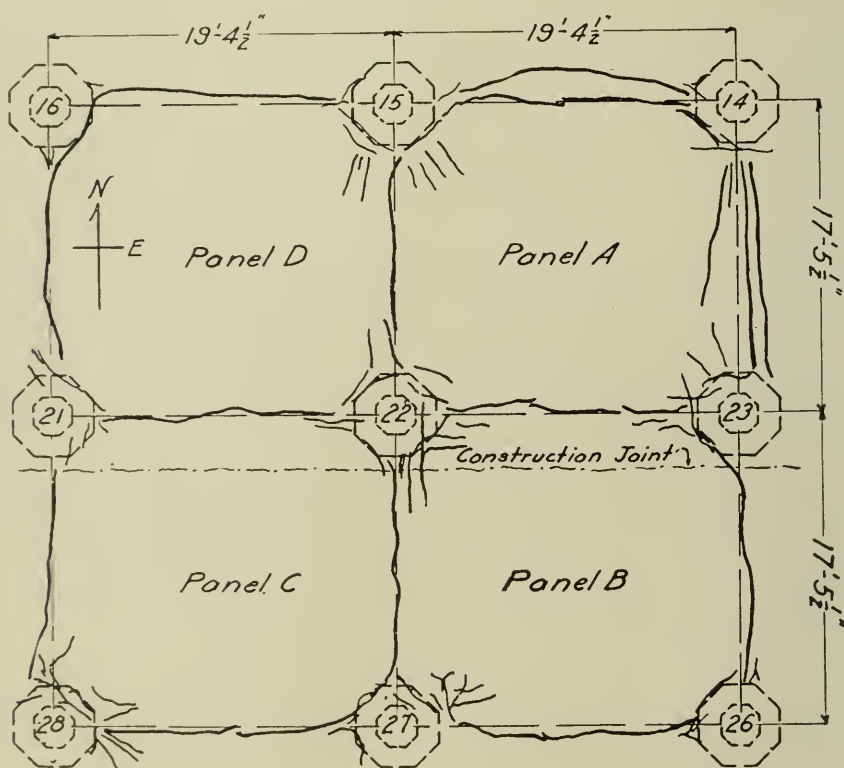


FIG. 9. LOCATION OF MAIN CRACKS IN UPPER SIDE OF SLAB

fully as wide as those at the capital of the central column. The cracks along the panel edges were generally as pronounced as those around the capitals.

Fig. 10 shows the location of the more important cracks on the lower side of the slab in panels A and C, the panels in which the principal tension gage lines on the lower side of the floor were placed. In panels B and D, in addition to those noted in the figure, there were cracks over the panels in positions similar to those noted in panels A

and C. In panel D the cracks were not so wide nor so numerous as in the other panels, even at the maximum load. Panel B had larger and more numerous cracks than the other panels. Most of the cracks on the lower side of the slab were found in bands extending in rectangular directions. In a small area at the panel centers cracks extended in diagonal directions. No cracks or checks had been noted on the lower side of the slab before loading was begun. The first hair cracks here were observed at a load of 234 lb. per sq. ft. At a load of 425 lb. per sq. ft., the cracks were fairly well defined and for higher

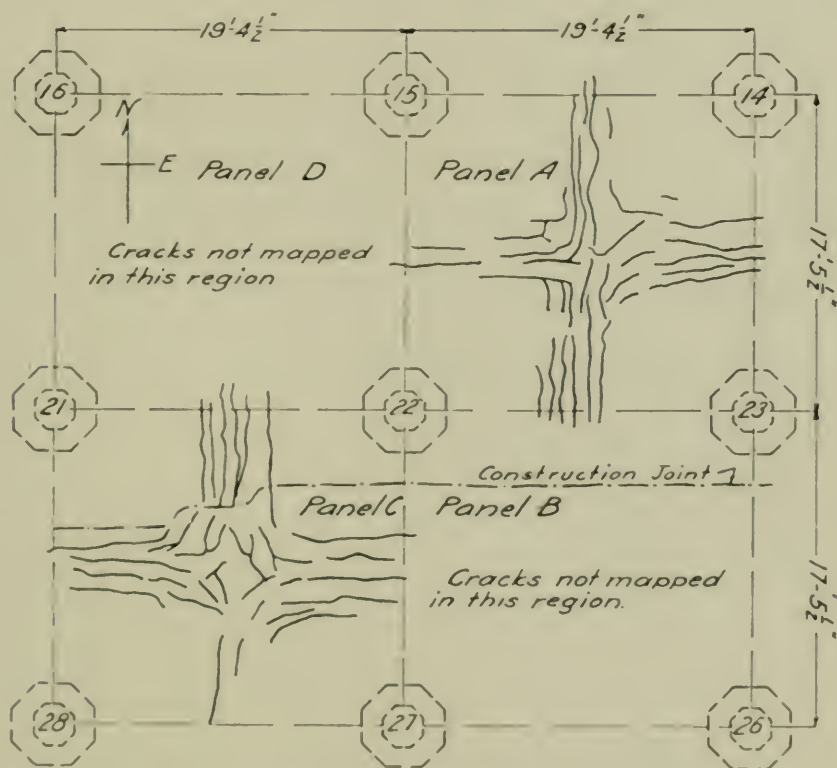


FIG. 10. LOCATION OF MAIN CRACKS IN LOWER SIDE OF SLAB

loads they gradually opened up and extended. The main cracks on the lower side of the slab did not open so much as the main cracks on the upper side of the slab. The construction joint shown in Fig. 9 and 10 opened appreciably under the application of load.

Upon the removal of the load the cracks closed, giving the slab an appearance similar to that which it had before the load was applied.

7. *Load-strain Diagrams.*—In Figs. 11 to 13 the load-strain diagrams are given for gage lines on the upper and lower sides of the

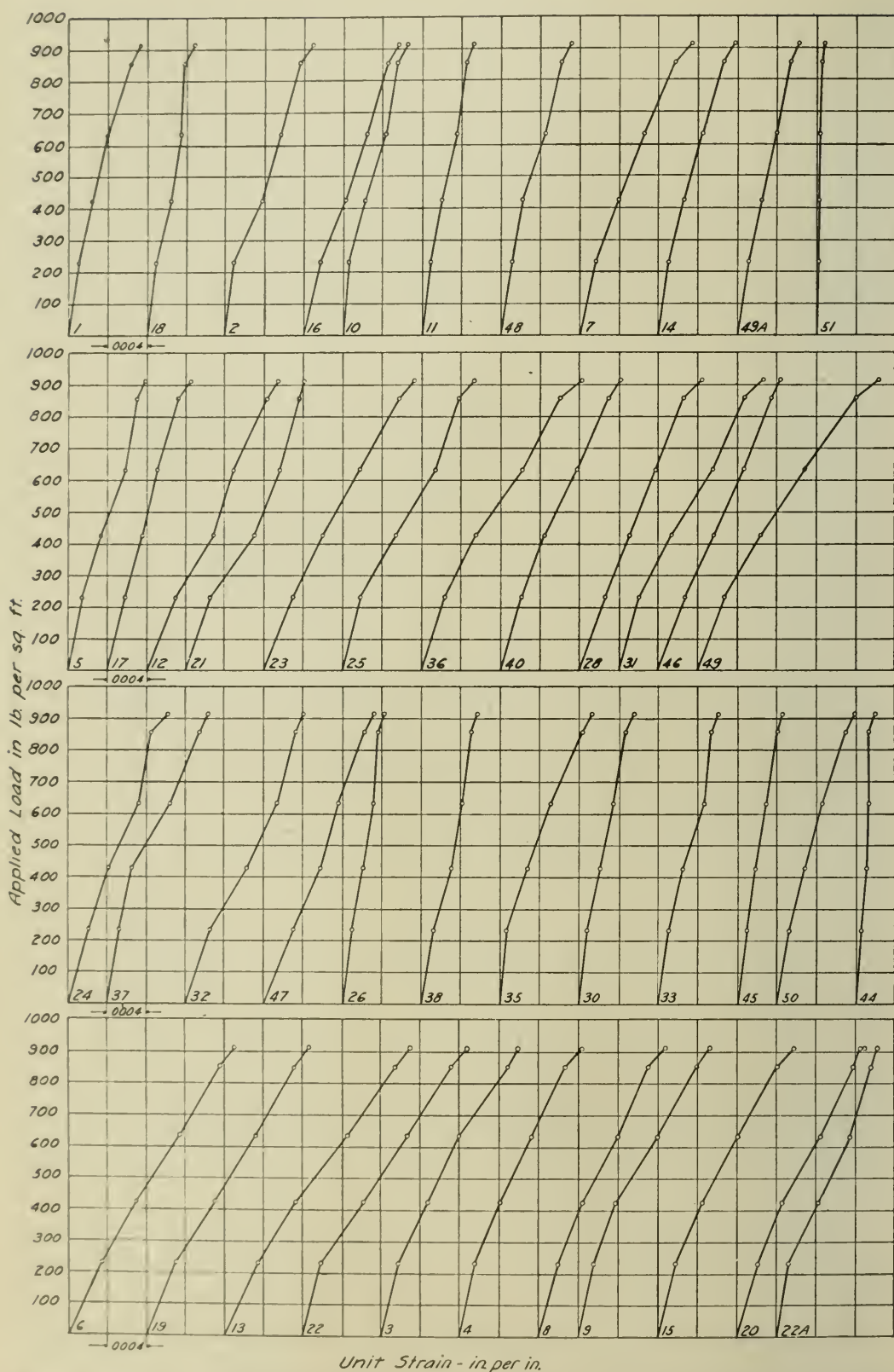


FIG. 11. LOAD-STRAIN DIAGRAMS FOR GAGE LINES ON REINFORCING BARS ON UPPER SIDE OF SLAB

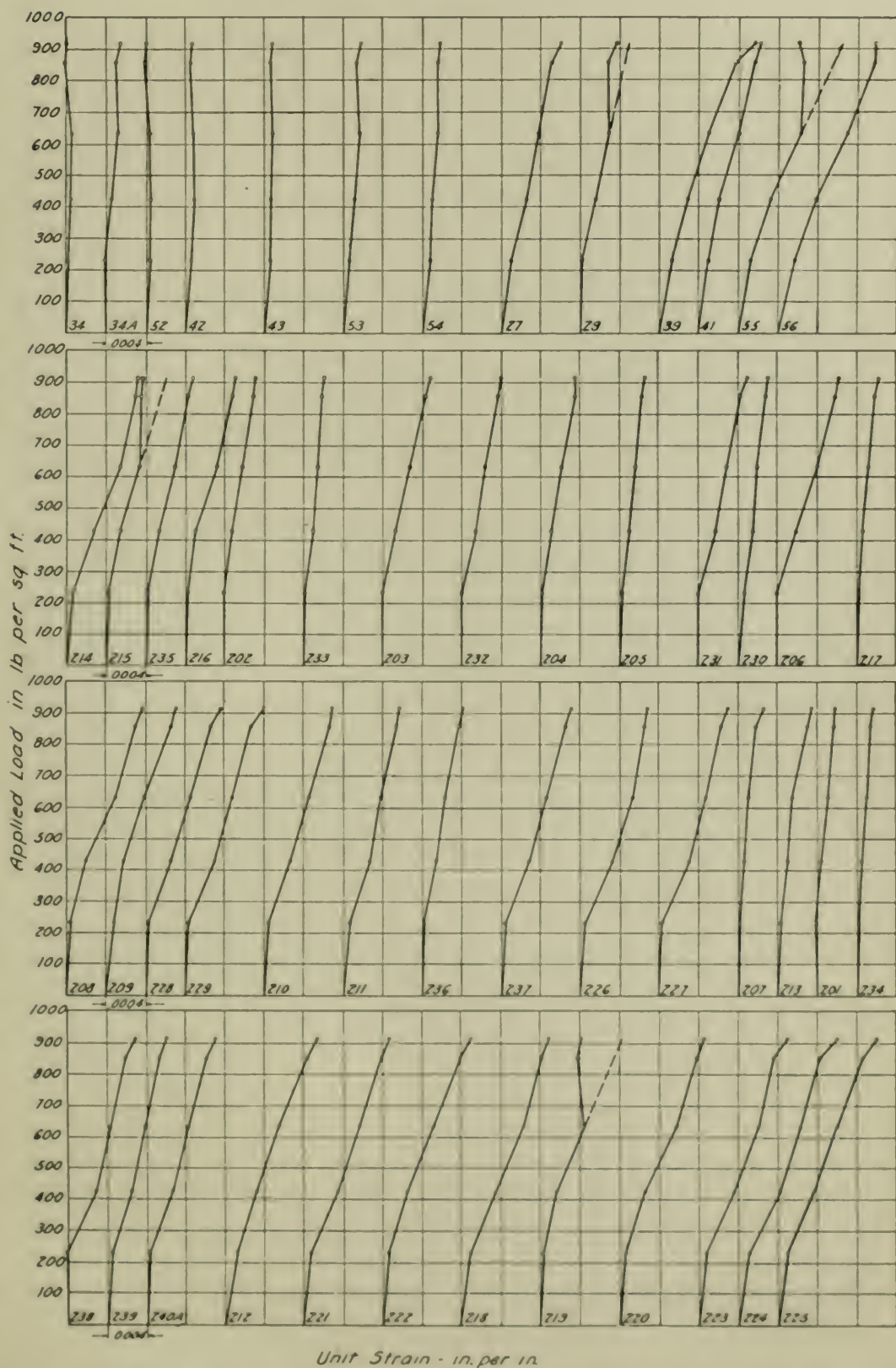


FIG. 12. LOAD-STRAIN DIAGRAMS FOR GAGE LINES ON REINFORCING BARS ON UPPER AND LOWER SIDES OF SLAB

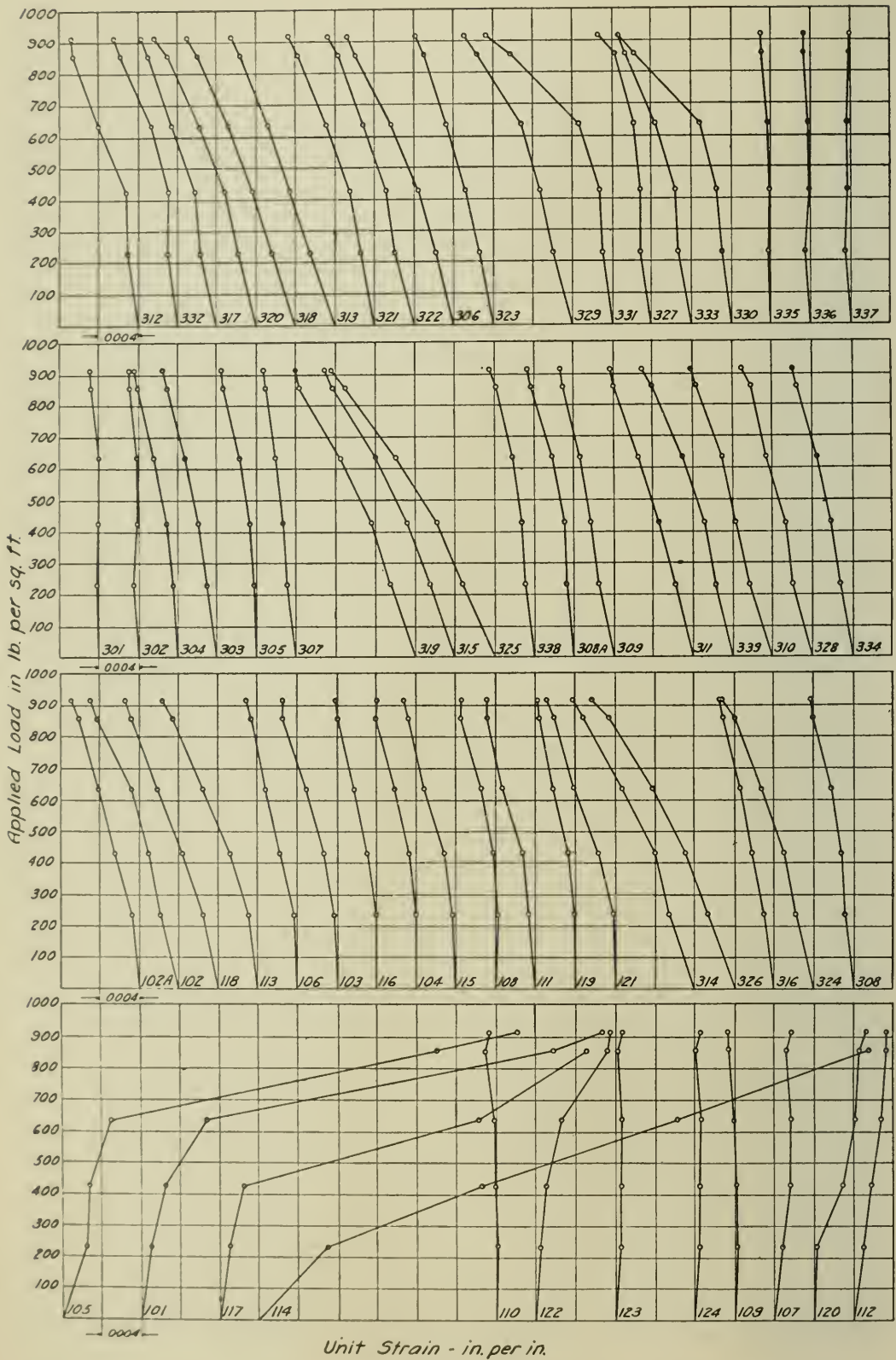


FIG. 13. LOAD-STRAIN DIAGRAMS FOR GAGE LINES IN CONCRETE ON UPPER AND LOWER SIDES OF SLAB

slab. In these diagrams tensile strains are plotted to the right and compressive strains to the left of the axis. Gage lines having similar positions on the floor have been grouped. Averages of the several readings were used in computing the strains. It will be noted that for gage lines on bars in the upper side of the slab the diagrams show deformations of considerable amount at the load of 234 lb. per sq. ft.; the diagrams for gage lines on bars on lower side show markedly smaller deformations. For loads greater than 234 lb. per sq. ft. the deformations in bars on the lower side of the slab increased fairly uniformly with increase of load; it has already been noted that the first cracks on the lower side appeared at this load. The diagrams for gage lines on the concrete on both upper and lower sides of the slab show deformations of considerable amount at the first load and a fairly uniform increase in deformation for higher loads.

8. *Stresses in Reinforcing Bars in Upper Side of Slab.*—The stresses in the reinforcing bars at the several loads may be computed from the strains given in Fig. 11 and 12. For convenience of comparison the stresses corresponding to the strains for the maximum load (913 lb. per sq. ft.) are given in Fig. 14. The values shown at points around the columns are in bars at the upper side of the slab; the others are in bars at the lower side.

For bars on the upper side of the slab the greatest stresses were found at gage lines located on diagonal bars at the edge of the column capitals. At column 22 (the central column of the loaded area) the stresses in diagonal bars over the edge of the column capital ranged from 49 200 to 57 300 lb. per sq. in. The stresses in diagonal bars at column 22 at gage lines located some distance from the column capital ranged from 15 000 to 34 500 lb. per sq. in. The stresses in diagonal bars at the columns bordering the loaded area ranged from 36 600 to 54 900 lb. per sq. in. In general, the stresses in the diagonal bars at the columns bordering the loaded area were nearly as great as those observed at corresponding gage lines at the central column. The stresses given do not include the stress due to the load of the slab itself; allowing for this, it is apparent that the yield point of the steel was not reached in even the most highly stressed bars.

The stresses in the east and west rectangular band at column 22 averaged about 41 400 lb. per sq. in. The stresses in the north and south rectangular band at column 22 ranged from 23 700 lb. per sq. in.

at the edge of the column capital to 40 500 lb. per sq. in. at gage lines near the edge of the band on either side of the column. The bars in this band were lapped at column 22, the bars ending about 80 inches north and south of the column center. It will be noted that at gage lines near the edges of the rectangular bands the stresses were greater

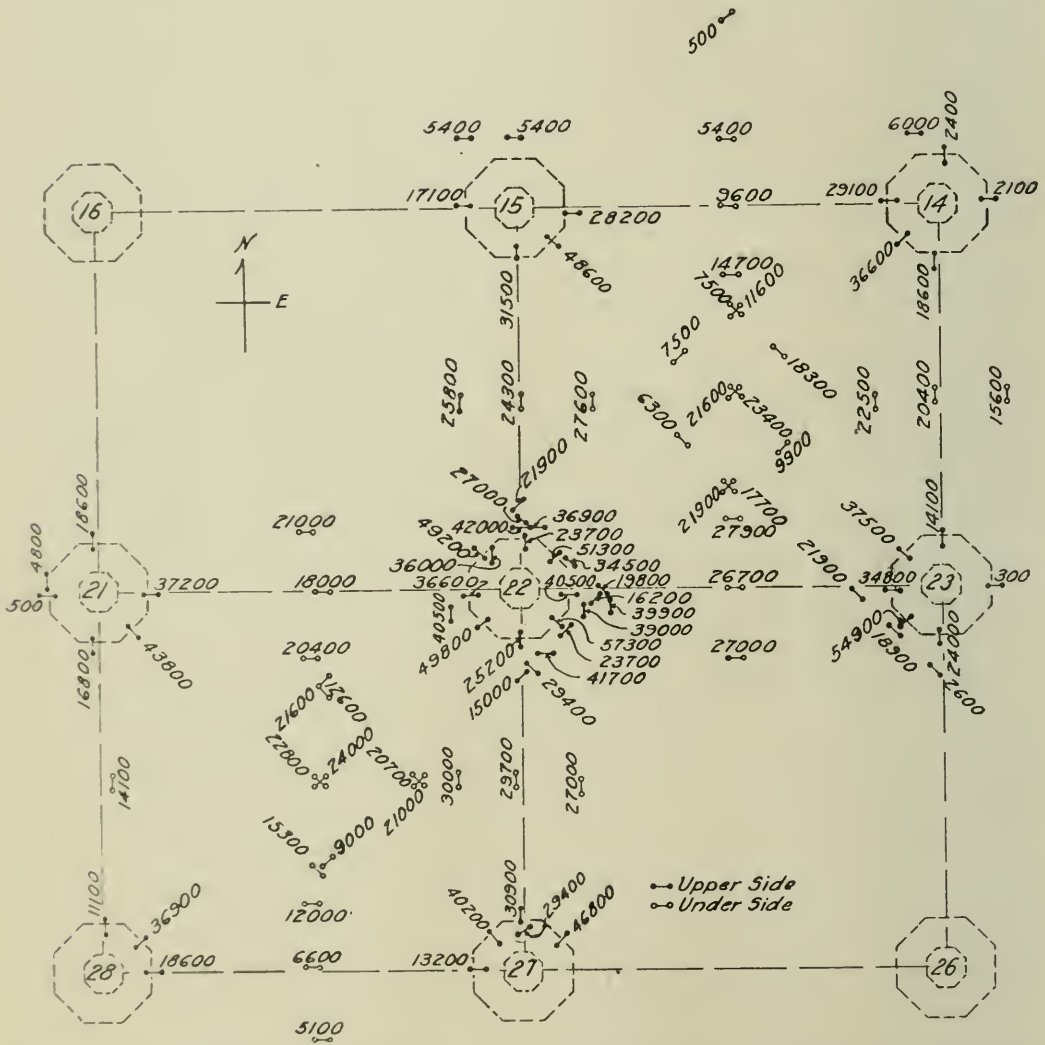


FIG. 14. STRESSES IN REINFORCING BARS IN POUNDS PER SQUARE INCH AT LOAD OF 913 POUNDS PER SQUARE FOOT

than in bars at the middle of the band. In bars outside the loaded area and near the edge of a rectangular band stresses were found as great as 6000 lb. per sq. in. It is evident that portions of the slab outside the loaded area contributed measurably to the resistance developed in the slab.

9. *Stresses in Reinforcing Bars in Lower Side of Slab.*—On the lower side of the slab the greatest stresses were found in the bars of the rectangular bands (Fig. 14). At the maximum load stresses from 24 300 to 30 000 lb. per sq. in. were observed in rectangular bands within the loaded area. In the one which had 30 bars instead of the usual 15, stresses from 18 000 to 21 000 lb. per sq. in. were found.

The stresses in the rectangular bands at the edge of the loaded area varied from 5100 lb. per sq. in. in a bar outside the loaded area to 22 500 lb. per sq. in. in a bar inside the loaded area. At one gage line outside the loaded area and near the edge of the band a stress of 15 600 lb. per sq. in. was found.

The stresses in the diagonal bands in the region of the center of the panels were smaller than those in bars of the rectangular bands in the region between columns. The stresses in diagonal bars at gage lines away from rectangular sections which pass through panel centers ranged from 6300 to 18 300 lb. per sq. in.

The effect of position of reinforcing bars with respect to surface of slab on the stress developed is discussed in another place.

10. *Strains in Concrete at Upper Surface of Slab.*—The unit-strains in the concrete at the several loads are plotted in Fig. 13. For convenience of comparison, the unit-strains at the maximum load are recorded in Fig. 15. The values for points around the columns are for gage lines on the lower side of the slab; a few gage lines which cross the panel edges between columns are also on the lower side. The remaining gage lines are on the upper side of the slab.

On the upper surface of the slab the greatest compressive strains were found at gage lines along the inner panel edges midway between columns. Strains from 0.00089 to 0.00097 in. per in. were observed at these gage lines. Assuming a modulus of elasticity of concrete of 4 000 000 lb. per sq. in. and a straight-line stress-strain relation the stresses in the concrete corresponding to these deformations would be 3560 and 3880 lb. per sq. in. Strains as great as 0.00054 in. per in. (corresponding stress in the concrete on the assumption just given, 2160 lb. per sq. in.) were found at gage lines at the panel centers.

11. *Strains in Concrete at Lower Surface of Slab.*—The greatest strains in concrete on the lower surface of the slab were found close to the edge of the capital of column 22. The strains at this column at

Near the edges of the capitals of columns bordering the loaded area strains from 0.00054 to 0.0011 in. per in. were observed at diagonal gage lines, and strains from 0.00044 to 0.00081 in. per in. at rectangular gage lines. With a modulus of elasticity of concrete of 4 000 000 lb. per sq. in. the stresses corresponding to these strains would range from 1760 to 4400 lb. per sq. in.

At the gage lines crossing the inner panel edges at a section of negative moment between columns 15 and 22, compressive strains about one-half those found near the column capital were observed at the lower loads, even though there was no tension reinforcement in the upper side of the slab in this region. For the highest load when the concrete at the capital had begun to crush there was a relatively great increase in the strains in this middle region, a value as great as 0.0012 in. per in. being found.

It should be noted that there were compressive strains of some amount in regions of negative moment outside the loaded area.

12. *Influence of Position of Bar.*—The stresses in Fig. 14 are given without reference to the position of the bar with respect to the surface of the slab and without reference to its position in the band. It may be expected that the stresses in the several layers of bars will vary. The average depths of the layers of bars at the edge of the capital of column 22 ranged from 0.90 in. below the upper surface of the slab for the upper layer to 3.65 inches for the lower layer. At this column the layers of bars in the order of their position with respect to the upper surface of the slab were as follows: (1) north and south rectangular bars (one lap), (2) diagonal bars running to northwest, (3) east and west rectangular bars, (4) diagonal bars running northeast, (5) north and south rectangular bars (second lap), (6) circumferential ring bars, (7) radial column bars. (See Fig. 20.)

The strains in steel and concrete at gage lines near the capital of column 22 for the load of 913 lb. per sq. ft. have been plotted in Fig. 16 to show the position of the neutral axis for the various layers of bars on which readings of deformation were taken. The strains found at gage lines 5 and 17 are markedly smaller than those found at gage lines 8, 9, 20, and 22A which were placed on bars of the same layer. Gage line 22A, like gage lines 5 and 17, was placed over the edge of

the capital and it will be noted that the deformation found at this gage line is more in harmony with the deformations found at other gage lines on bars of the same layer than are the deformations at 5 and 17.

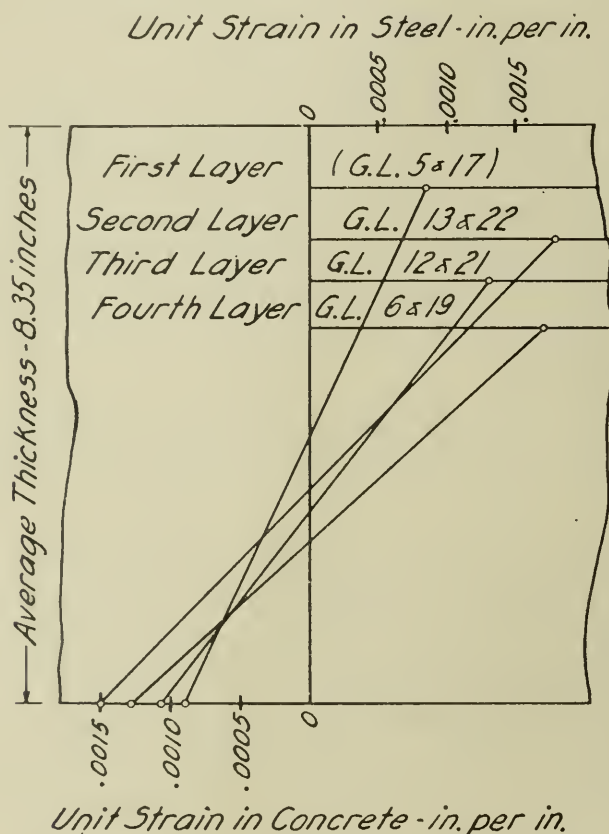


FIG. 16. POSITION OF NEUTRAL AXIS FOR THE SEVERAL LAYERS OF BARS AT COLUMN 22

No reason for the markedly smaller value in gage lines 5 and 17 is apparent. For the gage lines other than 5 and 17 the position of the neutral axis with respect to the under side of slab ranged from 0.43 to 0.46 of the effective depth of the several layers. The value of j (which represents the ratio of the distance between the bar and the centroid of compressive stresses to the distance between the bar and the face of the slab) for these bars is, therefore, about 0.85. The value of j found in a similar way for gage lines located at columns bordering the loaded area was 0.87. It will be noted in Fig. 16 that the strain in the lower layer of diagonal bars is nearly as great as that in the upper layer of diagonal bars. The strain in the layer of rectangular

bars between the two layers of diagonal bars was less than that found in the diagonal bars.

No measurements of strain were made on the rings nor on the column bars bent out radially into the slab. These bars were placed low in the slab and near where the neutral axis may be expected to be. It is probable that they were stressed somewhat. As has been stated elsewhere these bars were not taken into account in the calculations of resisting moment.

For the diagonal bars in the central area of the panels, the stresses in the bars of the lower layers were generally considerably greater than those in the upper layer. In the one case where the lower bar shows considerably less stress, the gage lines were near the end of lapped bars.

The bars in the rectangular bands at sections of positive moment were farther from the under surface of the slab than were the bars of the diagonal bands, but the stresses in the former were greater than in the latter except where laps occurred. It should be noted that the bars at the middle of the rectangular bands were farther from the under surface of the slab than were the outer bars. Differences in the magnitude of stresses in bars in similar places at sections of positive moment are partly accounted for by differences in the position of the bars; for example, the bar at gage line 236 was 2.7 in. from the lower surface and the one at 237 was 1.15 in. from the lower surface, while the stresses were 12 600 and 21 600 lb. per sq. in. respectively. Similarly, the bars for gage lines 219, 221, and 239 are higher in the slab than the corresponding bars at the edges of the band.

Gage lines 2, 7 and 10 were placed on the same diagonal bar. Similarly, gage lines 11, 14, and 18 were on another diagonal bar. The bar having gage lines 2, 7, and 10 was about 1.95 in. below the upper surface of the slab and the other bar about 3.15 in. below the upper surface. The stresses at gage lines 2, 7, and 10 were 27 000, 34 500 and 19 800 lb. per sq. in., respectively; the stresses at gage lines 11, 14, and 18 were 16 200, 23 700, 15 000 lb. per sq. in., respectively. It will be seen that the stresses at the gage lines opposite the column capital (7 and 14) were greater than those observed at the other gage lines.

13. *Resisting Moment Accounted for by Stresses in Reinforcing Bars.*—It will be of interest to find the magnitude of the resisting moments developed in sections of the slab and to compare the values

with the bending moment due to the external forces. As the part played by the tensile stresses in the concrete is unknown and uncertain, only that part of the resisting moment found by using the measured stress in the reinforcing bars can be considered. The resisting moment based on the tensile stresses in these bars may not be expected to account fully for the bending moment. The two sections considered will be (1) a section across the panels midway between columns (AB, Fig.

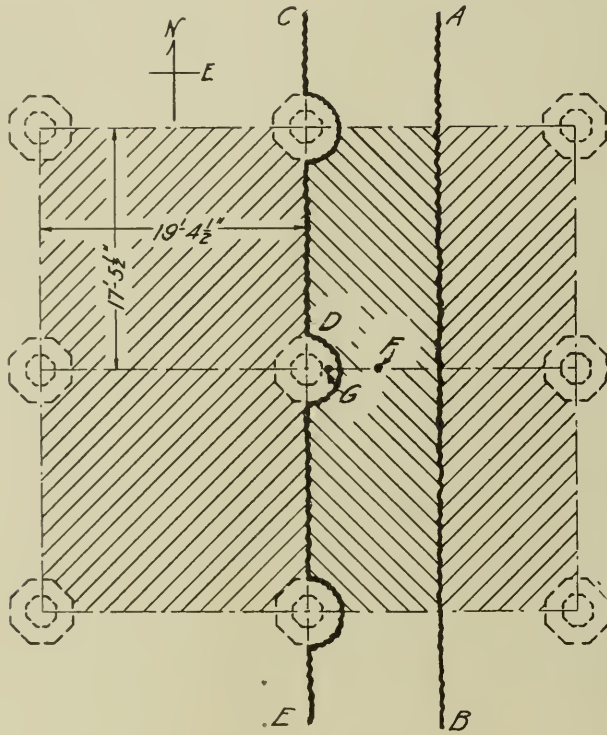


FIG. 17. SECTIONS OF POSITIVE MOMENT AND NEGATIVE MOMENT CONSIDERED IN THE CALCULATIONS

17) and (2) one along an edge of the panels parallel to the first section but skirting the part of the periphery of the column capitals at the corners of the panels (CDE, Fig. 17). It will be noted that the edges of the area here considered are along lines of zero shear, except around the column capitals. The external forces acting on the row of half panels are the load on the two half panels and the reaction or external shear at the three column capitals. The moment of the couple formed by these two external forces will be resisted by the numeri-

cal sum of the resisting moments developed in the two sections AB, and CDE. The moment of the internal stresses at the section of the panel midway between columns is referred to as the positive resisting moment and that at the edge of the panel as the negative resisting moment. The actual distribution of these resisting moments along the sections need not be considered in making the desired comparison.

The point of application of the resultant of the load on the two half panels is shown at F; that of the resultant of the reaction of the three supports at G. The moment of the couple formed by these two resultants is the bending moment due to the external forces and is the moment to be considered. Analysis shows that, for a uniformly distributed load, and round columns, the value of this bending moment for a load two panels wide is given quite closely by the equation

$$M = \frac{2}{8} Wl_l \left[1 - \frac{2}{3} \frac{c}{l_l} \right]^2 \dots \dots \dots (1)$$

for sections at right angles to the long way of the panel, and

$$M = \frac{2}{8} Wl_s \left[1 - \frac{2}{3} \frac{c}{l_s} \right]^2 \dots \dots \dots (2)$$

for sections at right angles to the short way of the panel where

- M = bending moment for a width of two panels
- W = load on one panel
- l_l = long side of an oblong panel measured from center to center of column
- l_s = short side of an oblong panel measured from center to center of column
- c = diameter of column capital.

With the load of 913 lb. per sq. ft. over four panels, the bending moment for a width of two panels, as obtained by equations (1) and (2), is 12 820 000 lb.-in. for the long way of the panels (resisted at north and south sections) and 11 060 000 lb.-in. for the short way (resisted at east and west sections).

The positive moment (the resisting moment at the section across the panels midway between columns) and the negative moment (the resisting moment at the section at the edge of the panels) together

must resist this bending moment. With a condition of ends of slab for which the tangent to the curve of flexure at the edges of the panels remains horizontal when the load is applied (usually termed fixed ends), the condition assumed in the analysis, the positive moment will be found by analysis to be one-third of the total resisting moment and the negative moment two-thirds, provided the slab is uniformly stiff throughout. If the tangents at the panel edges deflect somewhat, the positive moment will be greater than one-third and the negative moment less than two-thirds. In the comparisons to be made it will be assumed that the proportions given by analysis are one-third for the positive moment and two-thirds for the negative moment. As stated in "14, Bending of Columns," calculations of the resisting moment developed at sections of the slab at the boundaries of the loaded area as accounted for by the observed stresses in the reinforcing bars, the results of analytical determinations of the moments at sections at the edges and at the middle of the loaded area, and the amount of flexure in the columns all go to show that there is little inaccuracy in this assumption in the case under consideration. The analytical value of the positive resisting moment will be termed the analytical positive moment and that of the negative resisting moment the analytical negative moment. For the north and south sections the magnitude of these analytical moments becomes 4 270 000 and 8 550 000 lb.-in. respectively, and for the east and west sections 3 700-000 and 7 400 000 lb.-in. respectively.

The sections used in obtaining the resisting moment accounted for by the stresses observed in the reinforcing bars are shown in Fig. 18. QKC-BJT is the east and west section of positive moment used, and MNOP the east and west section of negative moment; similarly, IJO-NKL is the north and south section of positive moment, and ABCD and EFGH the north and south sections of negative moment. Sections of negative moment in the north and south direction are taken on two sides of the column capitals, because the available reinforcement differed in the two sections. The sections of positive moment were taken as shown because they cross the greatest number of gage lines.

In the calculation of the resisting moments developed, lapped bars were considered as contributing to the resisting moment wherever the bars extended beyond the section a sufficient distance to insure adequate anchorage with respect to the magnitude of the stress

developed in the bar. Many of the laps were made in such a way that the additional section may not be expected to contribute much to the resisting moment. As measurable stresses were found in bars near the edge of the band outside the loaded area, they were taken into account in the calculations. The ring rods around the columns and the column bars which were bent out into the slab were not included in the reinforcement, for no measurements of strain were made on these bars. For the diagonal bars the component of the stress was taken in a direction at right angles to the direction of the panel edge. The average of the stresses at the principal critical gage lines was generally used. For the bands at the edges of the loaded area the stresses were considered to vary over the band from gage line to gage

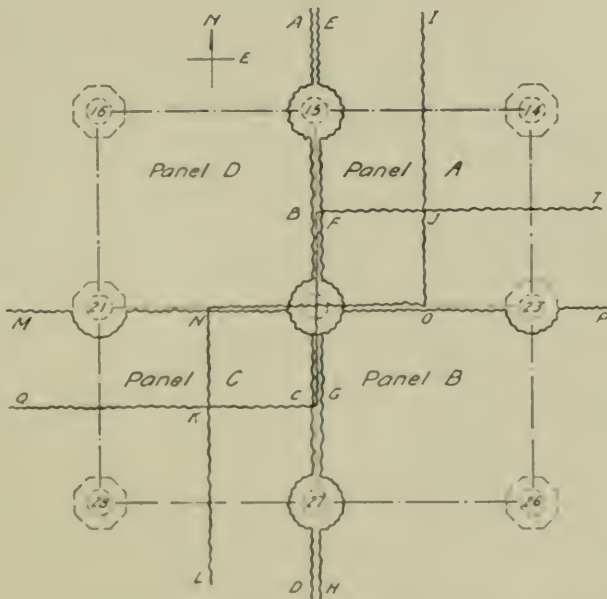


FIG. 18. SECTIONS OF POSITIVE AND NEGATIVE MOMENT CONSIDERED IN THE CALCULATION OF THE RESISTING MOMENTS ACCOUNTED FOR BY STRESSES IN THE REINFORCING BARS

line. Some judgment had to be used in determining the stress variation as well as the availability of lapped bars, but it is believed that the assumptions made give results well below the actual resistance developed in the slab. It should be noted that the maximum stresses observed in bars of negative moment were 15 per cent greater than the average of the stresses used in computing the resisting moment accounted for by these stresses, not counting bars in bands near the

edges of the loaded area, and similarly the maximum stresses in bars at sections of positive moment were 25 per cent greater than the average of the stresses used in computing the positive moment. The measured position of the bars with reference to the face of the slab and the measured thicknesses of the slab were used. The position of the neutral axis was determined from the strains measured in the reinforcing bars and in the concrete at the face of the slab; knowing this distance, the values of jd , the distance from the bar to the center of gravity of the compressive stresses, were computed in the usual way. The value of jd was generally about 0.87 of the distance from the center of gravity of the bars to the face of the slab.

TABLE 3

RESISTING MOMENTS ACCOUNTED FOR BY STRESSES OBSERVED IN REINFORCING BARS
(Moments are given in pound inches)

NORTH AND SOUTH SECTIONS			
Negative Moment		Positive Moment	
Width AB Fig. 18	2 760 000	Width IJ	560 000
Width BC	2 720 000	Width JO-NK	1 570 000
Width CD	1 690 000	Width KL	750 000
Total	7 170 000	Total	2 880 000
Width EF	2 950 000		
Width FG	2 890 000		
Width GH	2 030 000		
Total	7 870 000		
Average total negative moment	7 520 000		

EAST AND WEST SECTIONS			
Negative Moment		Positive Moment	
Width MN	2 190 000	Width QK	600 000
Width NO	2 950 000	Width KG-FJ	1 340 000
Width OP	1 680 000	Width JT	730 000
Total	6 820 000	Total	2 670 000

Table 3 gives the resisting moments accounted for by the stresses observed in the reinforcing bars as calculated by the method described. A comparison of these moments with the values of the analytical moments already given indicates that about 88 per cent of the analyti-

cal negative moment is accounted for by the stresses in the reinforcing bars in the case of the north and south section and 92 per cent in the case of the east and west section. For positive moments 68 per cent of the analytical positive moment is accounted for in the north and south section and 72 per cent in the east and west section. The average percentages for the sections in the two directions are 90 for the negative moment and 70 for the positive moment.

The reinforcing bars are not the only source of tensile resistance in the slab; tensile strength of concrete assists. It is evident that at sections of positive moment tension in the concrete may make up a considerable part of the resistance offered by the slab, and even at sections of negative moment it may have an effect. This tension will be especially influential in regions away from cracks. At points of the sections which are outside the loaded area and where the stresses in the bars are small it may be expected to form a not inconsiderable part of the total resisting moment developed. Of course, if all panels were loaded, the effect of tension in the concrete would be much less, since its greatest proportional effect is outside the loaded area.

In making comparisons, it must be kept in mind that the unit-strain computed from the measurements gives the average strain over the gage length and that at a crack the stress in the bar will be more than the average stress over the gage length.

Measurements made in beam tests in various laboratories generally show that up to loads near the ultimate load the measured stress in the reinforcing bars is less than that necessary to account for the full bending moment due to the applied load. At the lower stresses the deficiency is considerable; at stresses near the yield point of the steel it may not be much, and the measured stresses may even be larger than necessary to account for the bending moment. The effect is particularly noticeable in concrete of high quality.

On the whole the analytical values of the moments are closely approached, as closely as may be expected in tests of this kind. The negative moment of course is most fully accounted for. The positive moment is not wholly accounted for; but as the section of positive moment involves relatively low stresses the effect of the tensile resistance of the concrete on the measured stresses may be considerable, especially in the part of the slab outside the loaded area. It must not be overlooked, also, that the stress in the bars at the cracks will be larger than the average stress over the gage length.

It should be noted that in the case of negative moments the magnitudes of the moments found for the part of the sections adjacent to the central column are smaller than those for the corresponding parts of the sections adjacent to the outer columns. In the case of column 15, part of this difference is due to the additional amount of reinforcement furnished by the extra laps at this place, and it should be noted too that the larger number of laps at the outer columns makes this portion of the slab stiffer than the portion around the central column. In general, however, most of it appears to be due to the stresses developed in slab and reinforcing bars outside the loaded area, a part of the slab which has not usually been considered to contribute to the resisting moment. In the sections of positive moment, the magnitudes of the resisting moments found are less proportionally at the ends of the sections than at the middle, a condition which may be due to the greater proportional effect of the tensile resistance developed in the concrete in these regions.

The foregoing comparisons have been made on the basis of the full analytical value of the bending moment and by considering one-third of it as positive moment and two-thirds as negative moment. The Joint Committee on Concrete and Reinforced Concrete recommended for the sum of the positive and negative moments a value which is about 85 per cent of the analytical value heretofore used and recommended that the distribution be three-eighths positive moment and five-eighths negative moment. It may be of interest to note that the sum of the positive and negative moments accounted for by the measured stresses in the reinforcing bars has nearly the same value as the sum of the moments recommended by the Joint Committee. The negative moments so accounted for are 13 per cent higher than the value recommended by the Joint Committee, and the positive moments are about 73 per cent of the committee's values. In judging of the results, it should be remembered that at the section of positive moments tensile stresses in the concrete have a considerable influence at the stage of the loading indicated by the stresses in the bars in the loaded area and outside of it. The reference to the tensile resistance of the concrete as contributing to the resisting moment of the slab in the test should not be taken to mean that it will be effective as a resistance when the ultimate load is reached. It is apparent, also, that requirements less than those of the Joint Committee will not provide for all the moment developed by the bars of the slab.

In making a comparison with methods used in design it should be borne in mind that the principal observed maximum stresses were from 15 to 25 per cent greater than the average of the observed stresses which were used in computing the resisting moments accounted for by the stresses in the bars; in designing, a uniform stress over the section is assumed.

14. *Bending of Columns.*—A few gage lines were placed on columns bordering the loaded area. In Fig. 19 the location of these is shown, together with the load-strain diagrams. Although for some

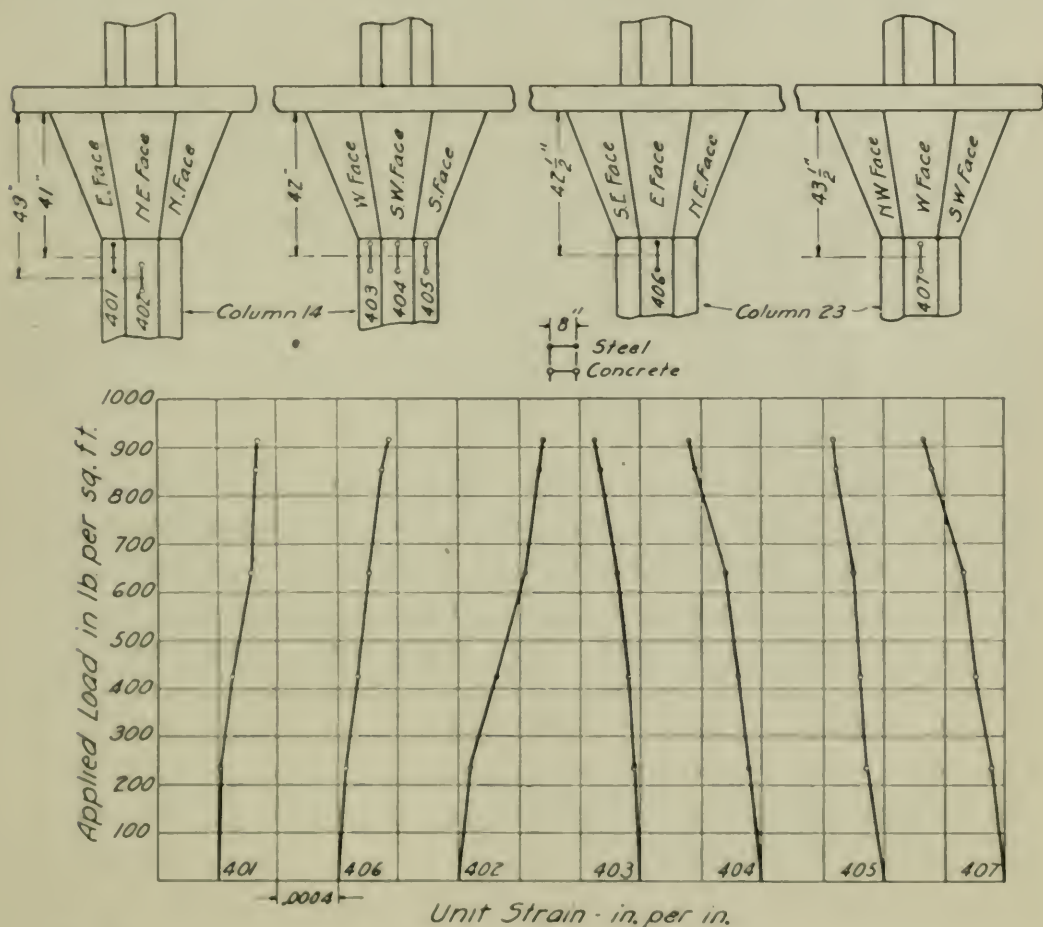


FIG. 19. LOAD-STRAIN DIAGRAMS FOR GAGE LINES LOCATED ON COLUMNS

of the gage lines the strains show as tensile strains, it is probable that for the gage lines given as in tension the compressive strain in the column due to the weight of the floors above was sufficient to overcome the tensile strain measured by the instrument.

The direction of the bending was in the direction which would be expected from the condition of loading. For gage line 406 on a column reinforcing bar the deformation at the load of 913 lb. per sq. ft. corresponded to a flexural tensile stress of 10 500 lb. per sq. in. and for gage line 407 on the concrete on the opposite face of the column the strain was 0.00054 in. per in. corresponding to a stress of 2160 lb. per sq. in. with a modulus assumed as 4 000 000 lb. per sq. in. Similarly at gage line 401 on a column reinforcing bar at the east face of column 14 a stress of 8100 lb. per sq. in. was found, and for gage line 403 on the concrete on the opposite face of the column the compressive stress was 1200 lb. per sq. in. based on the same modulus of elasticity of concrete. The highest compressive strain on column 14 was found at gage line 404 on the southwest face of the column. Gage line 402 was placed across an opening which had been cut into the concrete of the column in an unsuccessful effort to expose column bars at this point. A fine crack formed across this gage line. It is seen that the deformation at this gage line is large.

Fine cracks formed on the tension side of the columns at or near the juncture of the column shaft and capital. At column 15 a crack was found about half way up the column capital. All the cracks noted were fine cracks—much smaller than those observed on the lower surface of the slab.

The amount of flexure in the columns was apparently not sufficient to give more than a slight reduction in the proportion of resisting moment carried by the slab at the sections at the edges of the loaded area, nor more than a slight increase in the moment carried at the sections through the middle of the loaded area. This view is borne out by calculations of the resisting moment developed at sections of the slab at the boundaries of the loaded area, as accounted for by the observed stresses in the reinforcing bars, and also by the results of analytical determinations of the moments at sections at the edges and the middle of the loaded area.

15. *Time Effect of Load and Recovery upon Removal of Load.*—As has been stated, two or more sets of strain gage and deflection readings were generally taken after an increment of load had been applied,—the first set immediately after the last of the load had been placed and another set 12 hours thereafter. A third set was taken 66 hours after the load of 913 lb. per sq. ft. had been applied.

The effect of time on the strains in the steel and concrete was very small; the reading in a few of the gage lines increased slightly after 12 hours, but generally not enough to allow two points to be plotted on Fig. 11 to 13 and not more than may be considered to be within the error of observation. This was true for both steel and concrete even in the most highly stressed places at the maximum load after a period of 66 hours had elapsed.

The deflection readings were affected but little through the 12-hour period until a load of 637 lb. per sq. ft. was reached; Fig. 8 shows that at this load a marked change in deflection occurred at the centers of panels B, C, and D in the 12 hours' time. At the maximum load the increase in deflection through the 66-hour period was small.

Four days were consumed in removing the load. Four hours after the last of the load was removed, readings were taken on the gage lines and deflection points, and twenty hours later another set was taken. The cracks in both upper and lower surfaces of the slab had closed so that they were not easily traced. The recovery in deflection is given by the plotted points at the bottom of the diagrams in Fig. 8. The recovery at the centers of the panels was about 75 per cent; at other points it was generally greater. The recovery in strains in steel and concrete was also large. Even where high stresses and open cracks had been observed the residual strain in the steel was not more than that to be expected from the lack of interlocking of particles at the cracks. In the region of high compressive stress in the concrete the recovery at the principal gage lines was 75 per cent or more; in the reinforcing bars the recovery averaged about 80 per cent. In general the action of the slab was that of concrete of high quality.

16. *General Comments.*—Although there were differences in the stresses and deflections found at corresponding points in the four loaded panels, these differences can generally be accounted for by (1) difference in the arrangement and number of the reinforcing bars, (2) differences in position of the bars with respect to the surfaces of the slab, and (3) differences in strength and stiffness of concrete in the four panels. The large amount of steel in the region of the column capitals bordering the loaded area (where there were two or three bands lapped) added greatly to the stiffness of the slab in this

region. The effect of this added stiffness was to cause higher relative stresses to be developed at the columns bordering the loaded area, and relatively large negative resisting moments were developed at columns where lapped bars were numerous. Another effect of the lapping of bars at columns and in certain of the rectangular bands was to decrease the deflection of the slab in the panels affected by the laps. The influence of the position of the reinforcing bars has already been discussed. Higher stresses in reinforcing bars and greater deflections in the slab were found in the panels for which the compression tests showed weaker and less stiff concrete. Cracks, also, were wider and more numerous in these panels. It is apparent that the tensile resistance of the concrete at sections of positive moment, particularly near the outer edges of the loaded panels, contributed to the resistance of the slab even at the maximum load.

With reference to the design of the slab; it may be noted that the slab was strongly reinforced, though the reinforcing bars were not distributed to the best advantage and the laps were not placed so as to be fully effective. Taking the total reinforcement found over the loaded area and immediately outside, and including such lapped bars as had sufficient anchorage beyond critical sections to be effective (there were many lapped bars which were not counted as effective), and considering the thinness of the slab, the amount of reinforcement for negative moment was on the average as much as that required for the negative moments recommended by the Joint Committee on Concrete and Reinforced Concrete. The amount of reinforcement available for positive moment was on the average more than 50 per cent greater than that required for the positive moments recommended by the same committee. The distribution of the reinforcing bars was, however, quite different from that recommended by this committee. It may be noted also that the amount of reinforcement was greater than that required by most building regulations.

Although the nominal thickness of the slab ($8\frac{1}{2}$ in.) was less than that required by building regulations, the slab fulfilled the common requirements for compressive and shearing stresses in concrete of the high quality shown in the tests of prisms taken from the slab. The provisions of the Joint Committee on Concrete and Reinforced Concrete for bending moments and working stresses in concrete of 3000 lb. per sq. in. strength give a thickness about the same as the designed thickness of the slab.

In view of the large number of lapped bars not shown on the building plans, the use of high-carbon steel instead of the mild steel specified, and the unexpectedly high strength of the concrete, it is not strange that the floor carried a much higher load than was anticipated when the test was begun.

17. *Wrecking of the Floor.*—As soon as the load was removed, portions of the floor in the four panels of the loaded area were broken into to uncover the reinforcing bars at important sections. Measurements of the position and location of the reinforcing bars were made for use in calculation. The thickness of the floor was measured at a number of points to check the values obtained from level readings. Photographs were taken to give a record of the position and location of reinforcing bars and their laps; Fig. 21 is a sample. As the wrecking of the floor by the contractor progressed, further measurements of the position of the reinforcing bars and their laps, including those in the bands outside of the loaded area, were taken.

The wrecking of the building by the contractor was an interesting operation. Fires were first built around the columns on the floor below the one to be wrecked; the effect on the concrete at the base of the column after several hours application of heat was to crack and loosen the concrete shell and expose the reinforcement. To assist in cracking the concrete and separating it from the steel, in many cases water was thrown over the columns after they were well heated. A heavy iron pear-shaped weight (about 1600 pounds) was dropped on the floor immediately over the column capital close to where the column of the story above had been. After the column capital had been sheared and shattered by this operation, the portion of the floor surrounding the column and that directly between columns was broken up with the weight and with sledge hammers. After the concrete of sections of the floor had been removed in this way, the reinforcing bars were cut with the oxyacetylene flame. Many of the bars were taken out in good condition. The process was continued until the entire floor was wrecked. The longitudinal reinforcing bars in the column were then cut near the floor below and the columns were pulled down on the floor and broken up with the heavy weight and hammers.

18. *Summary.*—The following deductions have been drawn in the foregoing presentation of the results of the test:

(1) The tests of samples of the concrete from the slab, as well as the hardness and toughness of the concrete observed in breaking up the slab, indicate that the concrete was of unusually good quality and that it had high strength and stiffness. The action of the slab under load was that to be expected with high grade, well-seasoned concrete. The effect of time on the stresses in steel and concrete and on the deflection of the slab under a sustained load was slight, even over a period of 66 hours under the maximum load of 913 lb. per sq. ft.,—conditions which would not exist at an early age of concrete. Upon removal of load, the recovery in deflection at the centers of the panels was about 75 per cent of that under load, and at other points generally more; the recovery in strains in steel and concrete was as large.

(2) The position of the important cracks on both upper and lower sides of the slab may be expected to indicate the region of high tensile stresses in the reinforcing bars; it is also an indication of the general action of the slab in flexure. The cracks on the upper side at the load of 913 lb. per sq. ft. opened to a width of 0.02 to 0.06 in.; those on the lower side were not so wide. Upon removal of the load the cracks closed, leaving the surfaces of the slab with the appearance which they had before the load was applied.

(3) For reinforcing bars in the upper side of the slab in the regions of negative moment, the stresses in bars of diagonal bands were greater than those in bars of rectangular bands, a stress of 57 300 lb. per sq. in. being observed in a diagonal bar and one of 42 000 lb. per sq. in. in a rectangular bar at the maximum load. Stresses were found in both rectangular and diagonal bars at the columns bordering the loaded area nearly as great as those at corresponding points at the central column. Stresses of some magnitude were found in bars outside the loaded area. The stresses given do not include the stress due to the load of the slab itself.

(4) For reinforcing bars in the lower side of the slab in the regions of positive moment, the stresses in bars of rectangular bands were greater than those in bars of diagonal bands even though the former were farther above the lower surface of the slab than the latter; in the one apparent exception, the presence

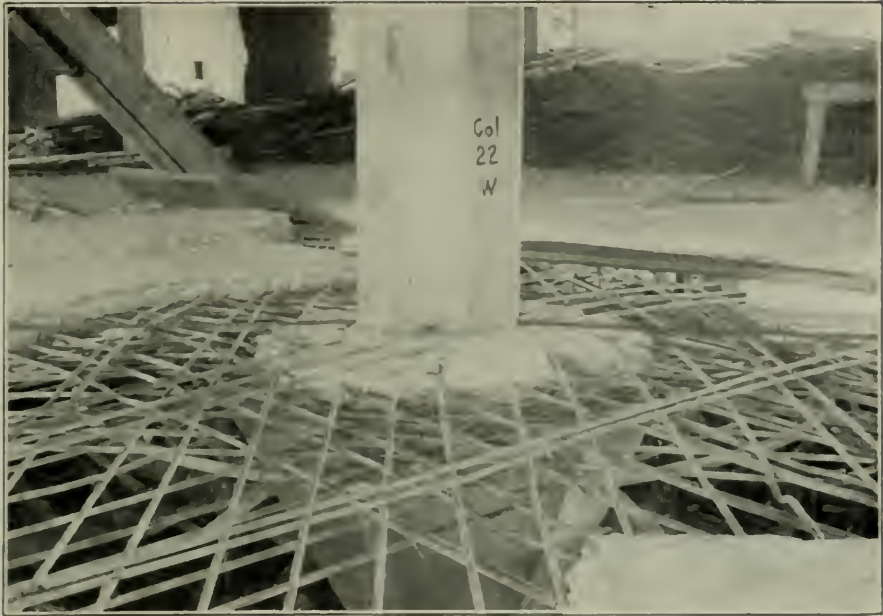


FIG. 20. VIEW SHOWING ARRANGEMENT OF REINFORCEMENT AT COLUMN 22



FIG. 21. VIEW OF SLAB AFTER REINFORCING BARS WERE EXPOSED

of laps doubled the usual number of bars. At the maximum load stresses of 24 000 to 30 000 lb. per sq. in. were observed in bars of rectangular bands and of 20 000 to 24 000 lb. per sq. in. in bars of diagonal bands. A stress of 15 600 lb. per sq. in. was observed in a bar outside the loaded area at the edge of a rectangular band.

(5) On the upper surface of the slab the greatest compressive strains were found at gage lines along the inner panel edges midway between columns, ranging from 0.0009 to nearly 0.001 in. per in. at the maximum load; at the centers of the panels values about half as great were found. On the lower surface the greatest strains were found at the middle column; these ranged from 0.0012 to 0.0016 in. per in., values which are as great as the strains found at failure in the tests of the concrete prisms cut from the slab and as great as are ordinarily found in compression tests of concrete at the ultimate load. In some places there was chipping and spalling of the concrete. It is evident that the action of the surrounding concrete assisted in preventing failure. At gage lines crossing the inner panel edges high compressive strains also were found, even though there was no tension reinforcement in the upper side of the slab in this region. It may be noted that the maximum compressive strains on the upper surface of the slab were one-half to three-fifths those found on the lower surface. The intensity of the strains at various points along sections of positive moment and negative moment will give some measure of the distribution of intensity of moments along those sections.

(6) The observed stresses in the reinforcing bars accounted for about 90 per cent of the analytical negative moment and about 70 per cent of the analytical positive moment, as given by the methods used. It should be noted that the observed stresses used are average stresses over the gage length, and the stress at a crack may be expected to be greater than the average over the gage length. It seems probable that tensile resistance of the concrete contributed to the resistance of the slab, particularly in the sections of positive moment and in regions near the edges of the loaded area. A similar influence of the tensile resistance of concrete, when the stresses in the steel are well below its yield point, has been observed in numerous beam tests. That the tensile re-

sistance of the concrete contributed to the resisting moment of the slab in the test should not be taken to mean that it will be effective in resisting moment when the ultimate load is reached. It may be noted also that the sum of the positive and negative moments accounted for by the measured stresses in the reinforcing bars has almost the same value as the sum of the positive and negative moments recommended by the Joint Committee on Concrete and Reinforced Concrete, and that the negative moments so accounted for are about 113 per cent and the positive moments about 73 per cent of the moments recommended by this committee. In making a comparison with methods used in designing it should be borne in mind that the principal maximum stresses were from 15 to 25 per cent greater than the average stresses which were used in computing the resisting moments accounted for by the stresses in the bars; in designing, a uniform stress over the section is assumed.

Although the arrangement of bars was not as recommended, the amount of reinforcement for negative moment, considering all available bars over the area used, was as much as that required for the negative moments recommended by the Joint Committee on Concrete and Reinforced Concrete, even though the slab was thinner than recommended for ordinary concrete. The amount of reinforcement for positive moment was more than 50 per cent greater than that required for the positive moments recommended by the committee. Although the nominal thickness of the slab was less than that required by building regulations, it fulfilled the provisions of the committee for bending moments and working stresses for concrete of a test strength of 3000 pounds per square inch.

(7) The action of the floor slab under test should give added confidence in the suitability and reliability of the flat slab as a load-carrying structure.

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ANALYSIS AND TESTS
OF RIGIDLY CONNECTED REINFORCED
CONCRETE FRAMES

BY

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CONTENTS

	I. INTRODUCTION	PAGE
1. Preliminary		9
2. The Use and the Advantages of the Rigidly Connected Reinforced Concrete Frame		10
3. Scope of Investigation and Acknowledgment		12
II. THE ANALYSIS OF RIGIDLY CONNECTED FRAMES		
4. Notation		14
5. Statically Determinate and Indeterminate Systems and Number of Statically Indeterminates		15
6. Principle of Least Work		18
7. The Effect of Work of Normal Force on the Magnitude of Statically Indeterminate Forces		20
8. Simple Frames under Vertical Load		25
9. Single Story Construction with Three Spans		30
10. Trestle Bent with Tie		34
11. Building Construction with Several Stories and Spans		35
12. Frame with Three Spans		39
13. Square Frame under Horizontal Load		44
14. The Nature of the Resulting Formulas; Relation between Horizontal Reactions in Frame under Uniform Load and under Concentrated Load		45
15. Effect of Variation in Moment of Inertia and Relative Height of Frame on Bending Moment in Horizontal Member		48
16. Effect of Variation in Moment of Inertia on Bending Moment in Vertical Member		52

III. TESTS OF RIGIDLY CONNECTED REINFORCED CONCRETE
FRAMES

	PAGE
17. Test Specimens	55
18. Materials	59
19. Making and Storage of Test Frames	60
20. Testing	61
21. Explanation of Tables and Diagrams	70
22. Phenomena of Frame Tests	71
23. Conditions Considered in the Comparison of Tests with Analysis	91
24. Comparison of Test Results with Analyses	92
25. Effect of End Condition of Column on Results	96
26. Distribution of Stress over the Cross-Section	96
27. Position of Point of Inflection in Columns	98
28. Continuity of the Composing Members of a Frame	98
29. Stresses at Corners	99
30. Conclusions and General Comments	100

LIST OF FIGURES

NO.	PAGE
1. Simple Hinged Frame with Concentrated Load	15
2. Illustrations of Degree of Indeterminateness by Means of Joint Bars . .	16
3. Types of Frames of Various Degrees of Indeterminateness	17
4. Type of Structure in which Effect of Normal Force is Great	18
5. Unsymmetrical Frame under Concentrated Load	20
6. Simple Frames under Vertical Load; Lower Ends of Columns Hinged .	24
7. Simple Frames under Vertical Load; Lower Ends of Columns Fixed . .	28
8. Rectangular Frame with Rigidly Connected Tie at Base of Columns . .	28
9. L-frame with Column Hinged at Support	29
10. L-frame with Column Fixed at Support	30
11. Continuous Single Story Spans; One Panel Loaded	31
12. Single Story Three-Span Frame, Having Middle Span Uniformly Loaded .	31
13. Single Story Three-Span Frame under Concentrated Load	34
14. A-frame Showing Form of Moment Curve	35
15. Cross-Section of Building Frame with Single Panel Loaded	36
16. Two-Story Three-Span Frame Having Middle Span Uniformly Loaded .	36
17. Coefficient of Bending Moments for Top of Lower Column and Foot of Upper Column for Two-Story Three-Span Frame Having All External Connections Fixed	38
18. Formulas for Single Story Three-Span Frame with Four Columns Having Columns Hinged at Lower End	40
19. Formulas for Single Story Three-Span Frame with Four Columns Having Columns Fixed at Lower End	41
20. Single Story Three-Span Frame with Four Columns Having Middle Span Uniformly Loaded	42
21. Formulas for Rectangular Frame under Horizontal Load and for Octago- nal and Rectangular Reservoirs	43
22. Horizontal Reactions for Distributed Loads and Concentrated Loads .	44
23. Locus of Intersection of Reaction Lines in Single Span Single Story Frame under Single Concentrated Load	47
24. Coefficient of Bending Moment at Center of Middle Span for Single Story Three-Span Frame Having Extreme Ends of Columns and Beams Hinged	49
25. Coefficient of Bending Moment at Center of Middle Span for Single Story Three-Span Frame Having Extreme Ends of Beams Hinged and Column Ends Fixed	50
26. Coefficient of Bending Moment at Center of Middle Span for Single Story Three-Span Frame Having All External Connections Fixed . .	51

NO.	PAGE
27. Coefficient of Bending Moment at Center of Middle Span for Single Story Three-Span Frame Having Lower Ends of Columns Hinged . . .	52
28. Coefficient of Bending Moment at Center of Middle Span for Single Story Three-Span Frame Having Lower Ends of Columns Fixed . . .	53
29. Coefficient of Bending Moment at Lower Ends of Interior Columns for Single Story Three-Span Frame Having Lower Ends of Columns Fixed	54
30. Size and Distribution of Reinforcing Bars in Frames 1 and 6	55
31. Size and Distribution of Reinforcing Bars in Frames 2 and 4	56
32. Size and Distribution of Reinforcing Bars in Frames 3 and 7	56
33. Size and Distribution of Reinforcing Bars in Frame 5	57
34. Size and Distribution of Reinforcing Bars in Frame 8	57
35. Stress-Deformation Diagrams for Cylinders	61
36. Observed and Computed Stresses in Frame 1	62
37. Observed and Computed Stresses in Frame 2	63
38. Observed and Computed Stresses in Frame 3	64
39. Observed and Computed Stresses in Frame 4	65
40. Observed and Computed Stresses in Frame 5	66
41. Observed and Computed Stresses in Frame 6	67
42. Observed and Computed Stresses in Frame 7	68
43. Observed and Computed Stresses in Frame 8	69
44. Points Used for Comparison of Computed and Observed Stresses . . .	76
45. Load-Deformation Diagrams for Additional Gage Lines, Frames 1, 2, 3	78
46. Location of Additional Gage Lines, Frames 1, 2, 3	79
47. Load-Deformation Diagrams for Additional Gage Lines, Frames 4, 5, 6, 7	80
48. Location of Additional Gage Lines, Frames 4, 5, 6, 7	81
49. View of Frame 1 in Testing Machine	83
50. View of Frame 5 after Test	83
51. View of Frame 1 after Test	84
52. View of Frame 2 after Test	84
53. View of Frame 3 after Test	85
54. View of Frame 4 after Test	85
55. View of Frame 6 after Test	86
56. View of Frame 7 after Test	86
57. View of Frame 8 after Test	89
58. Connection of Beam and Column Showing Typical Location of First Cracks	98
59. Rectangular Joint with and without Fillet	99

LIST OF TABLES

NO.		PAGE
1.	Elements Used in Conditional Equations for Solution of an Indeterminate Structure	21
2.	Elements Used in Conditional Equations for Solution of Inverted U-frame	26
3.	Formulas for Reactions, Bending Moments, and Points of Inflection for Single Story Three-Span Frames	32
4.	Coefficients of Bending Moment for Single Story Three-Span Frame	33
5.	Formulas for Two-Story Three-Span Frame	37
6.	Coefficients of Bending Moment for Two-Story Three-Span Frame with Ends of Columns and Beams Fixed	39
7.	Coefficients of Bending Moment for Single Story Three-Span Frame Having Four Columns	42
8.	Data of Test Frames	58
9.	Tension Tests of Reinforcing Steel	59
10.	Compression Tests of Concrete Cubes and Cylinders	60
11.	Values of Modulus of Elasticity of Concrete Used in Stress Computations	71
12.	General Data of Tests of Frames	72, 73
13.	Computed Stresses for Test Frames in Terms of Total Applied Load	74, 75
14.	Computed and Observed Stresses in Frames	94, 95
15.	Stresses in Bars at Front and Back of Members of Test Frames	97
16.	Observed Stresses at Sharp Corners in Frames 1, 3 and 6	100

ANALYSIS AND TESTS OF RIGIDLY CONNECTED REINFORCED CONCRETE FRAMES*

I. INTRODUCTION

1. *Preliminary.*—The rigidly connected frame has frequent applications in reinforced concrete construction. It is made use of in engineering structures in a variety of forms. With a wider knowledge of the analysis of rigidly connected frames of reinforced concrete it is believed that the frame as an element in design will become more important and will be utilized in various ways in gaining economies and in securing effective designs.

Notwithstanding the importance, the exact determination of stresses as they actually occur in a rigidly connected reinforced concrete frame is not usually attempted. In most cases the determination requires considerable time and labor to work out the formulas for statically indeterminate quantities which depend upon the number and the fixity of the supports. There are few publications in English which treat this subject systematically, and formulas for practical use are not generally available. In some quarters there may still be a lack of acceptance of the principle of continuity and the effect of rigidity of joints as applied to reinforced concrete construction.

In this bulletin formulas for moments and other indeterminate quantities for several types of indeterminate structures are presented. For the derivation of the formulas the method which involves the use of the principle of least work was chosen. To test the applicability and reliability of the methods of analysis test frames designed according to the formulas found by the analyses were fabricated and the deformations produced in various parts of the members under the action of the loads measured, and a study and comparison of the results of analysis

*This bulletin presents a *résumé* of the analytical treatment of rigidly connected reinforced concrete frames and the principal results of the experimental work on reinforced concrete frames which were given in the thesis of Mikishi Abe as presented in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Engineering in the Graduate School of the University of Illinois, June, 1914. The work of condensation and reformulation necessary to bring the text within the space available for the bulletin has been done by members of the department of Theoretical and Applied Mechanics. The complete thesis is on file in the library of the University of Illinois. The original data of the tests and more detailed work of the analyses are on file at the Laboratory of Applied Mechanics of the University of Illinois.

It may be helpful to note that Dr. Abe's name is pronounced in two syllables; thus, Ah'be.

and test have been made. The general rigidity at the places where the members are joined has been investigated. The work is presented in the thought that it will be helpful in bringing into wider use the principles applicable to the design of rigidly connected reinforced concrete constructions.

2. *The Use and the Advantages of the Rigidly Connected Reinforced Concrete Frame.*—Since about 1905 reinforced concrete frame construction has been extensively used in continental Europe. Many examples can be found in the German texts and magazines. In England also frame constructions of reinforced concrete viaducts and other structures have been built in recent years. There is also a tendency in America to use reinforced concrete frames for buildings and bridges.

The field of the application of rigid frames is almost unlimited, for most reinforced concrete structures are composed of elements of rigid frames. It covers such constructions as buildings, bridge structures, trestles and viaducts, culverts and sewers, subway construction, retaining walls, and reservoirs and water tanks. In these structures rigid connections are used between members and in many or most of them the bending moments are statically indeterminate.

It is clear that every building construction of reinforced concrete may be considered as a rigidly connected frame, for columns, girders, beams and slabs are all rigidly connected with each other, even though the effect of this condition is not fully considered in the design. In continental European countries it is most common to use frames in building constructions, such as roofs, balconies, towers, and the building as a whole. In the design the requirements and the advantages of the frame are taken into account.

Bridge structures are in the field of the rigid frame. Arches, beam and bent construction, and most bridge structures can be designed as frames on a rigid analytical basis. In highway bridges, for example, a spandrel-braced arch is frequently used. In such a case columns are rigidly connected to the arch ribs and to the superstructure, and therefore the design should be made as a rigidly connected frame. The designing of trestles and viaducts as a frame will secure safety and at the same time obtain the best proportioning of parts.

Box culverts and the box type of construction for subways give sections which are examples of the rigidly connected frame and which may not be rationally designed without a sufficient knowledge of rigid frames.

In water tanks and reservoirs of a rectangular or a polygonal form, the unknown negative bending moment due to a rigid connection of wall to wall or base to wall will exist at each corner. These moments are modified by the relative thickness of walls and the other dimensions of the structure. A knowledge of the rigid frame will suggest the proper method of solution.

It can thus be seen that most monolithic construction falls within the field of the rigid frame. A study of the rigid frame will assist in developing judgment for use in the design of such construction.

In building and structural design, insufficient attention is often given to the bending of columns caused by the rigidity of connections. The bending moment for a beam is frequently taken as an assumed fraction of Pl (where P is the load and l is the span) while bending moments at the ends and in the columns are disregarded entirely, thus leaving the structure inadequate or making one part stronger at the expense of the other.

The reinforced concrete frame is advantageous in that material can be saved and a much better result obtained from the theoretical and structural point of view. In ordinary concrete building construction the element of rigidity is usually not fully taken advantage of. With the concrete frame construction, however, the rigidity of the connection of the members may be used. The rigid frame is capable of exact design, and therefore the economical distribution of materials can be realized.

One reason why some engineers hesitate to use concrete frames extensively is that they hardly believe in the continuity of the parts of the structure and doubt the effect of the rigidity of the connection. The question is also naturally raised if the formulas deduced from the elastic work of deformation of a non-homogeneous material like reinforced concrete will hold good for such composite members with fair agreement; furthermore the secondary stresses may act to modify the results. Under actual conditions, as is well known, the fundamental assumptions which underlie the static considerations can seldom be more than partially fulfilled even under carefully prepared specifications and well executed designs. These things must be considered before coming to a conclusion as to the reliability of rigidly connected reinforced concrete frames. It is evident that reinforced concrete frames will be reliable if there is perfect continuity or complete rigidity of joint, close agreement between theory and experiment, and a small effect of stresses of a secondary character. It is not known that an experi-

mental study of this subject has before been made. It may be expected then that careful experiments and investigation will give information which will help to settle these questions.

3. *Scope of Investigation and Acknowledgment.*—In this bulletin formulas for several types of statically indeterminate structures which have been deduced by the use of the principle of least work are given. For vertical load the following cases have been analyzed: (1) single story, single span; (2) single story, three spans; (3) trestle bent with tie, single span; (4) building frame with several stories and several spans; and (5) bridge trestle. For horizontal load the following cases have been analyzed: (1) single story, single span; (2) octagonal reservoir or tank; and (3) rectangular reservoir or tank.

In order to put to practical test the reliability of these formulas for reinforced concrete structures, eight test frames designed according to the formulas found by the analyses were made, and the deformations produced in the various parts of the members by the series of test loads were measured. In the design of the frames requirements not touched upon by the analyses referred to were provided for in a practical way. The analyses and the results of the tests have been subjected to critical study and discussion. The specimens were made in November and December, 1913, and January, 1914, and were tested in January, February, and March, 1914. In making these tests the purpose was to obtain experimental information along the following lines which have a bearing on the design of rigidly connected reinforced concrete frames:

- (1) The amount and the distribution of stresses in the reinforcement and in the concrete
- (2) The continuity of the composing members of a frame
- (3) The location of sections of critical stress
- (4) The reliability of a reinforced concrete frame
- (5) The applicability of the theoretical formulas in the design of frames.

The experimental work was done as a research problem of the Engineering Experiment Station of the University of Illinois.

The work was under the charge of PROFESSOR ARTHUR N. TALBOT, to whom, as well as to other members of the staff, acknowledgment is due for valuable suggestions and aid.

The limits of space set for this bulletin will not permit publication of even a small part of the details of the derivation of the formulas

found nor of the observations, calculations, and other data of the tests. Instead the plan has been followed generally of giving the formulas found from the analyses without the details of the derivation and, in the case of the experimental work, of showing graphically the main stresses that were observed at the principal loads, and of not including details of the data. The original and the reduced data and more detailed work of the analyses are on file at the Laboratory of Applied Mechanics of the University of Illinois.

II. THE ANALYSIS OF RIGIDLY CONNECTED FRAMES

4. *Notation.*—The following notation is used generally throughout the bulletin:

A = area of cross-section of a member. Numerical suffixes are used for individual members when a frame is composed of members of different sizes.

A is also used as a coefficient to represent certain algebraic expressions.

a = distance from the left corner or axis of a frame to the point of application of a concentrated load on a top beam.

b = distance from the right corner or axis of a frame to the point of application of a concentrated load on a top beam.

E = modulus of elasticity (considered as constant) of the material.

H = horizontal reaction acting at the end of a column.

I = moment of inertia in general.

h = total vertical height of frame.

l = total length of horizontal span of frame.

s = length of an inclined member.

m = ratio of moment of inertia of horizontal member to that of vertical member.

$n = \frac{h}{l}$ = ratio of height of frame to length of span.

M = bending moment in general.

N = normal force or stress on a section (total internal force normal to the section).

P = a concentrated load.

p = intensity of a uniformly distributed load.

V = vertical reaction.

f_s = unit stress in steel in tension.

f'_s = unit stress in steel in compression.

f_c = unit stress in concrete in compression.

In the diagrams representing the forms of the frames analyzed, the ends of members are indicated by lower case letters and the symbols for the properties, forces, and moments use these letters as subscripts to indicate the members to which they apply as well as the point of

application. The method of use will be made clear by the following examples and by reference to Fig. 6:

- H_a = horizontal reaction at a.
 I_{ab} = moment of inertia of member ab.
 h_{bc} = vertical height of bc.
 l_{bc} = length of horizontal projection of bc.
 M_{bc} = bending moment at any point in bc.

5. *Statically Determinate and Indeterminate Systems and Number of Statically Indeterminates.*—A force is said to be statically determinate when its direction and magnitude and its point of application are known from the conditions of static equilibrium. The conditions of static equilibrium for any number of forces in a plane, as is generally well known, are three, that is to say,

- (1) $\Sigma Y = 0$, or the algebraic sum of all vertical forces acting on a body is equal to zero.
- (2) $\Sigma X = 0$, or the algebraic sum of all horizontal forces acting on a body is equal to zero.
- (3) $\Sigma M = 0$, or the algebraic sum of the moments of all forces is equal to zero.

The loads to which structures may be subjected are always given. The other external forces are the reactions due to the loads. The reactions are exerted by the supports of the structure, and in order that they may be determined from the statical conditions the total number of unknowns must not exceed three. The ordinary trusses without redundant members are always statically determinate if a frictionless pin is used at each joint and if in the determination the effect of the longitudinal deformation of members on the stresses is neglected. If a case in which two members meet at a joint is considered as shown in Fig. 1, two unknown forces exist at the joint, the vertical

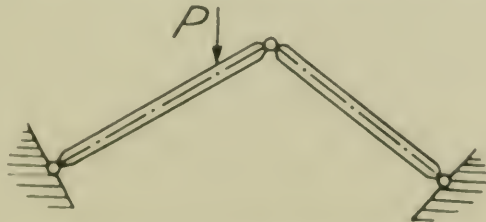


FIG. 1. SIMPLE HINGED FRAME WITH CONCENTRATED LOAD

and horizontal forces, and therefore the total number of unknown forces due to the external force P is six. But each member will give three statical conditions as stated before, and therefore this is a statically determinate system.

In studying the behavior of statically determinate and statically indeterminate systems, the conception of the connection of members by means of joint bars is a convenience. In Fig. 2 (a) the two members are not connected. They are entirely free to move horizontally and

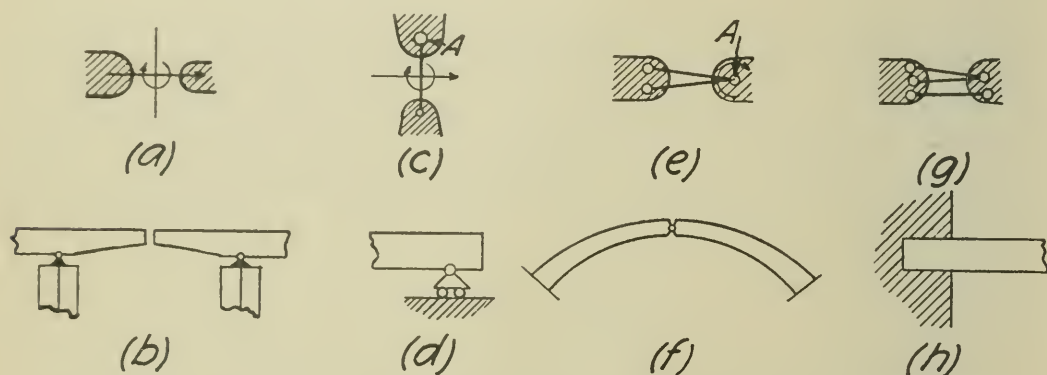


FIG. 2. ILLUSTRATIONS OF DEGREE OF INDETERMINATENESS BY MEANS OF JOINT BARS

vertically and also are free in rotation; accordingly it may be called the arrangement having three freedoms in motion. An example of this arrangement is a touching joint between the free ends of cantilever beams as shown in Fig. 2 (b). In Fig. 2 (c) two members are connected by a single bar, and they are prevented from moving vertically, but are free to move horizontally and to rotate about a point A. This may be called the arrangement having two freedoms in motion. An example of this arrangement is the frictionless roller end of a cantilever as shown in Fig. 2 (d). In Fig. 2 (e) two members are connected by two connecting bars and have only one freedom in motion, that is, the rotation of a member about A, the intersecting point of two bars. The crown hinge of an arch, Fig. 2 (f), is an example. In Fig. 2 (g), two members are connected by three joint-bars, and may be called a rigid connection which allows no freedom of motion. The restrained end of a cantilever beam, Fig. 2 (h), is an example of this arrangement.

To make a rigid joint it is necessary to have three joint bars at each connection between members, and $3S$ conditions of equilibrium must be set up to determine $3S$ unknowns when the structure is composed of S members. If the structure is rigidly connected to the ground,

more conditions than $3S$ are required. Let a be the number of joint-bars needed to connect one member to another, and b be the number of joint bars needed to connect the member to the ground; then from the existing $3S$ conditions the following relation is necessary to make the structure statically determinate,

$$a+b=3S$$

When the members in the structure are all rigidly connected to each other, $a+b$ always exceeds $3S$ and therefore the case becomes a statically indeterminate system in which

$$a+b-3S=m$$

where m represents the number of the statically indeterminate forces. Such a system is called m -fold statically indeterminate, and m additional equations of condition are necessary to determine these unknowns.

Fig. 3 gives a few examples.

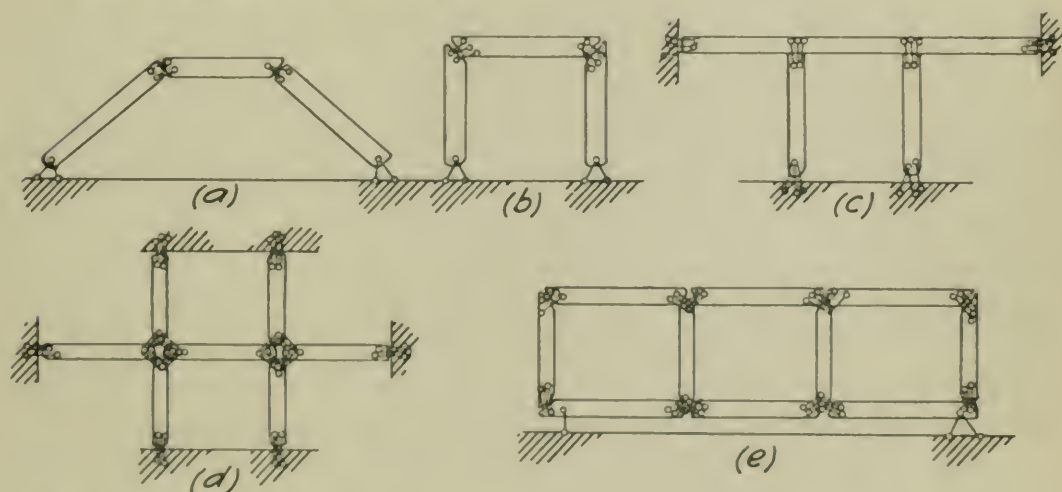


FIG. 3. TYPES OF FRAMES OF VARIOUS DEGREES OF INDETERMINATENESS

Case a. $a+b-3S=5+4-9=0$, therefore statically determinate.

Case b. $a+b-3S=6+4-9=1$, 1-fold statically indeterminate.

Case c. $a+b-3S=6+12-9=9$, 9-fold statically indeterminate.

Case d. $a+b-3S=18+18-21=15$, 15-fold statically indeterminate.

Case e. $a+b-3S=36+3-30=9$, 9-fold statically indeterminate.

In general the change in length of a member due to the direct stresses in the member will have an effect on the magnitude of the stresses developed. However, as is shown in a later paragraph, this effect is very slight in all ordinary forms of construction and has been neglected in stating the method of determining the degree of indeterminateness. Consequently in any member in which the change in length due to the direct stress has an appreciable effect on the stress developed, the criterion stated does not apply. Fig. 4 illustrates such a case.

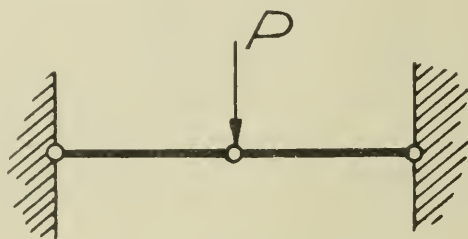


FIG. 4. TYPE OF STRUCTURE IN WHICH EFFECT OF NORMAL FORCE IS GREAT

6. *Principle of Least Work.*—By a law of nature, the principle of least work, the resisting forces will develop no more energy than the minimum which is necessary to maintain equilibrium with the external forces; or in other words, the external forces are so adjusted, among themselves, as to develop internal forces in the structure which will make the total internal work of resistance of the internal forces a minimum. When forces act upon an elastic system in which the deformations are proportional to the stresses, the principle of least work may be applied to determine the statically indeterminate forces.

The principle of least work has been known for a hundred years, but the first complete announcement of this theorem was given by Castigliano in 1879. Professor Cain expresses the principle in the following words:*

“The elastic forces experienced between the molecules after deformation correspond to a minimum of the work of deformation of the system, expressed as a function of certain stresses, taken with respect to these stresses successively, regarded as independent during the differentiation.”

Professor Hiroi[†] translates Castigliano's expression of the principle of least work in the following words: “The partial derivatives

*See Trans. Amer. Soc. Civ. Eng., Vol. XXIV, p. 291.

†See Statically Indeterminate Stresses, by I. Hiroi.

of the work of resistance with respect to statically indeterminate forces which are so chosen that the forces themselves perform no work are equal to zero."

The total internal work may be subdivided into the parts due to bending moment, normal stress, and shearing stress.

The total work due to the bending moment M in a member will be

$$w_1 = \int \frac{M^2 dx}{2EI}$$

For the total internal work due to a total normal stress* N on a section

$$w_2 = \int \frac{N^2 dx}{2EA}$$

If shearing stress S is uniform over the cross-section, the expression for the internal work due to shearing stress is

$$w = \int \frac{S^2 dx}{2GA}$$

where G expresses the shearing modulus of elasticity of a material. Since, however, the shearing stress is not uniform over the cross-sections, the expression for the internal work due to shear is modified, and

$$u_3 = \int \frac{KS^2 dx}{2GA}$$

where K is a known factor for a specified form of the cross-section. Therefore for the total work of resistance,

$$W = w_1 + w_2 + w_3 = \frac{1}{2} \int \frac{M^2 dx}{EI} + \frac{1}{2} \int \frac{N^2 dx}{EA} + \frac{1}{2} \int \frac{KS^2 dx}{GA}$$

Suppose that there are n statically indeterminate forces $P_1, P_2, \dots, P_n$ in an elastic system. According to the theorem of Castigliano

$$\frac{\partial W}{\partial P_1} = \int \frac{M}{EI} \frac{\partial M}{\partial P_1} dx + \int \frac{N}{EA} \frac{\partial N}{\partial P_1} dx + \int \frac{KS}{GA} \frac{\partial S}{\partial P_1} dx = 0$$

$$\frac{\partial W}{\partial P_2} = \int \frac{M}{EI} \frac{\partial M}{\partial P_2} dx + \int \frac{N}{EA} \frac{\partial N}{\partial P_2} dx + \int \frac{KS}{GA} \frac{\partial S}{\partial P_2} dx = 0$$

• • • • •

$$\frac{\partial W}{\partial P_n} = \int \frac{M}{EI} \frac{\partial M}{\partial P_n} dx + \int \frac{N}{EA} \frac{\partial N}{\partial P_n} dx + \int \frac{KS}{GA} \frac{\partial S}{\partial P_n} dx = 0$$

* Normal to a cross-section of the member.

These furnish as many equations of condition as there are unknown quantities. The solution of these equations will give the exact formulas for statically indeterminate quantities.

7. *The Effect of Work of Normal Force on the Magnitude of Statically Indeterminate Forces.*—In the general equation

$$\int \frac{M}{EI} \frac{\partial M}{\partial P} dx + \int \frac{N}{EA} \frac{\partial N}{\partial P} dx + \int \frac{KS}{GA} \frac{\partial S}{\partial P} dx = 0$$

the second term will disappear when $\frac{\partial N}{\partial P}$ is equal to zero or, in other words, when the normal force* does not contain any of the statically indeterminate quantities.

When a frame is fixed at its column ends, the vertical and horizontal reactions and the bending moment at the fixed column ends are statically indeterminate. Generally the normal force in any member contains these reactions as factors. But if a frame having a single span is symmetrical in form and in the manner of loading, the vertical reactions become statically determinate, and therefore the horizontal reaction is the only indeterminate term which enters into the expression of the normal force. If, at the same time, the columns are vertical, the horizontal reactions do not affect the normal forces in the columns, and in the horizontal members only are the normal forces affected by a statically indeterminate force, namely, the horizontal reaction.

From these statements it will be seen that the form of frame which will be largely affected by the normal force is that having a sloped column under a vertical load.

The frame shown in Fig. 5 is used to illustrate the method of

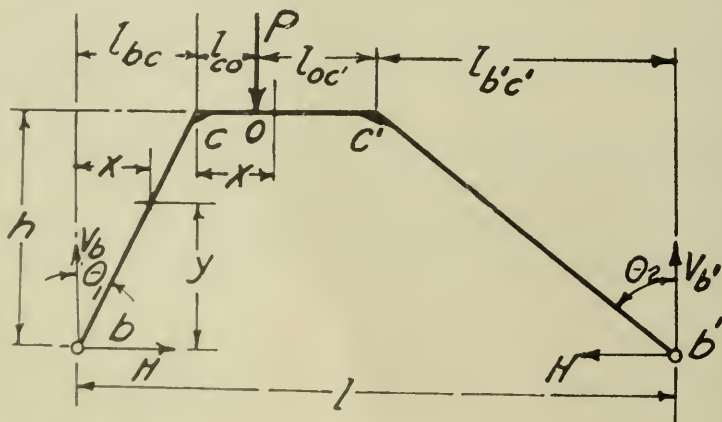


FIG. 5. UNSYMMETRICAL FRAME UNDER CONCENTRATED LOAD

*Normal to a cross-section of the member.

TABLE 1
ELEMENTS USED IN CONDITIONAL EQUATIONS FOR SOLUTION OF AN INDETERMINATE STRUCTURE

Member	Moment of Inertia	Limits of Integration	Bending Moment in the Member	Normal Forces in the Member	$\frac{\partial M}{\partial H}$	$\frac{\partial N}{\partial H}$
bc	I_{bc}	From zero to h	$V_b x - H y = V_b y \tan \theta_1 - H y$	$V_b \cos \theta_1 + \frac{H \sin \theta_1}{H}$	$-y$	$\sin \theta_1$
co	I_{co}	From zero to l_{co}	$V_b(l_{co} + x) - H h$	$\frac{H}{H}$	$-\frac{h}{h}$	$+1$
oc'	$I_{oc'}$	From l_{co} to $l_{oc'}$	$V_b(l_{co} + x) - H h - P(x - l_{co})$	$\frac{H}{H}$	$-\frac{h}{h}$	$+1$
$b'e'$	$I_{b'e'}$	From zero to h	$V_{b'} x - H y = V_{b'} y \tan \theta_2 - H y$	$V_{b'} \cos \theta_2 + \frac{H \sin \theta_2}{H}$	$-y$	$\sin \theta_2$

analysis and to bring out the effect of the work of the direct normal force on the magnitude of the statically indeterminate forces. In this case H is the only statically indeterminate force.

Taking the moment of all forces about b'

$$V_b l - P(l_{oc'} + l_{b'c'}) = 0 \text{ or } V_b = \frac{l_{b'c'} + l_{oc'}}{l} P = KP$$

and

$$V_{b'} = P - KP \text{ or } V_{b'} = (1 - K)P$$

K being used as a general coefficient.

In general the internal work due to shearing stress may be neglected, and the general equation becomes

$$\int \frac{M}{EI} \frac{\partial M}{\partial H} dx + \int \frac{N}{EA} \frac{\partial N}{\partial H} dx = 0$$

All necessary elements in forming this equation are arranged in Table 1.

Inserting these values in the general equation gives the following expression, in which it is assumed that E and I are constant:

$$\begin{aligned} & \frac{1}{EI_{bc}} \int_0^h (V_b \tan \theta_1 y - Hy) (-y) \sec \theta_1 dy + \frac{1}{EI_{cc'}} \int_0^{l_{co}} (V_b l_{bc} + V_b x - Hh) (-h) dx \\ & + \frac{1}{EI_{cc'}} \int_{l_{co}}^{l_{cc'}} \left\{ V_b l_{bc} + V_b x - Hh - P(x - l_{co}) \right\} (-h) dx \\ & + \frac{1}{EI_{b'c'}} \int_0^h (V_{b'} \tan \theta_2 y - Hy) (-y) \sec \theta_2 dy \\ & + \frac{1}{EA_{bc}} \int_0^h (V_b \cos \theta_1 + H \sin \theta_1) \sin \theta_1 \sec \theta_1 dy + \frac{1}{EA_{cc'}} \left\{ \int_0^{l_{co}} H dx + \int_{l_{co}}^{l_{cc'}} H dx \right\} \\ & + \frac{1}{EA_{b'c'}} \int_0^h \left\{ V_{b'} \cos \theta_2 + H \sin \theta_2 \right\} \sin \theta_2 \sec \theta_2 dy = 0 \end{aligned}$$

Integrating and simplifying this equation gives the following general expression for the statically indeterminate force H , in which the work of the normal force is fully counted:

$$H = \frac{K \left[\frac{l_{bc} S_{bc}}{3I_{bc}} + \frac{2l_{bc} l_{cc'} + l_{cc'}^2}{2I_{cc'}} - \frac{l_{b'e'} S_{b'e'}}{3I_{b'e'}} - \frac{\sin \theta_1}{A_{bc}} + \frac{\sin \theta_2}{A_{b'e'}} \right] - \left[\frac{(l_{cc'} - l_{oc'})^2}{2I_{cc'}} - \frac{S_{b'e'} l_{b'e'}}{3I_{b'e'}} + \frac{\sin \theta_2}{A_{b'e'}} \right]}{h \left[\frac{S_{bc}}{3I_{bc}} + \frac{l_{cc'}}{I_{cc'}} + \frac{S_{b'e'}}{3I_{b'e'}} \right] + \left[\frac{\sin \theta_1 \tan \theta_1}{A_{bc}} + \frac{\sin \theta_2 \tan \theta_2}{A_{b'e'}} + \frac{l_{oc'}}{h A_{cc'}} \right]}$$

where $K = \frac{l_{b'e'} + l_{oc'}}{l}$

In this formula, the terms which contain A_{bc} , $A_{cc'}$, and $A_{b'e'}$ enter because of taking into consideration the work of the normal forces represented by the second term of the general equation

$$\int \frac{M}{EI} \frac{\partial M}{\partial H} dx + \int \frac{N}{EA} \frac{\partial N}{\partial H} dx = 0$$

If the effect of the normal force is neglected, then

$$H = \frac{K \left[\frac{l_{bc} S_{bc}}{3I_{bc}} + \frac{2l_{bc} l_{cc'} + l_{cc'}^2}{2I_{cc'}} - \frac{l_{b'e'} S_{b'e'}}{3I_{b'e'}} \right] - \left[\frac{(l_{cc'} - l_{oc'})^2}{2I_{cc'}} - \frac{S_{b'e'} l_{b'e'}}{3I_{b'e'}} \right]}{h \left[\frac{S_{bc}}{3I_{bc}} + \frac{l_{cc'}}{I_{cc'}} + \frac{S_{b'e'}}{3I_{b'e'}} \right]} P$$

In most cases a value of θ greater than 30 degrees will not be used, because an increase in θ rapidly increases the horizontal reaction at the end of the column.

Assume a case in which $l_{bc} = l_{cc'} = l_{b'e'} = h = 120$ inches. $\theta_1 = \theta_2 = 45$ degrees, $A_{bc} = A_{cc'} = A_{b'e'} = 10$ by 12 inches. $I_{bc} = I_{cc'} = I_{b'e'} = 1,000$ in.⁴ $K = \frac{1}{2}$

When the effect of the work of the direct force is considered,

$$H = 0.5637P$$

If the effect of the work of the direct force is neglected,

$$H = 0.5643P$$

The difference $0.0006P$ ($0.0011H$) is inconsiderable.

In the foregoing example it is seen that the final formula is very much complicated by taking the direct force into consideration and that the effect of the internal work of all the direct stresses on the final value for statically indeterminate stresses is inconsiderable when compared with that of the bending moment. The work of the normal force is therefore disregarded in making the analyses of the frames treated in this bulletin.

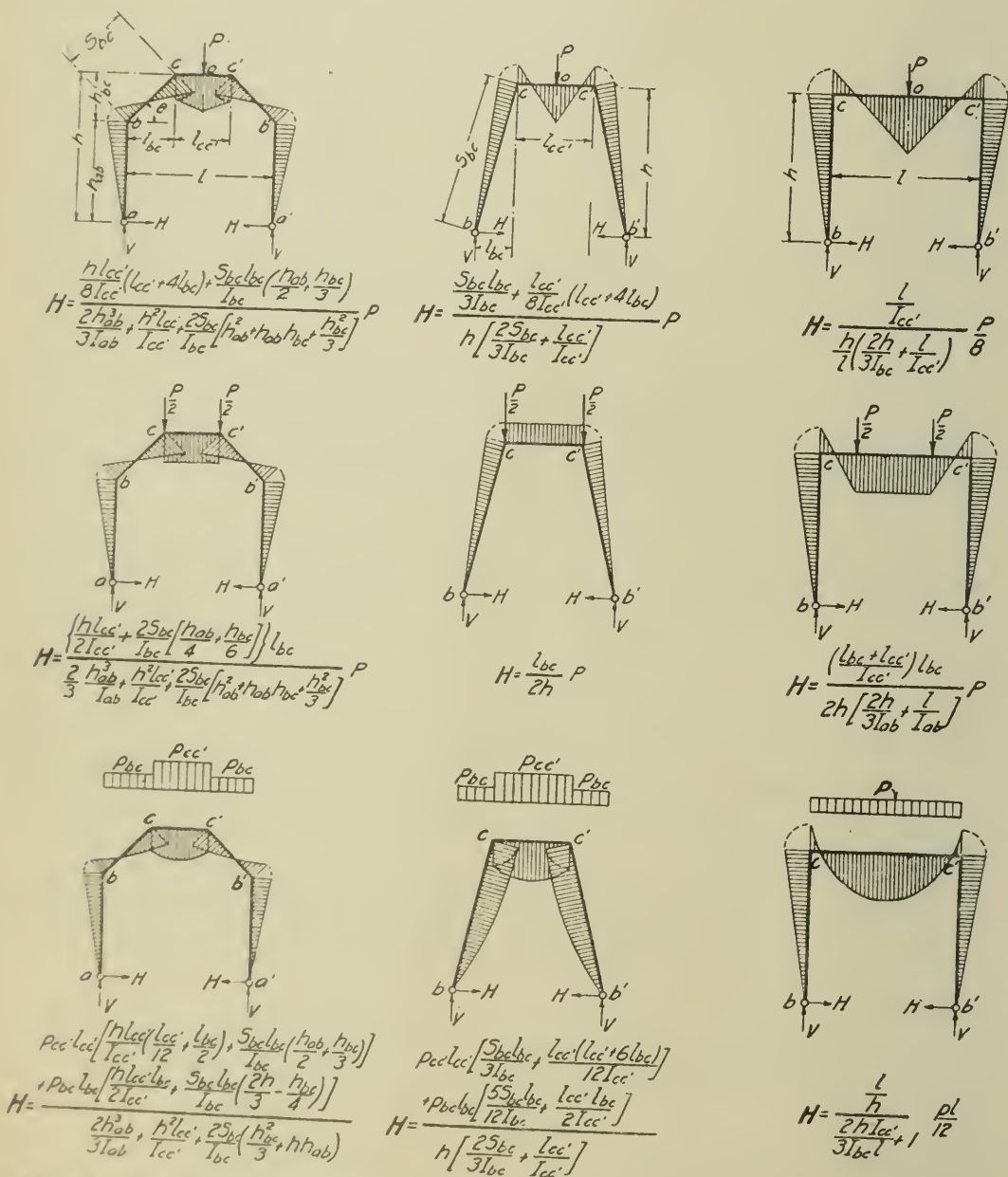


FIG. 6. SIMPLE FRAMES UNDER VERTICAL LOAD; LOWER ENDS OF COLUMNS HINGED

Attention is called to the fact that even though the effect of the internal work of the direct stresses may be neglected in determining the reactions the direct stresses themselves can not be neglected when calculating the total stress in any member. The direct stresses may be added algebraically after the statically indeterminate stresses are found.

The effect of the work of the deformation due to shear is generally so insignificant when compared with that due to the bending that it may be entirely neglected without sensible error in the calculation of the internal work.

8. *Simple Frames under Vertical Load.*—In Fig. 6 are given forms of a type of simple frame. The form at the left of the figure is the general form of the inverted U-frame, and the others are special cases of this frame. The column connections at the base are hinged and all other joints are rigid. Three forms of vertical loading are presented—a single concentrated load, two concentrated loads, and a uniform load. For this frame the statically indeterminate force is the horizontal reaction H . Formulas for H , derived by analysis using the principle of least work, are given in the figure for the three forms of frame and for the three loadings.

Knowing the horizontal reaction H , the bending moment and the forces at any section of the frame may be determined by the ordinary analytical method. For example, the bending moment at the middle of the top beam for the frames with two concentrated loads shown in Fig. 6 is equal to the algebraic sum of the moment of the horizontal reaction H with a moment arm equal to the height of the frame, the moment of the vertical reaction V about the section considered, and the moment of a load $\frac{P}{2}$ about the section.

As a specimen application of the method of using the principle of least work, the general solution of the frame and loading shown in the upper left-hand corner of Fig. 6 is given. The statically indeterminate force is H . The effect of normal forces being neglected, the general equation of condition is

$$\int \frac{M}{EI} \frac{\partial M}{\partial H} dx = 0$$

All quantities necessary in forming the conditional equation for this case are arranged in Table 2.

TABLE 2

ELEMENTS USED IN CONDITIONAL EQUATIONS FOR SOLUTION OF INVERTED U-FRAME

Member		I	M	$\frac{\partial M}{\partial H}$
<i>ab</i>	<i>a'b'</i>	<i>I_{ab}</i>	<i>-Hy</i>	<i>-y</i>
<i>bc</i>	<i>b'c'</i>	<i>I_{bc}</i>	$\frac{Px}{2} - H(h_{ab} + x \tan \theta)$	$-(h_{ab} + x \tan \theta)$
<i>co</i>	<i>co'</i>	<i>I_{cc'}</i>	$\frac{Px}{2} - Hh$	<i>-h</i>

Substituting the proper values in the general equation of condition

$$\begin{aligned} &\frac{2}{EI_{ab}} \int_0^{h_{ab}} Hy^2 dy - \frac{2 \sec \theta}{EI_{bc}} \int_0^{l_{bc}} \left[\frac{Px}{2} - H(h_{ab} + x \tan \theta) \right] (h_{ab} + x \tan \theta) dx \\ &\quad - \frac{2h}{EI_{cc'}} \int_{l_{bc}}^{l_{bc} + \frac{l_{cc'}}{2}} \left(\frac{Px}{2} - Hh \right) dx = 0 \quad \dots \quad (1) \end{aligned}$$

Integrating equation (1)

$$\begin{aligned} &\frac{2}{EI_{ab}} \left[\frac{Hy^3}{3} \right]_0^h - \frac{2 \sec \theta}{EI_{bc}} \left[\frac{Px^2 h_{ab}}{4} + \frac{Px^3}{6} \tan \theta - H(h_{ab}^2 x + h_{ab} x^2 \tan \theta + \frac{x^3}{3} \tan^2 \theta) \right]_0^{l_{bc}} \\ &\quad - \frac{2h}{EI_{cc'}} \left[\frac{Px^2}{4} - Hhx \right]_{l_{bc}}^{l_{bc} + \frac{l_{cc'}}{2}} = 0 \quad \dots \quad (2) \end{aligned}$$

Substituting the limits in equation (2)

$$\begin{aligned} &\frac{2Hh^3}{3I_{ab}} - \frac{2 \sec \theta}{I_{bc}} \left[\frac{Pl_{bc}^2 h_{ab}}{4} + \frac{Pl_{bc}^3}{6} \tan \theta - H(h_{ab}^2 l_{bc} + h_{ab} l_{bc}^2 \tan \theta + \frac{l_{bc}^3}{3} \tan^2 \theta) \right] \\ &\quad - \frac{2h}{I_{cc'}} \left[\frac{P}{4} \left(l_{bc} l_{cc'} + \frac{l_{cc'}^2}{4} \right) - Hh \frac{l_{cc'}}{2} \right] = 0 \quad \dots \quad (3) \end{aligned}$$

Collecting the terms involving H and those involving P

$$\begin{aligned} \frac{2h_{ab}^3 H}{3I_{ab}} + \frac{2\sec\theta}{I_{bc}} \left(h_{ab}^2 l_{bc} + h_{ab} l_{bc}^2 \tan\theta + \frac{l_{bc}^3}{3} \tan^2\theta \right) H + \frac{2h_{cc'}^2 H}{2I_{cc'}} \\ = \frac{2\sec\theta}{I_{bc}} P \left[\frac{l_{bc}^2 h_{ab}}{4} + \frac{l_{bc}^3}{6} \tan\theta \right] + \frac{2h}{I_{cc'}} P \left(l_{bc} l_{cc'} + \frac{l_{cc'}^2}{4} \right) \quad (4) \end{aligned}$$

Solving for H ,

$$\begin{aligned} H = \frac{\frac{2\sec\theta}{I_{bc}} \left[\frac{l_{bc}^2 h_{ab}}{4} + \frac{l_{bc}^3}{6} \tan\theta \right] + \frac{2h}{4I_{cc'}} \left(l_{bc} l_{cc'} + \frac{l_{cc'}^2}{4} \right)}{\frac{2h_{ab}^3}{3I_{ab}} + \frac{2\sec\theta}{I_{bc}} \left(h_{ab}^2 l_{bc} + h_{ab} l_{bc}^2 \tan\theta + \frac{l_{bc}^3}{3} \tan^2\theta \right) + \frac{h_{cc'}^2}{I_{cc'}}} P \quad (5) \end{aligned}$$

Substituting $l_{bc} \sec\theta = S_{bc}$ and $h_{bc} = l_{bc} \tan\theta$

$$\begin{aligned} H = \frac{\frac{h l_{cc'}}{8I_{cc'}} \left(4l_{bc} + l_{cc'} \right) + \frac{S_{bc} l_{bc}}{I_{bc}} \left[\frac{h_{ab}}{2} + \frac{h_{bc}}{3} \right]}{\frac{2h_{ab}^3}{3I_{ab}} + \frac{h_{cc'}^2}{I_{cc'}} + \frac{2S_{bc}}{I_{bc}} \left(h_{ab}^2 + h_{ab} h_{bc} + \frac{h_{bc}^2}{3} \right)} P \quad (6) \end{aligned}$$

Fig. 7 gives sketches of the inverted U-frame having the lower ends of the columns fixed. Equations of the statically indeterminates, the horizontal reaction, and the bending moment at the lower end of the column, as determined from analyses, are given in the figure.

Knowing these indeterminates, the moments, and the forces at any section of the frame may be determined by ordinary analysis. Thus the moment at the middle of the top beam for the frame in the upper left-hand corner of Fig. 7 is equal to the algebraic sum of the bending moment at the lower end of the column, M_a , the moment of the horizontal reaction H with a moment arm equal to the height of the frame, the moment of the vertical reaction V about the section considered, and the moment of one load $\frac{P}{2}$ about the section. Fig. 7 indicates the manner in which the moment varies along the members composing the frame.

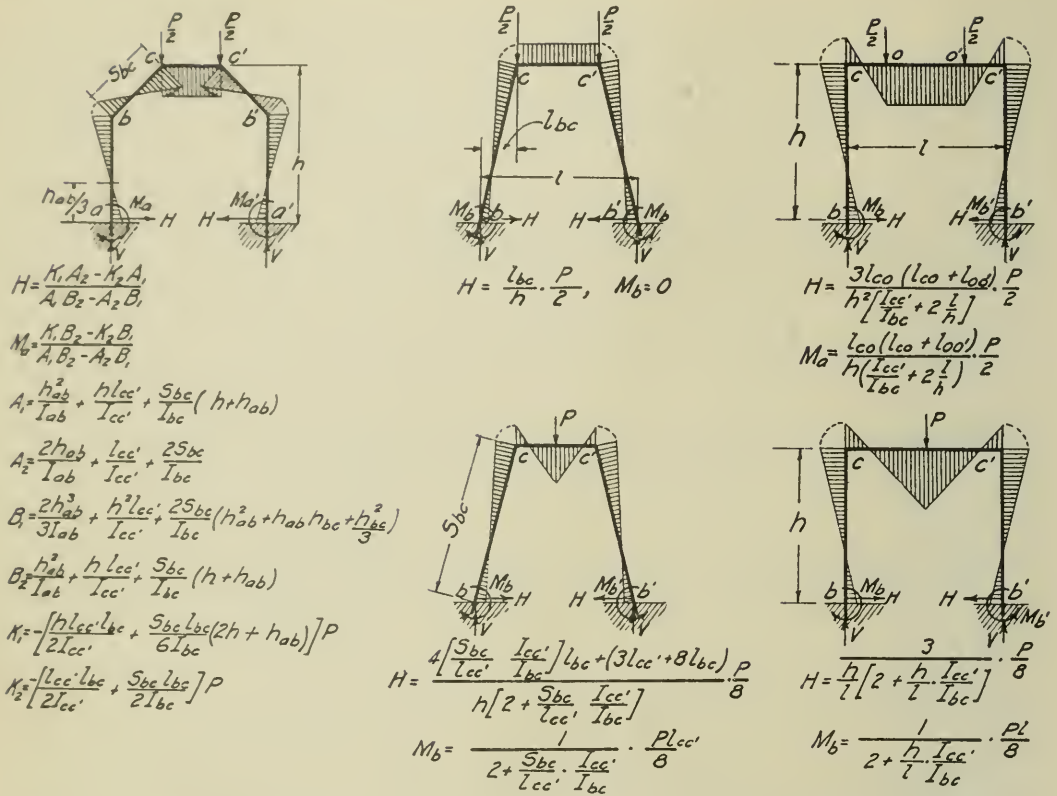


FIG. 7. SIMPLE FRAMES UNDER VERTICAL LOAD; LOWER ENDS OF COLUMNS FIXED

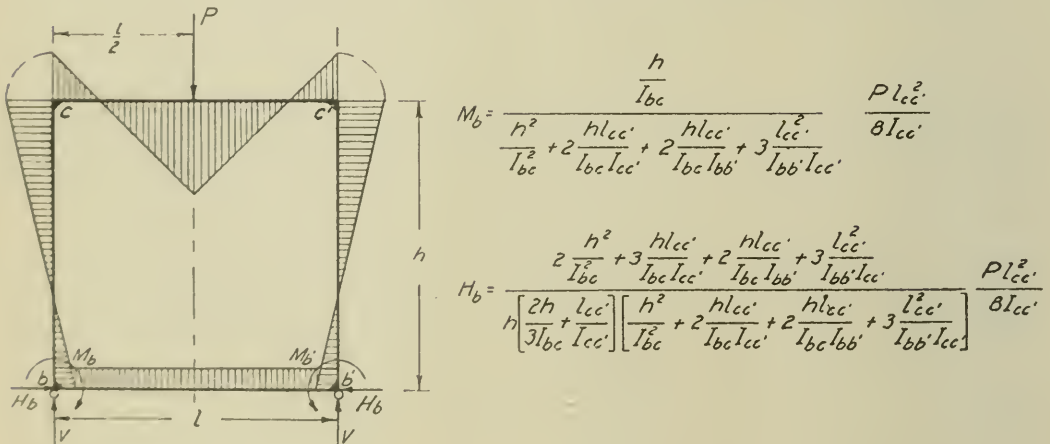


FIG. 8. RECTANGULAR FRAME WITH RIGIDLY CONNECTED TIE AT BASE OF COLUMNS

Fig. 8 shows a rectangular frame in which all the joints are rigid, the horizontal cross tie at the bottom of the frame having rigid connection to the columns. The equations of the indeterminates M_b and H_b

for this frame are given in the figure. The figure indicates the manner in which the moment varies along the members.

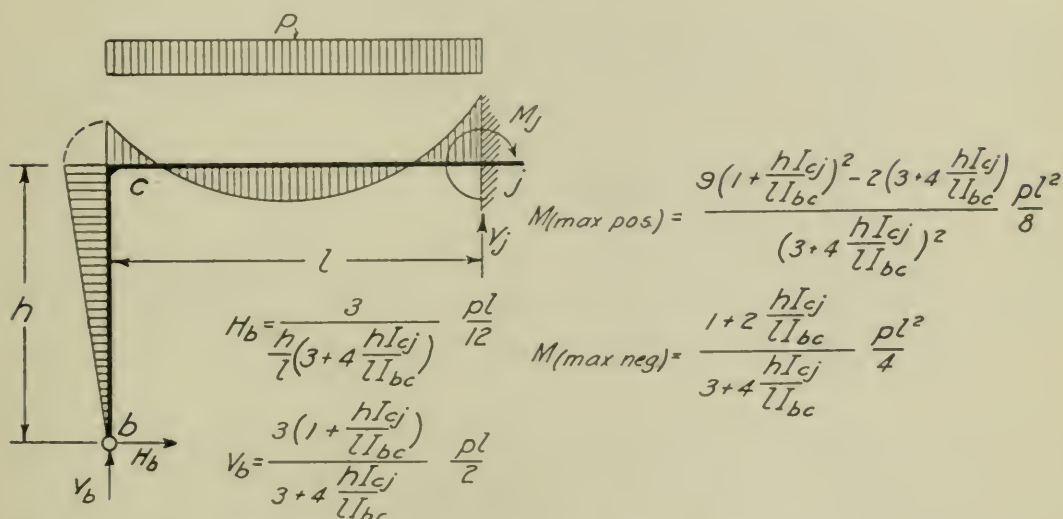


FIG. 9. L-FRAME WITH COLUMN HINGED AT SUPPORT

Fig. 9 shows an unsymmetrical frame, here termed an L-frame, under uniform load and with the column hinged at the support. For this frame the horizontal and vertical reactions H_b and V_b at the column end are given in the figure.

The maximum positive bending moment in this frame occurs at the distance x from c where

$$x = \frac{3(1 + \frac{hI_{cj}}{lI_{bc}})}{3 + \frac{4hI_{cj}}{lI_{bc}}} \frac{l}{2}$$

The value of the maximum positive bending moment is given in the figure. The maximum bending moment is the negative moment at the wall, which is also given in the figure.

Fig. 10 shows an L-frame in which the column is fixed at the support. On account of the fixity of the column there are three statically indeterminates for this frame, H_b , V_b , and M_b or M_j . The values of these are given in the figure.

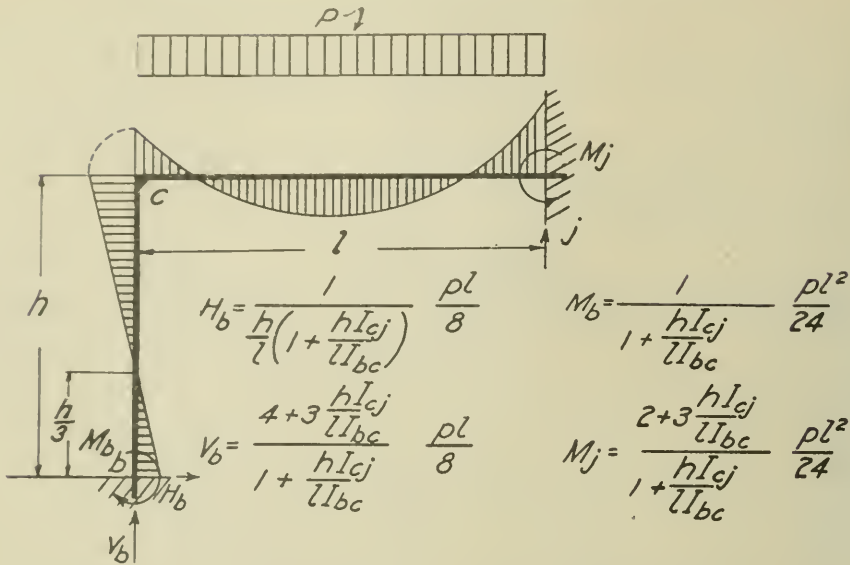


FIG. 10. L-FRAME WITH COLUMN FIXED AT SUPPORT

The formulas for the maximum positive moment in the beam and the distance of the section of maximum positive moment from c are

$$M_{max\ pos} = \frac{3 \left(1 + \frac{3hI_{cj}}{4lI_{bc}} \right)^2 - 2 \left(1 + \frac{hI_{cj}}{lI_{bc}} \right)}{\left(1 + \frac{hI_{cj}}{lI_{bc}} \right)^2} \frac{pl^2}{24}$$

$$x = \frac{1 + \frac{3hI_{cj}}{4lI_{bc}}}{1 + \frac{hI_{cj}}{lI_{bc}}} \frac{l}{2}$$

The manner in which the bending moment varies along the members composing the frames is indicated in Fig. 9 and 10.

9. *Single Story Construction with Three Spans.*—In the design of a beam-and-girder or a flat-slab construction of a single story, many engineers do not take the effect of the bending of columns on the moments in other portions of the structure into consideration. Authors also have tried to analyze the stress distribution without taking this bending into account. Obviously a bending in the columns will allow an increased bending moment at the center of the span of a girder or slab loaded unsymmetrically with respect to the column, and stresses in the slab will be modified by variations in the ratio of the moment of inertia of the girder or slab to that of the column and of the column

height to the span length. Tests have shown that columns may be subjected to severe bending, and it will be seen that this will be more important for a single story structure than for others.

In actual cases, there may be twenty or more spans in succession, with different span lengths and different cross-sections of members, and consequently an exact analysis is hardly possible with any assumption.

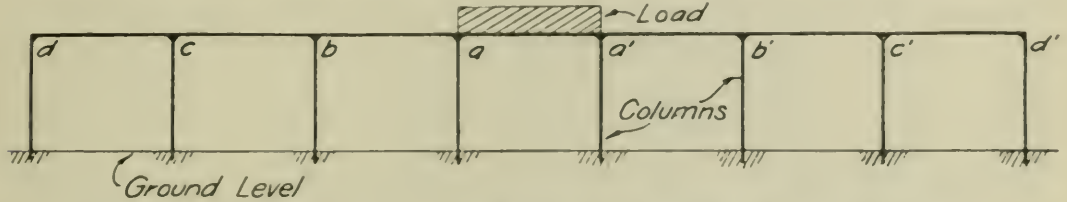


FIG. 11. CONTINUOUS SINGLE STORY SPANS; ONE PANEL LOADED

If panel aa' in Fig. 11 is loaded, the bending moments in slab bc , cd , $b'c'$, and $c'd'$ are so small as to be negligible in actual cases. It may be assumed, therefore, that the end condition of slabs or beams ab and $a'b'$ will, perhaps, be between the hinged and the fixed state at b and b' , the degree of fixity depending upon the ratio of moments of inertia of the column and the slab at that joint. Formulas will, therefore, be given for both conditions.

Fig. 12 shows the manner in which the bending moment varies along the members composing the frame for four combinations of end

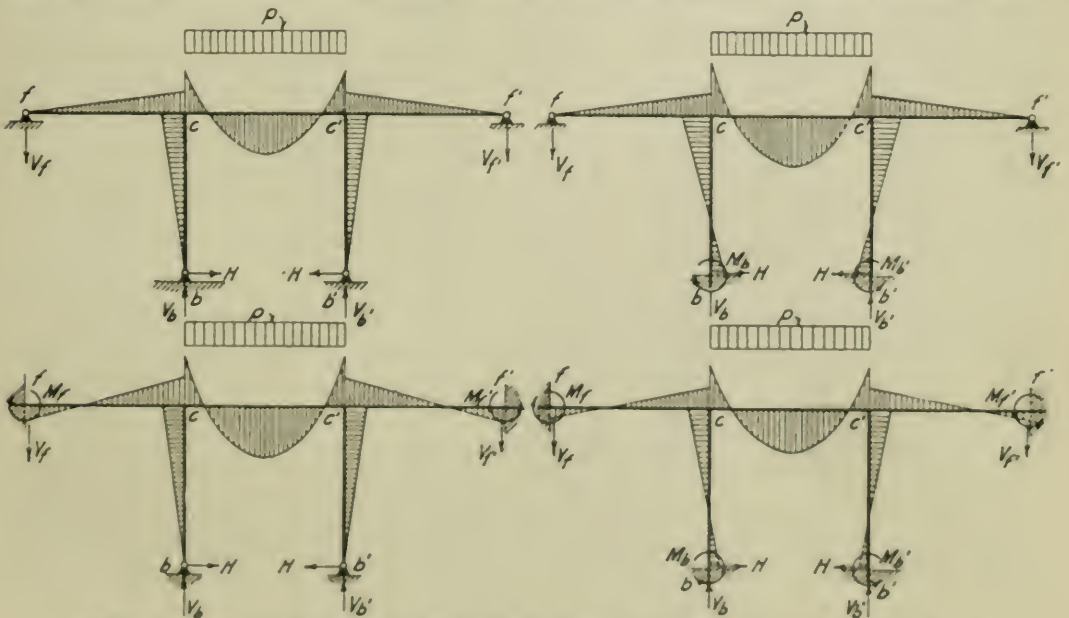


FIG. 12. SINGLE STORY THREE-SPAN FRAME, HAVING MIDDLE SPAN UNIFORMLY LOADED

conditions of beams and columns outside the loaded panel. The formulas which have been derived for these four cases are given in Table 3.

TABLE 3

FORMULAS FOR REACTIONS, BENDING MOMENTS, AND POINTS OF INFLECTION FOR SINGLE STORY THREE-SPAN FRAMES

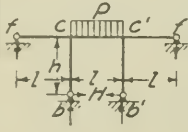
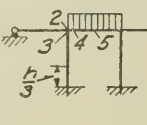
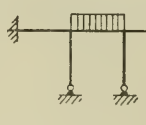
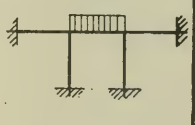
				
H Hor. reaction at col. ends	$\frac{1}{4n(3+5mn)}pl$	$\frac{1}{2n(4+5mn)}pl$	$\frac{1}{12n(1+2mn)}pl$	$\frac{1}{4n(2+3mn)}pl$
V_f Vert. reaction at beam ends	$\frac{mn}{4(3+5mn)}pl$	$\frac{mn}{4(4+5mn)}pl$	$\frac{mn}{6(1+2mn)}pl$	$\frac{mn}{4(2+3mn)}pl$
V_b Vert. reaction at col. ends	$\frac{6+11mn}{4(3+5mn)}pl$	$\frac{8+11mn}{4(4+5mn)}pl$	$\frac{3+7mn}{6(1+2mn)}pl$	$\frac{4+7mn}{4(2+3mn)}pl$
M_f Moment at beam ends	0	0	$\frac{mn}{(1+2mn)} \frac{pl^2}{18}$	$\frac{mn}{(2+3mn)} \frac{pl^2}{12}$
M_b Moment at column ends	0	$\frac{1}{(4+5mn)} \frac{pl^2}{6}$	0	$\frac{1}{(2+3mn)} \frac{pl^2}{12}$
M_2 Moment at point 2	$\frac{-mn}{(3+5mn)} \frac{pl^2}{4}$	$\frac{-mn}{(4+5mn)} \frac{pl^2}{4}$	$\frac{-mn}{(1+2mn)} \frac{pl^2}{9}$	$\frac{-mn}{(2+3mn)} \frac{pl^2}{6}$
M_3 Moment at point 3	$\frac{-1}{(3+5mn)} \frac{pl^2}{4}$	$\frac{-1}{(4+5mn)} \frac{pl^2}{3}$	$\frac{-1}{(1+2mn)} \frac{pl^2}{12}$	$\frac{-1}{(2+3mn)} \frac{pl^2}{6}$
M_4 Moment at point 4	$\frac{-(1+mn)}{(3+5mn)} \frac{pl^2}{4}$	$\frac{-(4+3mn)}{(4+5mn)} \frac{pl^2}{12}$	$\frac{-(3+4mn)}{3(1+2mn)} \frac{pl^2}{12}$	$\frac{-(1+mn)}{(2+3mn)} \frac{pl^2}{6}$
M_5 Moment at point 5	$\frac{(1+3mn)}{(3+5mn)} \frac{pl^2}{8}$	$\frac{(4+9mn)}{(4+5mn)} \frac{pl^2}{24}$	$\frac{(3+10mn)}{3(1+2mn)} \frac{pl^2}{24}$	$\frac{(2+5mn)}{(2+3mn)} \frac{pl^2}{24}$
Height of point of inflection in col.	0	$\frac{h}{3}$	0	$\frac{h}{3}$
Distance from column to point of inflect. in cent. span	$\frac{l}{2} \left[\frac{1 \pm \sqrt{1 - \frac{2(1+mn)}{(3+5mn)}}}{2} \right]$	$\frac{l}{2} \left[\frac{1 \pm \sqrt{1 - \frac{2(4+3mn)}{3(4+5mn)}}}{2} \right]$	$\frac{l}{2} \left[\frac{1 \pm \sqrt{1 - \frac{2(3+4mn)}{9(1+2mn)}}}{2} \right]$	$\frac{l}{2} \left[\frac{1 \pm \sqrt{1 - \frac{4(1+mn)}{3(2+3mn)}}}{2} \right]$
$m = \frac{I_{cc'}}{I_{bc}} \qquad n = \frac{h}{l}$				

TABLE 4

COEFFICIENTS OF BENDING MOMENT FOR SINGLE STORY THREE-SPAN FRAME

 α = coefficient of pl^2 for bending moment at end of middle span.

 β = coefficient of pl^2 for bending moment at top of column.

	Coefficient	Values of m $m = \frac{I_{cc'}}{I_{bc}}$	Values of n $n = \frac{h}{l}$					
			0.20	0.50	0.75	1.00	1.25	1.50
Extreme Ends of Columns and Beams Hinged	$\alpha = \frac{1+mn}{(3+5mn) \cdot 4}$	0.5	.0785	.0735	.0705	.0682	.0664	.0648
		1.0	.0750	.0682	.0648	.0625	.0608	.0595
		1.5	.0722	.0648	.0615	.0595	.0582	.0570
		2.0	.0700	.0625	.0595	.0577	.0564	.0555
		2.5	.0682	.0608	.0582	.0564	.0554	.0546
		3.0	.0667	.0595	.0570	.0556	.0546	.0540
	$\beta = \frac{1}{(3+5mn) \cdot 4}$	0.5	.0714	.0588	.0513	.0455	.0408	.0370
		1.0	.0625	.0455	.0370	.0312	.0270	.0238
		1.5	.0556	.0370	.0290	.0238	.0202	.0175
		2.0	.0500	.0312	.0238	.0192	.0161	.0139
		2.5	.0455	.0270	.0202	.0161	.0134	.0115
		3.0	.0417	.0238	.0175	.0139	.0115	.0098
Ext. Ends of Beams Hinged Lower Ends of Columns Fixed	$\alpha = \frac{4+3mn}{(4+5mn) \cdot 12}$	0.5	.0796	.0754	.0728	.0706	.0687	.0672
		1.0	.0767	.0706	.0672	.0647	.0630	.0616
		1.5	.0743	.0672	.0637	.0616	.0600	.0587
		2.0	.0723	.0648	.0616	.0595	.0581	.0572
		2.5	.0706	.0630	.0600	.0581	.0568	.0559
		3.0	.0690	.0616	.0588	.0571	.0559	.0550
	$\beta = \frac{1}{(4+5mn) \cdot 3}$	0.5	.0741	.0635	.0575	.0513	.0468	.0430
		1.0	.0667	.0513	.0430	.0371	.0326	.0290
		1.5	.0607	.0430	.0346	.0290	.0250	.0219
		2.0	.0555	.0371	.0290	.0238	.0202	.0175
		2.5	.0513	.0326	.0250	.0202	.0170	.0147
		3.0	.0477	.0290	.0219	.0175	.0147	.0126
Extreme Ends of Columns and Beams Fixed	$\alpha = \frac{1+mn}{(2+3mn) \cdot 6}$	0.5	.0798	.0757	.0732	.0714	.0700	.0686
		1.0	.0774	.0714	.0686	.0667	.0652	.0640
		1.5	.0745	.0686	.0658	.0640	.0628	.0619
		2.0	.0728	.0667	.0640	.0624	.0614	.0606
		2.5	.0714	.0652	.0626	.0614	.0604	.0598
		3.0	.0702	.0640	.0619	.0606	.0598	.0591
	$\beta = \frac{1}{(2+3mn) \cdot 6}$	0.5	.0695	.0556	.0476	.0416	.0370	.0333
		1.0	.0595	.0416	.0333	.0278	.0238	.0208
		1.5	.0521	.0333	.0256	.0208	.0175	.0151
		2.0	.0463	.0278	.0208	.0167	.0139	.0119
		2.5	.0417	.0238	.0175	.0139	.0115	.0098
		3.0	.0379	.0208	.0151	.0119	.0098	.0077

In Table 4 are given values of the bending moment coefficients for both hinged and fixed ends at top of column and at end of middle span for six ratios of moments of inertia.

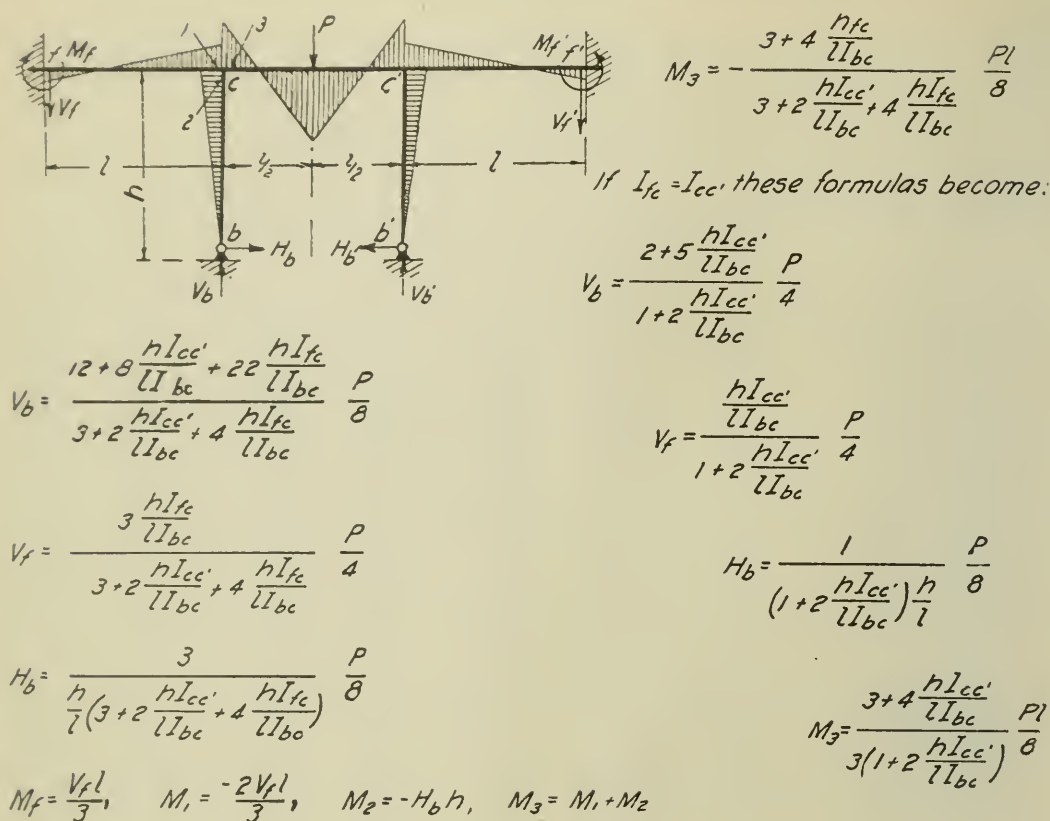


FIG. 13. SINGLE STORY THREE-SPAN FRAME UNDER CONCENTRATED LOAD

For a three-span frame with concentrated load at the center of the middle panel, Fig. 13 shows the manner in which the moment varies along the several members composing the frame.

Assuming symmetry about the vertical center line of this frame the formulas for the horizontal and vertical reactions are given in Fig. 13, and also formulas for bending moments at top of column and at end of middle span.

10. *Trestle Bent with Tie*.—The frame shown in Fig. 14, frequently termed the A-frame, may be used in trestle construction. Formulas for $M_{cc'}$, H_b , and H_c are given in the figure.

Knowing the values of $M_{cc'}$, H_c , and H_b , the stresses at any section of the frame may be computed. When $\theta = 0$, $I_{bc} = 0$, and l_{bc} approaches 0, making $h_{cd} = h$ (that is when $k = 1$), this frame becomes the same as that shown in Fig. 8 and the formulas for $M_{cc'}$ and H_c reduce to the same form as those given in Fig. 8.

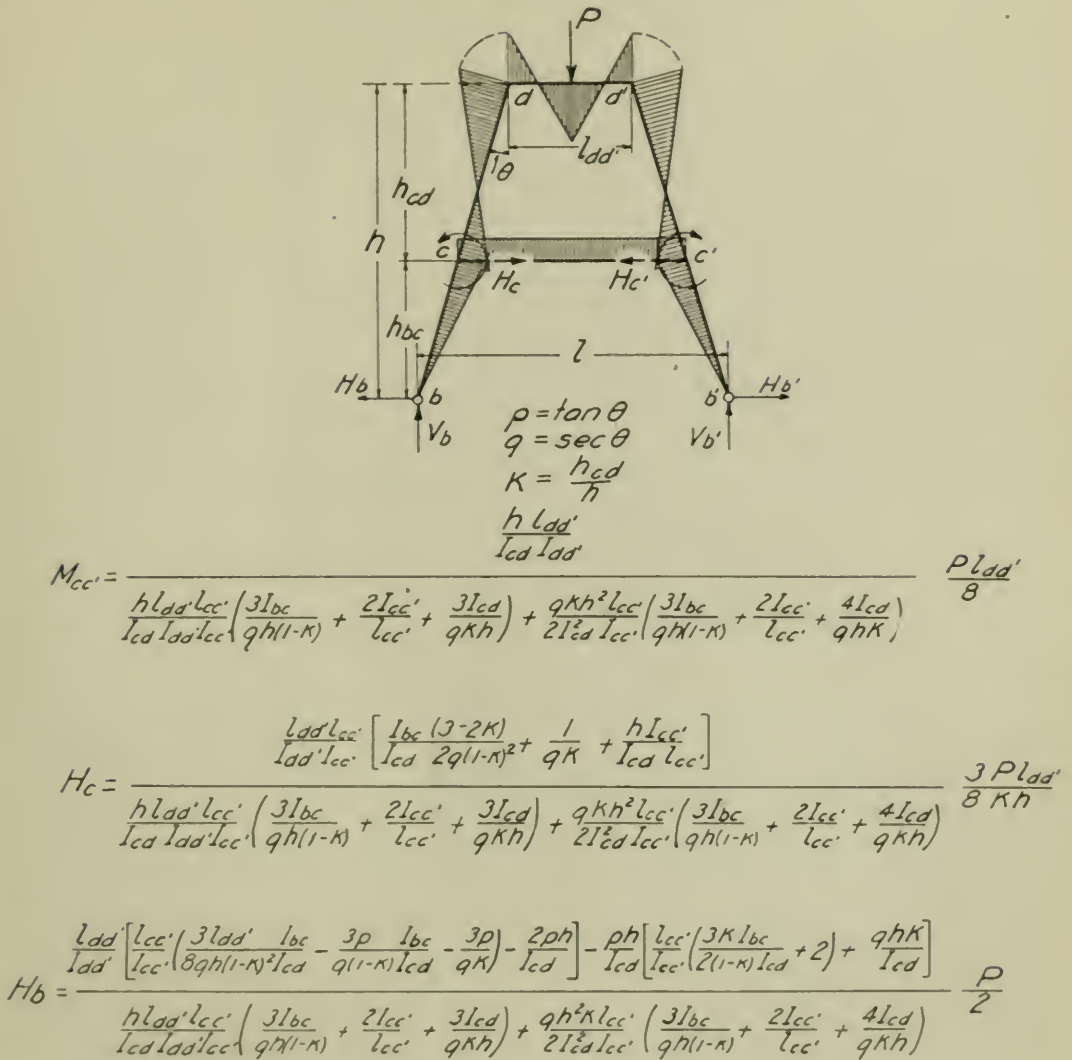


FIG. 14. A-FRAME SHOWING FORM OF MOMENT CURVE

11. *Building Construction with Several Stories and Spans.*—In the actual construction of buildings of reinforced concrete, it is common to use a continuous slab for floors supported by a number of columns. A loading which produces serious bending in columns is, of course, an eccentric arrangement, such as is shown in Fig. 15. The moments of inertia of columns are sometimes smaller than those of slabs. Accordingly, the bending moment in the floor slab $f'c'f'$ is greatly modified by the flexure of columns bc , $b'c'$, cd , and $c'd'$.

In present practice, frequently little attention is paid to this point, and columns are assumed to be rigid enough to resist the bending. This assumption may be approximately true for the lower stories,

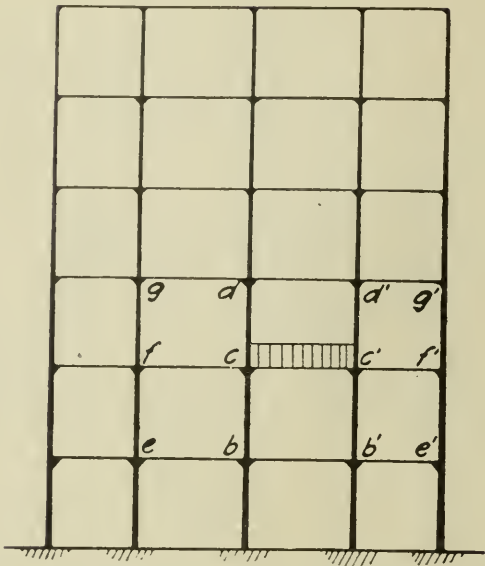


FIG. 15. CROSS-SECTION OF BUILDING FRAME WITH SINGLE PANEL LOADED

where the columns have large diameters, but it is not true for the upper stories, where the cross-section of columns is usually small, and serious bending stress may exist in the column due to eccentric loading. An exact analysis is hardly possible because there are many unknown conditions entering into the solution. From a practical standpoint, it is easily understood that the bending moment in floor slabs, $gdd'g'$ and $ebb'e'$, (see Fig. 15) due to the load on the floor cc' is so small as to be inconsiderable if the floors are of moderate thickness. That is, the columns cb , $c'b'$, cd , and $c'd'$ are practically fixed at b , b' , d , and d' , respectively. If the floor slabs are not thick enough to keep the

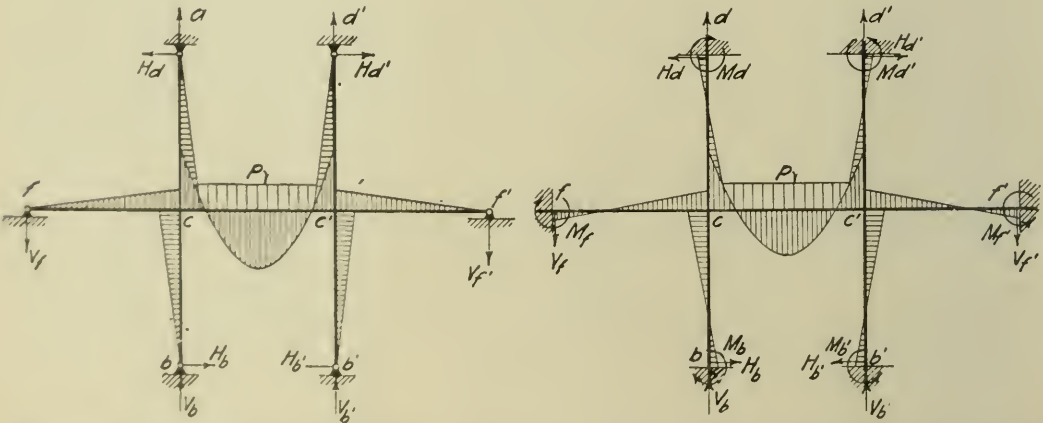


FIG. 16. TWO-STORY THREE-SPAN FRAME HAVING MIDDLE SPAN UNIFORMLY LOADED

column ends in a fixed condition, the end condition of the columns will be between the hinged and the fixed state. Using these assumptions an analysis which is almost exact is possible. The resulting formulas may be used in the design of buildings. For such a frame, Fig. 16 shows the manner in which the moment varies along the members composing the frame for two combinations of end conditions of beams and columns. The formulas for the horizontal and vertical reactions and the bending moments are given in Table 5 for ends of columns and beams hinged and for ends of columns and beams fixed, the end spans being equal in both cases. Formulas are also given for equal spans and equal moments of inertia of the beams and for equal spans, story heights, and moments of inertia of beams and columns. Fig. 17 gives numerical values of the coefficients of pl^2 for the case in which ends of beams and columns are fixed, α being the coefficient of the bending moment at the top of the lower column and β the coefficient at the foot of the upper column. In Table 6 are given values

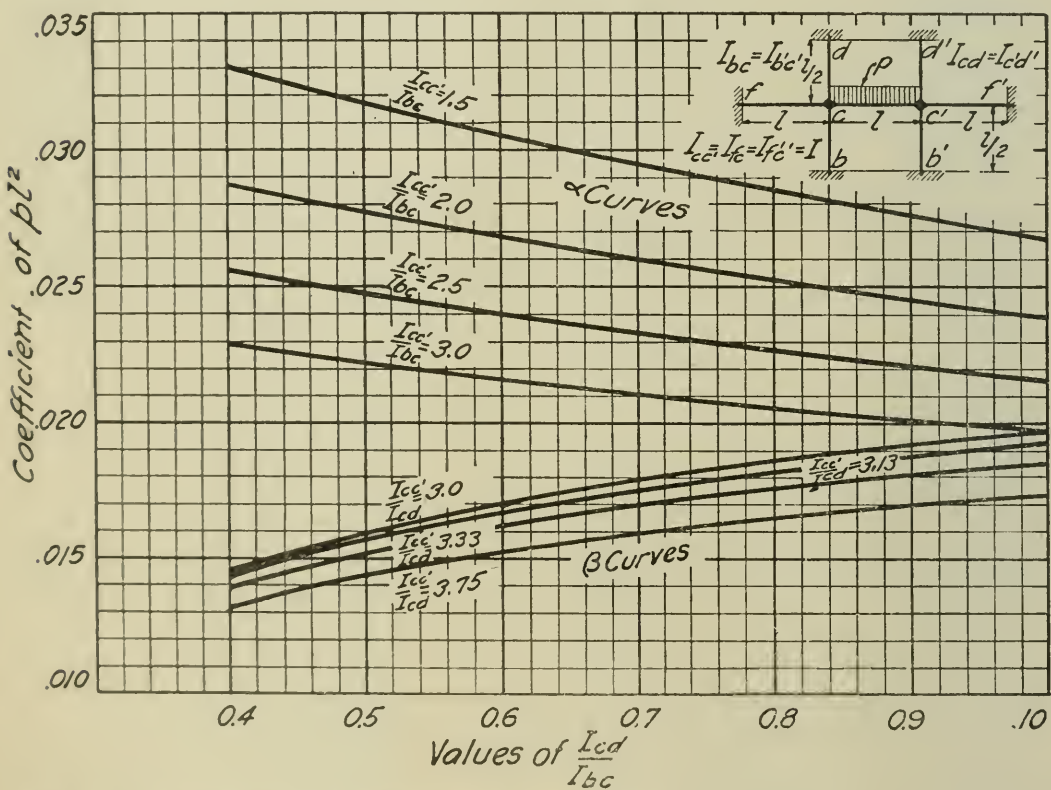


FIG. 17. COEFFICIENT OF BENDING MOMENTS FOR TOP OF LOWER COLUMN AND FOOT OF UPPER COLUMN FOR TWO-STORY THREE-SPAN FRAME HAVING ALL EXTERNAL CONNECTIONS FIXED

TABLE 6

COEFFICIENTS OF BENDING MOMENT FOR TWO-STORY THREE-SPAN FRAME WITH ENDS OF COLUMNS AND BEAMS FIXED

α = coefficient of pl^2 for bending moment at top of lower column.

β = coefficient of pl^2 for bending moment at foot of upper column.

	$\frac{I_{fc}}{I_{bc}}$	$\frac{I_{cd}}{I_{be}}$			
		0.4	0.6	0.8	1.0
α	1.5	.0330	.0306	.0285	.0267
	2.0	.0287	.0269	.0253	.0238
	2.5	.0255	.0240	.0227	.0215
	3.0	.0228	.0216	.0206	.0196
β	$\frac{I_{fc}}{I_{cd}}$				
	3.75	.0132	.0153	.0165	.0173
	3.33	.0139	.0162	.0175	.0185
	3.13	.0143	.0167	.0181	.0192
	3.00	.0145	.0170	.0185	.0196

of the bending moment coefficients α and β for four ratios of the moments of inertia. The frame with hinged ends has nine statically indeterminate quantities, while the frame with fixed ends has fifteen statically indeterminates, but the condition of symmetrical loading shown greatly reduces the number of these quantities. In the analyses it has been assumed that the vertical reactions at b and d (also at b' and d') are the same. This assumption may not be the real condition in actual cases, but no effect is produced on bending moments by it.

12. *Frame with Three Spans.*—In bridge or trestle construction across a wide stream or valley, several spans may be built continuously as a monolith. Because of the necessity of providing expansion joints, the number of spans thus connected is frequently limited to three.

Rigidly connected frames with three spans, equal or unequal, may advantageously be used for bridges of moderate spans.

No analytical formulas for such frames have, to the writer's knowledge, been published. Fig. 18 and 19 give the formulas for a three-span frame under various conditions of load. Fig. 20 shows the manner in which the moment varies along the members composing the frames. Table 7 gives values of the bending moment coefficients

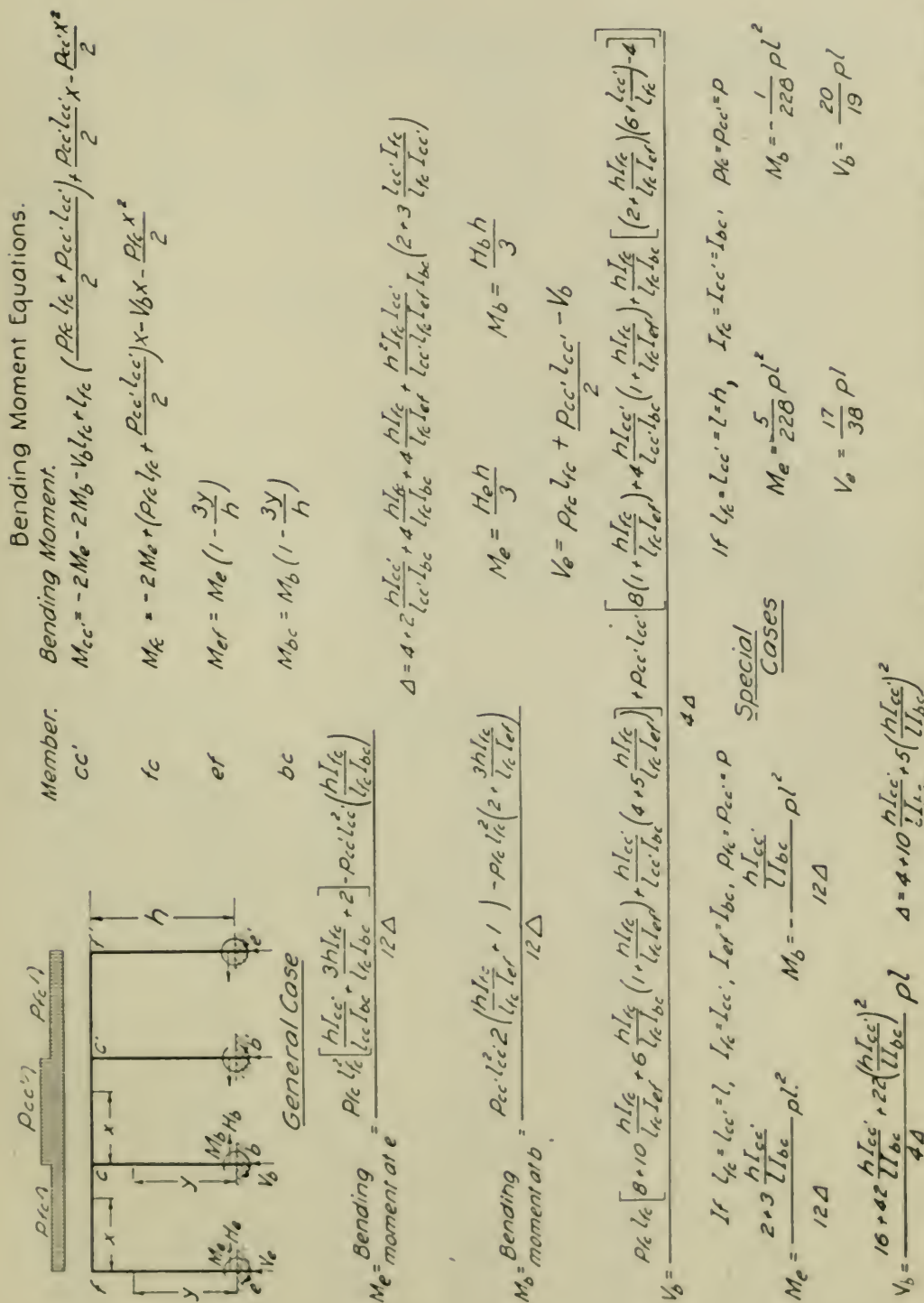


FIG. 19. FORMULAS FOR SINGLE STORY THREE-SPAN FRAME WITH FOUR COLUMNS HAVING COLUMNS FIXED AT LOWER END

$$H = \frac{k \left[3 + (3-k^2) \frac{h I_{cc'}}{l I_{bc}} \right] P}{2 \left[3 + 2 \frac{h I_{cc'}}{l I_{bc}} \right]}$$

$$l_i = \frac{3 + (3-k^2) \frac{h I_{cc'}}{l I_{bc}}}{2 \left[3 + 2 \frac{h I_{cc'}}{l I_{bc}} \right]} l$$

$$H = \frac{6 + 5 \frac{h I_{cc'}}{l I_{bc}}}{3 + 2 \frac{h I_{cc'}}{l I_{bc}}} \frac{ph}{8}$$

$$l_i = \frac{3}{4} \frac{2 + \frac{h I_{cc'}}{l I_{bc}}}{3 + 2 \frac{h I_{cc'}}{l I_{bc}}} l$$

$$M_a = - \frac{l_{aa}^3 + l_{ab}^3 \frac{I_{aa'}}{I_{ab}}}{l_{aa} + l_{ab} \frac{I_{aa'}}{I_{ab}}} \frac{p}{12}$$

$$M_a = - \frac{1 + k^3 \frac{I_{aa'}}{I_{ab}}}{1 + k \frac{I_{aa'}}{I_{ab}}} \frac{pl^2}{12}$$

FIG. 21. FORMULAS FOR RECTANGULAR FRAME UNDER HORIZONTAL LOAD AND FOR OCTAGONAL AND RECTANGULAR RESERVOIRS

at the top of the middle columns and at the end of the middle span for the case in which the three spans are equal, the moments of inertia of the beams are equal, and the moments of inertia of the columns are equal, a uniform load being applied over the middle span.

13. *Square Frame under Horizontal Load.*—Hitherto only the cases in which the load was applied vertically have been discussed. It is frequently necessary to solve for the statically indeterminate stresses due to a horizontal force, such as a wind pressure or the braking

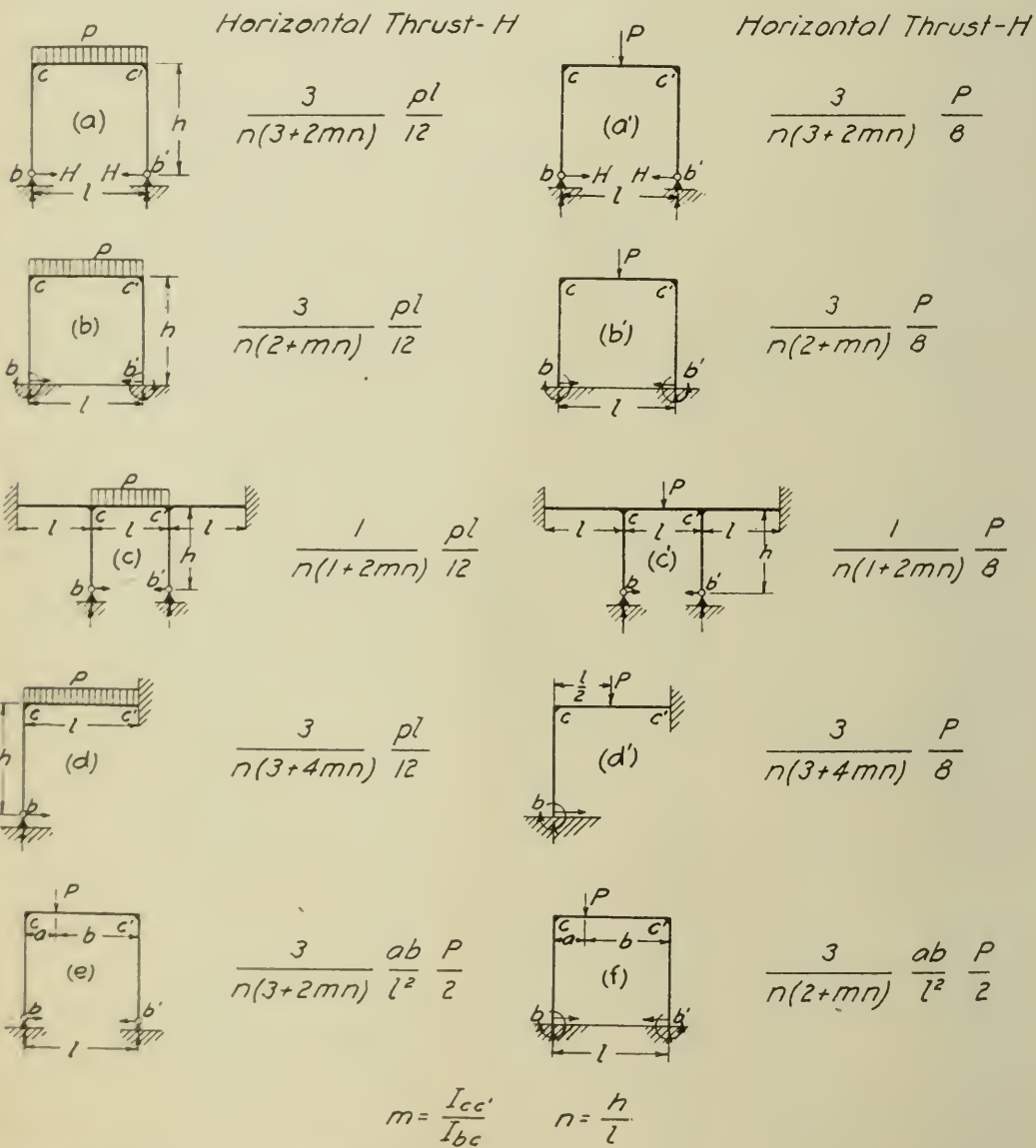


FIG. 22. HORIZONTAL REACTIONS FOR DISTRIBUTED LOADS AND CONCENTRATED LOADS

force of a locomotive. The method of determination of the statically unknowns is the same as that used for frames with vertical loads. A few cases have been taken as illustrations and the resulting equations are given in Fig. 21.

Another application is the water tank or reservoir subject to the static pressure of water, such as may be found in filter plants. Fig. 21 gives two examples of framed constructions of this character. It is seen that for a square tank having the same walls on the four sides the negative moment from the formula for rectangular tank becomes $\frac{1}{12} pl^2$ and the positive moment $\frac{1}{24} pl^2$, as is known from other sources.

14. *The Nature of the Resulting Formulas; Relation between Horizontal Reactions in Frame under Uniform Load and under Concentrated Load.*—It is interesting and important to note from the results of the foregoing analysis that there is a fixed relation between the horizontal reactions in the symmetrical frame under distributed loads and those in the same frame under concentrated loads. To show this relation a few cases have been selected as illustrative. These are shown in Fig. 22.

It has been stated previously that there is also a fixed relation between the horizontal thrust and the bending moment at the fixed column or beam ends, and the bending moment can be expressed in terms of the horizontal thrust. The bending moment at any section of a frame is a function of the horizontal thrust. Therefore, it may be stated that the statically indeterminate stresses in the *symmetrical* frame have a fixed relation under distributed and concentrated loads.

From the foregoing illustrations it will be seen that the horizontal thrusts at the column ends due to uniform and centrally concentrated loads may be expressed in the following forms:

$$\text{Uniform load,} \quad H = K \frac{pl^2}{12}$$

$$\text{Centrally concentrated load, } H = K \frac{Pl}{8}$$

The coefficient K is the same in both cases, but varies with the form of frame. The formula for the horizontal thrust in the frame under concentrated load may be written directly if the formula for thrust

in the frame under uniform load is known. An analysis of the statically indeterminate forces for a given case should first be made to find the form of the function.

The bending moment at the end of the span in these frames is:

$$\text{For Case a,} \quad M = \frac{3}{3+2mn} \frac{pl^2}{12} = K_1 \frac{pl^2}{12}$$

$$\text{For Case a',} \quad M = \frac{3}{3+2mn} \frac{Pl}{8} = K_1 \frac{Pl}{8}$$

$$\text{For Case b,} \quad M = \frac{2}{2+mn} \frac{pl^2}{12} = K_2 \frac{pl^2}{12}$$

$$\text{For Case b',} \quad M = \frac{2}{2+mn} \frac{Pl}{8} = K_2 \frac{Pl}{8}$$

It is known that when a beam is perfectly fixed at its ends the negative bending moments due to a distributed load and a centrally concentrated load are $\frac{pl^2}{12}$ and $\frac{Pl}{8}$, respectively. It is seen, therefore, that for these cases the bending moment at the end of the beam is obtained from the value of the end bending moment of a fixed beam by multiplying by K , a coefficient which depends upon the form of the frame, but is independent of whether the load is applied uniformly or is concentrated at the center of the span.

Returning to the nature of the formulas for the horizontal thrust at the lower column end of a frame, it is further seen from Fig. 22 that the given constant relation between the values of the horizontal thrusts for a frame under a distributed load and under a concentrated load still holds for the case in which a frame is subjected to a non-symmetrical load. These simple frames are sufficient to illustrate the general relation. It appears, therefore, that for the same frame the coefficient K remains constant and independent of the method of loading. This statement can easily be extended to the case of multiple concentrated loads, for then the horizontal thrust is the sum of the horizontal thrusts due to the individual concentrated loads. It will be found that this statement applies also to the non-symmetrical frames of Cases d and d' .

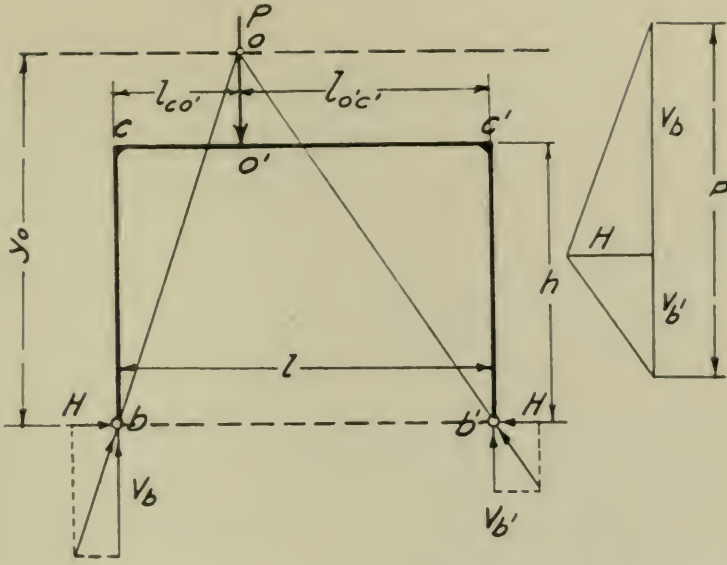


FIG. 23. LOCUS OF INTERSECTION OF REACTION LINES IN SINGLE SPAN SINGLE STORY FRAME UNDER SINGLE CONCENTRATED LOAD

For a concentrated load, it will be of interest to find the locus of y_o , the point of intersection of the lines of action of the reactions with the line of action of the load (see Fig. 23). In a complicated form of frame, there are, of course, many statically indeterminate quantities, but H is an important one. The remaining statically indeterminates have always the same factor in the denominator as H . Therefore, it is very interesting to know the form of the expression for H . It is evident that H (Fig. 23) is a function of l , h , I , and P in a given case.

Since the moments at b and b' (Fig. 23) are zero, the equilibrium polygon for the load P must pass through these points. Taking the moments of H and V_b about the point o

$$V_b l_{co'} - H y_o = 0 \quad y_o = \frac{V_b l_{co'}}{H}$$

H and V_b are known in this case when P and $l_{co'}$ are given, and

$$H = \frac{\left(\frac{l}{h}\right)^2}{2\left(\frac{2I_{cc'}}{3I_{bc}} + \frac{l}{h}\right)} \cdot \frac{l_{co'} l_{o'c'}}{l^2} P$$

$$V_b = \frac{l_{o'c'}}{l} P$$

Therefore

$$y_o = \frac{l_{co} l_{o'c'}}{l} \frac{P}{H} = \frac{2 \left(\frac{2I_{cc'}}{3I_{bc}} + \frac{l}{h} \right) l}{\left(\frac{l}{h} \right)^2}$$

or

$$y_o = 2h \left(1 + \frac{2hI_{cc'}}{3lI_{bc}} \right)$$

This equation is entirely free from l_{co} , $l_{o'c'}$, and P ; therefore, y_o is a constant quantity for a given frame and is not changed by the change of the point of application of a load P . Accordingly the locus of the point o is a straight line parallel to bb' .

In the case in which $\frac{I_{cc'}}{I_{bc}} = 1.0$ and $\frac{h}{l} = 1.0$, $y_o = \frac{10h}{3}$. For $\frac{I_{cc'}}{I_{bc}} = 2.0$ and $\frac{h}{l} = 1.0$, $y_o = \frac{14h}{3}$.

The equation for y_o permits the determination of the position of loads which gives the maximum reaction and stress in any member. The same method may be extended to any case, if it is remembered that when a column is fixed at its end the point of application of the reaction deviates from the neutral line of the column by $\frac{M_b}{V_b}$, where M_b is the end moment and V_b is the vertical reaction at that point.

15. *Effect of Variation in Moment of Inertia and Relative Height of Frame on Bending Moment in Horizontal Member.*—Fig. 24, 25, 26, 27, and 28 give bending moment coefficients for the beam of the central span for several cases of a three-span frame in which the spans are equal, the moments of inertia of the three beams are equal, and the moments of inertia of the columns are equal. The effect on the bending moment caused by variation in the relative values of the moments of inertia of members and in heights of frames is shown in these figures.

For a general comparison it is only necessary to consider the bending moment at the center of the span for load applied eccentrically with respect to the columns, since the effect on moments at other places

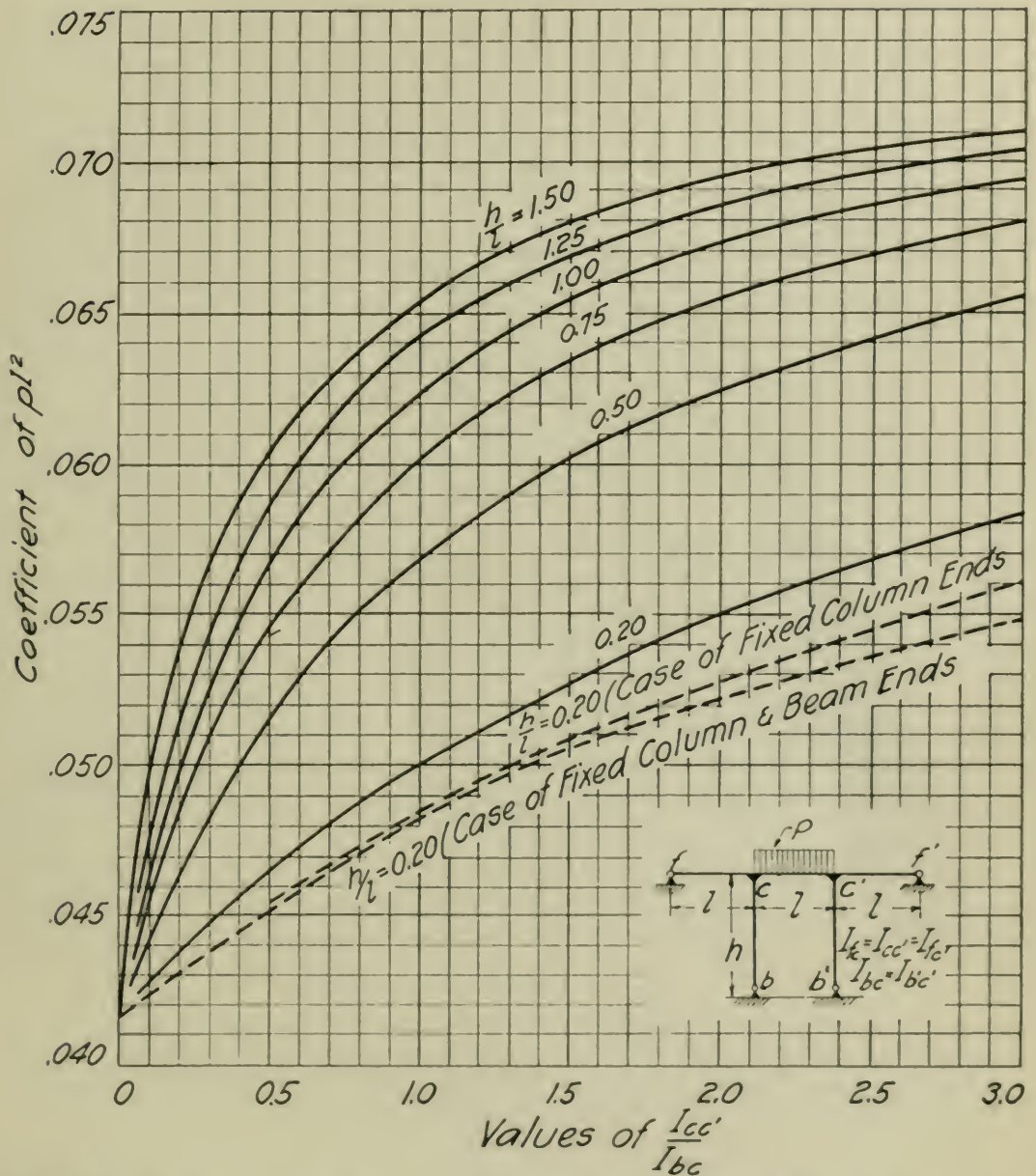


FIG. 24. COEFFICIENT OF BENDING MOMENT AT CENTER OF MIDDLE SPAN FOR SINGLE STORY THREE-SPAN FRAME HAVING EXTREME ENDS OF COLUMNS AND BEAMS HINGED

and on thrusts will be similar. From the general nature of the curves shown the following conclusions are drawn:

- (1) The bending moment is increased rapidly as the value of $\frac{I_{cc'}}{I_{bc}}$ increases from 0 to 1.5, but beyond that range the increase in bending moment is comparatively small.

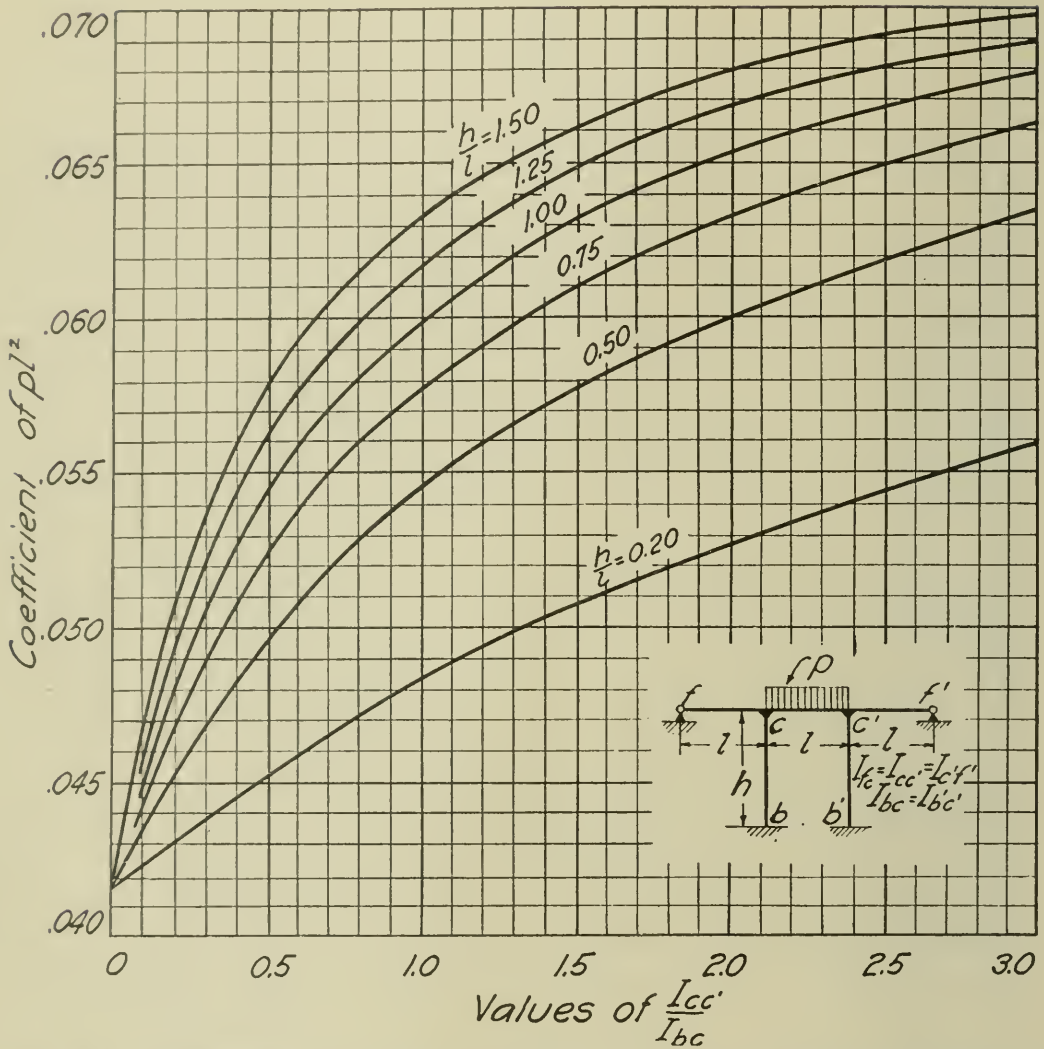


FIG. 25. COEFFICIENT OF BENDING MOMENT AT CENTER OF MIDDLE SPAN FOR SINGLE STORY THREE-SPAN FRAME HAVING EXTREME ENDS OF BEAMS HINGED AND COLUMN ENDS FIXED

(2) An increase in the height of the frame has an effect of the same nature on the bending moment as an increase in the ratio $\frac{I_{cc'}}{I_{bc}}$.

(3) The variation in coefficient of bending moment is wider in the frame hinged at ends of columns and beams than in the case of fixed ends.

(4) By the fixing of ends of columns and end beams the coefficient of positive bending moment is slightly decreased from that for hinged ends.

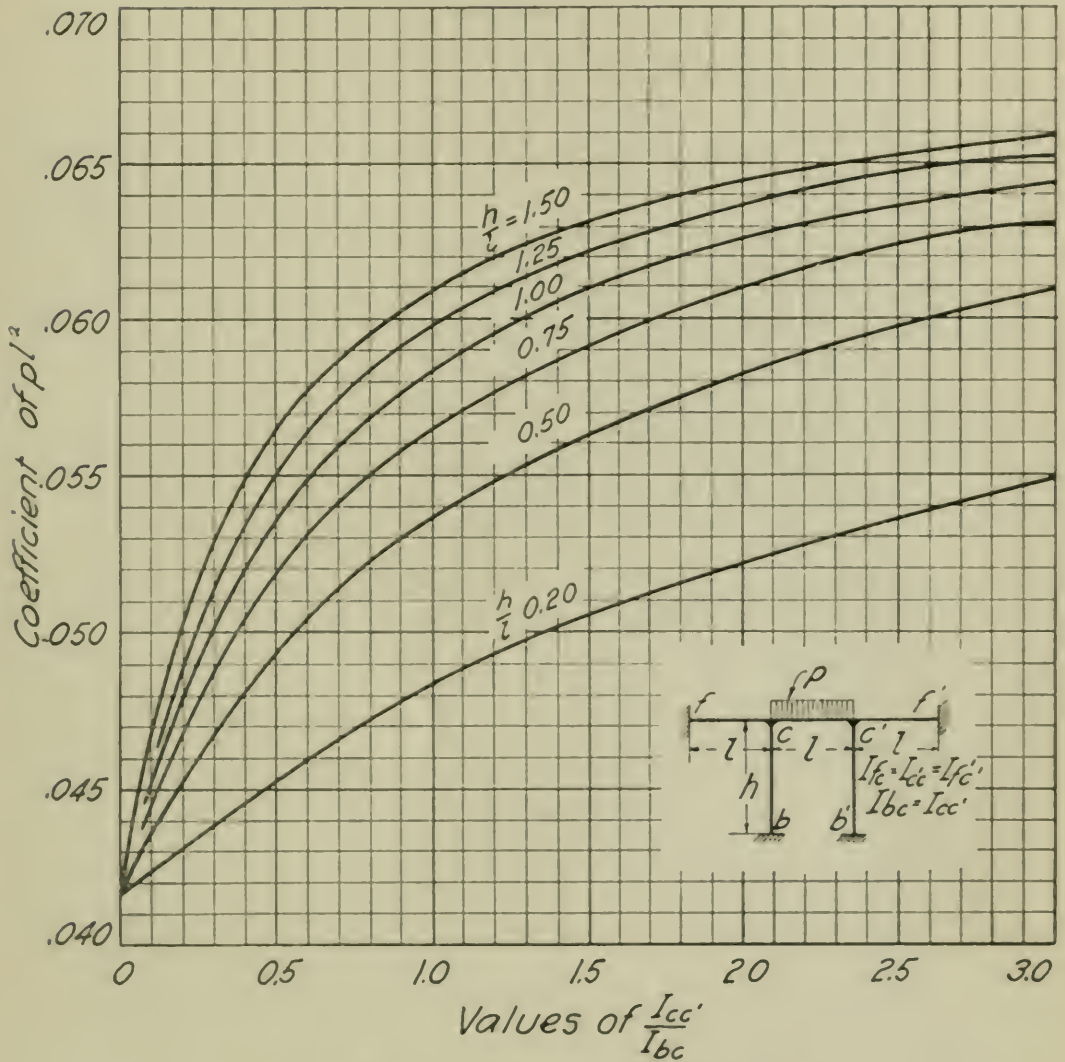


FIG. 26. COEFFICIENT OF BENDING MOMENT AT CENTER OF MIDDLE SPAN FOR SINGLE STORY THREE-SPAN FRAME HAVING ALL EXTERNAL CONNECTIONS FIXED

(5) In most common cases of panels under uniform load where the ratio $\frac{h}{l}$ is not far from 1.0 and $\frac{I_{cc'}}{I_{bc}}$ varies from 1.5 to 3.0, the bending moment at the center of the loaded span (case of equal spans) varies from about $\frac{1}{16}pl^2$ to about $\frac{1}{14}pl^2$, and may be conveniently assumed as $\frac{1}{15}pl^2$.

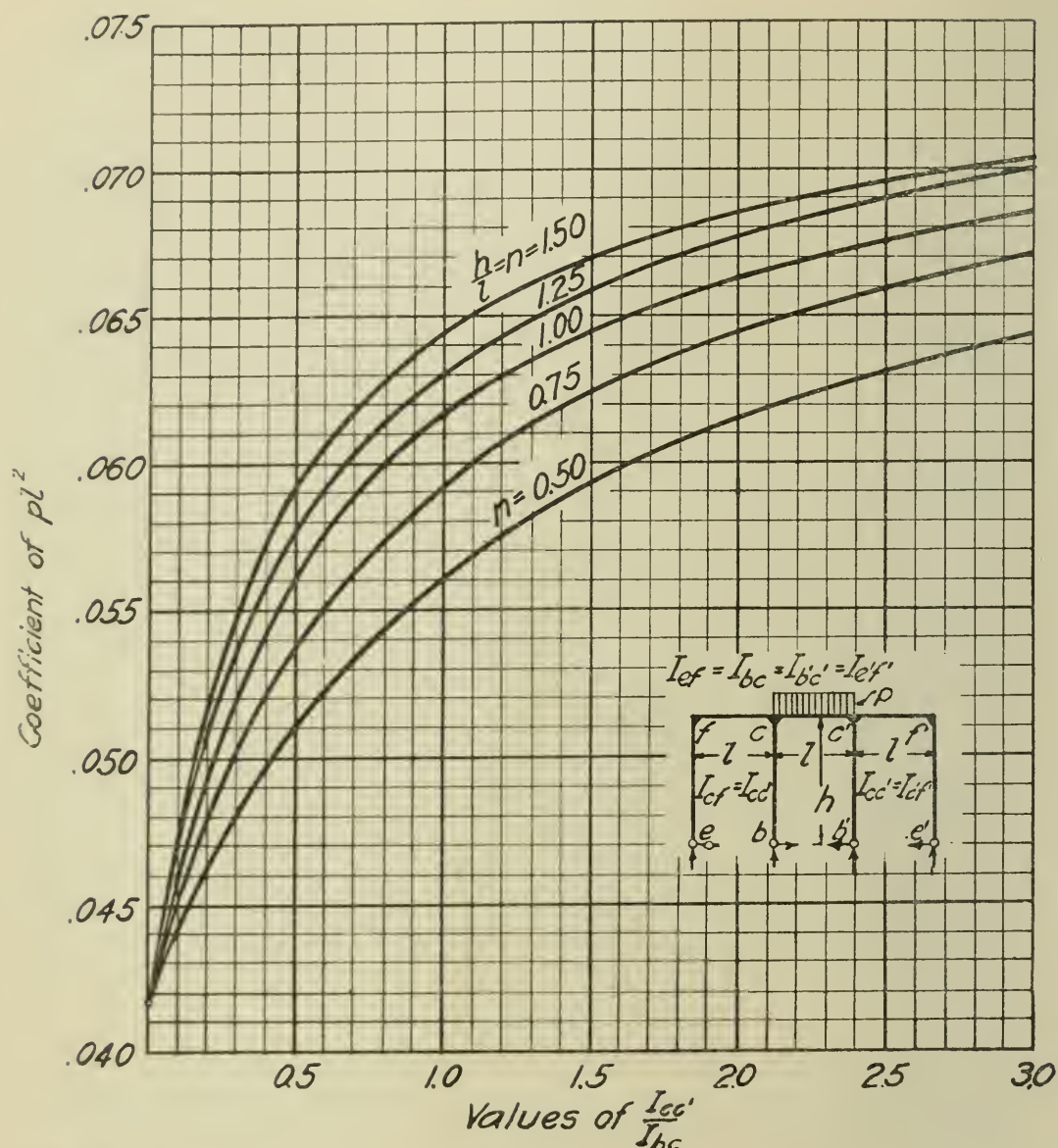


FIG. 27. COEFFICIENT OF BENDING MOMENT AT CENTER OF MIDDLE SPAN FOR SINGLE STORY THREE-SPAN FRAME HAVING LOWER ENDS OF COLUMNS HINGED

16. *Effect of Variation in Moment of Inertia on Bending Moment in Vertical Member.*—The variation in bending moments in column ends due to the variation in properties of the members for several cases of a three-span frame is shown in Fig. 17 and 29 and in Tables 4, 6, and 7. In Table 6 the three spans are taken as equal and the story height is taken equal to half the span.

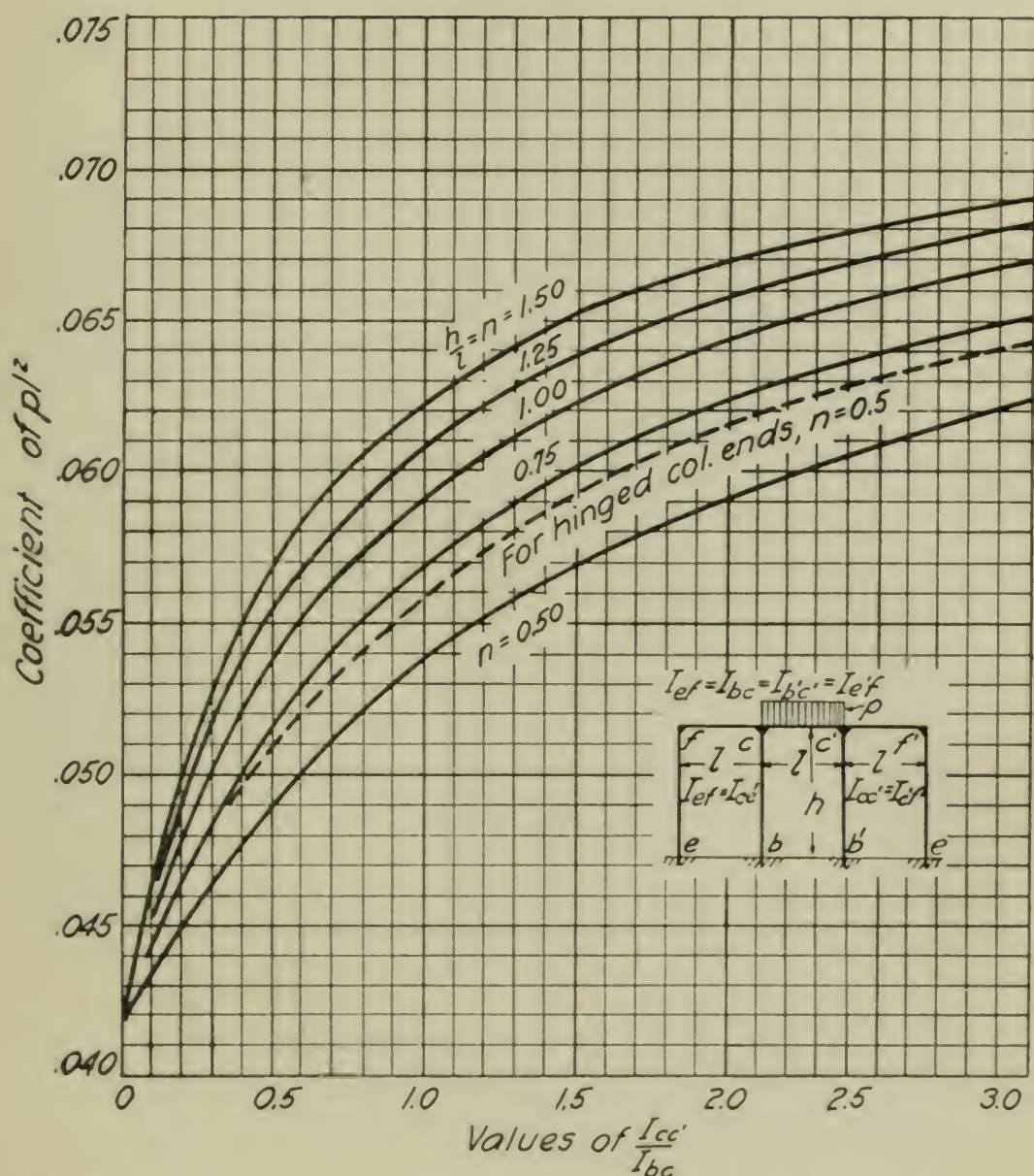


FIG. 28. COEFFICIENT OF BENDING MOMENT AT CENTER OF MIDDLE SPAN FOR SINGLE STORY THREE-SPAN FRAME HAVING LOWER ENDS OF COLUMNS FIXED

Values of coefficients of pl^2 for various values of $\frac{I_{cc'}}{I_{bc}}$ and $\frac{I_{cd}}{I_{bc}}$ are plotted in Fig. 17. It is seen from the diagram that for structures having the relations between span lengths and moments of inertia assumed in Table 6, higher bending stress will exist at the top of the

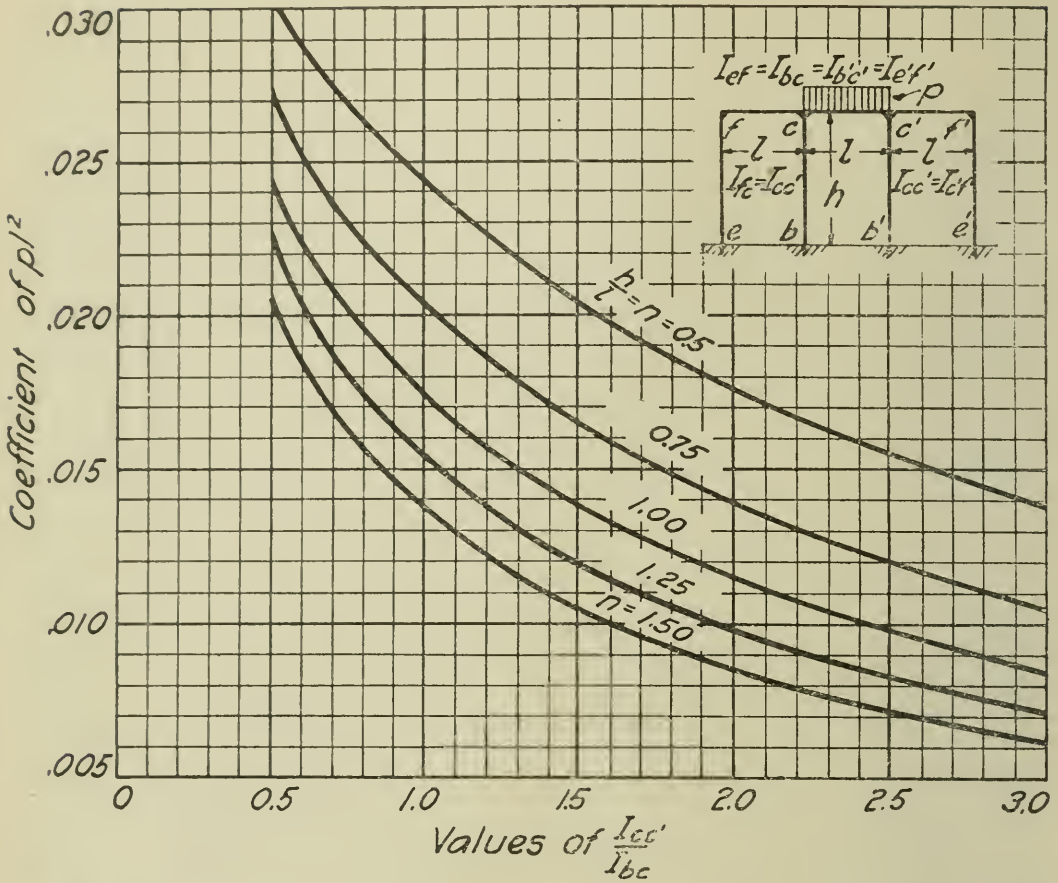


FIG. 29. COEFFICIENT OF BENDING MOMENT AT LOWER ENDS OF INTERIOR COLUMNS FOR SINGLE STORY THREE-SPAN FRAME HAVING LOWER ENDS OF COLUMNS FIXED

lower column than at the foot of the upper column and that the variations in moments of inertia assumed cause less variation in the moment in the upper columns than in the lower columns.

III. TESTS ON RIGIDLY CONNECTED REINFORCED CONCRETE FRAMES

17. *Test Specimens.*—Five types of frames were selected for the tests. The cross-section of the composing members varied from 8 by 8 in. to $8\frac{1}{2}$ by $17\frac{3}{8}$ in. The length of span of the frames was 6 ft. on centers except Frame No. 8, which had three spans of 4 ft. 8 in. The height of the frames varied from about 5 ft. to about 10 ft. The size and disposition of the reinforcing bars and the dimensions of the frames are shown in Fig. 30 to 34. Data of the frames are given in Table 8.

Care was taken in designing the test specimens to secure continuity of connected members and to obtain such proportions between moments of inertia and spans as would result in high bending stresses in the columns and beams at nearly the same time. The ends of the steel reinforcing bars were bent into hooks in the specimens having columns fixed at the ends. Bars continuous from one end to another were used for all frames. The radius of bends of the main rods was about 5 in. Several bars were welded and these welds were located at points where the bending moment was very small.

In the frames with stirrups, U-shaped or double U-shaped stirrups were used. They passed under the longitudinal bars and extended to the top of the beam. The size and spacing of the stirrups are given in Table 8.

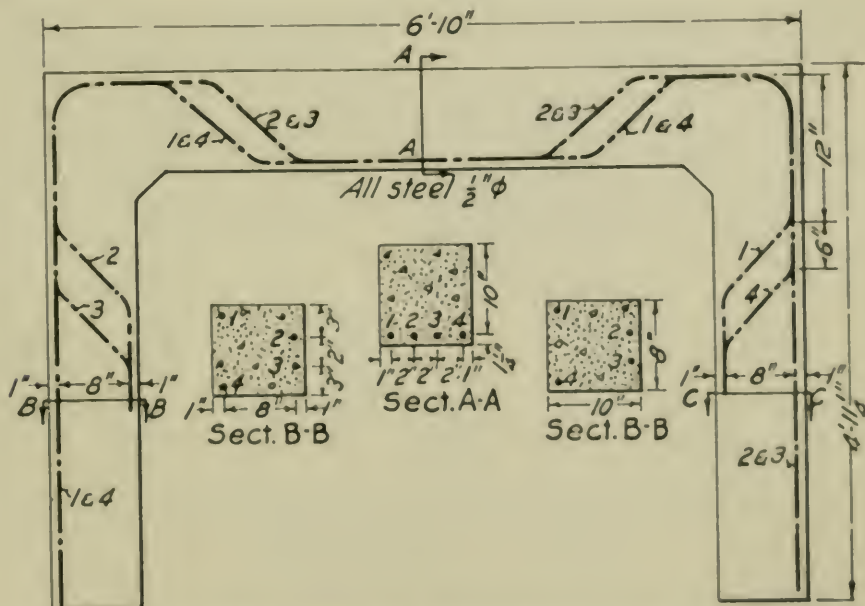
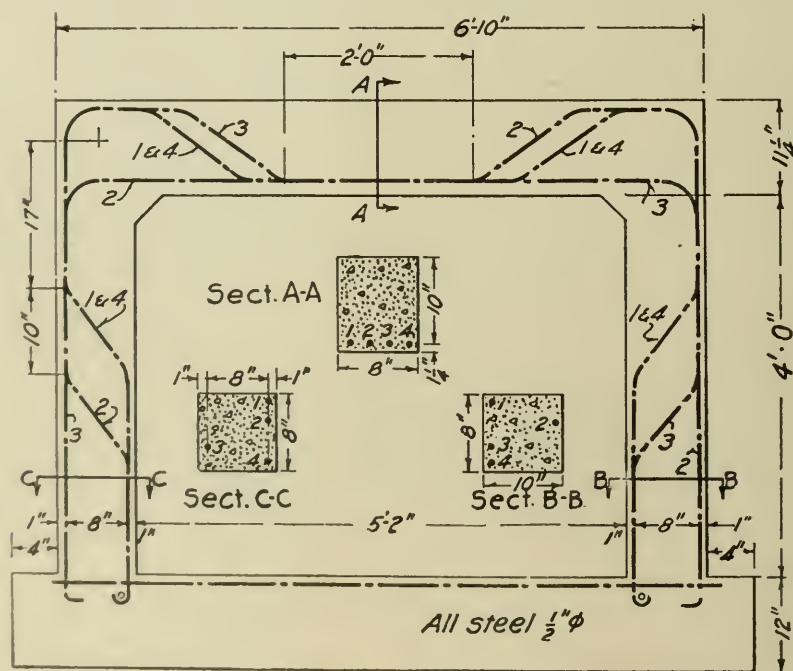
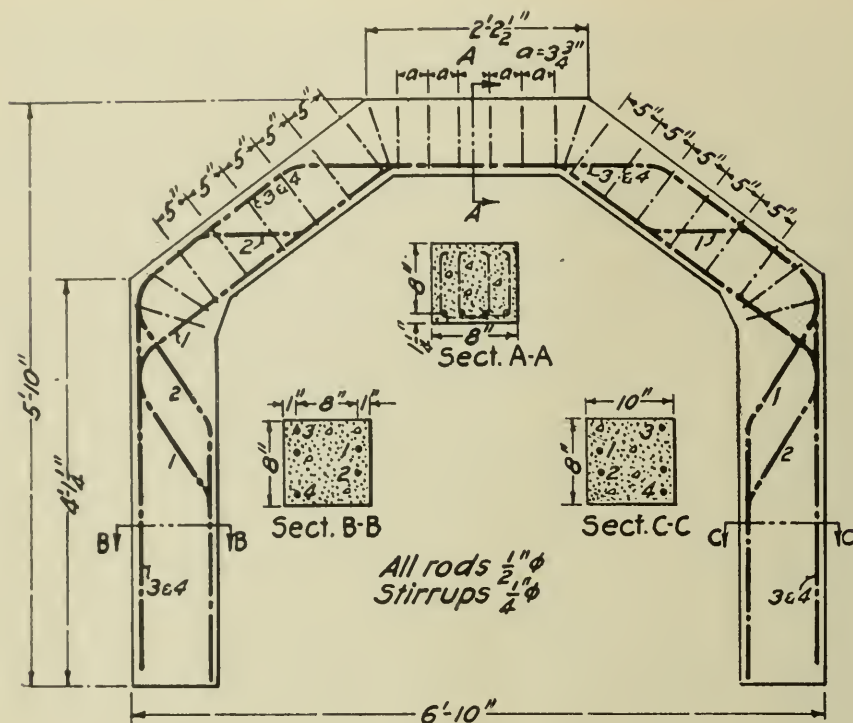


FIG. 30. SIZE AND DISTRIBUTION OF REINFORCING BARS IN FRAMES 1 AND 6



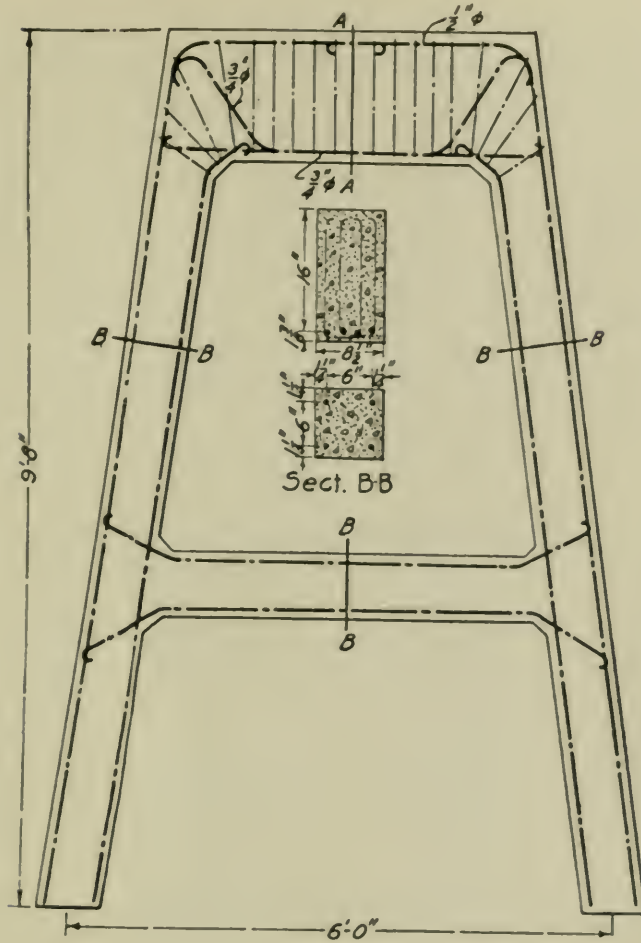


FIG. 33. SIZE AND DISTRIBUTION OF REINFORCING BARS IN FRAME 5

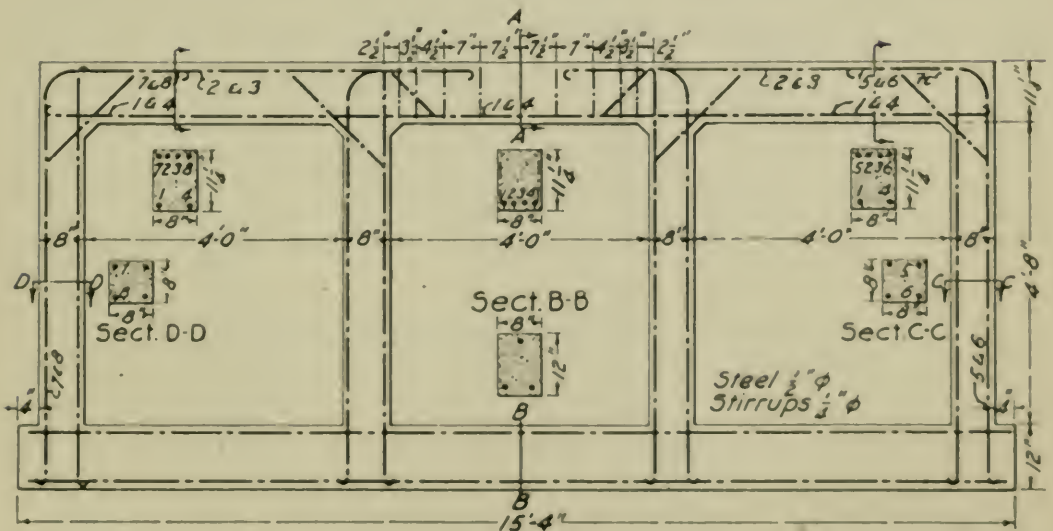


FIG. 34. SIZE AND DISTRIBUTION OF REINFORCING BARS IN FRAME 8

TABLE 8
DATA OF TEST FRAMES

Frame No.	Span Length c. to c.		Cross-Section		Longitudinal Reinforcement			Stirrups		Nominal Height of Frame ft.-in.
	At Top ft.-in.	At Bottom ft.-in.	Column in. x in.	Beam in. x in.	Column (top)		Beam (center)			
					Description	Per cent	Description	Per cent		
1	6-0	6-0	8 x 10	8 x 11 1/4	3-1/2 in.	0.82	4-1/2 in.	0.98	None	4-6
2	6-0	6-0	8 x 10	8 x 9 1/4	3-1/2 in.	0.82	4-1/2 in.	1.23	1/4-in.	6-0
3	6-0	6-0	8 x 10	8 x 11 1/4	4-1/2 in.	1.09	4-1/2 in.	0.98	None	4-6
4	6-0	6-0	8 x 10	8 x 9 1/4	4-1/2 in.	0.82	4-1/2 in.	1.23	1/4-in.	6-0
5	3-6	6-0	8 1/2 x 8 1/2	8 1/2 x 17 3/8	4-1/2 in.	1.28	4-3/4 in.	1.30	1/4-in.	9-0
6	6-0	6-0	8 x 10	8 x 11 1/4	3-1/2 in.	0.82	4-1/2 in.	0.98	None	4-6
7	6-0	6-0	8 x 10	8 x 11 1/4	4-1/2 in.	1.09	4-1/2 in.	0.98	None	4-6
8	4-8 2	4-8 2	8 x 8	8 x 11 1/4	4-1/2 in.	1.40	4-1/2 in.	0.98	1/4-in.	5-2 1/4

¹ Double loop.

² Three equal spans.

³ Only used in the middle span.

All reinforcement of plain round bars.

All concrete of 1-2-4 mix.

18. *Materials.*—The materials used in making the test frames were similar to those ordinarily used in reinforced concrete construction. The sand, stone, and cement were taken from the stock of the Laboratory of Applied Mechanics.

A good quality of crushed limestone ordered to pass over a $\frac{1}{4}$ -inch sieve and through a 1-inch sieve was used. The sand was of good quality, hard, sharp, well-graded, and generally clean.

The reinforcing bars were plain round rods of open hearth mild steel. Test pieces were taken from the test frames after the test. Table 9 gives the results of the tension tests of the steel.

TABLE 9
TENSION TESTS OF REINFORCING STEEL

Nominal Diameter inches	Yield Point lb. per sq. in.	Ultimate Strength lb. per sq. in.	Per Cent Elongation in 8 in.
$\frac{1}{2}$	36 200	55 100	26.3
$\frac{1}{2}$	36 200	54 200	26.9
$\frac{1}{2}$	36 900	54 700	28.7
$\frac{1}{2}$	36 700	54 600	26.9
$\frac{1}{2}$	37 700	54 200	25.0
$\frac{1}{2}$	37 400	55 900	30.0
Average	36 850	54 783	27.3

Universal Portland cement was used for all specimens. Standard briquettes of neat cement gave an average tensile strength of 575 lb. per sq. in. at 7 days and 670 lb. per sq. in. at 28 days, and standard briquettes of 1-3 mortar 207 lb. per sq. in. at 7 days and 303 lb. per sq. in. at 28 days. Briquettes of 1-3 mortar made with the sand used in the concrete gave a strength of 279 lb. per sq. in. at 7 days and 353 lb. per sq. in. at 28 days. Tests with the Vicat needle indicated that initial set occurred in 3 hours and 15 minutes and final set in 6 hours.

Men skilled in this kind of work were employed in making the concrete. Care was taken in measuring, mixing, and tamping to secure concrete as nearly uniform as possible. All the concrete was made in the proportions, 1 part cement, 2 parts sand, and 4 parts stone, by volume. The mixing was done with a concrete mixing machine.

The results of compression tests on 6-in. cubes made from the concrete used in the frames are given in Table 10. Tests were made

TABLE 10

COMPRESSION TESTS OF CONCRETE CUBES AND CYLINDERS

Frame No.	Age at Test days	Maximum Load lb. per sq. in.		Frame No.	Age at Test days	Maximum Load lb. per sq. in.	
		6 in. Cube	8x16 in. Cyl.			6 in. Cube	8x16 in. Cyl.
1	64	1780	1150	5	61	3070	2670
1	64	1750		5	61	3100	
1	64	1680		5	61	2580	
Average		1740		Average		2920	
2	62	2210	1850	6	62	2605	2310
2	62	2250		6	62	2445	
2	62	2540		6	62	2510	
Average		2330		Average		2520	
3	73	2860	2050	7	60	2140	1970
3	73	2820		7	60	2390	
3	73	2840		7	60	2220	
Average		2840		Average		2250	
4	66	2600	1910	8	63	3288	3060
4	66	2580		8	63	3900	
4	66	2570		8	63	3653	
Average		2580		Average		3614	

on one 8 by 16-in. cylinder for each frame, and the axial deformation was measured to give a means of judging of the modulus of elasticity of the concrete used in the frames. Fig. 35 gives the stress-deformation diagrams for these cylinders. Table 10 gives the compressive strength of the cylinders.

19. *Making and Storage of Test Frames.*—It had been hoped to make the frames in a vertical position similar to that in practice, but because of the difficulty and added expense in doing this, all the frames were built directly on the concrete floor of the laboratory in a horizontal position with a strip of building paper beneath the forms.

The forms were generally removed after seven days, and the frames were lifted from the horizontal position after thirty days and were kept in a vertical position in the laboratory where they were made until the day they were tested. They were dampened every morning for two weeks after making to prevent too rapid drying, and were

dampened occasionally after that time. The temperature of the room ranged from 55 to 70 degrees F.

20. *Testing.*—To develop high stresses in the beam and in the columns nearly at the same time one-third point loadings were used for many of the frames. In Frame 5 the centrally concentrated load was used to develop as high a flexural stress in the columns as possible.

In Frame 8 in order to see the effect of the eccentric load on the adjacent spans and at the same time to produce high bending stresses in the middle beam and in the central columns, a uniform load on the middle span was selected, the load being applied through a number of spiral springs.

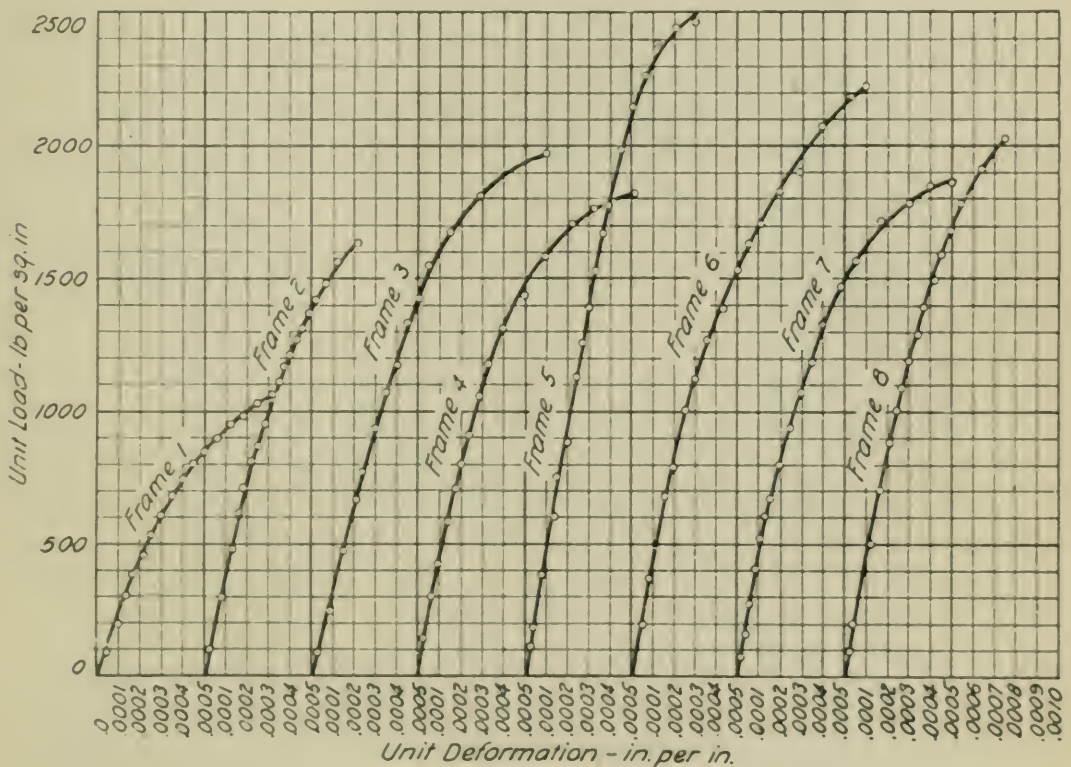


FIG. 35. STRESS-DEFORMATION DIAGRAMS FOR CYLINDERS

The positions of the loads for the different frames are shown in Fig. 36 to 43. The specimens were tested in the 600,000-lb. Riehle testing machine in the Laboratory of Applied Mechanics of the University of Illinois. Deflections were read on some of the frames. The deformations of the steel and of the concrete were measured at the various parts of the frames for each load applied.

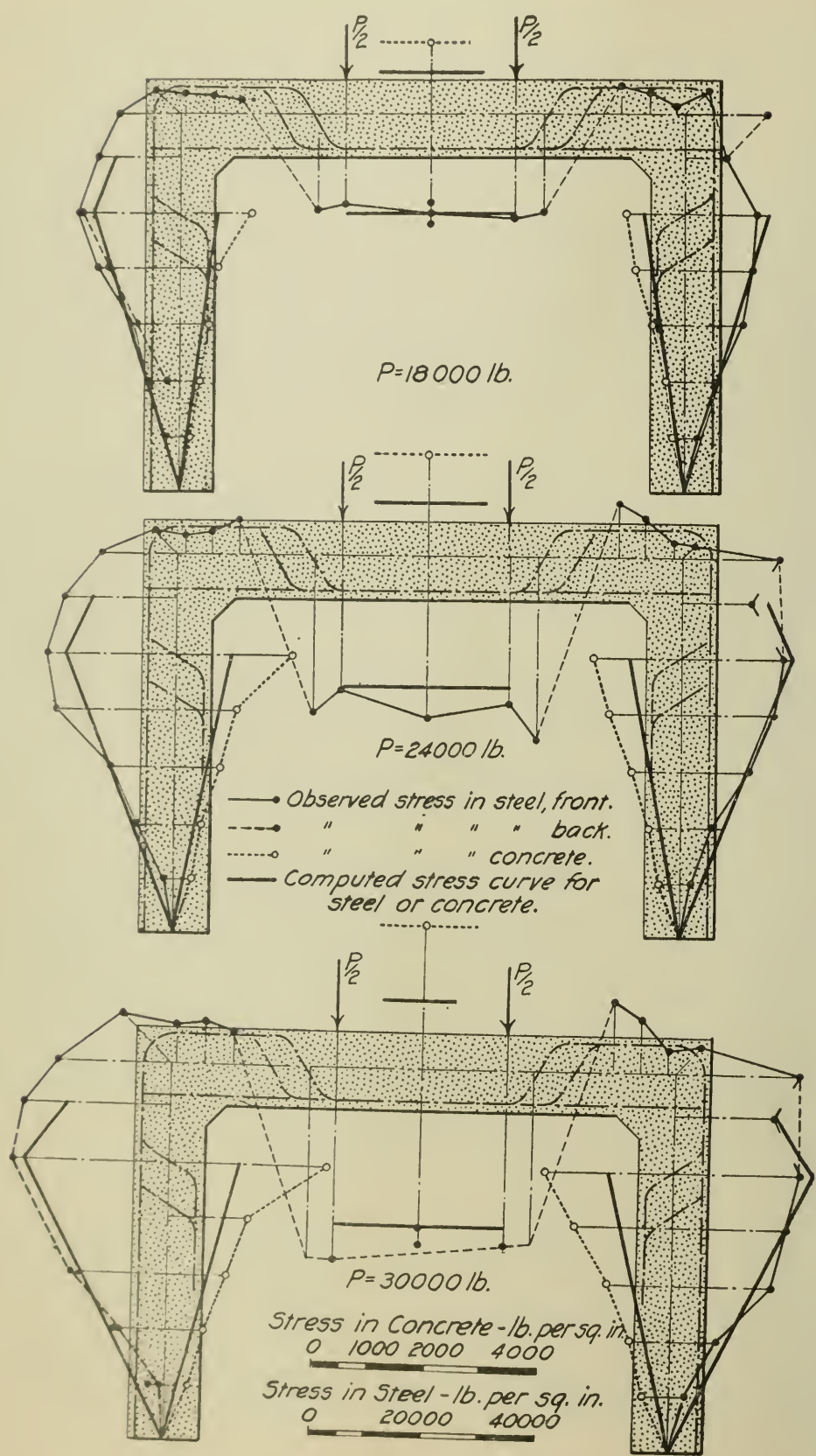


FIG. 36. OBSERVED AND COMPUTED STRESSES IN FRAME 1

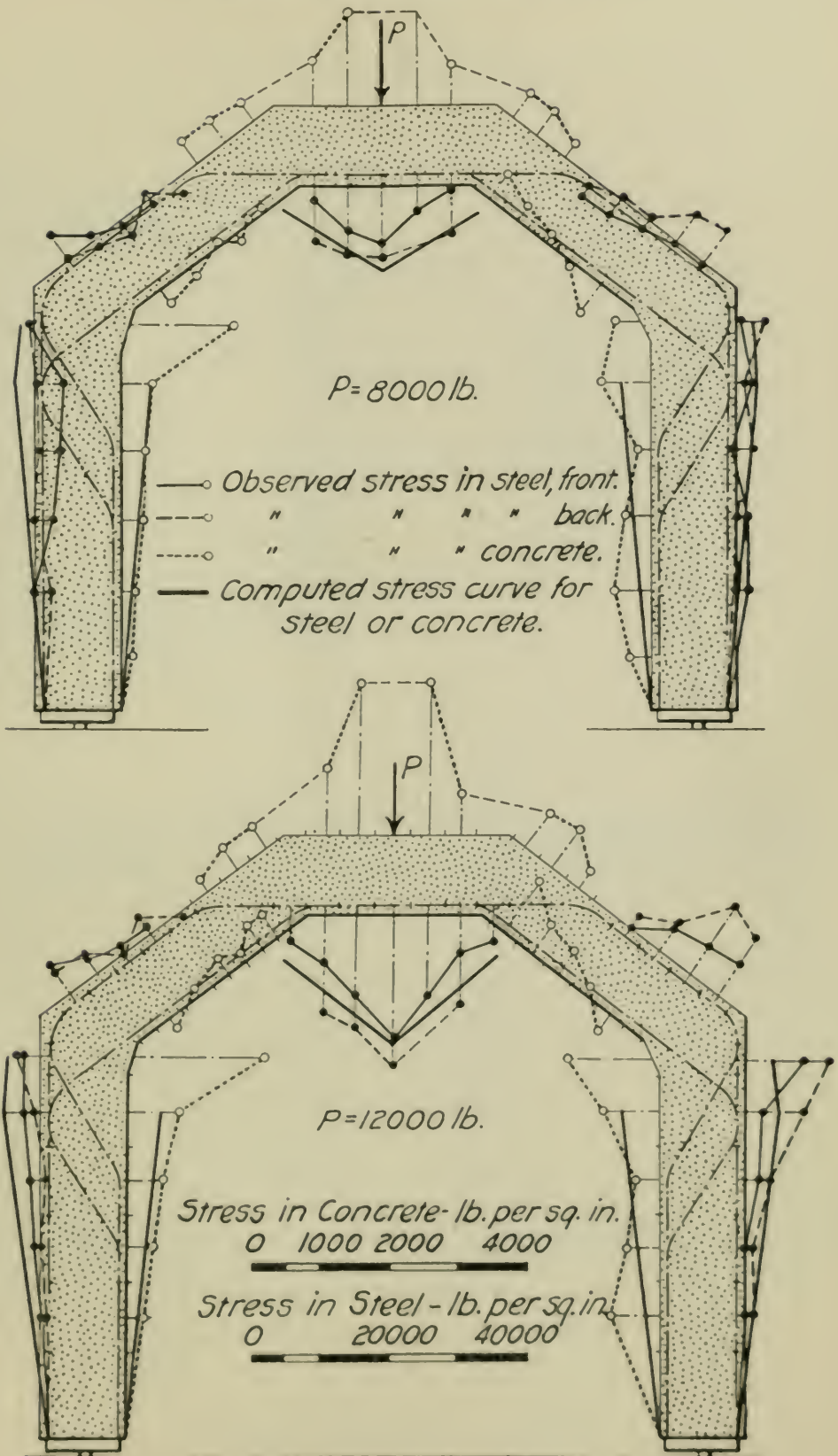


FIG. 37. OBSERVED AND COMPUTED STRESSES IN FRAME 2

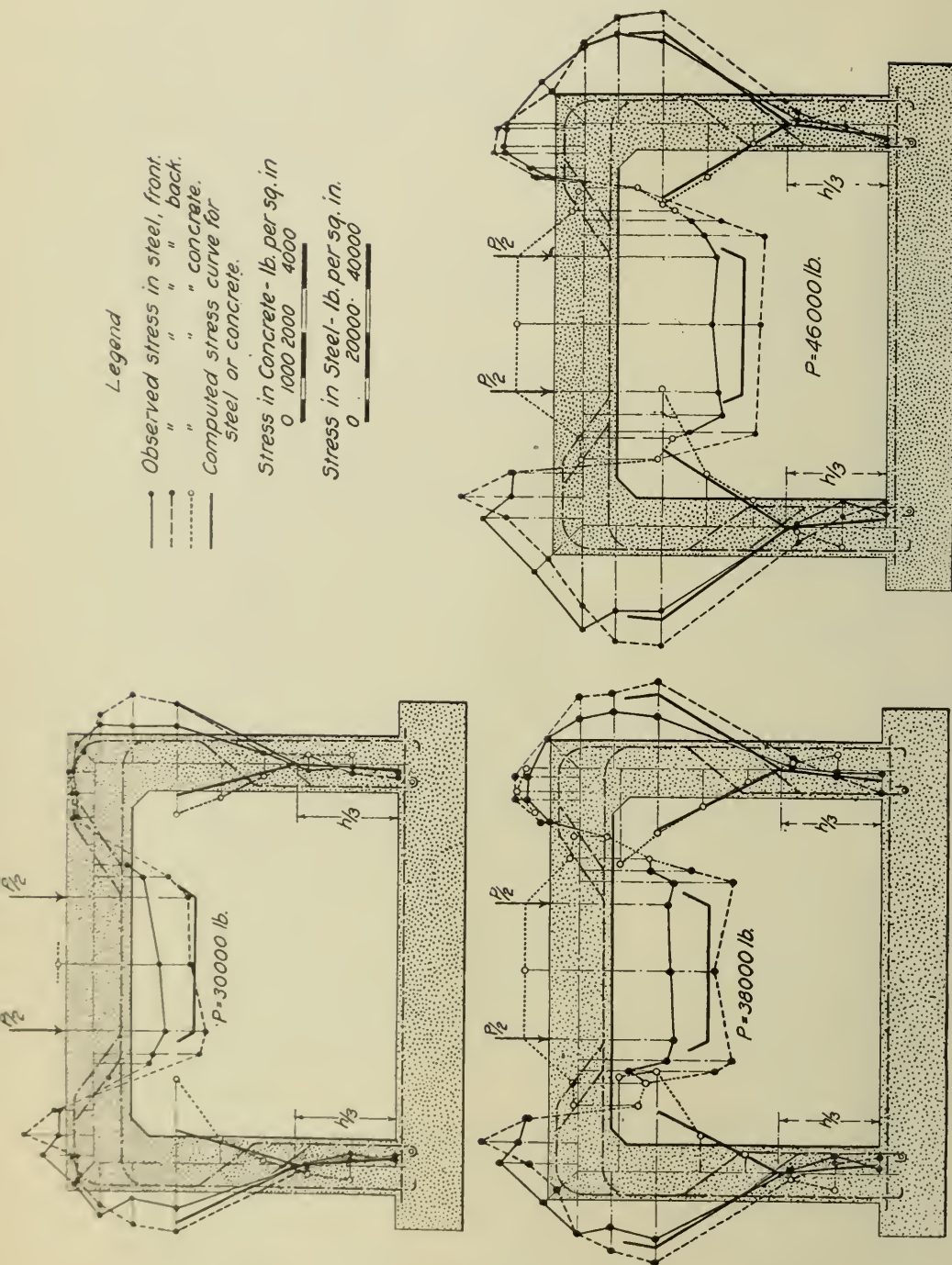


FIG. 38. OBSERVED AND COMPUTED STRESSES IN FRAME 3

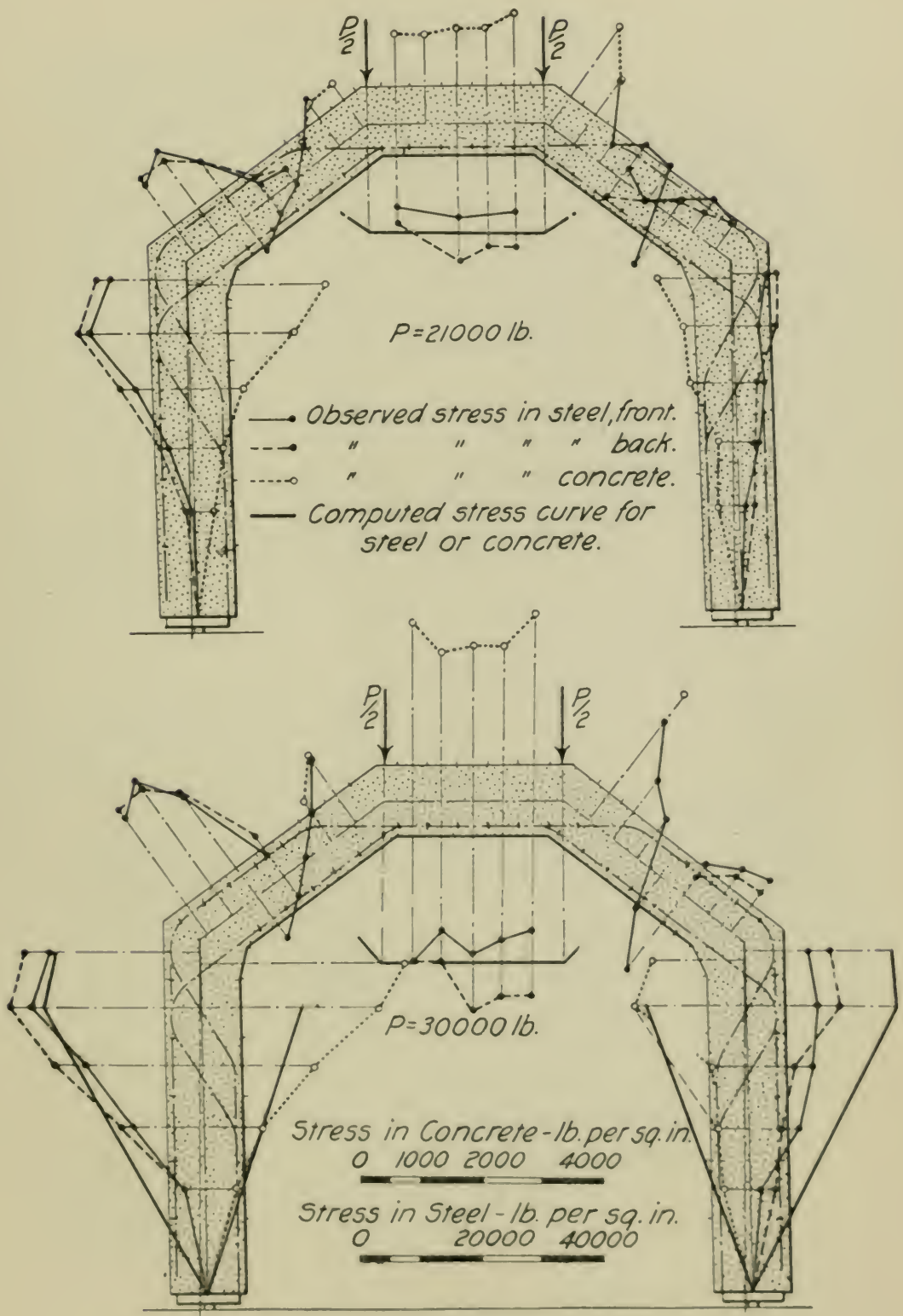


FIG. 39. OBSERVED AND COMPUTED STRESSES IN FRAME 4

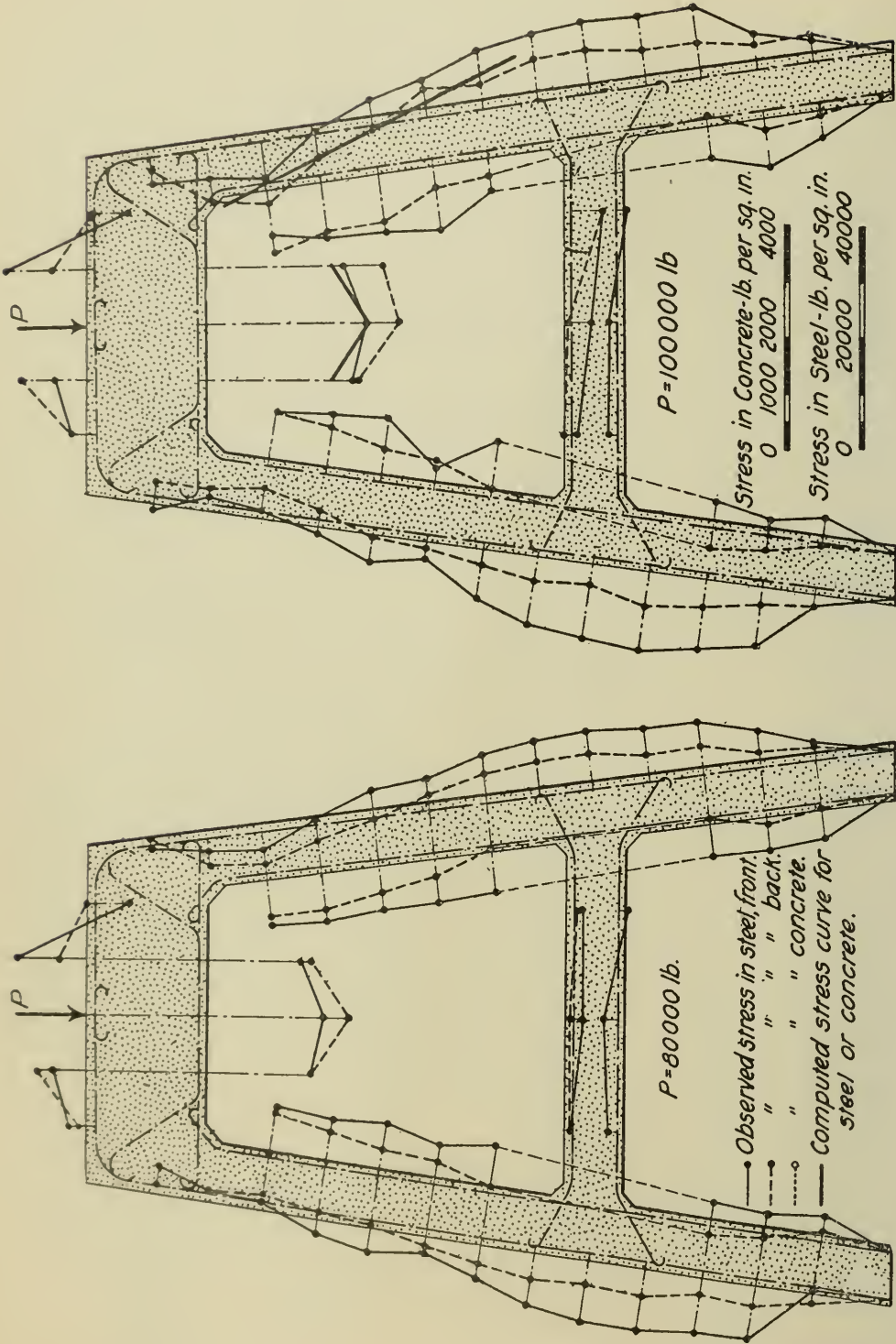


FIG. 40. OBSERVED AND COMPUTED STRESSES IN FRAME 5

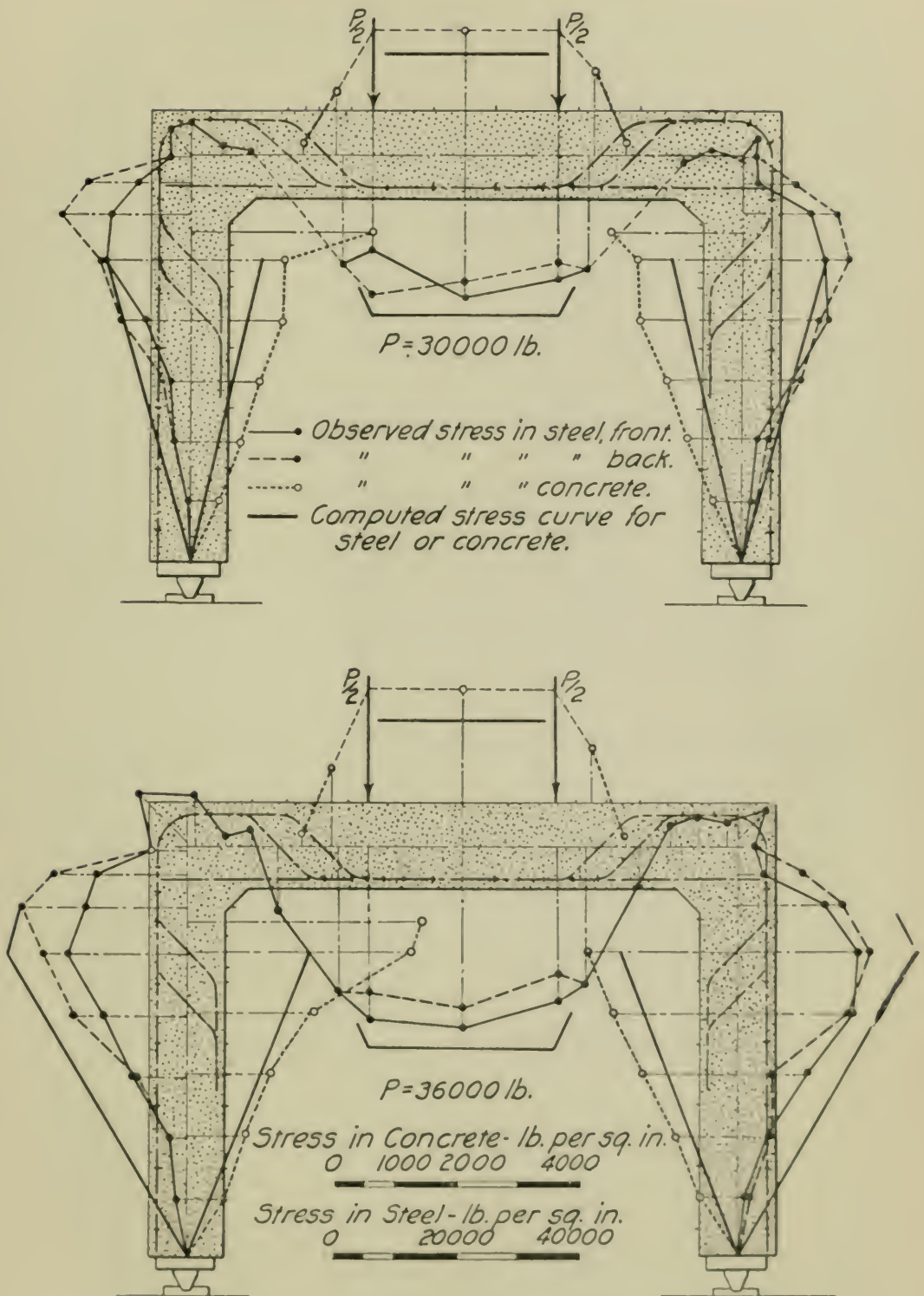


FIG. 41. OBSERVED AND COMPUTED STRESSES IN FRAME 6

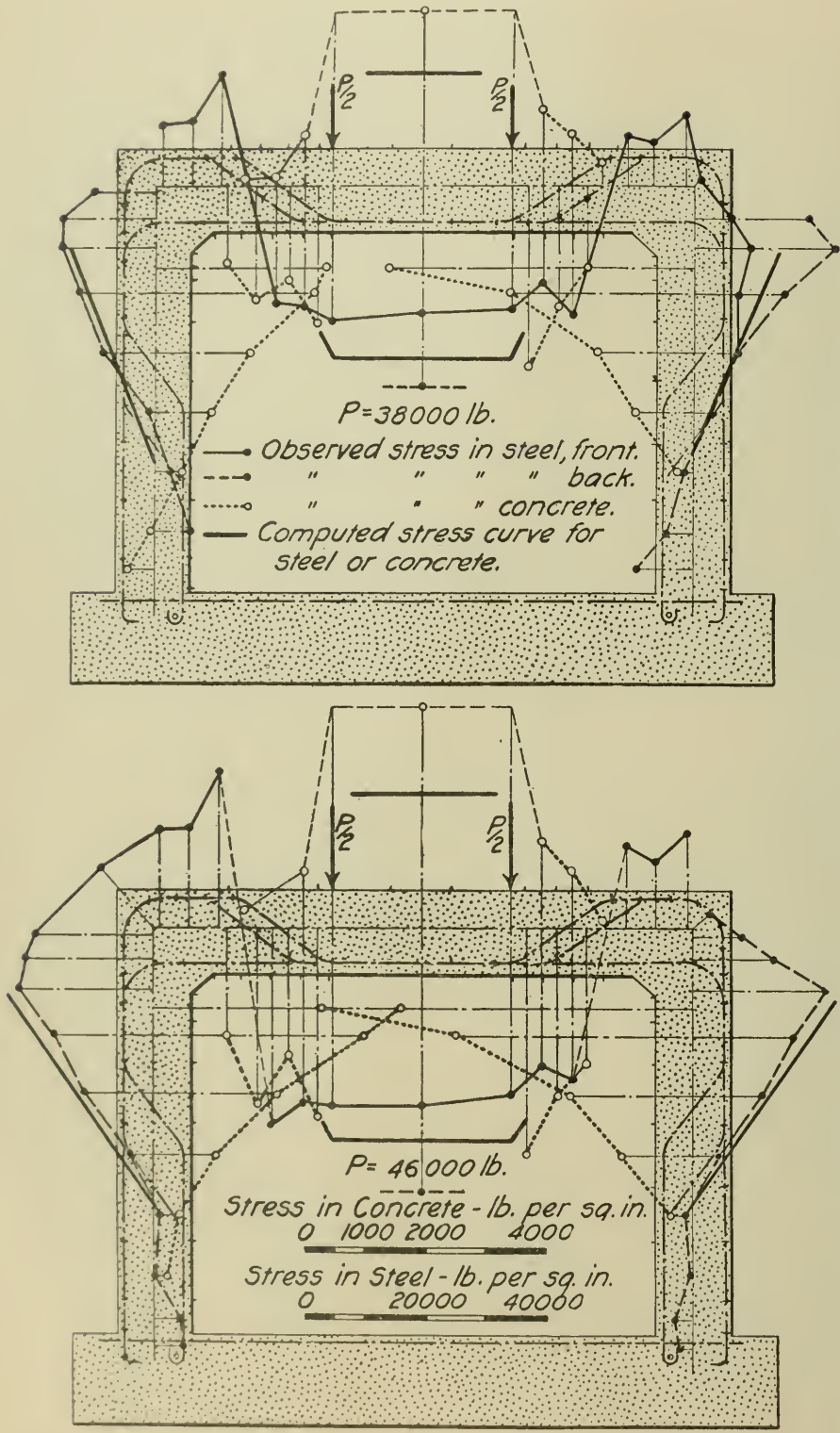


FIG. 42. OBSERVED AND COMPUTED STRESSES IN FRAME 7

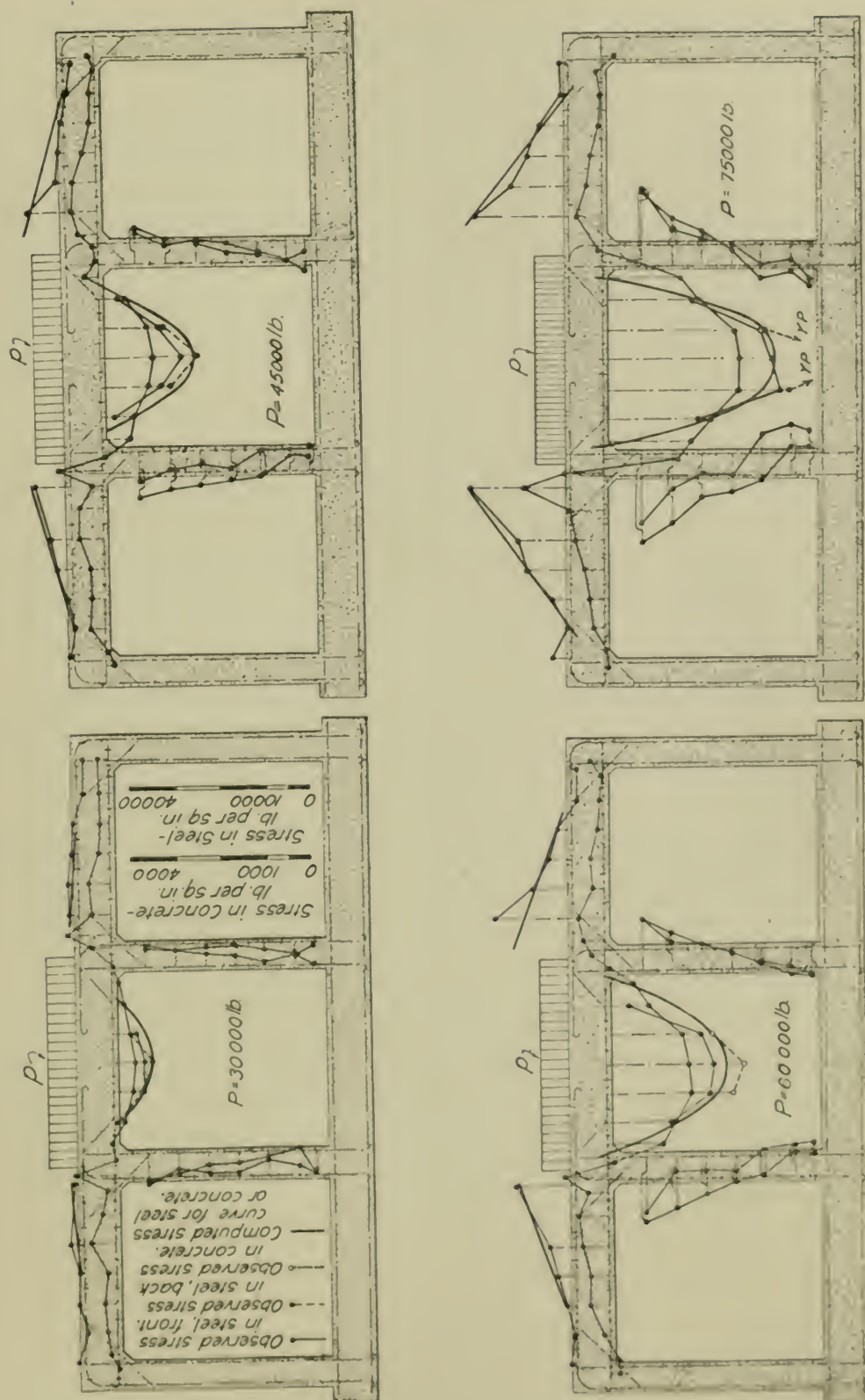


FIG. 43. OBSERVED AND COMPUTED STRESSES IN FRAME 8

Extensometers of the Berry type were used in measuring the deformations. The method of using these instruments is described in Bulletin 64 of the Engineering Experiment Station of the University of Illinois, "Tests of Reinforced Concrete Buildings under Load," and in a paper in Proceedings of the American Society for Testing Materials for 1913, "The Use of the Strain Gage in the Testing of Materials." Variation in temperature is sufficient to cause an appreciable change in the length of the instrument. Hence observations on an unstressed standard bar of invar steel were taken for the purpose of making temperature corrections. Small steel plugs, about one inch long, were set in plaster of paris in the concrete, where the concrete deformations were to be measured. Small gage holes, 0.055 in. in diameter, were drilled in the reinforcing bars and in the steel plugs. Two sets of initial readings were taken before the application of load. A complete set of observations of the deformations was taken at each increment of load. In reducing the strain gage readings to stress, temperature corrections were made. These were based on an assumed linear variation of length with time between successive readings on the standard bar.

The smallest number of gage lines on any frame was 75 on Frame 1; the greatest number, 163, on Frame 8. The gage lengths used were 2 in., 4 in., and 8 in. The average deformation over the gage length was used.

The faces of the test frames were whitewashed to enable the appearance of cracks and their growth to be more easily observed. The extent of the cracks at the several loads was marked on the specimen during the test.

21. *Explanation of Tables and Diagrams.*—The loads given in the various tables and figures are the loads applied by the testing machine and do not include the weight of the frame itself. The load at first crack is the load noted when the first fine crack was observed during the test. The ultimate load is the highest load applied to the specimen just before the load carried began to decrease slowly. The maximum tensile and compressive stresses are the highest stresses observed at the points specified. The vertical shearing stress was calculated with the ordinary formula $v = \frac{V}{bjd}$, where v represents the vertical shearing unit-stress in the concrete, V the total vertical shear at the end of the beam, b the breadth of the beam, and jd the distance

from the center of the steel to the center of the compressive stresses in the concrete. The bond stress in the beam was computed by means of the formula $u = \frac{V}{mojd}$, where u is the bond stress per unit of area of the surface of the reinforcing steel, m the number of reinforcing bars, and o the circumference or periphery of one reinforcing bar. The values of jd were selected with reference to the amount of reinforcing steel and the modulus of elasticity of the concrete.

Loads are given in pounds, unit-stresses in pounds per square inch, and moments in inch-pounds.

TABLE 11

VALUES OF MODULUS OF ELASTICITY OF CONCRETE USED IN STRESS COMPUTATIONS

Frame	E	Frame	E
1	2 100 000	5	3 000 000
2	3 600 000	6	3 900 000
3	2 070 000	7	3 700 000
4	3 600 000	8	3 300 000

The so-called observed stresses have been obtained from the observed deformations by using a modulus of elasticity of 30,000,000 lb. per sq. in. for the steel, and for the concrete the values given in Table 11.

Table 12 contains general data of the tests of the frames.

Table 13 gives computed stresses at the three points in each frame which are shown in Fig. 44, calculated by means of the formulas given in the preceding pages, the values being expressed in terms of the load applied to the frame. The values in columns marked I were obtained on the assumption that the concrete has full tensile strength; those in columns marked II on the assumption of no tensile strength. The division of the direct stress between concrete and steel was computed by the usual formulas for reinforced concrete columns.

Other explanations of tables and diagrams are made elsewhere.

22. *Phenomena of Frame Tests.*—As may be expected in reinforced concrete flexural members, the tensile stresses in the steel were very small at low loads. Undoubtedly this effect was largely due to the ability of the concrete to carry tensile stress. As soon as the con-

TABLE 12
GENERAL DATA OF TESTS OF FRAMES

Frame No.	Age days	Method of Loading	Load at First Crack pounds	Maximum Load at which Deformation was measured pounds	Ultimate Load at Failure pounds	Maximum Stress pounds per square inch								
						Tensile Stress in Steel					Compressive Stress in Concrete		Vertical Shearing Stress in Top Beam	Bond Stress in Top Beam
						Center of Top Beam	Ends of Top Beam	Bent up Bar in Beam	In Column	Center of Top Beam	In Column			
1	63	Concentrated Loads at Third Point	12 000	36 000	40 500	36 000	16 000	21 700	35 200	3 320 ¹	3 130 ¹	266	333	
2	62	Concentrated Load at Center	8 000	14 000		32 900	28 900		12 300	3 840 ¹	1 570	124	159	
3	63	Concentrated Loads at Third Point	21 000	46 000	61 000	39 500	20 300	15 700	23 800	1 500	3 050	326	415	
4	63	Concentrated Loads at Third Point	10 000	30 000	50 000	25 300	21 800		27 600	2 590	2 920	267	340	

TABLE 12—(CONTINUED)
 GENERAL DATA OF TESTS OF FRAMES

Frame No.	Age days	Method of Loading	Load at First Crack pounds	Maximum Load at which Deformation was measured pounds	Ultimate Load at Failure pounds	Maximum Stress pounds per square inch					
						Tensile Stress in Steel				Compressive Stress in Concrete	
						Center of Top Beam	Ends of Top Beam	Bent up Bar in Beam	In Column	Center of Top Beam	In Column
5	61	Concentrated Load at Center	40 000	100 000	146 000	36 400	5 700		11 800		12 200 ²
6	58	Concentrated Loads at Third Point	18 000	36 000	46 000	29 800	7 400	10 300	27 000	2 750	3 840 ¹
7	62	Concentrated Loads at Third Point	21 000	46 000	61 000	44 400	25 500	18 100	30 200	3 700 ¹	4 130 ¹
8	61	Uniform Load over Middle Span	45 000	60 000 ³	134 000	30 900 ¹	13 200		14 700	2 540 ⁴	14 700 ²
											425
											540

¹ Not reliable, elastic limit exceeded.

² Steel stress in compression side of column.

³ Not maximum load.

⁴ Stress at point one inch distant from extreme fiber.

TABLE 13
COMPUTED STRESSES FOR TEST FRAMES IN TERMS OF TOTAL APPLIED LOAD

No. of Frame	H	Point	M inch unit	p	Direct Stress		I			II		
					Steel	Concrete	k	f _s	f _c	k	f _s	f _c
1	0.082 P	A	-3.43 P	.0068	.070 P	.0049 P				0.35	0.82 P	.039 P
		B	-4.04 P	.0109	.070 P	.0049 P				0.41	0.60 P	.041 P
		C	+7.43 P	.0098	.012 P	.0009 P				0.41	1.09 P	.054 P
2	0.121 P	A	-4.84 P	.0082	.049 P	.0059 P	0.58	0.15 P	.041 P	0.31	0.97 P	.059 P
		B	+5.28 P	.0123	.015 P	.0018 P				0.36	0.93 P	.067 P
		C	+9.78 P	.0123	.015 P	.0018 P				0.36	1.75 P	.123 P
3	0.145 P	A	+1.37 P	$\begin{smallmatrix} p=.0054 \\ p'=.0027 \end{smallmatrix}$	.053 P	.0059 P	0.52	0.032 P	.016 P			
		B	-3.02 P	.0109	.052 P	.0058 P	0.54	0.11 P	.029 P	0.35	0.44 P	.036 P
		C	+5.40 P	.0098	.015 P	.0017 P	0.60	0.17 P	.032 P	0.35	0.76 P	.045 P
4	0.100 P	A	-4.30 P	.0082	.049 P	.0059 P	0.58	0.15 P	.041 P	0.31	0.85 P	.054 P
		B	+4.90 P	.0123	.013 P	.0015 P	0.61	0.20 P	.034 P	0.36	0.87 P	.063 P
		C	+4.90 P	.0123	.013 P	.0015 P				0.36	0.87 P	.063 P
5		A	0.23 P	$\begin{smallmatrix} p-p'=.0064 \\ p-p'=.0064 \\ p-p'=.0064 \end{smallmatrix}$	.060 P		0.50	0.047 P	.074 P ¹			
		A ₁	0.10 P		.060 P		0.50	0.055 P	.067 P ¹			
		B	0.62 P		.060 P		0.50	0.024 P	.097 P ¹	0.242	0.19 P	.066 P
6	0.077 P	A	-3.42 P	.0068	.040 P	.0052 P	0.58	0.10 P	.030 P	0.28	0.81 P	.047 P
		B	-4.04 P	.0109	.040 P	.0052 P	0.59	0.12 P	.033 P	0.34	0.60 P	.046 P
		C	+7.48 P	.0098	.007 P	.0009 P	0.59	0.22 P	.042 P	0.33	1.06 P	.065 P

TABLE 13—(CONTINUED)
COMPUTED STRESSES FOR TEST FRAMES IN TERMS OF TOTAL APPLIED LOAD

No. of Frame	H	Point	M inch unit	p	Direct Stress		I			II		
					Steel	Concrete	k	f _s	f _c	k	f _s	f _c
7	0.145 P	A	2.03 P	$\frac{p}{p'} = .0054$	.048 P	.0059 P	0.52	$\frac{f_s}{f'_s} = \frac{0.0281}{0.1781}$	.024 P	0.35	0.43 P	.038 P
		B	-3.02 P	.0109	.047 P	.0058 P	0.54	0.13 P	.029 P	0.35	0.76 P	.048 P
		C	+5.40 P	.0098	.014 P	.0017 P						
8	I H _s .009 P H _b .003 P V _s .047 P V _b .547 P	II A A ₁ B C		$\frac{p}{p'} = .0098$ $\frac{p}{p'} = .0073$ $\frac{p}{p'} = .0049$ $\frac{p}{p'} = \frac{p'}{p'} = .0062$ $\frac{p}{p'} = \frac{p'}{p'} = .0049$	Neglected		0.33	0.46 P	.028 P	0.33	0.51 P	.032 P
					Neglected		0.58	0.47 P	.007 P	0.28	$\frac{0.24}{0.34} \frac{P^2}{P^3}$	$\frac{.0102}{.0146} \frac{P^2}{P^3}$
					.070 P		0.50	0.166 P	.216 P ₁			
					Neglected		0.50	0.06 P	.06 P ₁	0.24	0.31 P	.047 P ₁

¹ Compressive stress in steel, f_s

³ Same conditions as I except beams and intermediate columns are considered as broken in tension.

² Same conditions as I except central beam is considered as broken in tension.

crete on the tension side of the member was sufficiently stretched, a vertical tension crack formed on the beam underneath the load and then a crack formed at the side near the juncture of the column and the beam, in most cases. After the formation of these cracks, the tension in these parts was taken mainly by the reinforcing bars. As the loads were increased the cracks developed and new cracks appeared on the tension side between the points of application of the load on the beam, and horizontal cracks formed at regular intervals in the columns.

The tensile stress due to the negative bending moment within the space occupied by the juncture of the beam and the column was small, and in these places tension cracks did not form in many frames until high loads were applied. The bent-up bars in the beam came into action as soon as tension cracks formed in their vicinity, and in the bent-up portions tensile stresses as high as 22,000 pounds per square inch were developed in several instances. The tensile stresses in the steel at the fixed ends of the columns were rather low. The tensile strength of the concrete in this part apparently reduced the tensile stress in the steel.

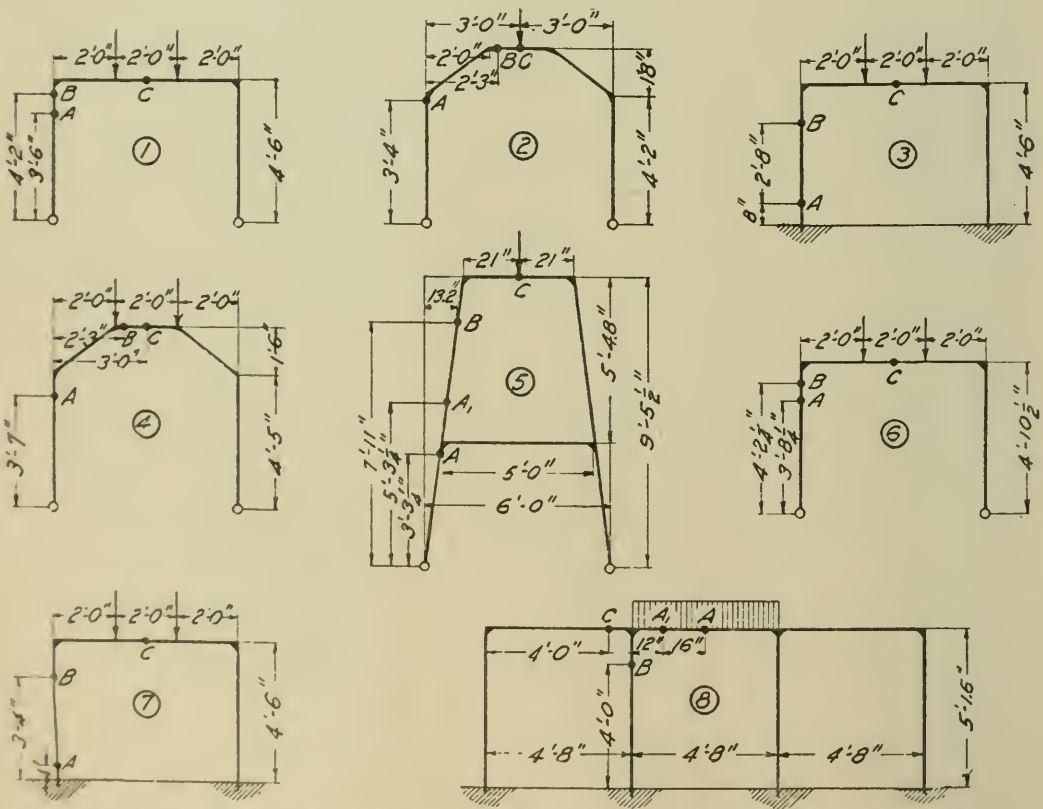


FIG. 44. POINTS USED FOR COMPARISON OF COMPUTED AND OBSERVED STRESSES

High compressive stresses were developed in the concrete in the upper portion of the columns below the intersection with the beam, and the maximum compression was observed along the sharp corner at the juncture of the beam and the column, as might be expected. This is due to the curved beam action at the rigid joint. In each frame the maximum load was higher than the load expected.

Views of the frames which show the location of cracks are given in Fig. 49 to 57.

The general phenomena of the tests of the individual frames are given in the following brief notes.

Frame 1—Square Frame with Columns Hinged at Lower End.—Nominal span length was 6 feet. Total height was 5 ft. 2 in. Frame was loaded at the one-third points of the span of the horizontal member. Fig. 49 shows the frame in the testing machine. The location of the cracks is shown in Fig. 51. At the 12,000-lb. load the first fine crack in the beam appeared directly under a load point and extended from the bottom of the member vertically 2 in. to the level of the reinforcement. At the same load the first noticeable cracks appeared in the columns, one in the outside edge of the column on a level with the bottom surface of the beam and one at 2 ft. 5 in. from the bottom of each column end. No crack appeared in the top side of the beam until the load was increased to 36,000 lb. At that load cracks appeared 8 in. from the top corner of the frame and extended vertically downward.

The frame carried 40,500 lb. and the load was held for a few minutes and then dropped very slowly. The cracks were well distributed in the tension zone of the frame and no crack due to diagonal tension was formed. The frame failed by tension in the reinforcement of the beam.

Frame 2—Inverted U-frame with Columns Hinged at Lower End.—Nominal span length and total height were each 6 ft. Load was applied at the center of the span of the horizontal member. The location of cracks is shown in Fig. 52. At 8,000 lb. two cracks appeared 2 in. on each side of the center of the top beam and extended upward 2 in. and 3 in., respectively. At 12,000 lb. these cracks had extended vertically 6 in. from the bottom surface of the beam, and a new crack appeared just inside the right-hand corner 10 inches from the center of the beam and extended diagonally toward the load point. At the same load four cracks appeared at the outer shoulders. At 14,000 lb.

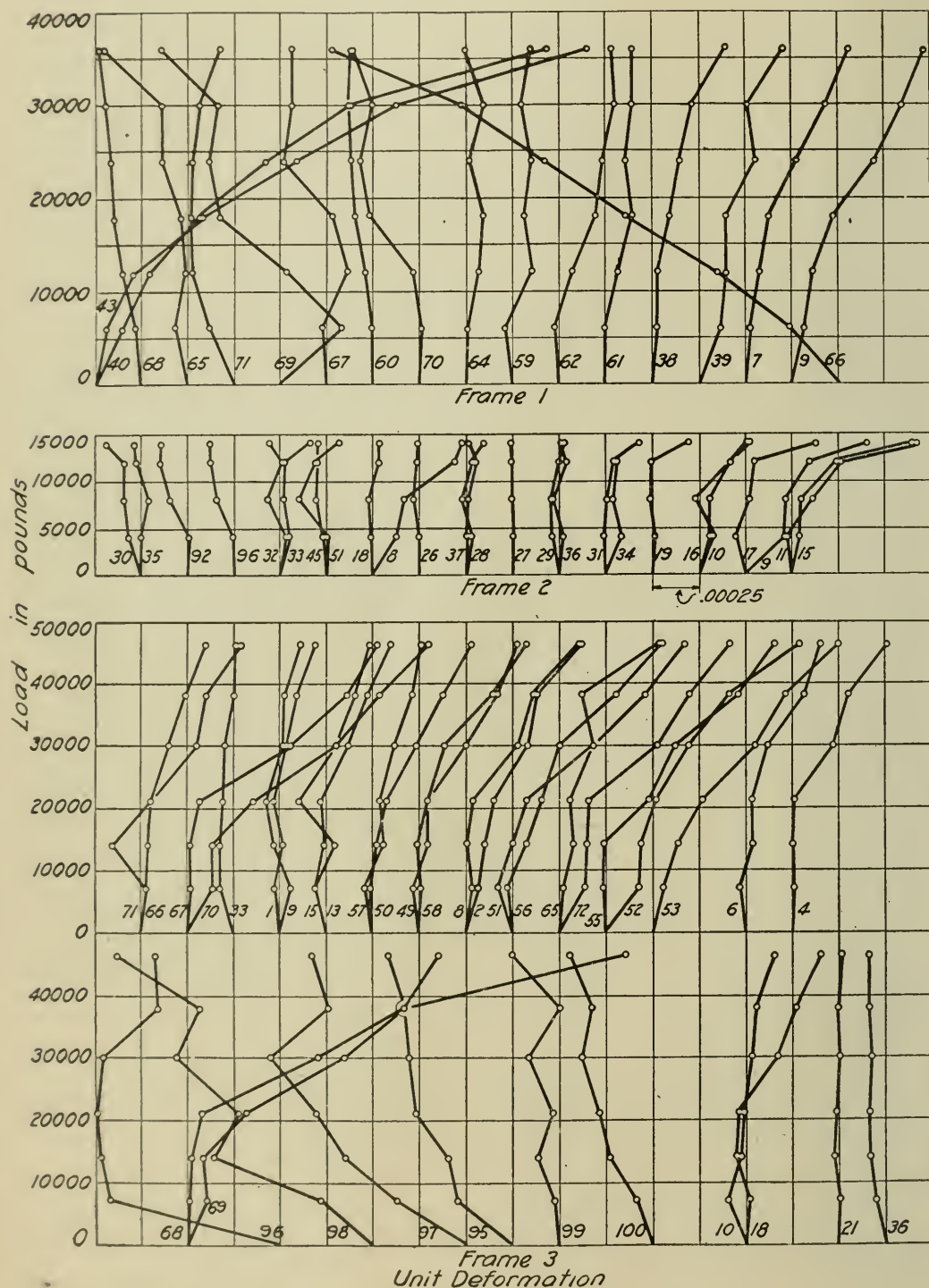


FIG. 45. LOAD-DEFORMATION DIAGRAMS FOR ADDITIONAL GAGE LINES, FRAMES 1, 2, 3

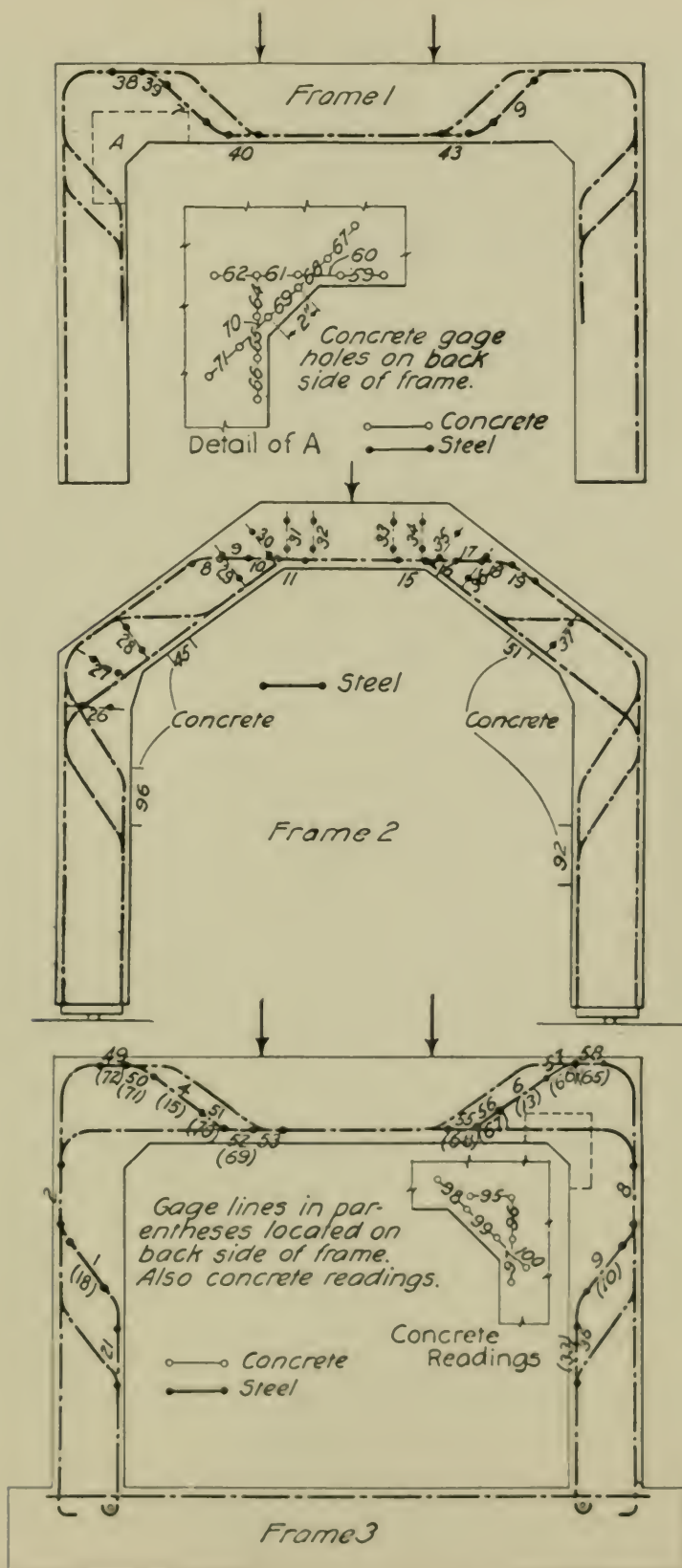


FIG. 46. LOCATION OF ADDITIONAL GAGE LINES, FRAMES 1, 2, 3

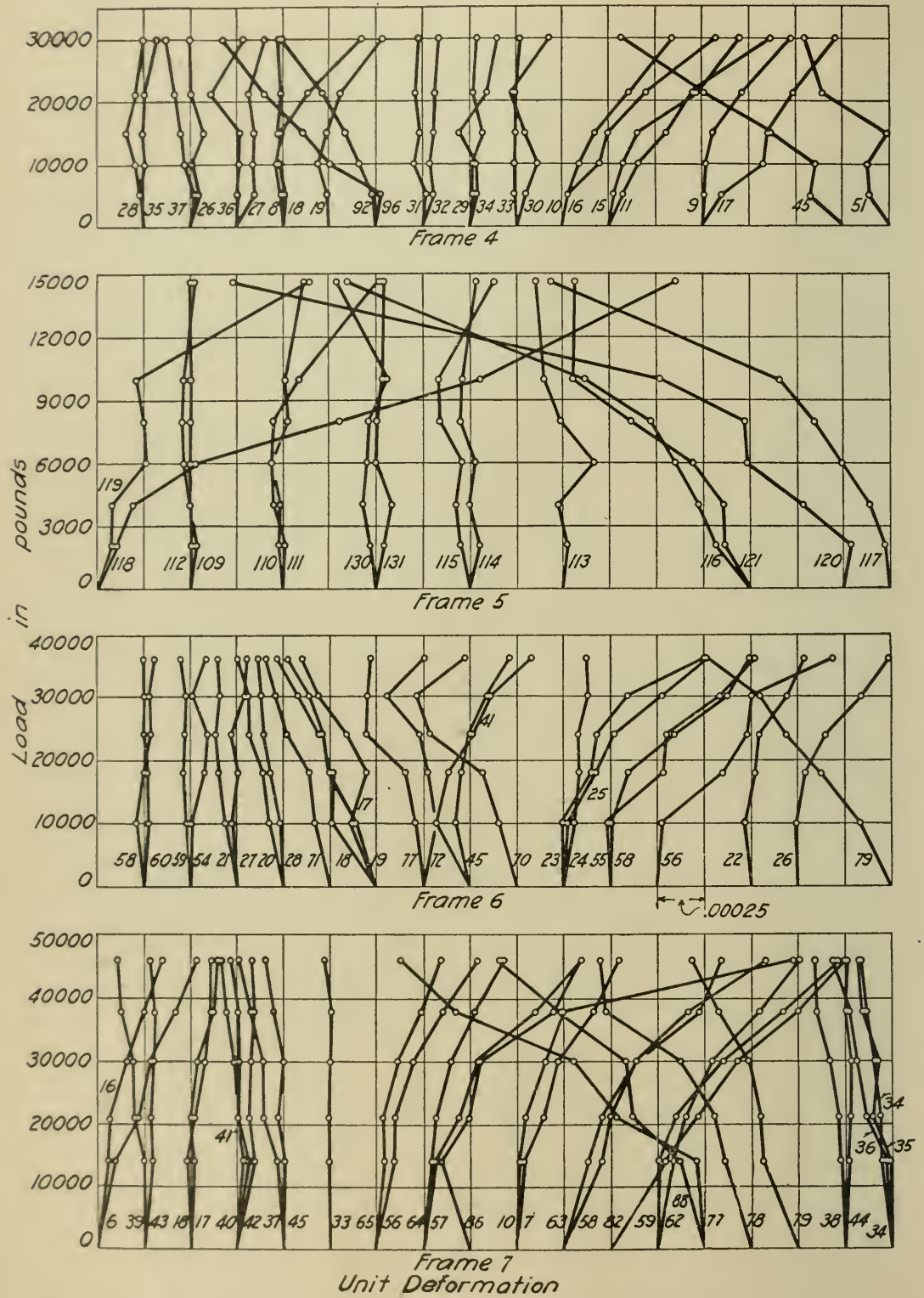


FIG. 47. LOAD-DEFORMATION DIAGRAMS FOR ADDITIONAL GAGE LINES, FRAMES 4, 5, 6, 7

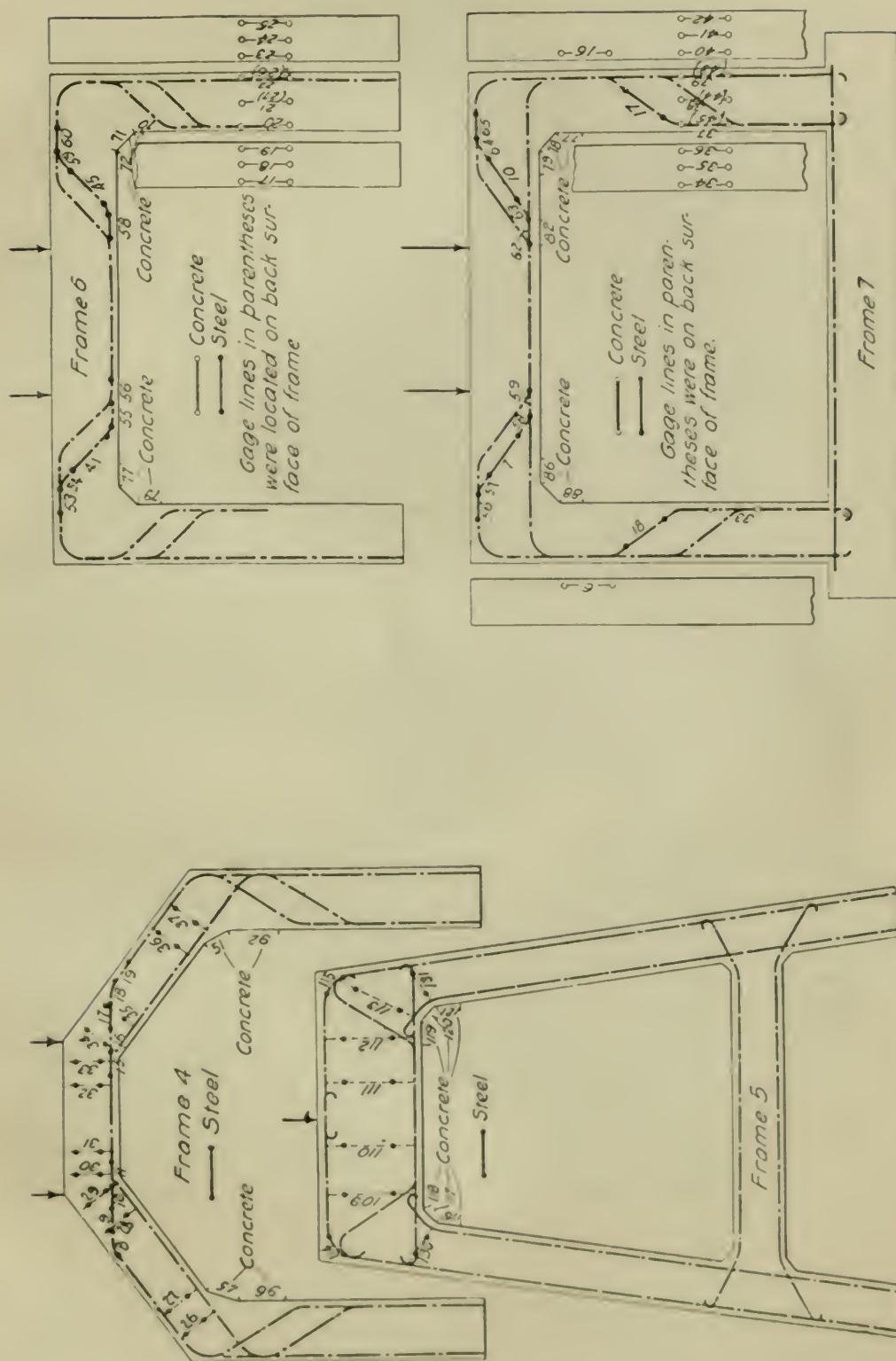


FIG. 48. LOCATION OF ADDITIONAL GAGE LINES, FRAMES 4, 5, 6, 7

the cracks were extending. Unfortunately at this load the foot of one of the columns slipped outward about $\frac{1}{4}$ in. due to the lack of sufficient friction to resist the horizontal thrust at the support. However, satisfactory information was obtained because very high tensile stress (32,900 lb. per sq. in.) had been developed at the center of the beam before the slipping occurred.

Frame 3—Square Frame with Columns Fixed at Lower End.—Nominal span length was 6 ft. Total height of frame from fixed column end was 4 ft. 11 in. Loads were applied at one-third points of span of horizontal member. No noticeable cracks appeared until 21,000 lb. had been applied, when three cracks appeared at the bottom between loads and several cracks appeared in both columns. At 30,000 lb. new cracks appeared in the beam and columns, and one crack appeared at the top of the beam. The location of the cracks is shown in Fig. 53. The cracks in the upper part of the columns were located within 14 in. downward from the extended line of the bottom face of the beam. No crack was observed at the fixed ends of the columns. The frame carried 60,000 lb. and the load was held for a few minutes, then dropped very slowly, and there appeared to be no danger of sudden failure. No diagonal tension crack appeared in the beam, and the frame failed by tension in the longitudinal steel of the beam.

Frame 4—Inverted U-frame with Columns Hinged at Lower End.—Nominal span length was 6 ft. Total height of frame was 6 ft. 3 in. from hinged end of columns. Loads were applied at one-third points. At 10,000 lb. the first noticeable crack appeared at the left-hand inside top corner, and extended $2\frac{1}{2}$ in. upward. Cracks are shown in Fig. 54. Accidentally the frame was built slightly out of form, the columns being out of plumb $1\frac{1}{4}$ in. in the height of 4 ft., and more stress was thrown to the left-hand column than to the other. The distribution of the cracks shows this clearly. The frame, however, carried a comparatively high load (50,000 lb.). Failure was by tension in the steel in the horizontal beam and at the rigid joint between the columns and the sloped beam.

Frame 5—Trestle Bent with Tie—(A-frame).—Span length center to center at the supported column ends was 6 feet. Total height from the base to the top of the frame was 10 ft. $1\frac{1}{2}$ in. Load was applied at the center of the top beam. The cross-section of the top beam



FIG. 49. VIEW OF FRAME 1 IN TESTING MACHINE



FIG. 50. VIEW OF FRAME 5 AFTER TEST

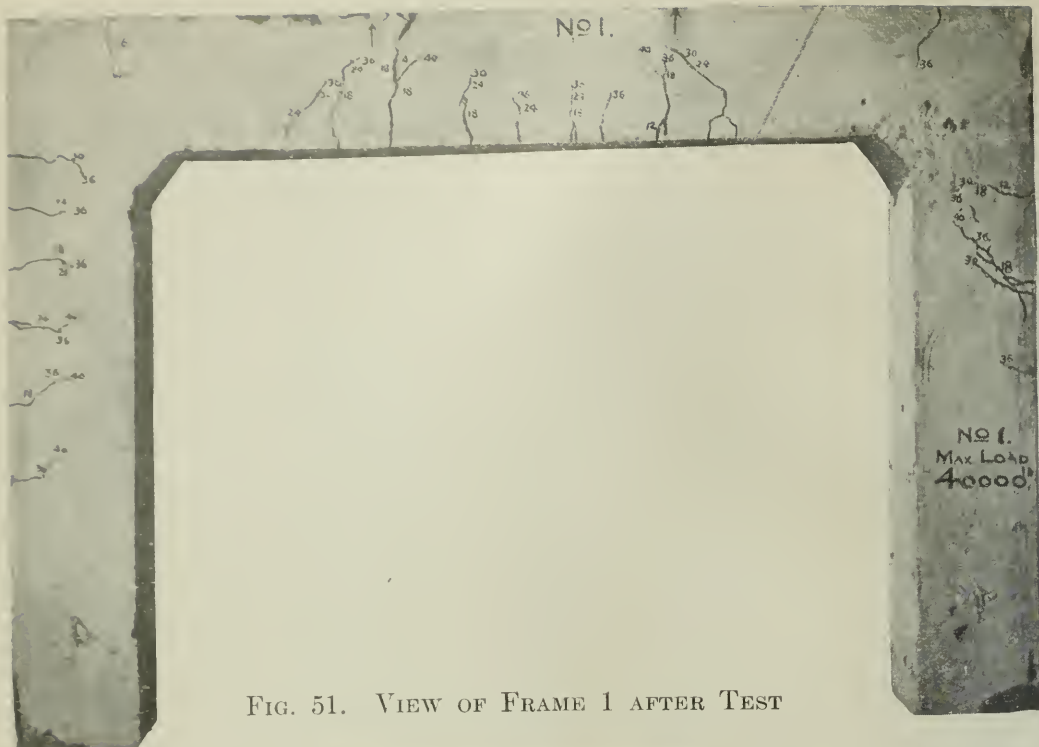


FIG. 51. VIEW OF FRAME 1 AFTER TEST

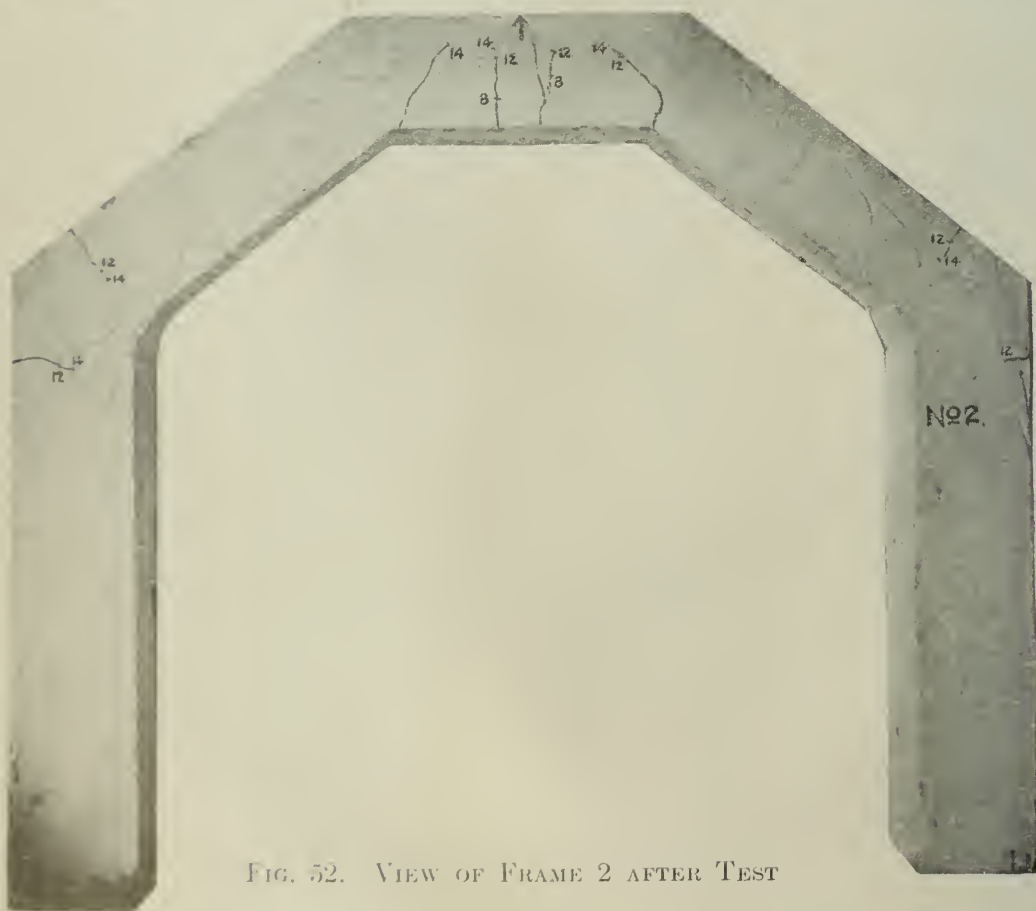


FIG. 52. VIEW OF FRAME 2 AFTER TEST



FIG. 53. VIEW OF FRAME 3 AFTER TEST



FIG. 54. VIEW OF FRAME 4 AFTER TEST

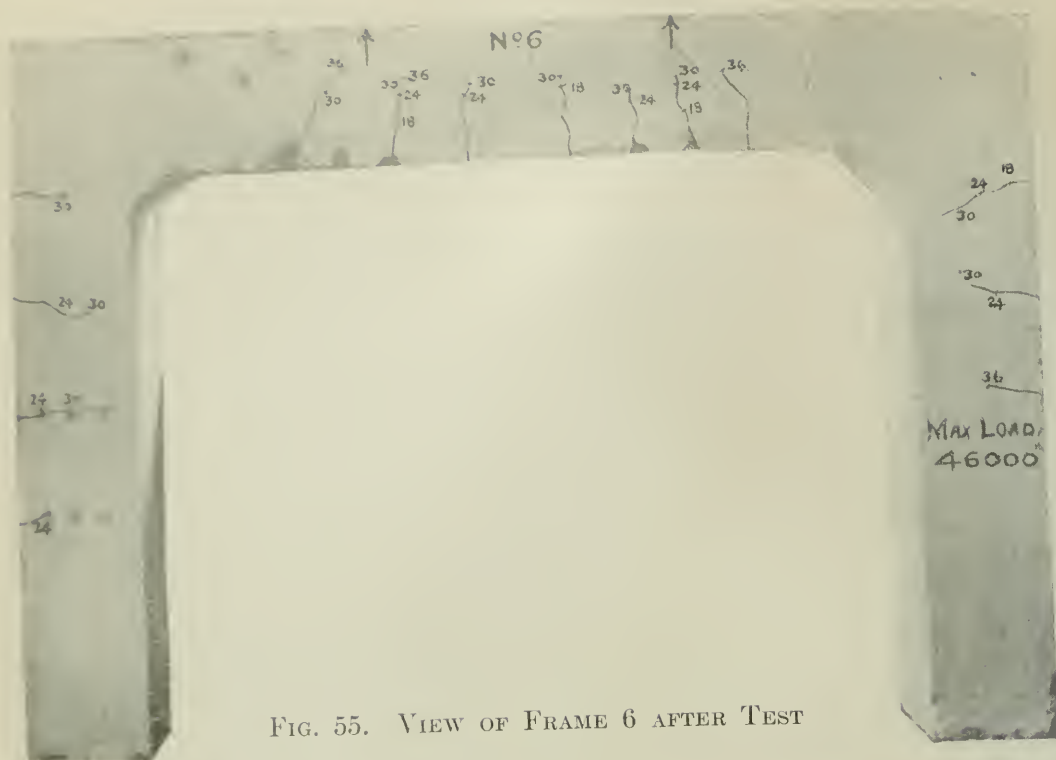


FIG. 55. VIEW OF FRAME 6 AFTER TEST



FIG. 56. VIEW OF FRAME 7 AFTER TEST

was $8\frac{1}{2}$ by 16 in. and the column section $8\frac{1}{2}$ by $8\frac{1}{2}$ in. At 40,000 lb. the first two noticeable cracks appeared. These were under the load points of the beam. For location of cracks see Fig. 50. The outer cracks of the beam finally extended diagonally nearly to the load point. At 100,000 lb. the first cracks appeared in the column at the outside near its juncture with the beam. At 140,000 lb. load a crack suddenly occurred at the right-hand rigid joint between the tie and the column with a breaking sound. The frame carried 146,000 lb. and the load gradually dropped. The maximum load was controlled by the failure of the top beam which failed by tension in the reinforcement.

Frame 6—Square Frame with Columns Hinged at Lower Ends. Same as Frame 1.—Load was applied at one-third points of span of horizontal member. Fig. 55, a view of the frame after the test, shows the appearance of the cracks. At 18,000 lb. four cracks appeared, two of them under the load points, one near the center of the beam, and one at the upper part of the right-hand column. At 24,000 lb. the cracks extended further and additional cracks appeared in the beam and columns at regular intervals. At 30,000 and 36,000 lb. new cracks appeared in the beam where the longitudinal bars were bent up and these cracks ran diagonally almost to the load points. No crack appeared on the top side of the beam. The frame carried 46,000 lb. and the load then dropped slowly. Failure was by tension in the longitudinal steel in the beam.

Frame 7—Square Frame with Columns Fixed at Lower Ends. Same as Frame 3.—Load was applied at one-third points of the span of the horizontal member. The location of the cracks is shown in Fig. 56. The first noticeable cracks appeared at 21,000 lb., three in the beam and three in the columns.

At 30,000 lb., the cracks had extended further and two cracks due to the negative bending moment appeared at the ends 8 in. from the outside face of the columns. At the same load three cracks formed in the bottom half of the beam. As the load increased, the crack located on the outside of the left-hand load point extended diagonally almost to the load point, and the cracks at both ends of the beam extended vertically downward nearly to the bottom side of the beam. The ultimate load carried by the frame was 61,000 lb. At this load sudden failure took place at both inside corners of the lower ends of the columns and the cracks extended horizontally and vertically almost through the

concrete base and almost through the columns. This fact shows that considerable positive bending was developed there. The frame also failed by tension in the longitudinal steel of the top beam.

Frame 8—Frame with Three Spans.—Span lengths were 4 ft. 8 in. center to center. Total height of frame was 6 ft. 7 $\frac{1}{4}$ in. The base of the frame was 15 ft. 4 in. in length, while the length of the base of the testing machine was 10 ft. 6 in. Consequently the ends of the frame projected beyond the base of the testing machine upon which the frame was bedded in plaster of paris. To observe the upward deflection of the ends of the frame under test an Ames dial was attached at each end of the frame. The movements of both ends were observed as the load increased. The maximum movements (upward) were observed at 60,000 lb. and the amounts were 1/263 in. at the east end and 1/300 in. at the west end. Therefore the steel stress in the beam of the side span may have been slightly modified by the movement, but the structure as a whole probably was not appreciably affected. Uniform load was applied to the upper horizontal member of the middle span. Fig. 57 shows the appearance of the frame after the test with the location of the cracks. No cracks were observed until the load had reached 45,000 lb., when three cracks appeared in the middle span, and one in each outer span on the top side of the horizontal member near the intermediate columns. The former are due to the positive bending moment and the latter are due to the negative moment. These cracks were located symmetrically and they extended vertically about 6 in. At 60,000 lb. they extended deeper. The frame was subjected to the load of 60,000 lb. over 20 hours, but the fall in the applied load was only 300 lb.

At 75,000 lb. several new cracks appeared at both ends of the middle span and also in the upper part of the intermediate columns. At this load the reinforcement at the bottom of the middle beam was stressed in tension beyond the elastic limit of the steel. The frame, however, carried an increasing load in good condition and the highest load was 134,000 lb. At this load the crack at the center of the middle span had opened considerably and the steel at this place had scaled, indicating failure by tension in the steel. At the same time the concrete at the top of the intermediate columns had crushed. Also the concrete base was cracked at the bottom end of the right-hand intermediate column. It is noted that the stresses in the outside columns were very low, even at the maximum load.

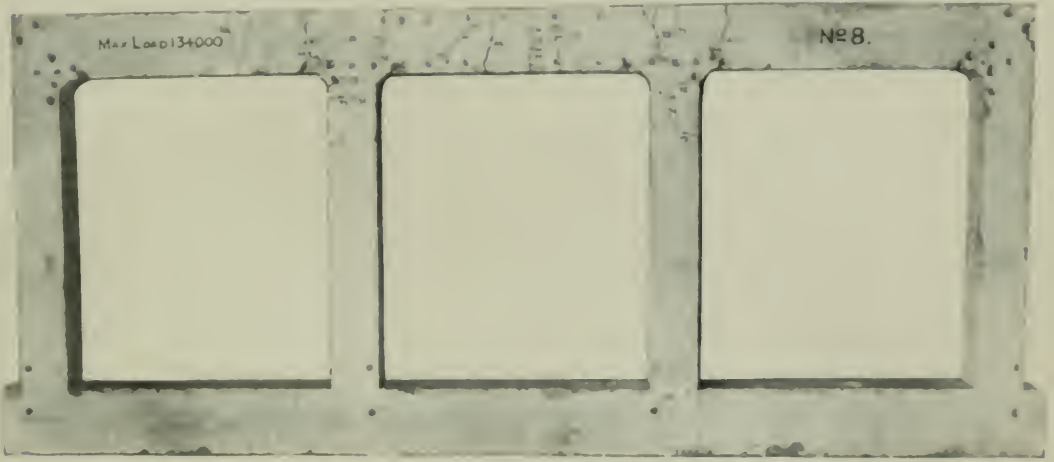


FIG. 57. VIEW OF FRAME S AFTER TEST

23. *Conditions Considered in the Comparison of Tests with Analysis.*—In the comparison of test results with analysis, the following modifying conditions have been taken into consideration:

(1) The quality of the concrete was not uniform over the cross-section of the member. The frames were made in a horizontal position on the floor of the laboratory, and the concrete on the rear side of the frame (the bottom side for the position in which the frame was made) seems to have been richer than that on the front side. The concrete on the rear side was stiffer and stronger than that on the front side and the distribution of steel stresses seem to have been modified by this fact. There was more stress in the steel at the rear side of the member than in the steel at the front side. It appears reasonable to take the average value of the observed stresses in the part in question for the purpose of comparison.

(2) The steel stresses are greatly modified by the presence of tension in the concrete for the low loads. Therefore in comparing the computed stresses with the observed stresses, two cases must be considered, one in which the concrete is considered to take tension and the other in which the concrete is considered to be broken in tension.

(3) The cross-sections of the test frames were designedly made larger in proportion to the span than would commonly be used in practice. In most of the test pieces, the column width occupied nearly one-seventh of the nominal span (distance center to center of the columns). In addition to this the corner at the juncture of the beam and the column was provided with a fillet. Under these conditions the bending moment at the center of the beam will be less than that calculated on the basis of the nominal span length—the difference being greater than that occurring with the dimensions found in practice. In the computations for the horizontal reactions the nominal span and height of the frames (distances center to center) were used. In finding the numerical values of the bending moment in the beam, and also of those in columns having fixed ends, the horizontal reactions computed as described previously were used, but when the equations involved further use of span lengths the nominal span length was replaced by the clear length of the span.

(4) The design of Frames 1 to 7 was such as to cause high stresses in columns and beams at about the same time. In Frame 8 the presence

of the members of unloaded panels caused the moments in the columns to be much smaller than in the loaded beam. The base also offered so much restraint as to give the column ends almost a fixed condition. Consequently loads which would cause high stresses in the beams would not produce cracks even in the intermediate columns although the moment there would be greater than in the outer columns. It will be seen then that for Frame 8 there are two cases to be considered in comparing the experimental results with the analyses, one in which tensile strength in the concrete is considered in all members and the other in which the beams and the intermediate columns are cracked on the tension side. The bending moment in the outside column is very small, and there is no chance for tension cracks. The moments in the intermediate columns and in the beams of the side spans are also small except at the extreme end, and only one crack appeared in this member. Therefore in the calculation of the moment of inertia of the cross-section for the second case it is not correct to neglect entirely the tensile strength of the concrete in these two members. A probable value for the moment of inertia of these members will be an average between that obtained by using the full cross-section and a section which neglects the part outside the tension rods. This assumption was made in the numerical computation of the moments and stresses for Frame 8.

In making the computations of stress in the concrete of the frames a constant modulus of elasticity was used, that is, a straight line stress-deformation relation was assumed. The selection of a proper value of the modulus of elasticity was somewhat dependent upon a knowledge of the qualities of the concrete of the various frames and a comparison of the behavior of the frames with that of the corresponding control cylinders. Naturally the modulus of elasticity used was generally less than the initial modulus. The values of modulus of elasticity of the concrete used in the computations of stress are given in Table 11.

24. *Comparison of Test Results with Analyses.*—The observed stresses, i. e., those obtained from the observed deformations by the process already described, have been plotted in Fig. 36 to 43. The light full line represents stress in the steel at the nearer side of the specimen; the dashed line the stress in the farther side of the specimen. The dotted line represents stress in the concrete, usually at the median plane. In general, the stresses have been plotted from the central longitudinal axis of the member in which they were observed. Because of the possibility of confusion resulting from this method an exception

was made in Frames 2, 5, and 8, where the stresses have been plotted from the line showing the location of the reinforcing bar. Tension in steel and compression in concrete when measured at or near the inner surface of a member have been plotted in, that is, toward the central part of the frame as a whole; tension in steel and compression in concrete when measured at or near the outer surface of a member have been plotted out. In only a few cases have tension in concrete and compression in steel been plotted. Both have been plotted out if measured on the inner surface of a member and in if measured on the outer surface. With very few exceptions, consequently, steel stresses represent tension regardless of whether in the diagrams they appear as positive or as negative with reference to the coördinate axes. Plotted concrete stresses, likewise, represent compression. The point at which the stress line crosses the coördinate axis represents the position of the point of inflection and not a change in the sign of the stress. The heavy full line represents the computed stresses for both steel and concrete which have been calculated for various points by means of the formulas given in preceding pages.

In Fig. 45 and 47 are given load-deformation curves for gage lines not represented in the diagrams of Fig. 36 to 43. The location of the gage lines is shown in Fig. 46 and 48.

In Table 14 are given values of both observed and computed stresses at three points for several loads for all the frames. Where I precedes the computed stress the calculation considered the tensile strength of the concrete; where II precedes it and where neither I nor II is given, the computed stress is based on the assumption of no tensile strength in the concrete.

A study of the tables and diagrams seems to justify the following statements:

(1) The experimental and the computed values of steel stress at the center of the loaded top beam are in fair agreement for each kind of frame tested except in a few instances in which the load was comparatively low or extremely high.

(2) The experimental and the computed values of the steel stress in the columns are also in fair agreement, but the maximum difference between experimental and theoretical values is higher than in the beams, owing to the fact that the direct stress is not equally distributed over the cross-section of the column.

TABLE 14

COMPUTED AND OBSERVED STRESSES IN FRAMES

Computed Values are given in Roman Type; Observed Stresses in Italics;
f_s is Unit Stress in Steel; f_c is Unit Stress in Concrete.

Frame and Load	A		B		C	
	f _s	f _c	f _s	f _c	f _s	f _c
Frame 1						
18 000	14 800 <i>16 100</i>	710 <i>1 140</i>	10 800 <i>8 900</i>	 <i>.....</i>	16 900 <i>17 900</i>	840 <i>1 240</i>
30 000	24 600 <i>25 300</i>	1 180 <i>2 560</i>	18 000 <i>17 400</i>	 <i>.....</i>	28 200 <i>28 600</i>	1 400 <i>2 500</i>
36 000	29 500 <i>31 900</i>	1 420 <i>.....</i>	21 600 <i>21 300</i>	 <i>.....</i>	33 800 <i>36 000</i>	 <i>.....</i>
Frame 2						
12 000 I	1 800	490				
II	11 600 <i>5 000</i>	710 <i>690</i>	11 200 <i>11 600</i>	800 <i>730</i>	21 000 <i>21 100</i>	 <i>.....</i>
Frame 3						
30 000 I	600	480				
II	 <i>1 400</i>	 <i>250</i>	13 200 <i>11 900</i>	1 080 <i>1 520</i>	22 800 <i>18 600</i>	1 350 <i>820</i>
38 000 I	900	610				
II	 <i>2 000</i>	 <i>340</i>	16 700 <i>16 700</i>	1 370 <i>1 840</i>	28 900 <i>25 700</i>	1 710 <i>1 190</i>
46 000 I	1 100	740				
II	 <i>2 800</i>	 <i>400</i>	20 200 <i>22 300</i>	1 660 <i>1 850</i>	35 000 <i>34 300</i>	2 070 <i>1 500</i>
Frame 4						
10 000 I	1 500	410	2 000	340		
II	8 500 <i>3 400</i>	540 <i>590</i>	8 700 <i>3 500</i>	630 <i>680</i>	8 700 <i>7 200</i>	630 <i>730</i>
21 000 I	3 200	860				
II	17 800 <i>11 700</i>	1 130 <i>1 550</i>	18 300 <i>16 300</i>	1 320 <i>1 670</i>	18 300 <i>19 300</i>	1 320 <i>1 540</i>
30 000	25 500 <i>21 400</i>	1 620 <i>2 350</i>	26 100 <i>23 700</i>	1 890 <i>.....</i>	26 100 <i>29 900</i>	1 890 <i>2 550</i>
Frame 5						
40 000 I	— 1 900	— 3 000 ¹	— 900	— 3 900 ¹		
II	 <i>— 2 000</i>	 <i>— 3 700¹</i>	 <i>+ 1 300</i>	 <i>— 2 200¹</i>	12 000 <i>10 100</i>	 <i>.....</i>
80 000 I	— 3 700	— 5 900 ¹	— 1 900	— 7 800 ¹		
II	 <i>— 2 400</i>	 <i>— 7 300¹</i>	<i>+15 200</i> <i>+ 4 500</i>	<i>— 5 300¹</i> <i>— 8 500¹</i>	24 000 <i>24 800</i>	 <i>.....</i>
100 000 I	— 4 700	— 7 400 ¹	— 2 400	— 9 700 ¹		
II	 <i>— 2 000</i>	 <i>— 9 400¹</i>	<i>+19 000</i> <i>+ 9 300</i>	<i>— 6 600¹</i> <i>—10 900¹</i>	30 000 <i>33 500</i>	 <i>.....</i>

TABLE 14—(CONTINUED)

COMPUTED AND OBSERVED STRESSES IN FRAMES

Computed Values are given in Roman Type; Observed Stresses in Italics;
 f_s is Unit Stress in Steel; f_c is Unit Stress in Concrete.

Frame and Load	A		B		C	
	f_s	f_c	f_s	f_c	f_s	f_c
Frame 6						
18 000 I	1 800	540	2 200	600	4 000	760
I					19 600 ²	1 170 ²
II			10 800	830	9 900	840
	<i>3 700</i>	<i>900</i>	<i>6 900</i>	<i>1 090</i>	<i>9 300</i>	<i>1 080</i>
30 000 I	3 000	900			31 800 ²	1 950 ²
II	24 300	1 320	18 000	1 380	26 500	1 680
	<i>14 800</i>	<i>1 670</i>	<i>17 700</i>	<i>2 480</i>	<i>21 900</i>	<i>2 050</i>
36 000 I					38 200 ²	2 340 ²
II	29 100	1 930	21 600	1 660	32 900	2 020
	<i>23 600</i>	<i>2 460</i>	<i>22 100</i>		<i>28 100</i>	<i>2 750</i>
Frame 7						
21 000 I	900	3 700 ¹	2 700	610		
II			9 000	800	16 000	1 000
	— <i>100</i>	<i>3 400</i> ¹	<i>4 900</i>	<i>1 350</i>	<i>13 100</i>	<i>1 530</i>
38 000 I	1 600	6 800 ¹	4 900	1 100		
II			16 300	1 440	28 900	1 830
	<i>400</i>	<i>6 000</i> ¹	<i>12 100</i>	 ³	<i>27 500</i>	<i>2 900</i>
46 000 I	1 900	8 200 ¹				
II			19 800	1 750	35 000	2 210
	<i>700</i>	<i>7 700</i> ¹	<i>16 500</i>	 ³	<i>37 000</i>	 ³
Frame 8						
30 000 I	8 900	850	5 000	6 500 ¹	1 800	1 800 ¹
	<i>7 000</i>	<i>1 490</i>	<i>1 300</i>	<i>5 100</i> ¹	<i>2 100</i>	<i>3 600</i> ¹
60 000 I	27 700 ²	1 700 ²	9 900	13 000 ¹	3 600	3 600 ¹
II					18 300	2 800 ¹
	<i>24 500</i>	<i>2 540</i>	<i>7 200</i>	<i>13 100</i> ¹	<i>12 800</i>	<i>6 200</i> ¹
75 000 I			12 400	16 200 ¹		
II	38 500	2 360			23 000	3 500 ¹
	<i>37 500</i>	<i>2 750</i>	<i>11 500</i>	<i>17 700</i> ¹	<i>19 100</i>	<i>6 700</i> ¹

¹ Compressive stress in steel.

² Concrete in beam under load considered as broken in tension.

³ Very high stress.

(3) The observed compressive stresses in the concrete at the low loads which developed a unit-stress up to about 800 lb. per sq. in. agree reasonably well with the computed values though in most of these cases the observed concrete stresses were somewhat higher than the computed stresses. In some instances the discrepancies ran up to 50 per cent, and for higher stresses the discrepancies were frequently even greater. Undoubtedly these differences are partly due to the fact that the modulus of elasticity used does not represent correctly the modulus of elasticity of the concrete in the frames. Other matters difficult of explanation probably cause further discrepancies.

25. *Effect of End Condition of Column on Results.*—The secondary stresses which would be expected as a result of the friction in the bearings at the free ends of the columns in Frames 1, 2, 4, and 6 seem to have been very small and may be neglected without appreciable error.

The concrete bases used for Frames 3, 7, and 8 to secure the fixity of the column ends were, of course, not entirely rigid, and a slight bending in the base due to a load may be expected to have an influence on the bending in the other members. Deformation readings at the middle point of the base were taken at each increase in the load. The results of these observations showed practically no bending stress for all loads except the ultimate load.

26. *Distribution of Stress over the Cross-Section.*—In the observations it was found that stress in the steel on bars near a front corner of a member differed from that on bars near a back corner of the member, the front of the member being the top side of the frame as poured and the back side being the bottom. In Table 15 are given stresses in bars at front and back at two places on the frame for one load generally near the maximum. It is seen that generally the stress in a bar near the front of the member (top of the member as poured) is less than that in a bar near the back (bottom of the member as poured). To investigate the distribution further, special measurements were made in the columns of Frames 6 and 7, the gage lines being placed where bending was not sufficiently large to produce tension cracks in the concrete. The gage lines were located on the four faces of the column, and the observations were made at each load. The front outer corner developed the lowest tensile stress and the front inner corner the highest compressive stress. The back outer corner developed the highest tensile

TABLE 15
STRESSES IN BARS AT FRONT AND BACK OF MEMBERS OF TEST FRAMES

Frame No.	At Center of Span				At Upper Part of Column			
	Observed Stress in Steel		Per cent	Difference	Observed Stress in Steel		Per cent	Difference
	Back Side	Front Side			Back Side	Front Side		
1	22 200	27 100	22.3	- 4 900	28 600	35 200	23.1	- 6 600
2	32 900	27 400	16.7	+ 5 500	11 300	9 500	15.9	+ 1 800
3	39 500	29 000	26.6	+10 500	25 500	18 900	25.9	+ 6 600
4	34 400	24 000	30.2	+10 400	31 000	27 600	11.0	+ 3 400
5	36 400	30 600	16.4	+ 5 800	35 600	22 700	36.2	+12 900
6	29 800	26 400	11.4	+ 3 400	23 500	19 300	17.9	+ 4 200
7	33 500	21 500	35.8	+12 500	22 300	10 500	52.9	+11 800
8	30 900	18 400	40.4	+12 500	21 400	15 200	29.0	+ 6 200

stress and the back inner corner the lowest compressive stress. The distribution was not much altered by the increase of load.

The observations indicate a lateral bending and twisting of the frame. It seems probable that the main source of the difference in stresses from front to back was the lack of homogeneity of the concrete, that at the bottom of the member as poured being stronger and stiffer than that at the top. A difference in stiffness would at least partially account for the phenomena.

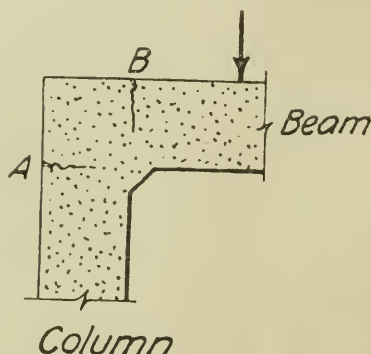


FIG. 58. CONNECTION OF BEAM AND COLUMN SHOWING TYPICAL LOCATION OF FIRST CRACKS

27. *Position of Point of Inflection in Columns.*—The position of the point of inflection in a member of a structure which is subject to flexure is an important element for use in designing the frame. To determine from observed deformations the position of the point of inflection for a member, it is necessary to separate the deformation into that caused by direct stress and that due to flexure of the member. The deformation in the columns of Frames 3 and 7 were thus separated, a straight line stress-deformation relation being assumed, and the position of the point of inflection found. The position of the point of inflection in the columns of these frames changed very little during the progress of the loading. For these frames the point of inflection was found to be almost exactly at one-third the height of the column, as is indicated by the analysis.

28. *Continuity of the Composing Members of a Frame.*—In the tests of the frames there was no sign of discontinuity of members whatever. It is apparent from the action of the frames and from the stresses

observed that the stresses and therefore the moments were well transmitted from member to member by the connection. From the results it is felt that there is every reason to have confidence in the rigidity of connections in frames that are properly designed.

In the frames free to turn at the lower column ends there was a tendency for a crack to form near the juncture at A (Fig. 58) at a lower load than that at which a crack appeared at B. In the frames with rigid connection at the lower column ends, the crack at B appeared at nearly the same time as that at A.

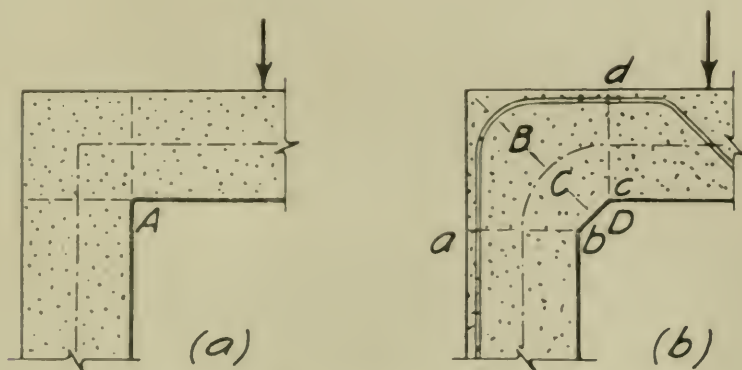


FIG. 59. RECTANGULAR JOINT WITH AND WITHOUT FILLET

29. *Stresses at Corners.*—In the design of a frame a square corner such as that shown at A in Fig. 59 (a) should be avoided for all connections, for it is well known that theoretically in resisting bending the material at the corner would develop excessively high stresses. It is therefore common to design such corners with fillets as shown in Fig. 59 (b). No attempt will be made to compute the stresses at the fillets. The observed deformations at gage lines in the neighborhood of the fillets and within the space occupied by the intersection of the two members are plotted in Fig. 45 and 47. These deformations are of interest. Some of these values have been converted into concrete stresses by the use of the moduli of elasticity of the concrete already assumed and are given in Table 16.

TABLE 16
OBSERVED STRESSES AT SHARP CORNERS IN FRAMES
1, 3 and 6
Stress in lb. per sq. in.

Load lb.	Frame 1 Gage Line 69	Frame 3 Gage Line 81	Frame 6 Gage Line 71
7 000		300	
10 000			310
12 000	80		
14 000		720	
18 000	670		450
21 000		760	
24 000	810		900
30 000		1 460	1 100
36 000	1 410		1 320
38 000		2 120	

30. *Conclusions and General Comments.*—Some of the conclusions which may be drawn from the tests and the discussion are as follows:

(1) Considering the errors involved in the measurement of the deformations and in the determination of the modulus of elasticity of the concrete, as well as those due to assumptions with reference to the distribution of stresses across the section and over the gage length, the results presented indicate a fair agreement between analyses and tests and justify the conclusion that the formulas given in the bulletin for statically indeterminate stresses as applied to reinforced concrete structures will give values for stresses in the members well within the limit of accuracy required in design.

(2) The elastic action of the frames under external load and the manner of stress distribution along the members of the frame agree fairly well with the analyses given.

(3) The location of the point of inflection in the members of the frames under load agrees closely with the location found by analyses.

(4) If a frame is carefully designed and well reinforced, there need be no anxiety as to the rigidity of a joint. Effective continuity of members has been found in the tests.

(5) No sudden failure took place in the frames tested. The increase in the deflection was uniform, indicating as great reliability for reinforced concrete frames as for steel structures.

(6) The load at which the first fine crack appears near the juncture of members is increased by fixing the lower column ends of a frame. This is obviously due to the increase in horizontal thrust at the lower column end over that developed when the lower end is free to turn.

(7) At sharp inside corners, high compressive stresses were developed in the concrete due to so-called curved beam action and in several cases local failure occurred by the crushing of the concrete at these corners under high loads.

(8) A slight deviation of the axis of vertical members from a vertical line, that is to say, a slight "out-of-form" of the vertical columns, produced an appreciable variation in the stress distribution in the frame.

(9) Owing to the existence of a horizontal thrust (which varies from $\frac{1}{8}P$ to $\frac{1}{18}P$ in most common cases of simple frames) at the ends of a vertical or inclined member, it is advisable to incline the member slightly toward the direction of the reaction at the end. Such arrangement will greatly reduce the bending stress in the member. If this arrangement is not practicable, a slight increase in the top width of a vertical member and a slight decrease in its bottom width, brought about by inclining the inner surface and making the outer surface vertical will add materially to the rigidity of a frame without a proportional increase in the amount of material used.

(10) For a frame having an inclined column, it may be possible to select the form of frame in such a way that the column will take no bending stress throughout its length.

(11) Due attention should be paid to the rigid joint of a tie member to insure the stiff connection with a main member. A marked tendency to cause a sudden breaking of such a joint accompanied an increase of bending moment in a main member.

(12) The use of a footing rigidly connected to the lower end of a vertical member is advisable, for it will reduce the bending moment at the juncture of the vertical and inclined members. A frame having such a footing is solvable analytically, since it approaches the case halfway between that of the hinged end and that of the fixed end of the vertical member, provided the foundation is sufficiently unyielding. A little consideration is needed to provide proper reinforcement at the juncture of the column and the footing.

(13) The formulas derived by analysis may be applied to a variety of forms of frames and are of wide applicability.

LIST OF PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION

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- Circular No. 1.* High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None available.*
- Bulletin No. 2.* Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*
- Circular No. 2.* Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*
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- Bulletin No. 3.* The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*
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- Bulletin No. 6.* Holding Power of Railroad Spikes, by Roy I. Webber. 1906. *None available.*
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- Bulletin No. 9.* An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Revised Edition 1912. *Fifty cents.*
- Bulletin No. 10.* Tests of Concrete and Reinforced Concrete Columns, Series of 1906, by Arthur N. Talbot. 1907. *None available.*
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BULLETIN No. 108

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ANALYSIS OF STATICALLY INDETER-
MINATE STRUCTURES BY THE
SLOPE DEFLECTION METHOD

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CONTENTS

	PAGE
I. PRELIMINARY	9
1. Object and Scope of Investigation	9
2. Acknowledgments	9

PART I

DERIVATION OF FUNDAMENTAL EQUATIONS

II. PROPOSITIONS UPON WHICH FUNDAMENTAL EQUATIONS ARE BASED	10
3. Statement of Propositions	10
4. Proof of Propositions	10
III. DERIVATION OF FUNDAMENTAL EQUATIONS	14
5. Conventional Signs	14
6. Derivation of Equations for Moments at Ends of Members in Flexure—Member Restrained at the Ends with No Intermediate Loads	14
7. Derivation of Equations for Moments at Ends of Members in Flexure—Member Restrained at the Ends with Any System of Intermediate Loads	17
8. Derivation of Equations for Moments at Ends of Members in Flexure—Member Restrained at the Ends, with Special Cases of Loading	20

PART II

DETERMINATION OF STRESSES IN STATICALLY INDETERMINATE STRUCTURES

9. Assumptions upon which the Analyses are Based	28
10. Notation	28
IV. GIRDERS HAVING RESTRAINED ENDS	30
11. Moments at the Ends of a Girder Having Fixed Ends—Both Supports on the Same Level	30
12. Moments at the End of a Girder Having One End Fixed and the Other End Hinged, Both Supports on the Same Level	30
13. Moments at the Ends of a Girder Having Ends Restrained but not Fixed	30
14. Moment at End of a Girder Having One End Hinged and the Other End Restrained but not Fixed	31

V. CONTINUOUS GIRDERS	32
15. Girder Continuous over Any Number of Supports and Carrying Any System of Vertical Loads. Supports All on the Same Level. General Equation of Three Moments—Two Intermediate Spans	32
16. Girder Continuous over Any Number of Supports and Carrying Any System of Vertical Loads. Supports All on the Same Level. General Equation of Three Moments—Two Adjacent Spans at One End. End of Girder Hinged	33
17. Girder Continuous over Any Number of Supports and Carrying Any System of Vertical Loads. Supports All on the Same Level. General Equation of Three Moments—Two Adjacent Spans at One End. End of Girder Restrained	34
18. Girder Continuous over Any Number of Supports and Carrying Any System of Vertical Loads. Supports All on the Same Level. General Equation of Three Moments—Girder Fixed at the Ends	35
19. Girder Continuous over Three Supports and Carrying Any System of Vertical Loads. Supports on Different Levels. General Equation of Three Moments	35
20. Girder Continuous over Three Supports and Carrying Any System of Vertical Loads. Supports on Different Levels. Various Conditions of Restraint of Ends	37
21. Girder Continuous over Four Supports and Carrying Any System of Vertical Loads. Supports on Different Levels. Various Conditions of Restraint of Ends	37
VI. TWO-LEGGED RECTANGULAR BENT. LEGS OF EQUAL LENGTH	49
22. Two-legged Rectangular Bent. Concentrated Horizontal Force at the Top—Legs Hinged at the Bases	49
23. Two-legged Rectangular Bent. Concentrated Horizontal Force at the Top—Legs Fixed at the Bases	50
24. Two-legged Rectangular Bent. Any System of Horizontal Loads on One Leg—Legs Hinged at the Bases	54

25. Two-legged Rectangular Bent. Any System of Horizontal Loads on One Leg—Legs Fixed at the Bases	55
26. Two-legged Rectangular Bent. Any System of Vertical Loads on the Top—Legs Hinged at the Bases	58
27. Two-legged Rectangular Bent. Any System of Vertical Loads on the Top—Legs Fixed at the Bases	59
28. Two-legged Rectangular Bent. External Moment at One Corner—Legs Hinged at the Bases	62
29. Two-legged Rectangular Bent. External Moment at One Corner—Legs Fixed at the Bases	63
30. Two-legged Rectangular Bent. Settlement of Foundations—Legs Hinged at the Bases	65
31. Two-legged Rectangular Bent. Settlement of Foundations—Legs Restrained at the Bases	67

VII. TWO-LEGGED RECTANGULAR BENT. ONE LEG LONGER THAN THE OTHER	71
32. Two-legged Rectangular Bent. One Leg Longer than the Other. Concentrated Horizontal Load at Top of Bent—Legs Hinged at the Bases	71
33. Two-legged Rectangular Bent. One Leg Longer than the Other. Concentrated Horizontal Load at Top of Bent—Legs Fixed at the Bases	72
34. Two-legged Rectangular Bent. One Leg Longer than the Other. Any Horizontal Load on Vertical Leg—Legs Hinged at the Bases	75
35. Two-legged Rectangular Bent. One Leg Longer than the Other. Any Horizontal Load on Vertical Leg—Legs Fixed at the Bases	78
36. Two-legged Rectangular Bent. One Leg Longer than the Other. Any System of Vertical Loads on Top of Bent—Legs Hinged at the Bases	81
37. Two-legged Rectangular Bent. One Leg Longer than the Other. Any System of Vertical Loads on Top of Bent—Legs Fixed at the Bases	83
38. Two-legged Rectangular Bent. One Leg Longer than the Other. External Moment at One Corner—Legs Hinged at the Bases	86

39. Two-legged Rectangular Bent. One Leg Longer than the Other. External Moment at One Corner—Legs Fixed at the Bases	87
40. Two-legged Rectangular Bent. One Leg Longer than the Other. Settlement of Foundations—Legs Hinged at the Bases	89
41. Two-legged Rectangular Bent. One Leg Longer than the Other. Settlement of Foundations—Legs Fixed at the Bases	90
VIII. TWO-LEGGED TRAPEZOIDAL BENTS. BENTS AND LOADING SYMMETRICAL ABOUT VERTICAL CENTER LINE	
42. Two-legged Trapezoidal Bent. Bent and Load Symmetrical about Vertical Center Line. Vertical Load on Top—Legs Hinged at the Bases	94
43. Two-legged Trapezoidal Bent. Bent and Load Symmetrical about Vertical Center Line. Vertical Load on Top—Legs Fixed at the Bases	95
44. Two-legged Trapezoidal Bent. Bent and Loads Symmetrical about Vertical Center Line. Loads Normal to Legs of Bent—Legs Hinged at the Bases	95
45. Two-legged Trapezoidal Bent. Bent and Loads Symmetrical about Vertical Center Line. Loads Normal to Legs of Bent—Legs Fixed at the Bases	96
46. Two-legged Trapezoidal Bent. Bent and Loads Symmetrical about Vertical Center Line. External Moments at Upper Corners of Bent—Legs Hinged at the Bases	97
47. Two-legged Trapezoidal Bent. Bent and Loads Symmetrical about Vertical Center Line. External Moments at Upper Corners of Bent—Legs Fixed at the Bases	98
IX. RECTANGULAR FRAMES	
48. Rectangular Frame. Horizontal Force at the Top	99
49. Rectangular Frame. Any System of Horizontal Forces on One Vertical Side	102
50. Rectangular Frame. Any System of Vertical Forces on the Top	105
51. Rectangular Frame. External Moment at Upper Corner	109

X. MOMENTS IN FRAMES COMPOSED OF A LARGE NUMBER OF RECTANGLES AS A SKELETON-CONSTRUCTION BUILDING FRAME	113
52. Effect of Restraint at One End of a Member upon Moment at Other End	113
53. Moment in a Frame Composed of a Number of Rectangles Due to Vertical Loads	116
54. Extremities of Members Hinged	118
55. Extremities of Members Fixed	119
56. Distribution of Loads for Maximum Moments in a Frame Composed of a Large Number of Rectangles	121
57. Eccentric Load at Top of Exterior Column of a Frame. Connections of Girders to Columns Hinged	124
58. Eccentric Load at Top of Exterior Column of a Frame. Connections of Girders to Columns Rigid	129
59. Eccentric Load at Middle Floor Level of Exterior Column of a Frame. Connections of Girders to Columns Hinged	133
60. Eccentric Load at Middle Floor Level of Exterior Column of a Frame. Connections of Girders to Columns Rigid	136
61. Effect of Settlement of One Column of a Frame Composed of a Number of Rectangles	142
XI. THREE-LEGGED BENT	147
62. Three-legged Bent. Lengths of All Legs Different. Vertical Load on Left-hand Span—Legs Hinged at the Bases	147
63. Three-legged Bent. Lengths of All Legs Different. Vertical Load on Left-hand Span—Legs Fixed at the Bases	149
64. Three-legged Bent. Lengths of All Legs Different. Any System of Loads	150
65. Three-legged Bent. Lengths and Sections of All Legs Equal. Lengths and Sections of Top Members Equal. Any System of Loads. Let $\frac{I}{l}$ of a Top Member be n times $\frac{I}{h}$ of a Leg—Legs Hinged at the Bases	162

66. Three-legged Bent. Lengths and Sections of All Legs Equal. Lengths and Sections of Top Members Equal. Any System of Loads—Legs Fixed at the Bases	163
XII. EFFECT OF ERROR IN ASSUMPTIONS	181
67. Error Due to Assumption that Axial Deformation is Zero	181
68. Error Due to Assumption that Shearing Deformation is Zero	187
69. Effect of Slip in the Connections upon the Moments in a Rectangular Frame Having a Single Horizontal Force at the Top	194
XIII. NUMERICAL PROBLEMS	199
70. Girder Restrained at the Ends. Supports on Different Levels. Any System of Vertical Loads	199
71. Girder Continuous over Four Supports. Supports on Different Levels. Any System of Vertical Loads	200
72. Girder Continuous over Five Supports. Supports on Different Levels. Any System of Vertical Loads. Ends of Girder Restrained	202
73. Two-legged Rectangular Bent with Legs Fixed at the Bases. Bent and Loading Symmetrical about a Vertical Center Line	204
74. Two-legged Rectangular Bent. One Leg Longer than the Other. Concentrated Horizontal Load at the Top. Legs Fixed at the Bases	205
75. Two-legged Rectangular Bent. Settlement of Foundation	207
76. Three-legged Bent. Lengths of All Legs Different. Loads on Top and Settlement of Foundations	208
XIV. CONCLUSIONS	212

NO.	PAGE
1. General Equations for the Moments at the Ends of a Member AB in Fig. 6	20
2. Values of Constants C and H for Different Systems of Loads to be Used in the Equations of Table 1	24, 25
3. Values of Constants C and H for Different Systems of Loads to be Used in the Equations of Table 1	26, 27
4. Continuous Girders	38
5. Girder Continuous over Three Supports	40, 41
6. Girder Continuous over Four Supports	42-47
7. Equations for the Two-legged Rectangular Bent of Fig. 28	57
8. Equations for the Two-legged Rectangular Bent Represented by Fig. 30	61
9. Equations for the Moments in the Rectangular Bent Represented by Fig. 39	79
10. Equations for the Moments in the Rectangular Bent Represented by Fig. 42	84
11. Equations for Rectangular Frame with Horizontal Force at the Top	101
12. Equations for the Moments in the Rectangular Frame Represented by Fig. 54	103
13. Equations for the Moments in the Rectangular Frame Represented by Fig. 55	106
14. Equations for the Moments in the Rectangular Frame Represented by Fig. 56	110
15. Moments in Frame Represented by Fig. 61	121
16. Moments in Frame Represented by Fig. 62	126
17. Moments in a Frame Composed of a Number of Rectangles, Due to Settlement of One Column	146
18. Equations for Three-legged Bent	149
19. Equations for Three-legged Bent	151
20. Fundamental Equations for Three-legged Bents	152, 153
21. Fundamental Equations for Three-legged Bents	156, 157
22. Equations for Three-legged Bent	160
23. Equations for Three-legged Bent	161
24. Fundamental Equations for Three-legged Bent	164, 165
25. Equations for Three-legged Bent	166
26. Moments Due to Various Systems of Loads on a Three-legged Bent	167-170
27. Fundamental Equations for Three-legged Bent	172, 173
28. Equations for Three-legged Bent	175
29. Moments Due to Various Systems of Loads on a Three-legged Bent	177-180
30. Error in Stresses in Rectangular Frames Due to Neglecting Axial Deformation	187
31. Error in Stresses in Rectangular Frames Due to Neglecting Shearing Deformations	193
32. Equations for the Three-legged Bent of Fig. 112	210
33. Moments in Three-legged Bent of Fig. 112	211

ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES BY THE SLOPE DEFLECTION METHOD

I. PRELIMINARY

1. *Object and Scope of Investigation.*—Frames composed of rectangular elements must in general be designed with stiff connections between the members at the joints, in order that loads may be carried. These connections must be capable of transferring not only direct axial tensile and compressive forces, but also bending moments. It follows that frames made up of rectangular elements are usually statically indeterminate; that is, the stresses in them can be found only by taking into account the relative stiffness and deformations of the various members. The common use of rectangular frames in engineering structures makes it highly desirable that the most convenient methods of analyzing their stresses should be developed. The stresses in a number of such rectangular frames have been analyzed by the writers. This bulletin describes the methods used and presents the formulas derived.

The bulletin is divided into two parts: the first part is devoted to the derivation of fundamental equations; in the second part, methods and equations are derived for use in determining moments, stresses, and deflections for a variety of typical structures.

2. *Acknowledgments.*—The investigation here reported was made under the auspices of the Department of Civil Engineering of which DR. F. H. NEWELL is the head. A portion of the work was done in 1915 in connection with the development of a thesis in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering by F. E. RICHART. Many of the analyses have been checked by W. L. PARISH, graduate student in Architectural Engineering, and YI LIU, graduate student in Civil Engineering, to whom the authors gratefully acknowledge their indebtedness.

PART I

DERIVATION OF FUNDAMENTAL EQUATIONS

II. PROPOSITIONS UPON WHICH FUNDAMENTAL EQUATIONS
ARE BASED

3. *Statement of Propositions.*—The fundamental equations used in these investigations are derived from the principal propositions of the moment-area method.* These may be expressed as follows:

(1) *When a member is subjected to flexure, the difference in the slope of the elastic curve between any two points is equal in magnitude to the area of the $\frac{M}{EI}$ diagram for the portion of the member between the two points.*

(2) *When a member is subjected to flexure, the distance of any point Q on the elastic curve, measured normal to initial position of member, from a tangent drawn to the elastic curve at any other point P is equal in magnitude to the first or statical moment of the area of the $\frac{M}{EI}$ diagram between the two points, about the point Q.*

The $\frac{M}{EI}$ diagram is a graph in which the ordinate at any point is obtained by dividing the resisting moment, M , by the product of modulus of elasticity of material, E , and the moment of inertia of the section, I , at that point. If E and I are constant, the diagram will be similar in shape to the moment diagram for the member.

4. *Proof of Propositions.*—The line AB , Fig. 1, represents the elastic curve of a member in flexure. Consider the elementary length, ds , of the member shown in Fig. 2. The angle between radii at the ends of ds will be denoted by $d\theta$. The linear deformation of a fibre at a distance c from the neutral surface is $cd\theta$, and the unit deformation of the same fibre is $\frac{cd\theta}{ds}$. From the well known flexure formula the

*The principles of the moment-area method were given in an article by O. Mohr, *Beiträge zur Theorie der Holz-und Eisenkonstruktionen*, Zeitschrift des Arch.-und Ing. Ver. zu Hannover, 1868, p. 19. About the same time the method was presented by C. E. Greene in lectures at the University of Michigan. Several modern textbooks on mechanics give the method; see, for instance, *Strength of Materials*, by J. E. Boyd, Second Ed., 1917.

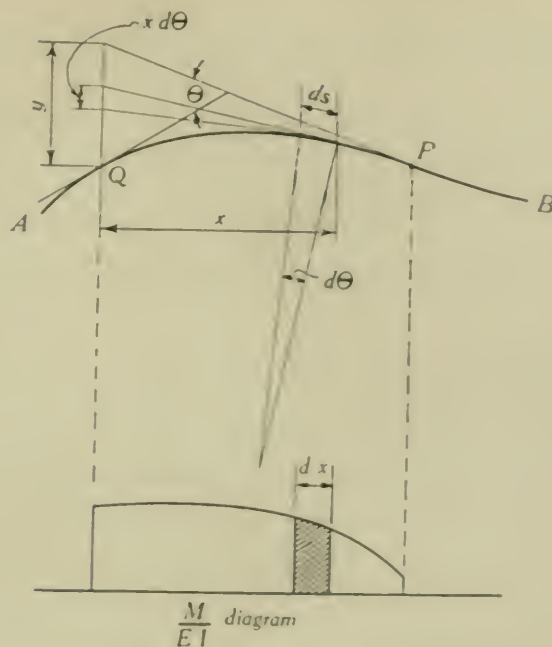


FIGURE 1

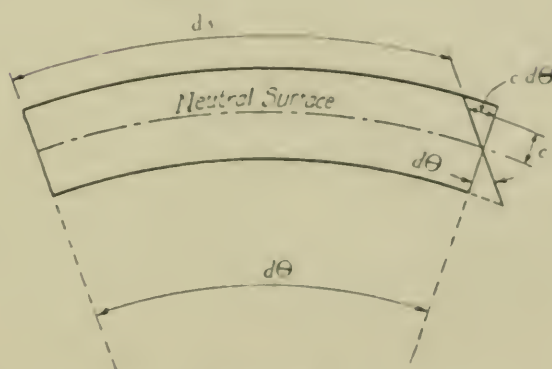


FIGURE 2

accompanying unit stress in the fibre is $S = \frac{Mc}{I}$, in which M is the resisting moment and I moment of inertia of section.

Since the modulus of elasticity is the ratio of unit stress to unit deformation, E is equal to $\frac{Mc}{I}$ divided by $\frac{cd\theta}{ds}$. Hence $d\theta = \frac{M}{EI} ds$. Since in a well designed beam, the curvature and slope are small, dx may be substituted for ds without material error, and $d\theta = \frac{M}{EI} dx$.

In the $\frac{M}{EI}$ diagram of Fig. 1, $\frac{M}{EI} dx$ represents the area of the diagram for the length dx . The area of the diagram between points Q and P on the elastic curve is then equal to $\int_Q^P \frac{M}{EI} dx$. But the difference of slope of the tangent to the elastic curve is also represented by

$$\theta = \int d\theta = \int_Q^P \frac{M}{EI} dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Hence proposition (1) of the preceding section is proved. It may be noted that if M is taken as the resisting moment acting on the portion of the member to the left of any section, by applying the conventions of section 5 the area of the $\frac{M}{EI}$ diagram is positive; also the direction of integration from Q to P is positive, and the difference in slope θ is positive. Other terms involved may be considered as scalar quantities. These conventions apply to any case, as, for instance, difference in slope from P to Q is negative, since the direction of integration is negative.

In Fig. 1 the tangents at the extremities of the element of the elastic curve, ds , are extended until they intersect the vertical line through the point Q . The intercept on this vertical line between the two consecutive tangents is $xd\theta$. The total vertical distance, y , of Q from the tangent drawn at P is the algebraic sum of all the intercepts between tangents for the portion of the curve between Q and P ;

that is, $y = \int_Q^P xd\theta$. Substituting the value of $d\theta$ found previously,

$$y = \int_Q^P \frac{M}{EI} x dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In the $\frac{M}{EI}$ diagram of Fig. 1, $\frac{M}{EI} dx$ represents the area of the diagram for the length dx , and $\frac{M}{EI} dx$ times x represents the moment of this area about the point Q . The moment of the entire area of the $\frac{M}{EI}$ diagram between points Q and P about the point Q may now be ex-

pressed by $\int_q^P \frac{M}{EI} x dx$. Since this expression is identical with the right-hand member of equation (2), proposition (2) of the preceding section is proved. The conventions of section 5 apply here as explained in the proof of proposition (1).

III. DERIVATION OF FUNDAMENTAL EQUATIONS

5. *Conventional Signs.*—The signs of the quantities used in the equations in this bulletin are determined by the following conventional rules:

When the tangent to the elastic curve of a member has been turned through a clockwise direction, measured *from* its initial position, the change in slope, or the angular deformation, is positive.

When the line joining the ends of a member is rotated, the movement of one end of the member relative to the other, measured perpendicular to the initial position of member is called a deflection, and is so used throughout the following discussion. The deflection is positive when such rotation is in a clockwise direction from the initial position of member.

The resisting moment or moment of the internal stresses on a section is positive when the internal or resisting couple acts in a clockwise direction upon *the portion of the member considered*. According to this rule the portion of the member considered must always be specified, and will be indicated by the subscripts used with the moments. For example, if C is a point on a member between the ends A and B , M_{CA} is equal to $-M_{CB}$.

The moment of an external force or couple is positive if it tends to cause a clockwise rotation.

6. *Derivation of Equations for Moments at Ends of Members in Flexure—Member Restrained at the Ends with No Intermediate Loads.*—The line AB in Fig. 3 represents the elastic curve of a member which is not acted upon by any external forces or couples except at the ends. The resisting moment at A is represented by M_{AB} and at B by M_{BA} . The change in the slope of the elastic curve at A from its initial position is represented by θ_A , and that at B by θ_B . The deflection of A from its original position A' is d . The distance of B from the tangent drawn to the curve at A is equal to $(d-l\theta_A)$.

From proposition (2), section 3, $(d-l\theta_A)$ may be expressed as the statical moment of the $\frac{M}{EI}$ diagram for member AB about the end B .

The quantities E and I will here be considered as constant throughout the length AB . If M represents the resisting moment on the portion of member to the left of a section, M is equal to $-M_{AB}$ at A , and to $+M_{BA}$ at B . The $\frac{M}{EI}$ diagram of Fig. 3 can best be treated as the

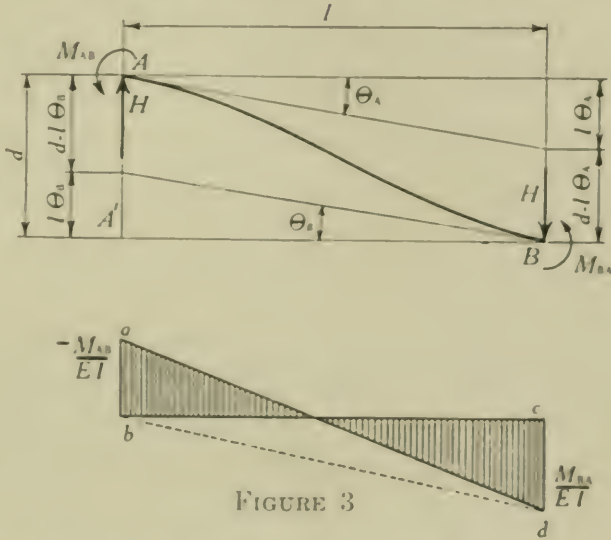


FIGURE 3

algebraic sum of the two triangles bad and bcd . Hence the statical moment of the $\frac{M}{EI}$ diagram about B is equal to the area of triangle bad times the distance to its centroid, $\frac{2}{3}l$, plus the area of triangle bcd times the distance to its centroid, $\frac{1}{3}l$. This gives

$$d - l\theta_A = \frac{-M_{AB}l^2}{3EI} + \frac{M_{BA}l^2}{6EI} \quad (3)$$

From proposition (1), section 3, $\theta_B - \theta_A$ is equal to the area of the $\frac{M}{EI}$ diagram for member AB , or the algebraic sum of areas bad and bcd . This gives

$$\theta_B - \theta_A = \frac{-M_{AB}l}{2EI} + \frac{M_{BA}l}{2EI} \quad (4)$$

Combining equations (3) and (4) to eliminate M_{BA} , letting $\frac{I}{l} = K$ and $\frac{d}{l} = R$, gives

$$M_{AB} = 2EK(2\theta_A + \theta_B - 3R) \quad (5)$$

Similarly combining equations (3) and (4) to eliminate M_{AB} gives

$$M_{BA} = 2EK(2\theta_B + \theta_A - 3R) \quad (6)$$

Since the signs of all quantities in equations (3) and (4) are independent of the sense of the quantities themselves it follows that equations (3) and (4) are general; and they give the sense as well as magnitude of the moments, no matter what the senses of θ_A , θ_B , and R may be, provided the method of determining signs given in section 5 is followed. As before noted, M_{AB} is the resisting moment acting at the end A of the member AB . The moment which AB exerts upon the support at A is equal in magnitude but opposite in sense to M_{AB} . A and B are not necessarily supports of a member but may be any two points along the length of a member, provided there is no intermediate load on the member between them.

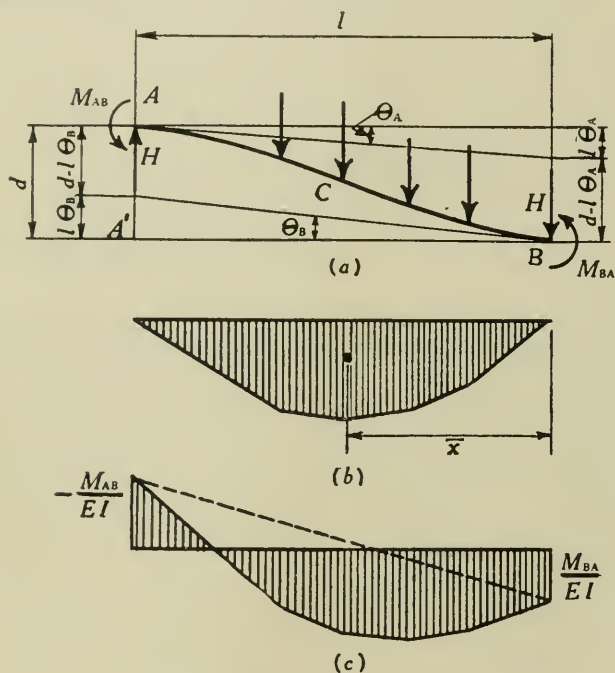


FIGURE 4

Equations (5) and (6) are fundamental equations.* They may be expressed as follows:—The moment at the end of any member carrying no intermediate loads is equal to $2EK$ times the quantity: Twice the change in slope at the near end plus the change in slope at the far end minus three times the ratio of deflection to length. E is the modulus

*The slope-deflection equations for a member acted upon only by forces and couples at the ends were deduced by Manderla in 1878. See Annual Report of the Technische Hochschule, Munich, 1879, and Allgemeine Bauzeitung, 1880. The use of these equations has been developed by several writers, among whom are:

Mohr, Otto, "Abhandlungen aus dem Gebiete der Technischen Mechanik," Second Ed., 1914.

Kunz, F. C. "Secondary Stresses," Engineering News, Vol. 66, p. 397, Oct. 5, 1911.

Wilson and Maney, "Wind Stresses in the Steel Frames of Office Buildings," Univ. of Ill. Eng. Exp. Sta., Bul. 80, 1915.

of elasticity of the material, and K is the ratio of moment of inertia to length of member.

7. *Derivation of Equations for Moments at Ends of Members in Flexure—Member Restrained at the Ends with Any System of Intermediate Loads.*—The line AB , Fig. 4-a, represents the elastic curve of the member of Fig. 3, but acted upon by a system of intermediate loads. The moments, slopes, and deflections at A and B are similar to those of Fig. 3. The $\frac{M}{EI}$ diagram, however, is affected by the intermediate loads. The quantity EI will again be considered constant. From well known principles of mechanics, the $\frac{M}{EI}$ diagram of Fig. 4-c may be obtained by superimposing the $\frac{M}{EI}$ diagram for a simple beam under the same intermediate loads (see Fig. 4-b) upon the $\frac{M}{EI}$ diagram of Fig. 3. This is merely the algebraic addition of the different moments at any section, just as in an algebraic analysis the moment at the end of a girder is combined with the moment of the shear at the end and of the external loads about the given section. Denote the area of the simple beam diagram of Fig. 4-b by F , and the distance of its centroid from B by $\bar{x}$. Then, using the propositions of section 3 as before, the

statical moment of the $\frac{M}{EI}$ diagram about B is equal to $(d - \theta_A l)$.

$$(d - \theta_A l) = \frac{-M_{AB}l}{3EI} + \frac{M_{BA}l}{6EI} - \frac{F\bar{x}}{EI} \quad \dots \quad (7)$$

The area of the $\frac{M}{EI}$ diagram is equal to $\theta_B - \theta_A$.

$$(\theta_B - \theta_A) = \frac{-M_{AB}l}{2EI} + \frac{M_{BA}l}{2EI} - \frac{F}{EI} \quad \dots \quad (8)$$

Combining equations (7) and (8) to eliminate M_{BA} , letting $\frac{I}{l} = K$ and $\frac{d}{l} = R$, gives

$$M_{AB} = 2EK (2\theta_A + \theta_B - 3R) - \frac{2F}{l^2} (3\bar{x} - l) \quad \dots \quad (9)$$

Similarly, combining equations (7) and (8) to eliminate M_{AB} gives

$$M_{BA} = 2EK (2\theta_B + \theta_A - 3R) + \frac{2F}{l^2} (2l - 3\bar{x}) \quad \dots \quad (10)$$

It is seen that equations (9) and (10) are identical with equations (5) and (6) except that they contain an additional term in the right-hand members of the equations. This additional term is independent of the slopes and deflections of the member, and depends solely upon the intermediate loads. Further significance is given to this term if the slopes and deflections are made equal to zero, as is true in a fixed beam with supports on same level. The last term then becomes the resisting moment acting on the end of the fixed beam. Hence it is seen that in general *the resisting moment at the end of a member with any system of intermediate loads can be expressed as the algebraic sum of the resisting moment at the end of a member with no intermediate loads, as given by equations (5) and (6), and the resisting moment at the end of a fixed beam with an equal span and carrying the same system of intermediate loads.*

If the resisting moment at the end of a fixed beam with supports on same level be expressed by C , with subscripts similar to those used for moments, M , equations (9) and (10) may be written in the following general form

$$M_{AB} = 2EK(2\theta_A + \theta_B - 3R) - C_{AB} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$M_{BA} = 2EK(2\theta_B + \theta_A - 3R) + C_{BA} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

These are the general *slope deflection* equations which apply to any condition of loading and restraint.

The sign of the constant C may be determined as follows: *In a fixed beam the sign of the resisting moment at the end of a member is opposite to that of the moment of external loads.* For instance, in Fig. 4 the moment of external loads about the end A is clockwise, so the resisting moment C_{AB} is counter clockwise or negative; and since the moment of the loads is counter clockwise about B , C_{BA} is clockwise or

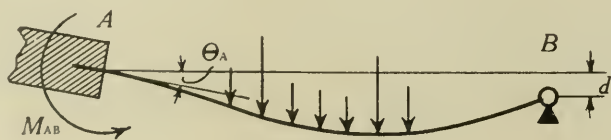


FIGURE 5

positive. If the loads were upward instead of downward, the signs of C_{AB} and C_{BA} would be reversed. With signs thus treated, C becomes merely a numerical, or scalar, quantity.

It has been noted that equations (11) and (12) apply to any condition of restraint of the ends of a member. Fig. 5 shows a member

restrained at A and hinged to the support at B , so that the resisting moment at B is zero. Equations (11) and (12) may be written:

$$\begin{aligned} M_{AB} &= 2EK(2\theta_A + \theta_B - 3R) - C_{AB} \\ 0 &= M_{BA} = 2EK(2\theta_B + \theta_A - 3R) + C_{BA} \end{aligned}$$

Combining these two equations to eliminate θ_B gives

$$M_{AB} = EK(3\theta_A - 3R) - (C_{AB} + \frac{C_{BA}}{2}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

If the beam is fixed at A and hinged at B , with the supports on the same level, θ_A and R in equation (13) are zero, and therefore the term $-(C_{AB} + \frac{C_{BA}}{2})$ represents the resisting moment at the end A , and can be readily calculated for any given loading.

By similar reasoning, when a beam is restrained at the end B and hinged to support at A , it is found that

$$M_{BA} = EK(3\theta_B - 3R) + (C_{BA} + \frac{C_{AB}}{2}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

For more convenient reference let the quantity $(C_{AB} + \frac{C_{BA}}{2})$ be denoted by H_{AB} , and the quantity $(C_{BA} + \frac{C_{AB}}{2})$ by H_{BA} .

Equations (13) and (14) then take the general form

$$M_{AB} = EK(3\theta_A - 3R) - H_{AB} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$M_{BA} = EK(3\theta_B - 3R) + H_{BA} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The term H represents the resisting moment at the fixed end of a beam which is fixed at one end and hinged to the support at the other, with supports at same level. The sign of H is determined in the same way as the sign of C in equations (11) and (12). That is, the sign of H is always opposite to the sign of the moment of the external loads about the fixed end of the member. If the external loads act upward instead of downward, the values of H in equations (15) and (16) must be reversed.

Equations (11) and (12) are the general equations for the ends of a member in flexure. Equations (15) and (16) are special forms of equations (11) and (12), applicable to members having one end hinged. For convenience in reference these four equations are given in Table 1 where they are denoted as equations (A), (B), (C), and (D), respectively.

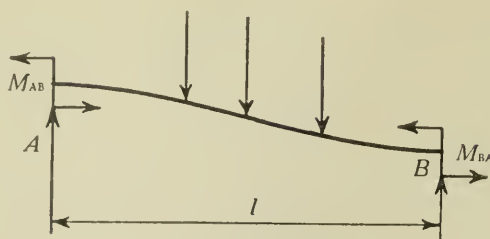


FIGURE 6

TABLE 1

GENERAL EQUATIONS FOR THE MOMENTS AT THE ENDS OF A MEMBER AB IN FIG

$$M_{AB} = 2EK(2\theta_A + \theta_B - 3R) \mp C_{AB} \quad (A)$$

$$M_{BA} = 2EK(2\theta_B + \theta_A - 3R) \pm C_{BA} \quad (B)$$

If end B is hinged,

$$M_{AB} = EK(3\theta_A - 3R) \mp H_{AB} \quad (C)$$

If end A is hinged,

$$M_{BA} = EK(3\theta_B - 3R) \pm H_{BA} \quad (D)$$

NOTE.—The signs of the quantities used in these equations are determined by the following rules.

 θ is positive (+) when the tangent to the elastic curve is turned in a clockwise direction. R is positive (+) when the member is deflected in a clockwise direction.

The moment of the internal stresses on a section is positive (+) when the internal couple acts in a clockwise direction upon the portion of the member considered.

If the moment of the external forces on the member about the end at which the moment is to be determined is positive (+), the sign before the constant is minus (—); if the moment of the external forces about the end at which the moment is to be determined is negative (—), the sign before the constant is plus (+). With the forces acting downward as shown in the sketch, for the moment at A, C_{AB} and H_{AB} are preceded by a minus (—) sign, but for the moment at B, C_{BA} and H_{BA} are preceded by a plus (+) sign.

8. *Derivation of Equations for Moments at Ends of Members in Flexure—Member Restrained at the Ends, with Special Cases of Loading.*—One method of determining the quantities C and H in equations (A), (B), (C), and (D) of Table 1 has been explained. To illustrate the method, some special cases will be considered here.

Fig. 7 shows a member restrained at the ends with a concentrated load at a distance a from A, and a distance b from B. In the simple beam moment diagram, the maximum ordinate is $\frac{-Pab}{l}$, the area is $\frac{-Pab}{2}$, and the distance $\bar{x}$ of the centroid of the area F from B is $\frac{1}{3}(l+b)$.

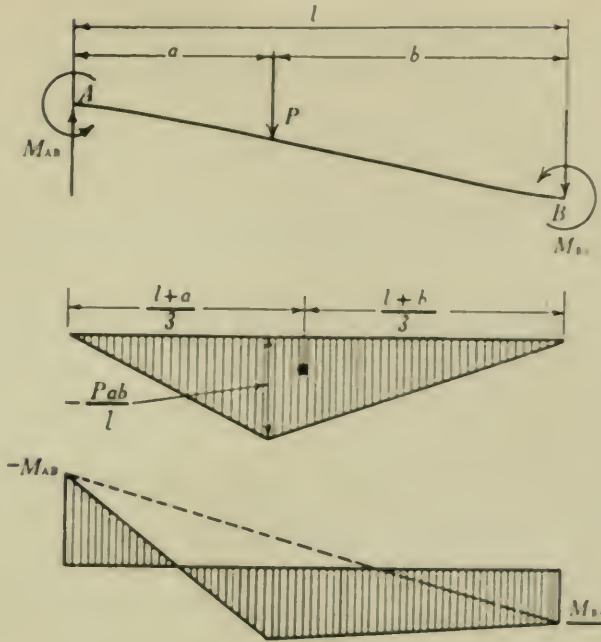


FIGURE 7

Hence putting these values in the last terms of equations (9) and (10) gives

$$-\frac{2F}{l^2}(3\bar{x}-l) = \frac{-Pab^2}{l^2} = -C_{AB} \quad (17)$$

and

$$\frac{2F}{l^2}(2l-3\bar{x}) = \frac{+Pa^2b}{l^2} = +C_{BA} \quad (18)$$

If the member had been hinged instead of being restrained at B , the value of H_{AB} could have been found from the last term of equation (13), in which

$$-(C_{AB} + \frac{C_{BA}}{2}) = \frac{-Pab}{2l^2}(l+b) = -H_{AB} \quad (19)$$

Similarly, if the member had been hinged at A and restrained at B , the value of H_{BA} could have been found from the last term of equation (14) in which

$$+(C_{BA} + \frac{C_{AB}}{2}) = \frac{Pab(l+a)}{2l^2} = H_{BA} \quad (20)$$

Another method for determining C and H may be readily applied to any kind of loading. For a member carrying a single concentrated load P , as shown in Fig. 7, the value of C_{AB} is $\frac{Pab^2}{l^2}$, and the value of C_{BA} is $\frac{Pa^2b}{l^2}$, as given in equations (17) and (18). If there are several

concentrated loads on the member, by summation $C_{AB} = \sum \frac{Pab^2}{l^2}$, and $C_{BA} = \sum \frac{Pa^2b}{l^2}$.

If there is a distributed load on the member the same method may be used, by performing an integration in place of the summation. Let w be the unit loading on an element of length dx , which is at a distance x from the left end, and a distance $l-x$ from the right end of the member. In the expression $\frac{Pab^2}{l^2}$, replace P by $w dx$, a by x , and

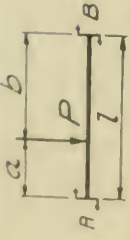
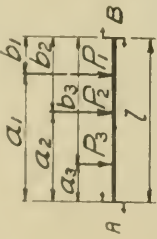
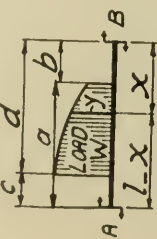
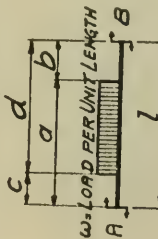
b by $l-x$, whence $C_{AB} = \int \frac{wx(l-x)^2 dx}{l^2}$. Similarly, $C_{BA} = \int \frac{wx^2(l-x) dx}{l^2}$.

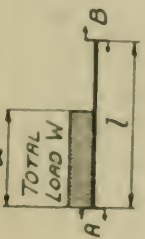
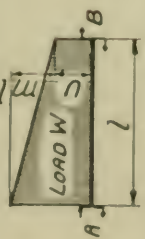
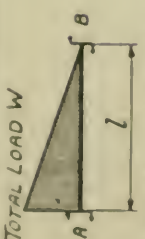
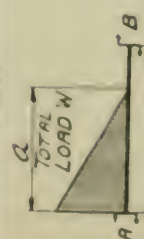
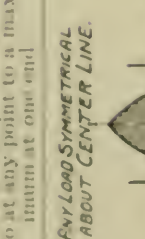
The limits of the definite integral are fixed by the length of the member under load.

If the unit load w is not constant, its variation may be expressed in terms of x , and the general value for the total load on a length dx thus found substituted for P in the given expression for a single concentrated load, after which the integration may be performed as just indicated.

Values of C and H for different systems of loads are given for reference in Tables 2 and 3.

TABLE 2
VALUES OF CONSTANTS C AND H FOR DIFFERENT SYSTEMS OF LOADS TO BE USED IN THE EQUATIONS OF TABLE 1

No.	Condition of Loading	C_{AB}	C_{BA}	H_{AB}	H_{BA}
1		0	0	0	0
2	No intermediate external loads. 	$\frac{Pab^2}{l^2}$	$\frac{Pba^2}{l^2}$	$\frac{Pab}{2l^2}(l+b)$	$\frac{Pab}{2l^2}(l+a)$
3	Single concentrated load at any point. 	$\frac{1}{l^2} \sum Pab^2$	$\frac{1}{l^2} \sum Pba^2$	$\frac{1}{2l^2} \sum Pab(l+b)$	$\frac{1}{2l^2} \sum Pab(l+a)$
4	Any number of concentrated loads. 	$\frac{1}{l^2} \int_b^d yx^2(l-x)dx$	$\frac{1}{l^2} \int_c^a yx(l-x)^2dx$	$\frac{1}{2l^2} \int_b^d yx(l^2-x^2)dx$	$\frac{1}{2l^2} \int_c^a yx(l-x)(2l-x)dx$
5	Any distributed load over any portion of the member. 	$\frac{w}{12l^2} [d^3(4l-3d) - b^3(4l-3b)]$	$\frac{w}{12l^2} [a^3(4l-3a) - c^3(4l-3c)]$	$\frac{w}{8l^2}(d^2-b^2)(2l^2-b^2-d^2)$	$\frac{w}{8l^2}(a^2-c^2)(2l^2-a^2-c^2)$
	Uniform load over any portion of the member.				

6	 Uniform load over portion of member at one end	$\frac{W a}{12 l^2} (3 a^2 - 8 a l + 6 l^2)$	$\frac{W a^2}{12 l^2} (4 l - 3 a)$	$\frac{W a}{8 l^2} (2 l^2 - a^2)$
7	 Uniformly varying load over entire member	$\frac{l^2}{60} (5 u + 3 m l)$	$\frac{l^2}{60} (5 u + 2 m l)$	$\frac{l^2}{120} (15 u + 7 m l)$
8	 Load varying uniformly from zero at one end to a maximum at the other end.	$\frac{W l}{10}$	$\frac{W l}{15}$	$\frac{7}{60} W l$
9	 Load varying uniformly from zero at any point to a maximum at one end	$\frac{W a}{30 l^2} (3 a^2 - 10 a l + 10 l^2)$	$\frac{W a^2}{30 l^2} (5 l - 3 a)$	$\frac{W a}{60 l^2} (10 l^2 - 3 a^2)$
10	 Any load symmetrical about center of member.	$\frac{F}{l}$	$\frac{F}{2} \cdot \frac{F}{l}$	$\frac{3}{2} \cdot \frac{F}{l}$

F is the area of the moment diagram of a simple beam having the same length and carrying the same load as the member in question.

Values of $\frac{F}{l}$ for different loads are given in Table 3.

TABLE 3

VALUES OF CONSTANTS C AND H FOR DIFFERENT SYSTEMS OF LOADS TO BE USED IN THE EQUATIONS OF TABLE 1

All Loads Symmetrical about Center of Member

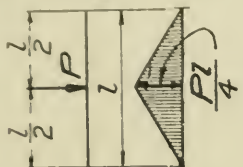
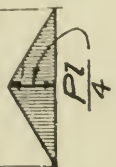
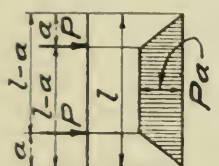
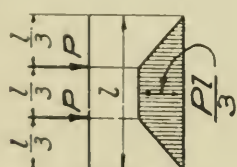
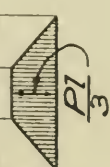
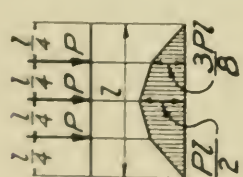
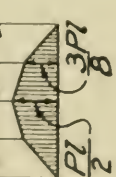
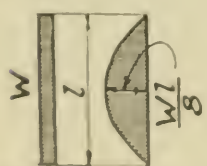
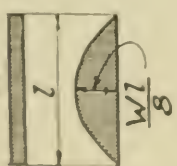
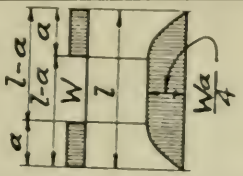
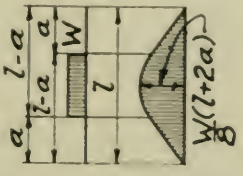
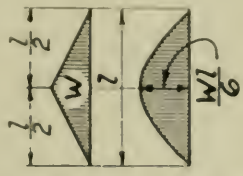
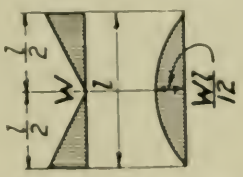
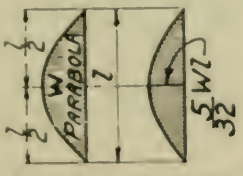
No.	Condition of Loading	Moment Diagram	$C_{AB} = C_{BA} = \frac{F}{l}$	$H_{AB} = H_{BA} = \frac{3}{2} \frac{F}{l}$
1	 Single load at the center.		$\frac{1}{8} Pl$	$\frac{3}{16} Pl$
2	 Two equal loads.		$\frac{Pa}{l} (l-a)$	$\frac{3}{2} \frac{Pa}{l} (l-a)$
3	 Equal loads at the third points.		$\frac{2}{9} Pl$	$\frac{1}{3} Pl$
4	 Equal loads at the quarter points.		$\frac{5}{16} Pl$	$\frac{15}{32} Pl$
5	 Uniform load over entire span.		$\frac{1}{12} Wl$	$\frac{1}{8} Wl$

TABLE 3—CONTINUED

No.	Condition of Loading	Moment Diagram	$C_{AB} = C_{BA} = \frac{F}{l}$	$H_{AB} = H_{BA} = \frac{3}{2} \frac{F}{l}$
6	 Equal uniform loads at the ends		$\frac{W a}{12 l} (3 l - 2 a)$	$\frac{W a}{8 l} (3 l - 2 a)$
7	 Uniform load at the center.		$\frac{W}{12 l} (l^2 + 2 a l - 2 a^2)$	$\frac{W}{8 l} (l^2 + 2 a l - 2 a^2)$
8	 Load increasing uniformly from zero at the ends.		$\frac{5}{48} W l$	$\frac{5}{32} W l$
9	 Load increasing uniformly from zero at the center.		$\frac{1}{16} W l$	$\frac{3}{32} W l$
10	 Load varying as the ordinates of a parabola.		$\frac{1}{10} W l$	$\frac{3}{20} W l$

PART II

DETERMINATION OF STRESSES IN STATICALLY INDETERMINATE STRUCTURES

9. *Assumptions upon which the Analyses are Based.*—The analyses in this bulletin are based upon the following assumptions:

- (1) That the connections are perfectly rigid.
- (2) That the length of a member is not changed by axial stress.
- (3) That the shearing deformation is zero.

Recent tests by Abe* show that the first assumption is approximately true for reinforced concrete frames, and tests by Wilson and Moore† show that this assumption is also approximately true for certain types of riveted connections of steel frames.

The error due to assumptions (2) and (3) depends upon the geometrical properties of the frame, but for frames of usual proportions the error is not large. These assumptions are discussed in detail in sections 67 and 68. The error due to slip in connections is discussed in section 69.

10. *Notation.*—The following notation has been used:

a = distance from end A of a member to a load.

b = distance from end B of a member to a load.

d = deflection of one end of a member with respect to the other end, measured perpendicular to initial position of member.

e = eccentricity of load.

h = vertical height of a structure.

k = error in resisting moment due to neglect of shearing strain.

l = length of a member.

m = change in the rate of loading in a unit distance.

n = ratio of K of top member to K of left-hand column for a four-sided frame.

p = ratio of K of top member to K of bottom member for a four-sided frame.

*Abe, Mitsuhiko, "Analysis and Tests of Rigidly Connected Reinforced Concrete Frames," Univ. of Ill. Eng. Exp. Sta., Bul. 107, 1918.

†Wilson, W. M., and Moore, H. F., "Tests to Determine the Rigidity of Riveted Joints of Steel Structures," Univ. of Ill. Eng. Exp. Sta., Bul. 104, 1917.

q = ratio of the length of the left-hand column to the length of the right-hand column of a two-legged bent.

s = ratio of K of top member to K of right-hand column for a four-sided frame.

u = load per unit of length (variable).

w = uniformly distributed load per unit of length.

A = area of section of a member.

C_{AB} = resisting moment at end A of a member AB fixed at both ends and having both ends at the same level.

E = modulus of elasticity in tension and compression.

F = area of the moment diagram of a simple beam.

G = modulus of elasticity in shear.

H = reaction.

H_{AB} = resisting moment at end A of a member AB fixed at A and hinged at B and having both ends at the same level.

I = moment of inertia of section of a member.

K = ratio of moment of inertia of section to length of a member.

M = moment of an external couple.

M_A = statical moment of external forces about point A .

M_{AB} = resisting moment acting at the end A of a member AB .

M_{BA} = resisting moment acting at the end B of a member AB .

N = restraint factor, depending on manner in which the ends of a member are held.

P = concentrated load.

$R = \frac{d}{l}$ = ratio of the deflection of one end of a member (with respect to the other end) to the length of the member.

S = shear.

W = total distributed load on a member.

$\alpha = n^2 + 2pn + 2n + 3p$, for a symmetrical four-sided frame.

$\beta = 6n + p + 1$, for a symmetrical four-sided frame.

$\Delta = 22(pns + ps + ns + np) + 2(p^2s + ps^2 + pn^2 + p^2n + s^2 + s + n^2 + n) + 6(n^2s + ns^2 + p^2 + p)$, for a rectangular frame.

$\Delta_o = 2[ns(4 + 3q + 4q^2) + (s^2 + s) + q^2(n^2 + n) + 3(q^2sn^2 + s^2n)]$, for a two-legged rectangular bent with unequal legs.

$\Delta_o = 2(3ns^2 + 11ns + s^2 + s + 3n^2s + n^2 + n)$, for a two-legged rectangular bent.

θ = change in the slope of the tangent to the elastic curve of a member.

IV. GIRDERS HAVING RESTRAINED ENDS

11. *Moments at the Ends of a Girder Having Fixed Ends—Both Supports on the Same Level.*—If a girder is fixed at the ends and if both supports are on the same level, θ_A , θ_B , and R of equations (A) and (B), Table 1, equal zero. This being the case, $M_{AB} = \mp C_{AB}$ and $M_{BA} = \pm C_{BA}$. Values of C_{AB} and C_{BA} for different systems of loads are given in Table 2.

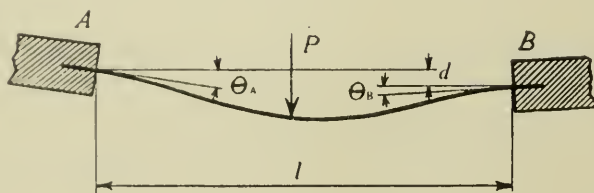


FIGURE 9

12. *Moments at the End of a Girder Having One End Fixed and the Other End Hinged, Both Supports on the Same Level.*—If a girder is fixed at one end, θ for that end equals zero. Likewise if both supports are on the same level, $R=0$. This being the case, the moment at the fixed end, as given by equations (C) and (D), Table 1, is $\mp H_{AB}$ or $\pm H_{BA}$. Values of H_{AB} and H_{BA} for different systems of loads are given in Table 2.

13. *Moments at the Ends of a Girder Having Ends Restrained but not Fixed.*—Fig. 9 represents a girder restrained at A and B. P represents the resultant of any system of forces on AB . The change in slope at A is θ_A and at B is θ_B . The deflection of B relative to A is d . $R = \frac{d}{l}$.

Applying equations (A) and (B), Table 1, gives

$$M_{AB} = 2EK(2\theta_A + \theta_B - 3R) - C_{AB} \quad (25)$$

$$M_{BA} = 2EK(2\theta_B + \theta_A - 3R) + C_{BA} \quad (26)$$

In order to determine M_{AB} and M_{BA} , θ_A , θ_B , and R must be known. As shown in Fig. 9, θ_A and R are positive and θ_B is negative. If P

had been upward instead of downward, C_{AB} would have been preceded by a plus (+) sign and C_{BA} by a minus (-) sign. Values of C_{AB} and C_{BA} for different systems of loads to be substituted in equations (25) and (26) are given in Table 2.

14. *Moment at End of a Girder Having One End Hinged and the Other End Restrained but not Fixed.*—Fig. 10 represents a girder hinged

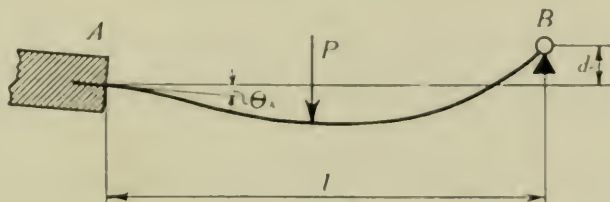


FIGURE 10

at B and restrained but not fixed at A. P represents the resultant of any system of forces on AB . The change in slope at A is θ_A , and the deflection of B relative to A is d . $R = \frac{d}{l}$.

Applying equation (C) of Table 1 gives

$$M_{AB} = EK(3\theta_A - 3R) - H_{AB} \quad (27)$$

As shown in Fig. 10, θ_A is positive (+) and R is negative (-). If P had been upward, H_{AB} would have been preceded by a plus (+) sign.

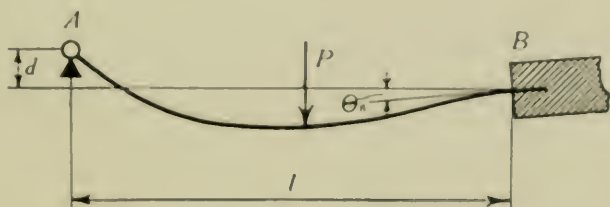


FIGURE 11

For the girder represented by Fig. 11

$$M_{BA} = EK(3\theta_B - 3R) + H_{BA} \quad (28)$$

in which R is positive (+) and θ_B is negative (-).

Values of H_{AB} and H_{BA} for different systems of loads to be used in equations (27) and (28) are given in Table 2.

Substituting the value of θ_B from equation (35) in equation (34), and substituting $-M_{BC}$ for M_{BA} gives

$$nM_{AB} + 2M_{BC}(n+1) + M_{CD} = -[n(2C_{BA} + C_{AB}) + (C_{CB} + 2C_{BC})] \quad (36)$$

In determining the values of C and H given in Table 3, it was found that

$$H_{AB} = \frac{2C_{AB} + C_{BA}}{2} \text{ and } H_{BA} = \frac{2C_{BA} + C_{AB}}{2}$$

Equation (36) can, therefore, be written in the form

$$nM_{AB} + 2M_{BC}(n+1) + M_{CD} = -2[nH_{BA} + H_{BC}] \quad (37)$$

This is the general form of the well-known "Equation of Three Moments."* It may be applied to a continuous girder having all supports on the same level, no matter what the type of loading to which the girder is subjected. As applied to two adjacent spans,

$K = \frac{I}{l}$ for the right-hand span and $\frac{K}{n} = \frac{I}{l}$ for the left-hand span; that is,

$n = \frac{I}{l}$ for the right-hand span divided by $\frac{I}{l}$ for the left-hand span.

Values of H for different types of loading are given in Table 2.

16. *Girder Continuous over Any Number of Supports and Carrying Any System of Vertical Loads. Supports All on the Same Level. General Equation of Three Moments—Two Adjacent Spans at One End. End of Girder Hinged.*—Fig. 13 represents the two spans at the left-hand end of a continuous girder. All supports are on the same level. P_0 represents the resultant of the loads on AB , and P_1 represents the

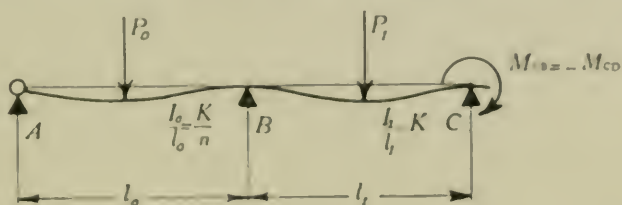


FIGURE 13

*The Equation of Three Moments was first deduced for a girder carrying uniform loads by Clapeyron, in 1857, and was published in *Comptes Rendus des Séances de l'Académie des Sciences*, Paris, Vol. 45, p. 1076. It has been extended and generalized for other loadings by Bresse, *Cours de Mécanique Appliquée*, Paris, 1862; Winkler, *Die Lehre von der Elasticität und Festigkeit*, Prague, 1867, and others.

resultant of the loads on BC . The girder is hinged at A . Equation (37), having been derived for the general case, is applicable. As the girder is hinged at A , $M_{AB}=0$. Equation (37), therefore, takes the form

$$2M_{BC}(n+1)+M_{CD}=-2[nH_{BA}+H_{BC}] \quad . \quad . \quad . \quad . \quad . \quad (38)$$

for two adjacent spans at the left-hand end of the girder when the left-hand end is hinged. Likewise,

$$nM_{AB}+2M_{BC}(n+1)=-2[nH_{BA}+H_{BC}] \quad . \quad . \quad . \quad . \quad . \quad (39)$$

for two adjacent spans at the right-hand end of the girder when the right-hand end is hinged.

17. *Girder Continuous over Any Number of Supports and Carrying Any System of Vertical Loads. Supports All on the Same Level. General Equation of Three Moments—Two Adjacent Spans at One End. End of Girder Restrained.*—Fig. 14 represents the two spans at the left-

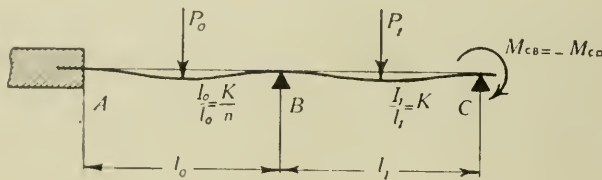


FIGURE 14

hand end of a continuous girder. All supports are on the same level. P_o represents the resultant of the loads on AB , and P_i represents the resultant of the loads on BC . The girder is restrained at A .

The values of the moments depend upon the restraint of the point A , and therefore the moments cannot be determined unless either the moment at A or the slope of the elastic curve of the girder at A is known. If the moment at A is known, equation (37) is applicable. If the slope at A is known, θ_A is a known quantity.

Substituting the value θ_B from equation (29) in equation (30) gives

$$2n M_{AB}=6EK\theta_A-n M_{BC}-2n H_{AB} \quad . \quad . \quad . \quad . \quad . \quad (40)$$

Substituting M_{AB} from equation (40) in equation (37) gives

$$M_{BC}(4+3n)+2M_{CD}=2n H_{AB}-4(nH_{BA}+H_{BC})-6EK \theta_A \quad . \quad (41)$$

Equation (41) is applicable to the two adjacent spans at the left-hand end of a continuous girder restrained at the left-hand end.

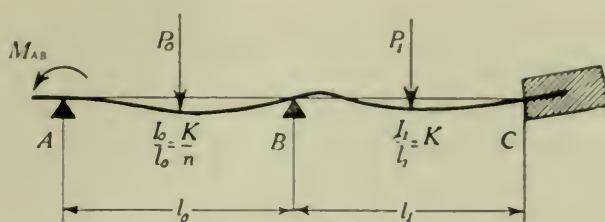


FIGURE 15

Fig. 15 represents the two spans at the right-hand end of a continuous girder. All supports are on the same level. P_o represents the resultant of the loads on AB , and P_i represents the resultant of the loads on BC . The girder is restrained at C .

From equation (37)

$$2nM_{AB} + 4M_{BC}(n+1) - 2M_{CB} = -4(nH_{BA} + H_{BC}) \quad (42)$$

Applying the equations of Table 1 gives

$$M_{BC} = 2EK(2\theta_B + \theta_C) - C_{BC} \quad (43)$$

$$M_{CB} = 2EK(2\theta_C + \theta_B) + C_{CB} \quad (44)$$

Eliminating θ_B from equations (43) and (44) gives

$$-2M_{CB} = -M_{BC} - 6EK\theta_C - 2H_{CB} \quad (45)$$

Substituting the value of M_{CB} from equation (45) in equation (42) gives

$$2nM_{AB} + M_{BC}(4n+3) = 2H_{CB} - 4[nH_{BA} + H_{BC}] + 6EK\theta_C \quad (46)$$

Equation (46) is applicable to the two adjacent spans at the right-hand end of a continuous girder.

18. *Girder Continuous over Any Number of Supports and Carrying Any System of Vertical Loads. Supports All on the Same Level. General Equation of Three Moments—Girder Fixed at the Ends.*—If the girder is fixed at the ends, θ_A of equation (41) and θ_C of equation (46) equal zero. Equations (41) and (46) then take the form

$$M_{BC}(4+3n) + 2M_{CB} = 2nH_{AB} - 4[nH_{BA} + H_{BC}] \quad (47)$$

$$2nM_{AB} + M_{BC}(4n+3) = 2H_{CB} - 4[nH_{BA} + H_{BC}] \quad (48)$$

Equation (47) is applicable to the two adjacent spans at the left-hand end of a continuous girder fixed at the left-hand end, and equation (48) is applicable to the two adjacent spans at the right-hand end of a continuous girder fixed at the right-hand end. Values of H for different systems of loads are given in Table 2.

19. *Girder Continuous over Three Supports and Carrying Any System of Vertical Loads. Supports on Different Levels. General*

Equation of Three Moments.—In section 15, the equations of Table 1 have been used to derive a general "Equation of Three Moments." The derivation is seen to be merely the combination of four linear equations involving slopes, deflections, and moments at the supports for each span into an equation involving the same quantities for any two adjacent spans.

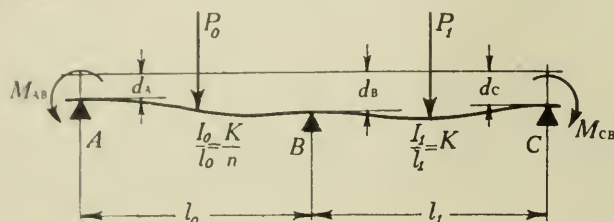


FIGURE 16

Fig. 16 represents two adjacent intermediate spans of a girder continuous over a number of supports. Some of the supports have settled. The vertical distances of the points A, B, and C from a horizontal base line are d_A , d_B , and d_C . P_o represents the resultant of all loads on AB, and P_i represents the resultant of all loads on BC.

The moments in the girder may be considered as made up of two parts, one part due to the loads represented by P_o and P_i when the supports are on the same level, and the other part due to the settlement of the supports, it being considered that the girder remains in contact with all supports.

Applying the equations of Table 1 gives

$$M_{AB} = \frac{2EK}{n}(2\theta_A + \theta_B - 3R_o) - C_{AB} \quad (49)$$

$$M_{BA} = \frac{2EK}{n}(2\theta_B + \theta_A - 3R_o) + C_{BA} \quad (50)$$

$$M_{BC} = 2EK(2\theta_B + \theta_C - 3R_1) - C_{BC} \quad (51)$$

$$M_{CB} = 2EK(2\theta_C + \theta_B - 3R_1) + C_{CB} \quad (52)$$

$$M_{BA} + M_{BC} = 0 \quad (53)$$

in which $\frac{I}{l}$ for the right-hand span equals K and $\frac{I}{l}$ for the left-hand span equals $\frac{K}{n}$.

$$\text{From Fig. 16 } R_o = \frac{d_B - d_A}{l_o} \text{ and } R_1 = \frac{d_C - d_B}{l_i}.$$

Equations (49) to (53) are identical with equations (29) to (33) of section 15, except for the additional terms R_o and R_1 . Hence the

method of solving for the moments is the same as in section 15, and will not be repeated here in detail. The general equation of three moments for a continuous girder carrying any system of vertical loads and with supports on different levels, together with some special forms of the equation, are given in Table 4.*

20. *Girder Continuous over Three Supports and Carrying Any System of Vertical Loads. Supports on Different Levels. Various Conditions of Restraint of Ends.*—Table 4, section 19, gives the general equation of three moments which is to be applied here. Various cases of restraint of the ends of girder and the effect upon the quantities in the general equation will be considered. It is seen that if the end of the girder is hinged, the resisting moment there is zero. If it is fixed, the slope at that point is zero. If the end is restrained, or partially fixed, the moments at the other supports can be found, if either the slope or the moment at the end is known. Modifications of the equations of Table 4, for these various cases of restraint, are given in Table 5.

21. *Girder Continuous over Four Supports and Carrying Any System of Vertical Loads. Supports on Different Levels. Various Conditions of Restraint of Ends.*—Table 4, section 19, gives the general equation of three moments which may be applied here. The method of using the equation of three moments is to apply it successively to each pair of adjacent spans in the continuous girder, thus deriving one less equation than the number of spans. If the conditions of restraint at the ends of the continuous girder are known, these equations may be solved simultaneously for the unknown moment at each support.

When the end of the girder is hinged, the moment there is zero. When the end is fixed, the slope there is zero. When the end is restrained, if either the slope or the moment at the end is known, the moments at the other supports may be found.

The equations of Table 4 have been applied to the girder described in this section, and the values of moments obtained for various cases of restraint of ends are given in Table 6.

*Johnson, Bryan, and Turneaure, "The Theory and Practice of Modern Framed Structures," Part II, p. 19, Ninth Ed. 1911, give the general Equation of Three Moments for a continuous girder with supports on different levels and carrying concentrated loads. The pocketbook, Die Huette, and Lanza, Applied Mechanics, have similar equations.

A. Ostenfeld, Teknisk Statik, Vol. 2, Second Ed., Copenhagen, 1913, pp. 98-162, gives a comprehensive treatment of continuous girders, including the case of varying moment of inertia from point to point, and of girder resting upon elastic supports. Both analytical and graphical methods are presented.

TABLE 4
CONTINUOUS GIRDERS
Equations of Three Moments.

Supports on Different Levels.¹

$$K = \frac{I}{l} \text{ for right-hand span.}$$

Any System of Vertical Loads.²

$$\frac{K}{n} = \frac{I}{l} \text{ for left-hand span.}$$

No.	Portion of Girder Considered	Equations of Three Moments
(a)	Fig. 16 Intermediate spans	$nM_{AB} + 2M_{BC}(n+1) + M_{CD} = \frac{6EK}{l_o l_1} \left[l_1(d_B - d_A) - l_o(d_C - d_B) \right] - 2 \left[H_{BC} + nH_{BA} \right]$
(b)	Fig. 17 Two adjacent spans at left-hand end. End of girder hinged.	$2M_{BC}(n+1) + M_{CD} = \frac{6EK}{l_o l_1} \left[l_1(d_B - d_A) - l_o(d_C - d_B) \right] - 2 \left[nH_{BA} + H_{BC} \right]$
(c)	Fig. 18 Two adjacent spans at right-hand end. End of girder hinged.	$nM_{AB} + 2M_{BC}(n+1) = \frac{6EK}{l_o l_1} \left[l_1(d_B - d_A) - l_o(d_C - d_B) \right] - 2 \left[nH_{BA} + H_{BC} \right]$
(d)	Fig. 19 Two adjacent spans at left-hand end. End of girder restrained.	If M_{AB} is known $2M_{BC}(n+1) + M_{CD} = \frac{6EK}{l_o l_1} \left[l_1(d_B - d_A) - l_o(d_C - d_B) \right] - 2 \left[H_{BC} + nH_{BA} \right] - nM_{AB}$ If θ_A is known $M_{BC}(4+3n) + 2M_{CD} = \frac{6EK}{l_o l_1} \left[3l_1(d_B - d_A) - 2l_o(d_C - d_B) \right] + 2nH_{AB} - 4 \left[H_{BC} + nH_{BA} \right] - 6EK \theta_A$ If the girder is fixed at A, $\theta_A = 0$
(e)	Fig. 20 Two adjacent spans at right-hand end. End of girder restrained.	If M_{CB} is known $nM_{AB} + 2M_{BC}(n+1) = \frac{6EK}{l_o l_1} \left[l_1(d_B - d_A) - l_o(d_C - d_B) \right] - 2 \left[H_{BC} + nH_{BA} \right] + M_{CB}$ If θ_C is known $2nM_{AB} + M_{BC}(4n+3) = \frac{6EK}{l_o l_1} \left[2l_1(d_B - d_A) - 3l_o(d_C - d_B) \right] - 4 \left[H_{BC} + nH_{BA} \right] + 2H_{CB} + 6EK \theta_C$ If the girder is fixed at C, $\theta_C = 0$

If there is no settlement of supports, let all values of d in these equations equal zero.

If there are no loads on girder except at supports, let all values of H in these equations equal zero.

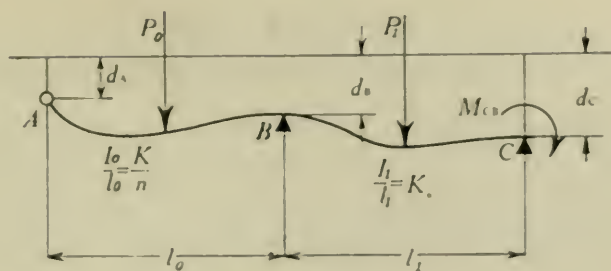


FIGURE 17

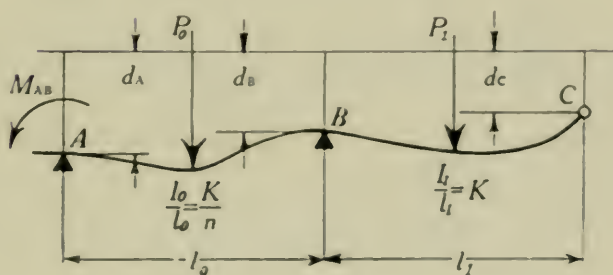


FIGURE 18

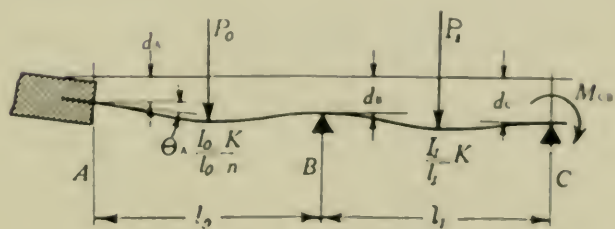


FIGURE 19

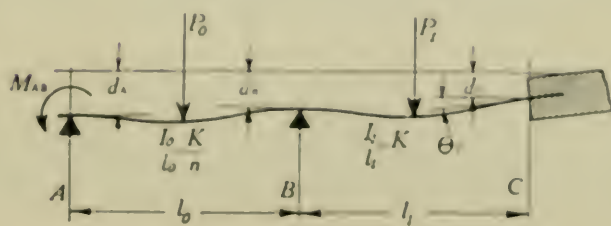


FIGURE 20

TABLE 5
GIRDER CONTINUOUS OVER THREE SUPPORTS

Supports on Different Levels.¹

$$K = \frac{I}{l} \text{ for right-hand span.}$$

Any System of Vertical Loads.²

$$\frac{K}{n} = \frac{I}{l} \text{ for left-hand span.}$$

(a)	Fig. 21 Ends of girder hinged	General Case
		$M_{BC} = \frac{6EK}{2l_0l_1(n+1)} \left[l_1(d_B - d_A) - l_0(d_C - d_B) \right] - \frac{2}{n+1} \left[nH_{BA} + H_{BC} \right]$ Spans Identical Except for Loads $M_{BC} = \frac{3EK}{2l^2} \left[l(2d_B - d_A - d_C) \right] - \left[H_{BA} + H_{BC} \right]$
(b)	Fig. 22 Ends of girder restrained	General Case
		$\theta_A \text{ and } M_{CB} \text{ known}$ $M_{BC} = \frac{1}{3n+4} \left\{ \frac{6EK}{l_0l_1} \left[3l_1(d_B - d_A) - 2l_0(d_C - d_B) \right] + 2nH_{AB} \right. \\ \left. - 4(H_{BC} + nH_{BA}) - 6EK\theta_A + 2M_{CB} \right\}$ $M_{AB} = \frac{3EK}{n} \left(\theta_A - \frac{d_B - d_A}{l_0} \right) - \frac{M_{BC}}{2} - H_{AB}$
		$\theta_C \text{ and } M_{AB} \text{ known}$ $M_{BC} = \frac{1}{4n+3} \left\{ \frac{6EK}{l_0l_1} \left[2l_1(d_B - d_A) - 3l_0(d_C - d_B) \right] - 4(H_{BC} + nH_{BA}) \right. \\ \left. + 2H_{CB} + 6EK\theta_C - 2nM_{AB} \right\}$ $M_{CB} = \frac{M_{BC}}{2} + 3EK \left[\theta_C - \frac{d_C - d_B}{l_1} \right] + H_{CB}$
		$\theta_A \text{ and } \theta_C \text{ known}$ $M_{BC} = \frac{1}{n+1} \left\{ \frac{6EK}{l_0l_1} \left[l_1(d_B - d_A) - l_0(d_C - d_B) \right] - nC_{BA} - C_{BC} \right. \\ \left. - 2EK(\theta_A - \theta_C) \right\}$ $M_{AB} = \frac{3EK}{n} \left[\theta_A - \frac{d_B - d_A}{l_0} \right] - \frac{M_{BC}}{2} - H_{AB}$ $M_{CB} = 3EK \left[\theta_C - \frac{d_C - d_B}{l_1} \right] + \frac{M_{BC}}{2} + H_{CB}$

¹ If there is no settlement of supports, let all values of d in these equations equal zero.

² If there are no loads on girder except at supports, let all values of H and C in these equations equal zero.

If an end is fixed, θ for that end is zero.

If an end is hinged, M for that end is zero.

TABLE 5—CONTINUED

(b)	Fig. 22	General Case—Continued
Ends of girder restrained		M_{AB} and M_{CB} known
		$M_{BC} = \frac{1}{2(n+1)} (M_{CB} - nM_{AB}) + \frac{3EK}{l_0 l_1 (n+1)} \left[l_1 (d_B - d_A) - l_0 (d_C - d_B) \right] - \frac{nH_{BA} + H_{BC}}{n+1}$
		Spans Identical Except for Load
		θ_A and M_{CB} known
		$M_{BC} = \frac{1}{7} \left\{ \frac{6EK}{l} \left[3(d_B - d_A) - 2(d_C - d_B) \right] + 2H_{AB} - 4(H_{BC} + H_{BA}) - 6EK\theta_A + 2M_{CB} \right\}$
		$M_{AB} = 3EK \left[\theta_A - \frac{d_B - d_A}{l} \right] - \frac{M_{BC}}{2} - H_{AB}$
		θ_C and M_{AB} known
		$M_{BC} = \frac{1}{7} \left\{ \frac{6EK}{l} \left[2(d_B - d_A) - 3(d_C - d_B) \right] - 4(H_{BC} + H_{BA}) + 2H_{CB} + 6EK\theta_C - 2M_{AB} \right\}$
		$M_{CB} = \frac{M_{BC}}{2} + 3EK \left[\theta_C - \frac{d_C - d_B}{l} \right] + H_{CB}$
		θ_A and θ_C known
		$M_{BC} = \frac{1}{2} \left\{ \frac{6EK}{l} \left[2d_B - d_C - d_A \right] - C_{BA} - C_{BC} - 2EK(\theta_A - \theta_C) \right\}$
		$M_{AB} = 3EK \left[\theta_A - \frac{d_B - d_A}{l} \right] - \frac{M_{BC}}{2} - H_{AB}$
		$M_{CB} = 3EK \left[\theta_C - \frac{d_C - d_B}{l} \right] + \frac{M_{BC}}{2} + H_{CB}$
		M_{AB} and M_{CB} known
		$M_{BC} = \frac{1}{4} \left[M_{CB} - M_{AB} \right] + \frac{3EK}{2l} \left[2d_B - d_A - d_C \right] - \frac{H_{BA} + H_{BC}}{2}$

TABLE 6
GIRDER CONTINUOUS OVER FOUR SUPPORTS
Supports on Different Levels,¹ Any System of Vertical Loads.²

$$\frac{l}{l} \text{ for span } AB = \frac{K_1}{n_o}, \quad \frac{l}{l} \text{ for span } BC = \frac{K_2}{n_1}, \quad \frac{l}{l} \text{ for span } CD = K_2$$

(a)	Fig. 23 Ends of girder hinged	General Case
		$M_{BC} = \frac{6EK_1 \left[2(n_1+1)l_1l_2(d_B-d_A) - (3n_1+2)l_o l_2(d_C-d_B) + n_1l_1(d_D-d_C) \right]}{l_o l_1 l_2 \left[4(n_o+1)(n_1+1) - n_1 \right]}$
		$M_{CD} = \frac{6EK_2 \left[(2n_o+3)l_o l_2(d_C-d_B) - 2(n_o+1)l_o l_1(d_D-d_C) - l_1l_2(d_B-d_A) \right]}{l_o l_1 l_2 \left[4(n_o+1)(n_1+1) - n_1 \right]}$
		$- \left[\frac{4(n_1+1)(n_o H_{BA} + H_{BC}) - 2(n_1 H_{CB} + H_{CD})}{4(n_o+1)(n_1+1) - n_1} \right]$
		All Spans Identical Except for Loads
		$M_{BC} = \frac{2EK}{5l} (9d_B - 6d_C + d_D - 4d_A) - \frac{2}{15} \left[4(H_{BA} + H_{BC}) - (H_{CB} + H_{CD}) \right]$
		$M_{CD} = \frac{2EK}{5l} (9d_C - 6d_B - 4d_D + d_A) - \frac{2}{15} \left[4(H_{CB} + H_{CD}) - (H_{BA} + H_{BC}) \right]$

¹ If there is no settlement of supports, let all values of d in these equations equal zero.

² If there are no loads on the girder except at supports, let all values of H in these equations equal zero.

If an end is fixed, θ for that end is zero. If an end is hinged, M for that end is zero.

TABLE 6—CONTINUED

(b)	Fig. 24 Ends of girder restrained	General Case
		$M_{BC} = \frac{\theta_A \text{ and } M_{DC} \text{ known}}{6EK_1 \left[3(n_1+1)l_1l_2(d_B-d_A) - (3n_1+2)l_o l_2(d_C-d_B) + n_1l_o l_1(d_D-d_C) \right]}$ $- \frac{l_o l_1 l_2 \left[3(n_o+1)(n_1+1)+1 \right]}{2 \left[(n_1+1)(2n_o H_{BA} - n_o H_{AB} + 2H_{BC} + 3EK_1 \theta_A) - (n_1 H_{CB} + H_{CD}) \right] + M_{DC}}$
		$M_{CD} = \frac{3EK_2 \left[3(n_o+2)l_o l_2(d_C-d_B) - (3n_o+4)l_o l_1(d_D-d_C) - 3l_1l_2(d_B-d_A) \right]}{l_o l_1 l_2 \left[3(n_o+1)(n_1+1)+1 \right]}$ $+ \frac{2n_1(2n_o H_{BA} - n_o H_{AB} + 2H_{BC} + 3EK_1 \theta_A) - (3n_o+4)(2n_1 H_{CB} + 2H_{CD} - M_{DC})}{2 \left[3(n_o+1)(n_1+1)+1 \right]}$
		$M_{AB} = - \frac{M_{BC}}{2} - \frac{H_{AB} + 3EK_1}{n_o d_o} \left[l_o \theta_A - (d_B - d_A) \right]$
		$\theta_D \text{ and } M_{AB} \text{ known}$ $M_{BC} = \frac{3EK_1 \left[(4n_1+3)l_1l_2(d_B-d_A) - 3(2n_1+1)l_o l_2(d_C-d_B) + 3n_1l_o l_1(d_D-d_C) \right]}{l_o l_1 l_2 \left[3(n_o+1)(n_1+1)+n_o n_1 \right]}$ $- \frac{\left[(4n_1+3)(2n_o H_{BA} + 2H_{BC} + n_o M_{AB}) + 2(H_{DC} - 2H_{CD} - 2n_1 H_{CB} + 3EK_2 \theta_D) \right]}{2 \left[3(n_o+1)(n_1+1)+n_o n_1 \right]}$

TABLE 6—CONTINUED
GIRDER CONTINUOUS OVER FOUR SUPPORTS
Supports on Different Levels. Any System of Vertical Loads.

$\frac{l}{l}$ for span $AB = \frac{K_1}{n_o}$. $\frac{l}{l}$ for span $BC = \frac{K_2}{n_1}$. $\frac{l}{l}$ for span $CD = K_2$

(b) Fig. 24 Ends of girder restrained (Continued)	General Case—Continued
	$M_{CD} = \frac{6EK_2 \left[(2n_o+3)l_o l_2 (d_C - d_B) - 3(n_o+1)l_o l_1 (d_D - d_C) - l_1 l_2 (d_B - d_A) \right]}{l_o l_1 l_2 \left[3(n_o+1)(n_1+1) + n_o n_1 \right]} + \frac{2(n_o+1)(H_{DC} - 2H_{CD} - 2n_1 H_{CB} + 3EK_2 \theta_D) + n_1(2n_o H_{BA} + 2H_{BC} + n_o M_{AB})}{3(n_o+1)(n_1+1) + n_o n_1}$
	$M_{DC} = \frac{M_{CD}}{2} + H_{DC} + 3EK_2 \left[\theta_D - \frac{(d_D - d_C)}{l_2} \right]$
	$\theta_A \text{ and } \theta_D \text{ known}$
	$M_{BC} = \frac{6EK_1 \left[(4n_1+3)l_1 l_2 (d_B - d_A) - 2(2n_1+1)l_o l_2 (d_C - d_B) + 2n_1 l_o l_1 (d_D - d_C) \right]}{l_o l_1 l_2 \left[4(n_o+1)(n_1+1) - n_o \right]}$
	$2 \left\{ (4n_1+3)(2n_o H_{BA} - n_o H_{AB} + 2H_{BC}) + 2(H_{DC} - 2H_{CD} - 2n_1 H_{CB}) + 3EK_1 \left[(4n_1+3)\theta_A + 2n_1 \theta_D \right] \right\}$
	$M_{CD} = \frac{6EK_2 \left[2(n_o+2)l_o l_2 (d_C - d_B) - (3n_o+4)l_o l_1 (d_D - d_C) - 2l_1 l_2 (d_B - d_A) \right]}{l_o l_1 l_2 \left[4(n_o+1)(n_1+1) - n_o \right]}$
	$2 \left\{ (3n_o+4)(H_{DC} - 2H_{CD} - 2n_1 H_{CB}) + 2n_1(2n_o H_{BA} - n_o H_{AB} + 2H_{BC}) + 3EK_2 \left[(3n_o+4)\theta_D + 2\theta_A \right] \right\}$
	$+ \frac{3 \left[4(n_o+1)(n_1+1) - n_o \right]}{3 \left[4(n_o+1)(n_1+1) - n_o \right]}$

TABLE 6—CONTINUED

(b)	Fig 24 Ends of girder restrained (Continued)	$M_{AB} = -\frac{M_{BC}}{2} - H_{AB} + \frac{3EK_1}{n_o} \left[\theta_A - \frac{(d_B - d_A)}{l_o} \right]$ $M_{DC} = \frac{M_{CD}}{2} + H_{DC} + 3EK_2 \left[\theta_D - \frac{(d_D - d_C)}{l_2} \right]$ M_{AB} and M_{DC} known $M_{BC} = \frac{6EK_1 \left[2(n_1 + 1)l_1l_2(d_B - d_A) - (3n_1 + 2)l_o l_2(d_C - d_B) + n_1l_o l_1(d_D - d_C) \right]}{l_o l_1 l_2 \left[4(n_o + 1)(n_1 + 1) - n_1 \right]}$ $- \left[\frac{4(n_1 + 1)(n_o H_{BA} + H_{BC}) - 2(n_1 H_{CB} + H_{CD}) + 2n_o(n_1 + 1)M_{AB} + M_{DC}}{4(n_o + 1)(n_1 + 1) - n_1} \right]$ $M_{CD} = \frac{6EK_2 \left[(2n_o + 3)l_o l_2(d_C - d_B) - 2(n_o + 1)l_o l_1(d_D - d_C) - l_1 l_2(d_B - d_A) \right]}{l_o l_1 l_2 \left[4(n_o + 1)(n_1 + 1) - n_o \right]}$ $- \left[\frac{4(n_o + 1)(n_1 H_{CB} + H_{CD}) - 2n_1(n_o H_{BA} + H_{BC}) - 2(n_o + 1)M_{DC} - n_o n_1 M_{AB}}{4(n_o + 1)(n_1 + 1) - n_1} \right]$
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TABLE 6—CONTINUED
GIRDER CONTINUOUS OVER FOUR SUPPORTS
Supports on Different Levels. Any System of Vertical Loads.
 $\frac{l}{l}$ for span $AB = \frac{K_1}{n_0}$ $\frac{l}{l}$ for span $BC = \frac{K_2}{n_1}$ $\frac{l}{l}$ for span $CD = K_2$.

(b)	General Case—Continued	
	All Spans Identical Except for Loads	
Fig. 24 Ends of girder restrained (Continued)	θ_A and M_{DC} known	
	$6EK(11d_B - 6d_A - 6d_C + d_D) - 2l \left[2(2H_{BA} - H_{AB} + 2H_{BC} + 3EK\theta_A) - (H_{CB} + H_{CD}) \right] - LM_{DC}$	13l
	$M_{BC} =$	
	$3EK(16d_C - 7d_D - 12d_B + 3d_A) + l \left[(2H_{BA} - H_{AB} + 2H_{BC} + 3EK\theta_A) - \frac{7}{2}(2H_{CB} + 2H_{CD} - M_{DC}) \right]$	13l
	$M_{CD} =$	
	$M_{AB} = -\frac{M_{BC}}{2} - H_{AB} + \frac{3EK}{l} \left[l\theta_A - d_B + d_A \right]$	
	θ_D and M_{AB} known	
	$3EK(16d_B - 7d_A - 12d_C + 3d_D) - l \left[\frac{7}{2}(2H_{BA} + 2H_{BC} + M_{AB}) + H_{DC} - 2H_{CD} - 2H_{CB} + 3EK\theta_D \right]$	13l
	$M_{BC} =$	
	$6EK(11d_C - 6d_D - 6d_B + d_A) + l \left[4(H_{DC} - 2H_{CD} - 2H_{CB} + 3EK\theta_D) + 2H_{BA} + 2H_{BC} + M_{AB} \right]$	13l
	$M_{CD} =$	
	$M_{DC} = \frac{M_{CD}}{2} + H_{DC} + 3EK \left[\theta_D - \frac{l(d_D - d_C)}{l} \right]$	

TABLE 6—CONTINUED

(b)	Fig. 24 Ends of girder restrained (Continued)	θ_A and θ_D known
		$M_{BC} = \frac{2EK}{5l} (13d_B - 8d_C - 7d_A + 2d_D) - \frac{2}{45} \left[7(2H_{BA} - H_{AB} + 2H_{BC}) + 2(H_{DC} - 2H_{CD} - 2H_{CB}) + 3EK(7\theta_A + 2\theta_D) \right]$
		$M_{CD} = \frac{2EK}{5l} (13d_C - 8d_B - 7d_D + 2d_A) + \frac{2}{45} \left[7(H_{DC} - 2H_{CD} - 2H_{CB}) + 2(-H_{AB} + 2H_{BA} + 2H_{BC}) + 3EK(7\theta_D + 2\theta_A) \right]$
		$M_{AB} = -\frac{1}{2} M_{BC} - H_{AB} + 3EK \left[\theta_A - \frac{(d_B - d_A)}{l} \right]$
		$M_{DC} = \frac{1}{2} M_{CD} + H_{DC} + 3EK \left[\theta_D - \frac{(d_D - d_C)}{l} \right]$
		M_{AB} and M_{DC} known
		$M_{BC} = \frac{2EK}{5l} (9d_B - 6d_C - 4d_A + d_D) - \frac{1}{15} \left[8(H_{BA} + H_{BC}) - 2(H_{CB} + H_{CD}) + 4M_{AB} + M_{DC} \right]$
		$M_{CD} = \frac{2EK}{5l} (9d_C - 6d_B - 4d_D + d_A) - \frac{1}{15} \left[8(H_{CB} + H_{CD}) - 2(H_{BA} + H_{BC}) - 4M_{DC} - M_{AB} \right]$

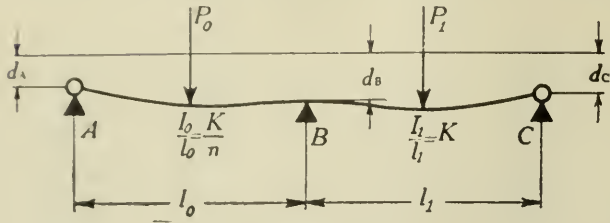


FIGURE 21

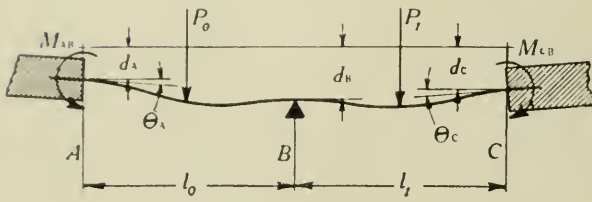


FIGURE 22

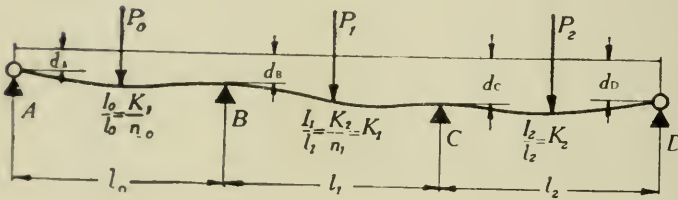


FIGURE 23

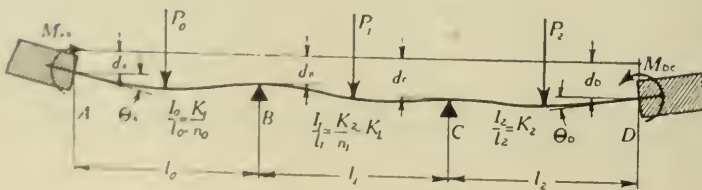


FIGURE 24

VI. TWO-LEGGED RECTANGULAR BENT. LEGS OF EQUAL LENGTH

22. *Two-legged Rectangular Bent. Concentrated Horizontal Force at the Top—Legs Hinged at the Bases.*—Fig. 25 represents a two-legged rectangular bent having legs hinged at the bases. A single force P is applied at the top of the bent.

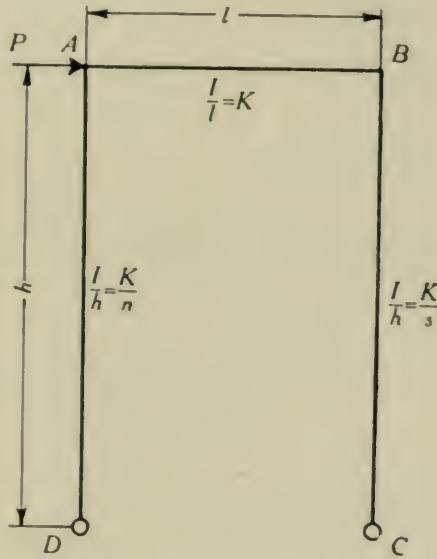


FIGURE 25

Applying the equations of Table 1

$$M_{AB} = 2EK(2\theta_A + \theta_B) \quad (54)$$

$$M_{AD} = \frac{EK}{n}(3\theta_A - 3R) \quad (55)$$

$$M_{BA} = 2EK(2\theta_B + \theta_A) \quad (56)$$

$$M_{BC} = \frac{EK}{8}(3\theta_B - 3R) \quad (57)$$

[illegible]

In these equations, from the definition of resisting moment, $M_{AB} = -M_{AD}$ and $-M_{BA} = +M_{BC}$. Hence M_{AB} and M_{BC} may be considered as the two unknown moments to be found. Elimination may be done as follows:

Subtracting equation 56 from equation 54 gives

$$M_{AB} + M_{BC} = 2EK(\theta_A - \theta_B) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

Subtracting s times equation 57 from n times equation 55 gives

$$-nM_{AB} - sM_{BC} = EK(3\theta_A - 3\theta_B) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

Combining equations 59 and 60 to eliminate the quantity $(\theta_A - \theta_B)$ gives

$$M_{AB}(3+2n) + M_{BC}(3+2s) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

Combining equation 61 with equation 58 gives

$$M_{AB} = \frac{Ph}{2} \left(\frac{3+2s}{3+n+s} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

$$M_{BC} = -\frac{Ph}{2} \left(\frac{3+2n}{3+n+s} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

If $n=s$, that is, if the sections of AD and BC have the same moments of inertia, equations 62 and 63 take the form

$$M_{BC} = -M_{AB} = -\frac{Ph}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

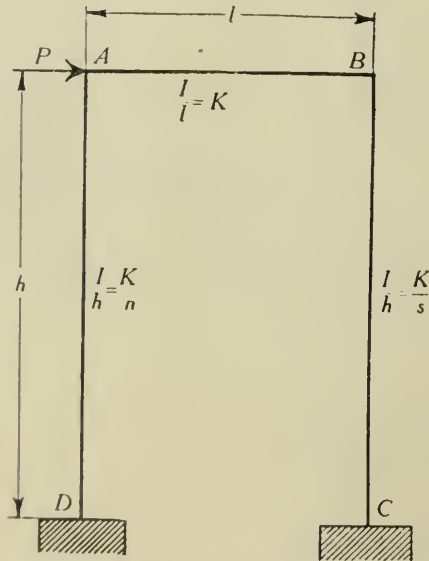


FIGURE 26

23. *Two-legged Rectangular Bent. Concentrated Horizontal Force at the Top—Legs Fixed at the Bases.*—Fig. 26 represents a two-legged rectangular bent having legs fixed at the bases. A single force P is applied at the top of the bent.

Applying equation (A), Table 1,

$$M_{AB} = 2EK(2\theta_A + \theta_B) \quad (65)$$

$$M_{AD} = \frac{2EK}{n}(2\theta_A - 3R) \quad (66)$$

$$M_{DA} = \frac{2EK}{n}(\theta_A - 3R) \quad (67)$$

$$M_{BA} = 2EK(2\theta_B + \theta_A) \quad (68)$$

$$M_{BC} = \frac{2EK}{s}(2\theta_B - 3R) \quad (69)$$

$$M_{CB} = \frac{2EK}{s}(\theta_B - 3R) \quad (70)$$

$$M_{AD} + M_{DA} + M_{BC} + M_{CB} + Ph = 0 \quad (71)$$

$$M_{AB} + M_{AD} = 0 \quad (72)$$

$$M_{BA} + M_{BC} = 0 \quad (73)$$

Substituting the values of M_{AB} and M_{AD} from equations 65 and 66 in equation 72 and simplifying gives

$$\frac{2\theta_A}{R}(1+n) + \frac{n\theta_B}{R} = 3 \quad (74)$$

Substituting the values of M_{BA} and M_{BC} from equations 68 and 69 in equation 73 and simplifying gives

$$\frac{2\theta_B}{R}(1+s) + \frac{s\theta_A}{R} = 3 \quad (75)$$

Substituting the values for the moments in equation 71 and simplifying gives

$$\frac{s\theta_A}{R} + \frac{n\theta_B}{R} = 2(s+n) - \frac{nsPh}{6EK} \cdot \frac{1}{R} \quad (76)$$

Solving equation 74 for $\frac{\theta_B}{R}$ gives

$$\frac{\theta_B}{R} = \frac{3}{n} - \frac{2\theta_A}{R} \left(\frac{1+n}{n} \right) \quad (77)$$

Substituting the value of $\frac{\theta_B}{R}$ from equation 77 in equation 75 gives

$$\frac{\theta_A}{R} = \frac{3(2s+2-n)}{3ns+4n+4s+4} \quad (78)$$

Substituting the value of $\frac{\theta_A}{R}$ from equation 78 in equation 77 and simplifying gives

$$\frac{\theta_B}{R} = \frac{3(2n+2-s)}{3ns+4n+4s+4} \quad \dots \quad (79)$$

Substituting the values of $\frac{\theta_A}{R}$ and $\frac{\theta_B}{R}$ from equation 78 and equation 79 in equation 76 gives

$$\frac{3s(2s+2-n)}{3ns+4n+4s+4} + \frac{3n(2n+2-s)}{3ns+4n+4s+4} = 2(s+n) - \frac{nsPh}{6EK} \cdot \frac{1}{R}$$

Solving this equation for $\frac{1}{R}$ gives

$$\frac{1}{R} = \frac{6EK}{nsPh} \frac{2(3ns^2+11ns+s^2+s+3n^2s+n^2+n)}{3ns+4n+4s+4} \quad \dots \quad (80)$$

Substituting the value of $\frac{1}{R}$ from equation 80 in equation 79 and solving for θ_B gives

$$\theta_B = \frac{Ph}{12EK} \frac{3ns(2n+2-s)}{3ns^2+11ns+s^2+s+3n^2s+n^2+n} \quad \dots \quad (81)$$

Substituting the value of $\frac{1}{R}$ from equation 80 in equation 78 and solving for θ_A gives

$$\theta_A = \frac{Ph}{12EK} \frac{3ns(2s+2-n)}{3ns^2+11ns+s^2+s+3n^2s+n^2+n} \quad \dots \quad (82)$$

Substituting the values of θ_B and θ_A from equations 81 and 82 in equation 65 gives

$$M_{AB} = \frac{Ph}{2} \frac{3ns(s+2)}{3ns^2+11ns+s^2+s+3n^2s+n^2+n} \quad \dots \quad (83)$$

Substituting the values of θ_B and θ_A from equations 81 and 82 in equation 68 and substituting $-M_{BC}$ for M_{BA} gives

$$M_{BC} = -\frac{Ph}{2} \frac{3ns(n+2)}{3ns^2+11ns+s^2+s+3n^2s+n^2+n} \quad \dots \quad (84)$$

From equation 67

$$M_{DA} = \frac{2EKR}{n} \left[\frac{\theta_A}{R} - 3 \right] \quad (85)$$

Substituting the value of R from equation 80 and of $\frac{\theta_A}{R}$ from equation 78 gives

$$M_{DA} = -\frac{Ph}{2} \frac{s(2s+2+5n+3ns)}{3ns^2+11ns+s^2+s+3n^2s+n^2+n} \quad (86)$$

From equation 70

$$M_{CB} = \frac{2EKR}{s} \left(\frac{\theta_B}{R} - 3 \right)$$

Substituting the value of R from equation 80 and of $\frac{\theta_R}{R}$ from equation 79 gives

$$M_{CB} = -\frac{Ph}{2} \frac{n(2n+2+5s+3ns)}{(3ns^2+11ns+s^2+s+3n^2s+n^2+n)} \quad (87)$$

Letting Δ_o represent $2(3ns^2+11ns+s^2+s+3n^2s+n^2+n)$, the equation for the moments in the frame are

[illegible]

[illegible]

$$M_{CB} = - \frac{Ph}{\Delta_c} n(2n+2+5s+3ns) \quad . \quad . \quad . \quad . \quad . \quad . \quad (90)$$

$$M_{DA} = -\frac{Ph}{\Delta_o}s(2s+2+5n+3ns) \quad . \quad . \quad . \quad . \quad . \quad . \quad (91)$$

If $n=s$, that is, if the section of AD has the same moment of inertia as the section of BC , equations 88 to 91 take the form

$$M_{AD} = M_{BC} = -\frac{Ph}{2} \frac{3n}{6n+1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (92)$$

$$M_{CB} = M_{DA} = -\frac{Ph}{2} \frac{3n+1}{6n+1} \quad \dots \dots \dots (93)$$

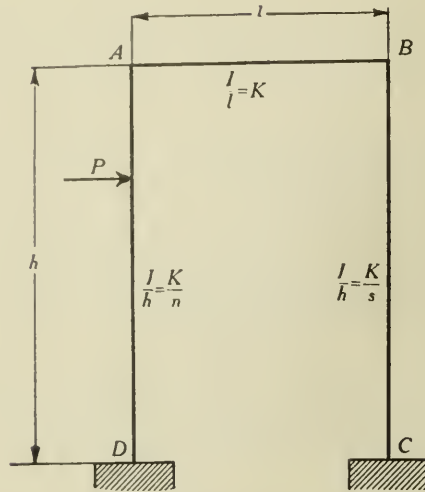


FIGURE 28

legged rectangular bent with any horizontal load on the leg AD . The legs are fixed at D and C . P represents the resultant of all the forces on AD .

If M_D represents the moment of P about D ,

$$M_{AD} + M_{DA} + M_{BC} + M_{CB} + M_D = 0$$

Applying the equations of Table 1 gives

$$M_{AB} = 2EK(2\theta_A + \theta_B) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (108)$$

$$n M_{AD} = 2EK(2\theta_A - 3R) + n C_{AD} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (109)$$

$$n M_{DA} = 2EK(\theta_A - 3R) - n C_{DA} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (110)$$

$$M_{BA} = 2EK(2\theta_B + \theta_A) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (111)$$

$$s M_{BC} = 2EK(2\theta_B - 3R) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (112)$$

$$s M_{CB} = 2EK(\theta_B - 3R) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (113)$$

$$M_{AD} + M_{DA} + M_{BC} + M_{CB} + M_D = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (114)$$

The equations 108 to 114 are similar to equations 65 to 73 of section 23. The method of solving for the four unknown moments will be done in a different way from that of section 23. The equations will first be combined to eliminate θ and R , then the resulting equations solved simultaneously for the moments.

Adding equation 112 and two times equation 110 and subtracting equation 109 and two times equation 113 to eliminate θ_A , θ_B , and R gives

$$n M_{AB} + s M_{BC} - 2s M_{CB} + 2n M_{DA} = -n(C_{AD} + 2C_{DA}) \quad . \quad (115)$$

Adding equation 109 and two times equation 112 and subtracting equations 110 and 111 and two times equation 113 gives

$$-n M_{AB} + (2s+1)M_{BC} - 2s M_{CB} - n M_{DA} = n(C_{AD} + C_{DA}) \quad (116)$$

Adding equations 108, 110 and 112, and subtracting equations 109, 111 and 113 gives

$$(n+1)M_{AB} + (s+1)M_{BC} - s M_{CB} + n M_{DA} = -n(C_{AD} + C_{DA}) \quad (117)$$

Equations 114 to 117 are rewritten in Table 7. In this table the unknown moments are written at the heads of the columns and the coefficients are written below.

TABLE 7

EQUATIONS FOR THE TWO-LEGGED RECTANGULAR BENT OF FIG. 28

No. of Equation	Left-hand Member of Equation				Right-hand Member of Equation	How Equation was Obtained
	M_{AB}	M_{BC}	M_{CB}	M_{DA}		
114	-1	1	1	1	$-M_D$	114
115	n	s	$-2s$	$2n$	$-n(C_{AD} + 2C_{DA})$	$-(109) + (112) + 2(110) - 2(113)$
116	$-n$	$2s+1$	$-2s$	$-n$	$n(C_{AD} + C_{DA})$	$-(111) + 2(112) - 2(113) - (110) + (109)$
117	$n+1$	$s+1$	$-s$	n	$-n(C_{AD} + C_{DA})$	$(108) - (109) + (110) - (111) + (112) - (113)$

Solving these equations simultaneously and letting

$$\Delta_o = 2(11sn + 3sn^2 + 3s^2n + s^2 + s + n^2 + n) \text{ gives}$$

$$M_{AD} = -\frac{n}{\Delta_o} \left[3s(s+2)(M_D - C_{DA}) - C_{AD}(6ns + 3s^2 + 5s + 2n) \right] \quad (118)$$

$$M_{BC} = -\frac{n}{\Delta_o} \left[3s(n+2)(M_D - C_{DA}) + C_{AD}(3ns - n - 4s) \right] \quad (119)$$

$$M_{CB} = -\frac{n}{\Delta_o} \left[(3ns + 2n + 5s + 2)(M_D - C_{DA}) + C_{AD}(3ns + 3n - 3s - 1) \right] \quad (120)$$

$$M_{DA} = -\frac{1}{\Delta_o} \left[s(3ns + 2s + 5n + 2)(M_D - C_{DA}) + n C_{AD}(3s^2 + 12s + 1) \right] - C_{DA} \quad (121)$$

If $n=s$ equations 118 and 121 take the form

$$M_{AD} = -\frac{n}{2} \left[\frac{3(M_D - C_{DA})}{6n+1} - C_{AD} \left(\frac{1}{n+2} + \frac{3}{6n+1} \right) \right] \quad (122)$$

$$M_{BC} = -\frac{n}{2} \left[\frac{3(M_D - C_{DA})}{6n+1} + C_{AD} \left(\frac{1}{n+2} - \frac{3}{6n+1} \right) \right] \quad (123)$$

$$M_{CB} = -\frac{1}{2} \left[\frac{(3n+1)(M_D - C_{DA})}{6n+1} - C_{AD} \left(\frac{1}{n+2} - \frac{3n}{6n+1} \right) \right] \quad (124)$$

$$M_{DA} = -\frac{1}{2} \left[\frac{(3n+1)(M_D - C_{DA})}{6n+1} + C_{AD} \left(\frac{1}{n+2} + \frac{3n}{6n+1} \right) \right] - C_{DA} \quad (125)$$

If both legs of the bent are loaded and if the bent and loads are symmetrical about a vertical center line

$$M_{AB} = M_{BC} = -\frac{n}{n+2} C_{AD} \quad (126)$$

$$M_{CB} = -M_{DA} = \frac{1}{n+2} C_{AD} + C_{DA} \quad (127)$$

Values of C_{AD} and C_{DA} to be used in equations 118 to 127 are given in Table 2.

26. *Two-legged Rectangular Bent. Any System of Vertical Loads on the Top—Legs Hinged at the Bases.*—Fig. 29 represents a two-legged rectangular bent having any system of vertical loads on AB . The legs are hinged at D and C . P represents the resultant of the loads on AB .

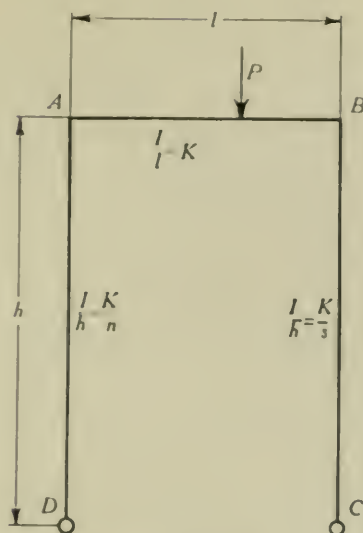


FIGURE 29

Applying the equations of Table 1 gives

$$M_{AB} = 2EK(2\theta_A + \theta_B) - C_{AB} \quad (128)$$

$$M_{AD} = \frac{EK}{n}(3\theta_A - 3R) \quad (129)$$

$$M_{BA} = 2EK(2\theta_B + \theta_A) + C_{BA} \quad (130)$$

$$M_{BC} = \frac{EK}{s}(3\theta_B - 3R) \quad (131)$$

$$M_{AD} + M_{BC} = 0 \quad (132)$$

Eliminating values of θ and R as in section 22 gives

$$M_{BC} = -\frac{3}{2} \left[\frac{C_{AB} + C_{BA}}{n + s + 3} \right] \quad (133)$$

$$M_{AD} = \frac{3}{2} \left[\frac{C_{AB} + C_{BA}}{n + s + 3} \right] = -M_{BC} \quad (134)$$

Values of C_{BA} and C_{AB} to be used in equations 133 and 134 are given in Table 2.

27. *Two-legged Rectangular Bent. Any System of Vertical Loads on the Top—Legs Fixed at the Bases.*—Fig. 30 represents a two-legged rectangular bent with any system of vertical loads on AB . The legs are fixed at C and D . P represents the resultant of all the loads on AB .

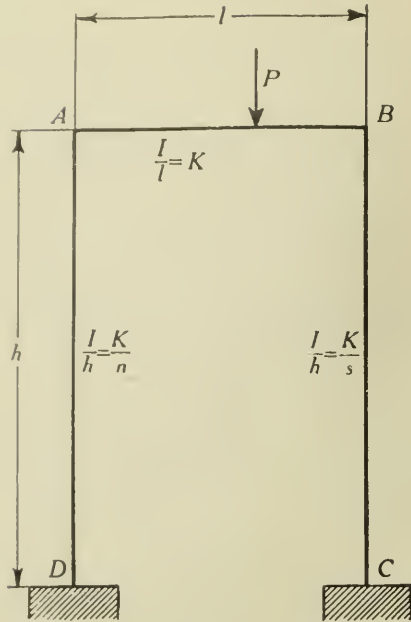


FIGURE 30

Applying the equations of Table 1 gives

$$n M_{DA} = 2EK(\theta_A - 3R) \quad (135)$$

$$n M_{AB} = -2EK(2\theta_A - 3R) \quad (136)$$

$$M_{AB} = 2EK(2\theta_A + \theta_B) - C_{AB} \quad (137)$$

$$M_{BC} = -2EK(2\theta_B + \theta_A) - C_{BA} \quad (138)$$

$$s M_{BC} = 2EK(2\theta_B - 3R) \quad (139)$$

$$s M_{CB} = 2EK(\theta_B - 3R) \quad (140)$$

$$-M_{AB} + M_{BC} + M_{CB} + M_{DA} = 0 \quad (141)$$

Combining these equations as indicated in the table to eliminate θ_A , θ_B , and R gives the equations in Table 8.

Solving these equations simultaneously gives

$$M_{AB} = -\frac{1}{\Delta_o} \left\{ C_{BA}(10ns + s^2) + C_{AB}(11ns + 2s^2 + 2s + 2n) \right\} \quad (146)$$

$$M_{BC} = -\frac{1}{\Delta_o} \left\{ C_{BA}(11ns + 2n^2 + 2n + 2s) + C_{AB}(10ns + n^2) \right\} \quad (147)$$

TABLE 8

EQUATIONS FOR THE TWO-LEGGED RECTANGULAR BENT REPRESENTED BY FIG. 30

No. of Equa- tion	Left-hand Member of Equation				Right-hand Member of Equation	How Equation Was Obtained
	M_{AB}	M_{BC}	M_{CB}	M_{DA}		
142	-1	1	1	1	0	(141)
143	n	s	$-2s$	$2n$	0	(136) + (139) + 2[(135) - (140)]
144	$-n$	$2s+1$	$-2s$	$-n$	$-C_{BA}$	(138) + 2[(139) - (140)] - [(135) + (136)]
145	$n+1$	$s+1$	$-s$	n	$-(C_{AB} + C_{BA})$	(135) + (136) + (137) + (138) + (139) - (140)

$$M_{CB} = -\frac{1}{\Delta_o} \left\{ C_{BA}(7ns - 2n^2 - 2n + s) + C_{AB}(8ns - n^2 + 3n) \right\} \quad (148)$$

$$M_{DA} = \frac{1}{\Delta_o} \left\{ C_{BA}(8ns - s^2 + 3s) + C_{AB}(7ns - 2s^2 - 2s + n) \right\} \quad (149)$$

in which $\Delta_o = 2(11sn + 3sn^2 + 3s^2n + s^2 + s + n^2 + n)$

If the load is symmetrical, that is, if $C_{BA} = C_{AB} = \frac{F}{l}$, equations 146 to 149 take the form

$$M_{AB} = -\frac{F}{l\Delta_o} (21ns + 3s^2 + 2s + 2n) \quad (150)$$

$$M_{BC} = -\frac{F}{l\Delta_o} (21ns + 3n^2 + 2n + 2s) \quad (151)$$

$$M_{CB} = -\frac{F}{l\Delta_o} (15ns - 3n^2 + n + s) \quad (152)$$

$$M_{DA} = \frac{F}{l\Delta_o} (15ns - 3s^2 + s + n) \quad (153)$$

If the bent is symmetrical about a vertical center line, that is, if $n = s$, equations 146 to 149 take the form

$$M_{AB} = -\frac{1}{2} \left\{ C_{BA} \left[\frac{2}{n+2} - \frac{1}{6n+1} \right] + C_{AB} \left[\frac{2}{n+2} + \frac{1}{6n+1} \right] \right\} \quad (154)$$

$$M_{BC} = -\frac{1}{2} \left\{ C_{BA} \left[\frac{2}{n+2} + \frac{1}{6n+1} \right] + C_{AB} \left[\frac{2}{n+2} - \frac{1}{6n+1} \right] \right\} \quad (155)$$

$$M_{CB} = -\frac{1}{2} \left\{ C_{BA} \left[\frac{1}{n+2} - \frac{1}{6n+1} \right] + C_{AB} \left[\frac{1}{n+2} + \frac{1}{6n+1} \right] \right\} \quad (156)$$

$$M_{DA} = \frac{1}{2} \left\{ C_{BA} \left[\frac{1}{n+2} + \frac{1}{6n+1} \right] + C_{AB} \left[\frac{1}{n+2} - \frac{1}{6n+1} \right] \right\} \quad (157)$$

If the bent and the loading are symmetrical about a vertical center line, that is, if $n=s$ and $C_{BA}=C_{AB}=\frac{F}{l}$, equations 146 to 149 take the form

$$M_{AB}=M_{BC} = -\frac{2}{(n+2)} \frac{F}{l} \quad (158)$$

$$M_{CB}=-M_{DA} = -\frac{1}{n+2} \frac{F}{l} \quad (159)$$

28. *Two-legged Rectangular Bent. External Moment at One Corner—Legs Hinged at the Bases.*—Fig. 31 represents a two-legged

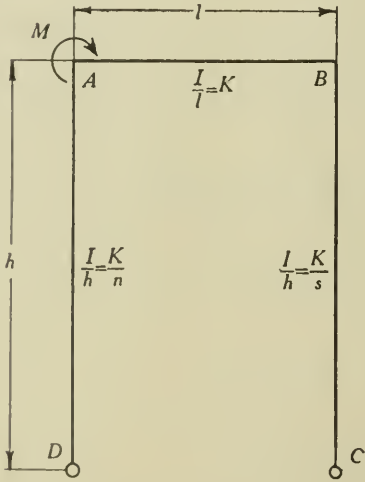


FIGURE 31

rectangular bent having legs hinged at the bases. An external couple whose moment is represented by M is applied at A .

Applying the equations of Table 1 to this bent gives four equations which are identical with the first four equations of section 22. Also, for equilibrium at A ,

$$M_{AD}+M_{AB}=M \quad (160)$$

Solving these equations for the moments gives

$$M_{AB} = -\frac{M}{\Delta_o}(11ns + 2s^2 + 2s + 2n) + M \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (170)$$

$$M_{BC} = -\frac{nM}{\Delta_o}(10s + n) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (171)$$

$$M_{CB} = -\frac{nM}{\Delta_o}(8s - n + 3) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (172)$$

$$M_{DA} = \frac{M}{\Delta_o}(7ns - 2s^2 - 2s + n) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (173)$$

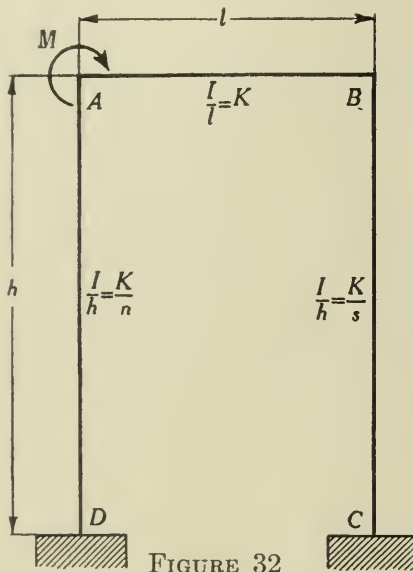


FIGURE 32

$$M_{AD} = \frac{M}{\Delta_o}(11ns + 2s^2 + 2s + 2n) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (174)$$

in which

$$\Delta_o = 22sn + 2(s^2 + s + n^2 + n) + 6(sn^2 + s^2n)$$

If the bent is symmetrical about a vertical center line $n = s$. Equations 170, 171, 172, 173, and 174 then take the form

$$M_{AB} = \frac{M}{2} \left[1 + \frac{n}{n+2} - \frac{1}{6n+1} \right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (175)$$

$$M_{BC} = \frac{M}{2} \left[\frac{n}{n+2} - \frac{6n}{6n+1} \right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (176)$$

$$M_{CB} = -\frac{M}{2} \left[\frac{1}{n+2} + \frac{1}{6n+1} \right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (177)$$

$$M_{DA} = \frac{M}{2} \left[\frac{1}{n+2} - \frac{1}{6n+1} \right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (178)$$

[illegible]

If $n=s=1$, equations 170, 171, 172, 173, and 174 take the form

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

30. *Two-legged Rectangular Bent. Settlement of Foundations—Legs Hinged at the Bases.*—Fig. 33 represents a two-legged rectangular bent. The legs of the bent are hinged at D and C . The unstrained position of the bent is represented by the broken line $DA'B'C'$. Due to settlement of the foundations the point C' has moved to C . The motion of C can be considered as made up of two parts, as follows: first, without being strained the bent rotates about D until C' is at C'' , a point on CD ; secondly, the bent is strained by applying a force at C'' acting along DC which moves C'' to C ; that is, no matter what motion of C relative to D takes place, the stress in the bent depends only upon the change in the distance of C from D .

The change in the distance from D to C is represented by d . d is made up of two parts, d_1 due to the deflection of AD , and d_2 due to the deflection of BC .

Applying the equations of Table 1 gives

[illegible]

$$M_{AB} = 2EK(2\theta_A + \theta_B) \quad (186)$$

31. *Two-legged Rectangular Bent. Settlement of Foundations—Legs Restrained at the Bases.*—Fig. 34 represents a two-legged rectangular bent. The legs of the bent are restrained at C and D . The unstrained position of the bent is represented by the broken line $DA'B'C'$. Due to settlement of the foundations C' has moved to C .

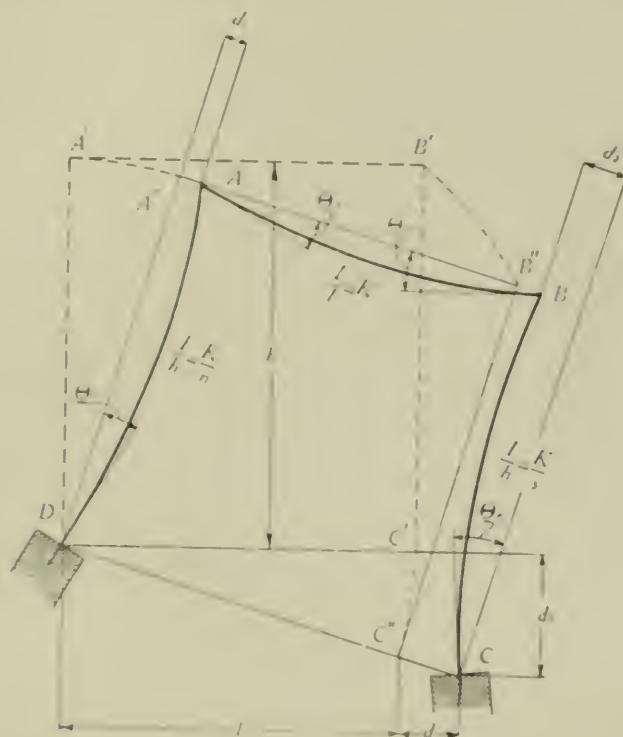


FIGURE 34

The foundations, moreover, have tipped so that whereas the tangents to the neutral axes of the columns at the bases were originally vertical now they are inclined. The motion of the bent can be considered as made up of two parts, as follows: first, without being strained the bent rotates about D until C' is at C'' , a point on DC . At the same time the supports rotate so that the tangents to the neutral axes of the columns at the bases are normal to DC . These motions produce no stress in the bent; secondly, C'' moves to C , the support D rotates through the angle θ_D , and the support C rotates through the angle θ_C ; that is, no matter what motion of C relative to D takes place, the stress in the bent depends only upon the change in the distance of C from D and upon the rotations of the supports at D and C relative to the line DC' (not relative to DC).

Applying the equations of Table 1 gives

$$M_{AB} = -\frac{2EK}{n} (2\theta_A + \theta_D - 3R_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (193)$$

$$M_{AB} = 2EK(2\theta_A + \theta_B) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (194)$$

$$M_{BC} = -2EK(2\theta_B + \theta_A) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (195)$$

$$M_{BC} = \frac{2EK}{s} (2\theta_B + \theta_C - 3R_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (196)$$

$$M_{CB} = \frac{2EK}{s} (2\theta_C + \theta_B - 3R_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (197)$$

$$M_{DA} = \frac{2EK}{n} (2\theta_D + \theta_A - 3R_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (198)$$

For the columns to be in equilibrium

$$M_{DA} - M_{AB} + M_{BC} + M_{CB} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (199)$$

Combining these equations and solving for the moments gives

$$M_{AB} = \frac{2EK}{\Delta_o} \left[3(6ns + 2n - s) \frac{d}{h} + (9ns + 8n - s) \theta_C - (6ns + 2n - 7s - 3s^2) \theta_D \right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (200)$$

$$M_{BC} = \frac{2EK}{\Delta_o} \left[3(6ns - n + 2s) \frac{d}{h} + (6ns - 7n + 2s - 3n^2) \theta_C - (9ns - n + 8s) \theta_D \right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (201)$$

$$M_{CB} = \frac{2EK}{\Delta_o} \left[3(6ns + 5n + 2s + 1) \frac{d}{h} + (12ns + 22n + 4s + 3 + 3n^2) \theta_C - (9ns + 7n + 7s + 3) \theta_D \right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (202)$$

$$M_{DA} = -\frac{2EK}{\Delta_o} \left[3(6ns + 2n + 5s + 1) \frac{d}{h} + (9ns + 7n + 7s + 3) \theta_C - (12ns + 4n + 22s + 3 + 3s^2) \theta_D \right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (203)$$

in which

$$\Delta_o = 22ns + 2(s^2 + s + n^2 + n) + 6(n^2s + s^2n)$$

If the bent is symmetrical about a vertical center line $n = s$. Equations 200, 201, 202, and 203 then take the form

$$M_{AB} = EK \left[\frac{d}{h} \frac{3}{n+2} + \frac{\theta_C - \theta_D}{n+2} + \frac{3(\theta_C + \theta_D)}{6n+1} \right] \quad (204)$$

$$M_{BC} = EK \left[\frac{d}{h} \frac{3}{n+2} + \frac{\theta_C - \theta_D}{n+2} - \frac{3(\theta_C + \theta_D)}{6n+1} \right] \quad (205)$$

$$M_{CB} = EK \left[\frac{d}{h} \frac{3(n+1)}{n(n+2)} + \frac{2n+3}{n(n+2)} (\theta_C - \theta_D) + \frac{3(\theta_C + \theta_D)}{6n+1} \right] \quad (206)$$

$$M_{DA} = -EK \left[\frac{d}{h} \frac{3(n+1)}{n(n+2)} + \frac{2n+3}{n(n+2)} (\theta_C - \theta_D) - \frac{3(\theta_C + \theta_D)}{6n+1} \right] \quad (207)$$

If $n = s = 1$

$$M_{AB} = \frac{EK}{21} \left[21 \frac{d}{h} + 16\theta_C + 2\theta_D \right] \quad (208)$$

$$M_{BC} = \frac{EK}{21} \left[21 \frac{d}{h} - 2\theta_C - 16\theta_D \right] \quad (209)$$

$$M_{CB} = \frac{EK}{21} \left[42 \frac{d}{h} + 44\theta_C - 26\theta_D \right] \quad (210)$$

$$M_{DA} = -\frac{EK}{21} \left[42 \frac{d}{h} + 26\theta_C - 44\theta_D \right] \quad (211)$$

In all these equations θ is measured from a line normal to the line CD .

If the foundations settle without tipping it is more convenient to measure θ from a line normal to the original position DC' . It is then necessary to consider the vertical settlement d_3 .

Proceeding as before, if the tangents to the neutral axes of the columns at their bases remain vertical, if the bases are separated

horizontally by an amount represented by d , and if C settles vertically by an amount represented by d_3 , it can be proved that

$$M_{AB} = \frac{2EK}{\Delta_o} \left[3(6ns + 2n - s) \frac{d}{h} - 3(ns + 2n + 2s + s^2) \frac{d_3}{l} \right] \quad (212)$$

$$M_{BC} = \frac{2EK}{\Delta_o} \left[3(6ns - n + 2s) \frac{d}{h} + 3(ns + 2n + 2s + s^2) \frac{d_3}{l} \right] \quad (213)$$

$$M_{CB} = \frac{2EK}{\Delta_o} \left[3(6ns + 5n + 2s + 1) \frac{d}{h} - 3(ns + 5n - s + n^2) \frac{d_3}{l} \right] \quad (214)$$

$$M_{DA} = -\frac{2EK}{\Delta_o} \left[3(6ns + 2n + 5s + 1) \frac{d}{h} + 3(ns - n + 5s + s^2) \frac{d_3}{l} \right] \quad (215)$$

If the bent is symmetrical about a vertical center line, $n = s$. Equations 212, 213, 214, and 215 then reduce to the form

$$M_{AB} = EK \left[\frac{d}{h} \frac{3}{n+2} - \frac{d_3}{l} \frac{6}{6n+1} \right] \quad (216)$$

$$M_{BC} = EK \left[\frac{d}{h} \frac{3}{n+2} + \frac{d_3}{l} \frac{6}{6n+1} \right] \quad (217)$$

$$M_{CB} = EK \left[\frac{d}{h} \frac{3(n+1)}{n(n+2)} - \frac{d_3}{l} \frac{6}{6n+1} \right] \quad (218)$$

$$M_{DA} = -EK \left[\frac{d}{h} \frac{3(n+1)}{n(n+2)} + \frac{d_3}{l} \frac{6}{6n+1} \right] \quad (219)$$

If $n = s = 1$

$$M_{AB} = EK \left[\frac{d}{h} - \frac{6}{7} \frac{d_3}{l} \right] \quad (220)$$

$$M_{BC} = EK \left[\frac{d}{h} + \frac{6}{7} \frac{d_3}{l} \right] \quad (221)$$

$$M_{CB} = EK \left[\frac{2d}{h} - \frac{6}{7} \frac{d_3}{l} \right] \quad (222)$$

$$M_{DA} = -EK \left[\frac{2d}{h} + \frac{6}{7} \frac{d_3}{l} \right] \quad (223)$$

VII. TWO-LEGGED RECTANGULAR BENT. ONE LEG LONGER THAN THE OTHER

32. *Two-legged Rectangular Bent. One Leg Longer than the Other. Concentrated Horizontal Load at Top of Bent—Legs Hinged at the Bases.*—Fig. 35 represents a two-legged rectangular bent having one leg longer than the other. The legs are hinged at C and D . Let q equal the ratio of the length AD to the length BC .

The horizontal deflections of A and B are equal and are represented by d . R_{AD} is $\frac{d}{h}$ and R_{BC} is $\frac{d}{h} q$. If R_{AD} is represented by R , $R_{BC} = q R$.

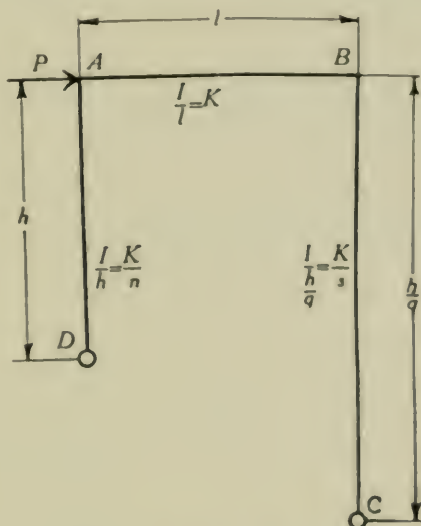


FIGURE 35

Applying the equations of Table 1 gives

[illegible]

$$M_{AD} = \frac{EK}{n}(3\theta_A - 3R) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (225)$$

[illegible]

$$M_{BC} = \frac{EK}{S} (3\theta_B - 3qR) \quad . \quad . \quad . \quad . \quad (227)$$

$$M_{AD} + q M_{BC} + Ph = 0 \quad . \quad . \quad . \quad (228)$$

Substituting the value of θ_B from equation 224 in equation 226 and substituting $-M_{BC}$ for M_{BA} gives

$$\theta_A = \frac{1}{6EK}(M_{BC} + 2M_{AB}) \quad (229)$$

Substituting the value of R from equation 225 and the value of θ_B from equation 224 in equation 227 and substituting $-M_{AB}$ for M_{AD} gives

$$\theta_A = \frac{1}{2EK(6+3q)} \left[M_{AB}(3-2nq) - 2s M_{BC} \right] \quad (230)$$

Equating the right-hand members of equations 229 and 230 gives $M_{BC}(2+q+2s) + M_{AB}(1+2nq+2q) = 0$ (231)

Substituting $-M_{AB}$ for M_{AD} in equation 228 and eliminating M_{AB} from equations 228 and 231 gives

$$M_{BC} = -Ph \left[\frac{2nq+2q+1}{q(2nq+2q+1) + (2s+2+q)} \right] \quad (232)$$

$$M_{AB} = Ph \left[\frac{2s+2+q}{q(2nq+2q+1) + (2s+2+q)} \right] \quad (233)$$

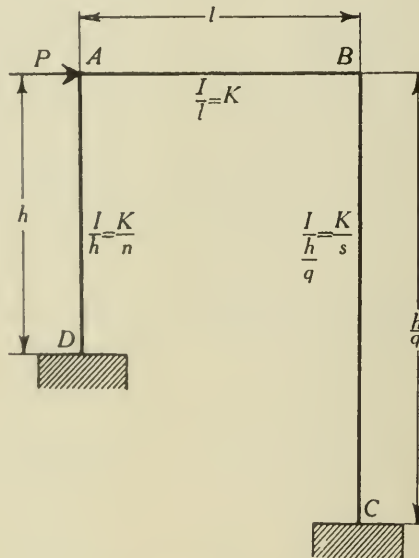


FIGURE 36

33. *Two-legged Rectangular Bent. One Leg Longer than the Other. Concentrated Horizontal Load at Top of Bent—Legs Fixed at the Bases.—*

Substituting the value of M_{BC} from equation 260 in equation 261 gives

$$M_{AB} = \frac{1}{2} \frac{M_D(2s+2+q) - 2q^2n H_{AD}}{q^2n + s + 1 + q + q^2} \quad (262)$$

Substituting the value of M_{AB} in equation 261 gives

$$M_{BC} = -\frac{1}{2} \frac{M_D(2qn+2q+1) + 2qn H_{AD}}{q^2n + s + 1 + q + q^2} \quad (263)$$

If $n=s$, equations 262 and 263 take the form

$$M_{AB} = \frac{1}{2} \frac{M_D(2n+2+q) - 2q^2n H_{AD}}{q^2n + n + 1 + q + q^2} \quad (264)$$

$$M_{BC} = -\frac{1}{2} \frac{M_D(2qn+2q+1) + 2qn H_{AD}}{q^2n + n + 1 + q + q^2} \quad (265)$$

If $n=s=1$, equations 262 and 263 take the form

$$M_{AB} = \frac{1}{2} \frac{M_D(4+q) - 2q^2 H_{AD}}{2+q+2q^2} \quad (266)$$

$$M_{BC} = -\frac{1}{2} \frac{M_D(1+4q) + 2q H_{AD}}{2+q+2q^2} \quad (267)$$

Values of H_{AD} for different systems of loadings to be used in equations 262 to 267 are given in Table 2.

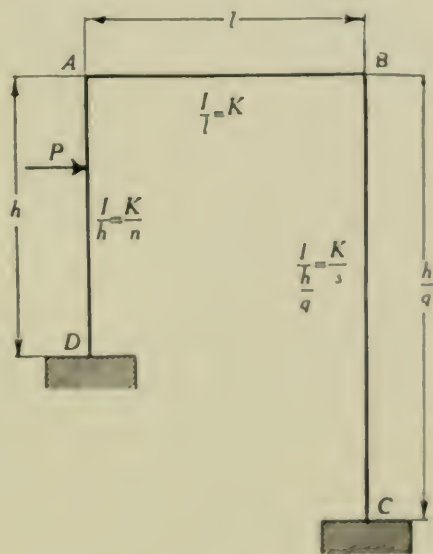


FIGURE 39

TABLE 9
EQUATIONS FOR THE MOMENTS IN THE RECTANGULAR BENT REPRESENTED BY FIG. 39

No. of Equa- tion	Left-hand Member of Equation				Right-hand Member of Equation	How Equation Was Obtained
	M_{AB}	M_{BC}	M_{CD}	M_{DA}		
274	-1	q	$+q$	1	$-M_D$	(274)
275	$1+2q-n(2+q)$	$2(s+1)(2+q)$	$-s(1+2q)$	$-2n(1+2q)$	$n[C_{AD}(2+q)+C_{DA}(2+4q)]$	$[2+q][2(270)-(268)+2(271)]$ $+ [1+2q][(269)-2(273)$ $- (272)]$
276	1	$2+3s$	$-3s$	0	0	$(269)+2(270)+3(271)$ $-3(272)$
277	$2(1+n)(1+2q)$	$(2+q)-s(1+2q)$	$2s(2+q)$	$n(2+q)$	$-n[C_{AD}(2+4q)+C_{DA}(2+q)]$	$[2+q][(270)+2(272)$ $+ (273)] + [1+2q]$ $[2(269)-(271)+2(268)]$

$$M_{DA} = -\frac{n}{\Delta_o} \left\{ (M_D - C_{DA}) (3n^2 + 6n + qn + 2) + C_{AD} [3n^2 + 4n (1 + q + q^2) + q^2] \right\} - C_{DA} \quad (285)$$

$\Delta_o = 2n \left\{ 3n^2(1 + q^2) + n(5 + 3q + 5q^2) + 1 + q^2 \right\}$ for equations 282 to 285 inclusive.

If $n = s = 1$, equations 282 to 285 take the form

$$M_{AB} = \frac{1}{\Delta_o} \left\{ (M_D - C_{DA}) (7 + 2q) - C_{AD} (8q^2 + q + 7) \right\} \quad (286)$$

$$M_{BC} = -\frac{1}{\Delta_o} \left\{ (M_D - C_{DA}) (7q + 2) + C_{AD} (q - 2 - q^2) \right\} \quad (287)$$

$$M_{CB} = -\frac{1}{\Delta_o} \left\{ (M_D - C_{DA}) (11q + 1) + C_{AD} (2q - 1 + q^2) \right\} \quad (288)$$

$$M_{DA} = -\frac{1}{\Delta_o} \left\{ (M_D - C_{DA}) (11 + q) + C_{AD} (7 + 4q + 5q^2) \right\} - C_{DA} \quad (289)$$

$\Delta_o = 6(3 + q + 3q^2)$ for equations 286 to 289 inclusive.

Values of C_{AD} and C_{DA} are given in Table 2.

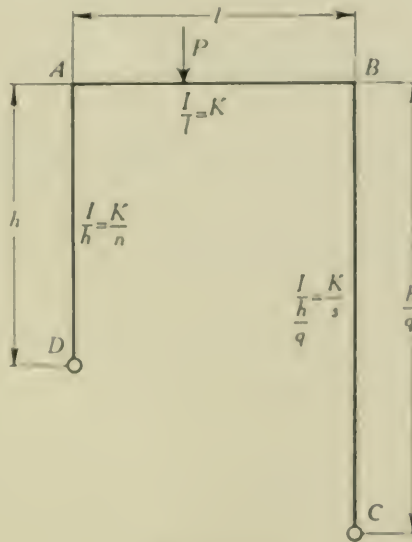


FIGURE 41

36. *Two-legged Rectangular Bent. One Leg Longer than the Other. Any System of Vertical Loads on Top of Bent—Legs Hinged at the Bases.*—Fig. 41 represents a two-legged bent having one leg longer

than the other. P represents the resultant of any system of vertical loads applied to AB . The legs of the bent are hinged at C and D .

Applying the equations of Table 1 gives

$$n M_{AB} = -EK(3\theta_A - 3R) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (290)$$

$$M_{AB} = 2EK(2\theta_A + \theta_B) - C_{AB} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (291)$$

$$M_{BC} = -2EK(2\theta_B + \theta_A) - C_{BA} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (292)$$

$$s M_{BC} = EK(3\theta_B - 3qR) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (293)$$

Since the sum of the shears in the two legs equals zero,

$$-M_{AB} + q M_{BC} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (294)$$

Combining equations 290 to 293 as follows

$$-\left[\frac{2+q}{3}\right]\left[2(290) - 3(292) - 4(293)\right] + \left[\frac{1+2q}{3}\right]\left[4(290) + 3(291) - 2(293)\right]$$

(the numerals in the parentheses are the equation numbers) gives

$$M_{AB}(2qn + 2q + 1) + M_{BC}(2s + 2 + q) = -[C_{BA}(2 + q) + C_{AB}(1 + 2q)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (295)$$

Substituting M_{BC} from equation 295 in equation 294 gives

$$M_{AB} = -\frac{q}{2} \frac{C_{BA}(2 + q) + C_{AB}(1 + 2q)}{q^2n + s + 1 + q + q^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (296)$$

Substituting M_{AB} from equation 296 in equation 294 gives

$$M_{BC} = -\frac{1}{2} \frac{C_{BA}(2 + q) + C_{AB}(1 + 2q)}{q^2n + s + 1 + q + q^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (297)$$

If the load is symmetrical about the center of AB , $C_{BA} = C_{AB} = \frac{F}{l}$ and equations 296 and 297 take the form

$$M_{AB} = -\frac{3}{2} q \frac{F}{l} \frac{1 + q}{q^2n + s + 1 + q + q^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (298)$$

$$M_{BC} = -\frac{3}{2} \frac{F}{l} \frac{1 + q}{q^2n + s + 1 + q + q^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (299)$$

Values of C_{AB} , C_{BA} , and $\frac{F}{l}$ are given in Tables 2 and 3.

37. *Two-legged Rectangular Bent. One Leg Longer than the Other. Any System of Vertical Loads on Top of Bent—Legs Fixed at the Bases.*—Fig. 42 represents a two-legged bent having legs of unequal length.

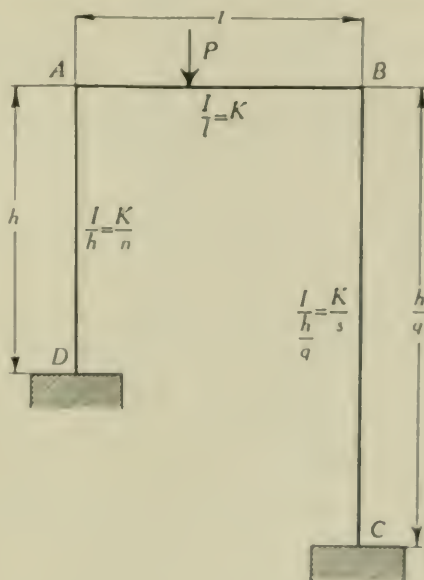


FIGURE 42

P represents the resultant of any system of vertical loads on AB . The legs are fixed at D and C .

Applying the equations of Table 1 gives

$$nM_{AB} = -2EK(2\theta_A - 3R) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (300)$$

$$M_{AB} = 2EK(2\theta_A + \theta_B) - C_{AB} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (301)$$

$$M_{BC} = -2EK(2\theta_B + \theta_A) - C_{BA} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (302)$$

$$sM_{BC} = 2EK(2\theta_B - 3qR) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (303)$$

$$sM_{CB} = 2EK(\theta_B - 3qR) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (304)$$

$$nM_{DA} = 2EK(\theta_A - 3R) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (305)$$

Since the sum of the shears in the two legs equals zero,

$$-M_{AB} + qM_{BC} + qM_{CB} + M_{DA} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (306)$$

Combining equations 300 to 306 as indicated gives the equations of Table 10.

TABLE 10
EQUATIONS FOR THE MOMENTS IN THE RECTANGULAR BENT REPRESENTED BY FIG. 42

No. of Equa- tion	Left-hand Member of Equation				Right-hand Member of Equation	How Equation Was Obtained
	M_{AB}	M_{BC}	M_{CB}	M_{DA}		
306	-1	q	q	1	0	(306)
307	$1+2q-n(2+q)$	$2(s+1)(2+q)$	$-s(1+2q)$	$-2n(1+2q)$	$-[2C_{BA}(2+q)+C_{AB}(1+2q)]$	$[2+q][2(302)-(300)+(2303)]$ $+[1+2q][(301)-2(305)$ $-(304)]$
308	1	$2+3s$	$-3s$	0	$-[2C_{BA}+C_{AB}]$	$(301)+3[(303)-(304)]$ $+2(302)$
309	$\frac{+2(1+n)}{(1+2q)}$	$\frac{(2+q)}{-s(1+2q)}$	$2s(2+q)$	$n(2+q)$	$-[C_{BA}(2+q)+2C_{AB}(1+2q)]$	$[2+q][(302)+2(304)+(305)]$ $+ [1+2q][2(301)-(303)+$ $2(300)]$

Solving these equations simultaneously gives

$$M_{AB} = -\frac{1}{\Delta_o} \left\{ C_{BA}[2qns(3+2q) + s^2] + C_{AB}[qns(3+8q) + 2s^2 + 2s + 2q^2n] \right\} \quad (310)$$

$$M_{BC} = -\frac{1}{\Delta_o} \left\{ C_{BA}[ns(8+3q)+2q^2n^2+2q^2n+2s] \right. \\ \left. + C_{AB}[2ns(2+3q)+q^2n^2] \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (311)$$

$$M_{CB} = -\frac{1}{\Delta_o} \left\{ C_{BA}[ns(4+3q) - 2q^2n^2 - 2q^2n + s] \right. \\ \left. + C_{AB}[2ns(1+3q) - q^2n^2 + 3qn] \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (312)$$

$$M_{DA} = \frac{1}{\Delta_o} \left\{ C_{BA} [2qns(3+q) - s^2 + 3qs] \right. \\ \left. + C_{AB} [qns(3+4q) - 2s^2 - 2s + q^2n] \right\}. \quad (313)$$

in which $\Delta_o = 2[ns(4 + 3q + 4q^2) + q^2n(3ns + n + 1) + s(3ns + s + 1)]$

If the load is symmetrical about the center of $A-B$, $C_{BA} = C_{AB} = \frac{F}{l}$, and equations 310 to 313 take the form

$$M_{AB} = -\frac{1}{\Delta_o} \frac{F}{l} [3qns(3+4q) + 3s^2 + 2s + 2q^2n] \quad . \quad . \quad . \quad (314)$$

$$M_{BC} = -\frac{1}{\Delta_o} \frac{F}{l} [3ns(4+3q) + 3q^2n^2 + 2q^2n + 2s] \quad . \quad . \quad . \quad (315)$$

$$M_{CB} = -\frac{1}{\Delta_o} \frac{F}{l} [3ns(2+3q) - 3q^2n^2 + qn(3-2q) + s] \quad . \quad . \quad (316)$$

$$M_{DA} = \frac{1}{\Delta_o} \frac{F}{l} [3qns(3+2q) - 3s^2 + s(3q-2) + q^2n] \quad . \quad . \quad (317)$$

If $n=s$ equations 310 to 313 take the form

$$M_{AB} = -\frac{n}{\Delta_o} \left\{ C_{BA}n(1+6q+4q^2) + C_{AB}[n(2+3q+8q^2) + 2+2q^2] \right\} \quad (318)$$

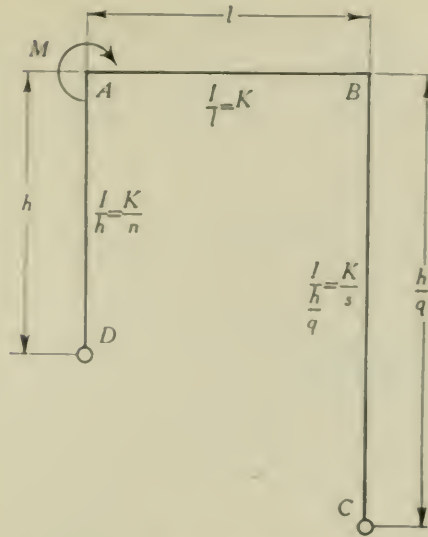


FIGURE 43

$$M_{BC} = -2EK(2\theta_B + \theta_A) \quad (328)$$

$$M_{BC} = \frac{EK}{s}(3\theta_B - 3qR) \quad (329)$$

$$M_{AD} + qM_{BC} = 0$$

$$M - M_{AB} + qM_{BC} = 0$$

$$M_{AB} - qM_{BC} = M \quad (330)$$

Combining these equations and solving for the moments gives

$$M_{AB} = \frac{M}{2} \frac{2s + 2 + q + 2q^2n}{q^2n + s + 1 + q + q^2} \quad (331)$$

$$M_{AD} = \frac{M}{2} \frac{q(1 + 2q)}{q^2n + s + 1 + q + q^2} \quad (332)$$

$$M_{BC} = -\frac{M}{2} \frac{1 + 2q}{q^2n + s + 1 + q + q^2} \quad (333)$$

39. *Two-legged Rectangular Bent. One Leg Longer than the Other. External Moment at One Corner—Legs Fixed at the Bases.*—Fig. 44 represents a two-legged rectangular bent having one leg longer than the other. An external couple with moment M is applied at A . The legs are fixed at C and D .

$$M_{DA} = + \frac{M}{\Delta_o} (3qns - 2s^2 - 2s + 4q^2ns + q^2n) \quad . \quad . \quad . \quad . \quad (344)$$

$$M_{AD} = + \frac{M}{\Delta_o} (3qns + 8q^2ns + 2s^2 + 2s + 2q^2n)$$

in which

$$\Delta_o = 2 \left[ns(4 + 3q + 4q^2) + s^2 + s + q^2n(n+1) + 3ns(q^2n + s) \right]$$

40. *Two-legged Rectangular Bent. One Leg Longer than the Other. Settlement of Foundations—Legs Hinged at the Bases.*—Fig. 45 represents a two-legged rectangular bent having one leg longer than the other. The legs of the bent are hinged at D and C . The unstrained position of the bent is represented by the broken line $DA'B'C'$. Due

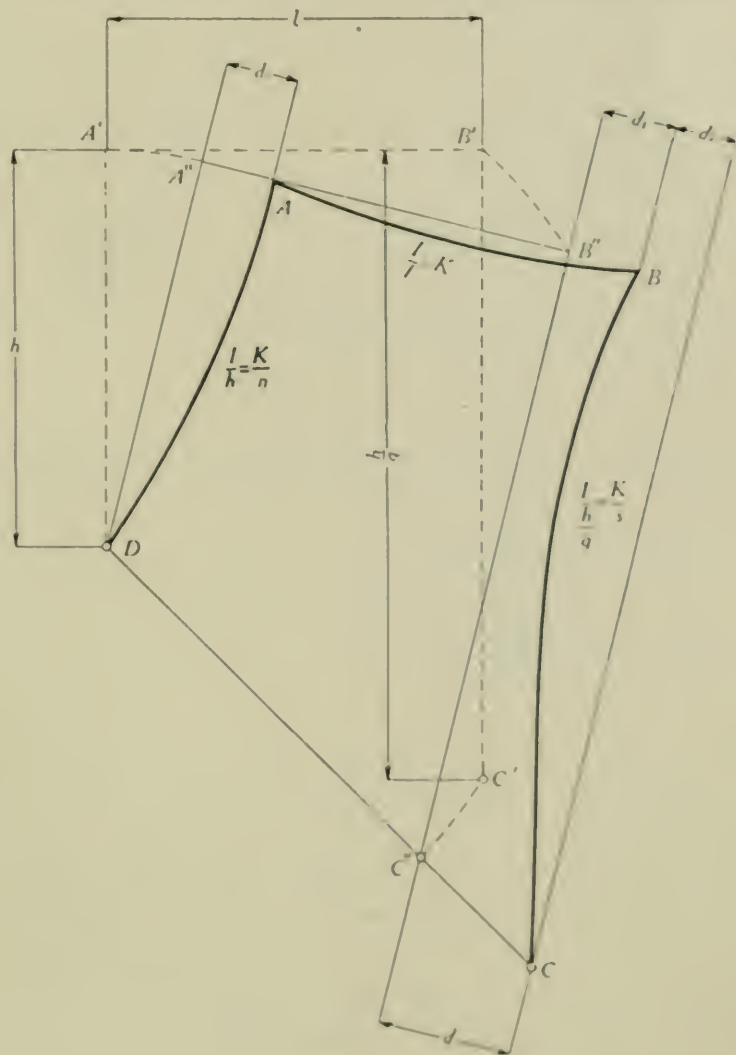


FIGURE 45

bent rotates about D until C' is at C'' , a point on DC ; secondly, the bent is strained by applying a force at C'' which moves C'' to C ; thirdly, couples are applied at D and C , still further straining the bent so that the tangents to the elastic curves at the bases of the legs make angles of θ_D and θ_C respectively with lines normal to DC . It is apparent that,

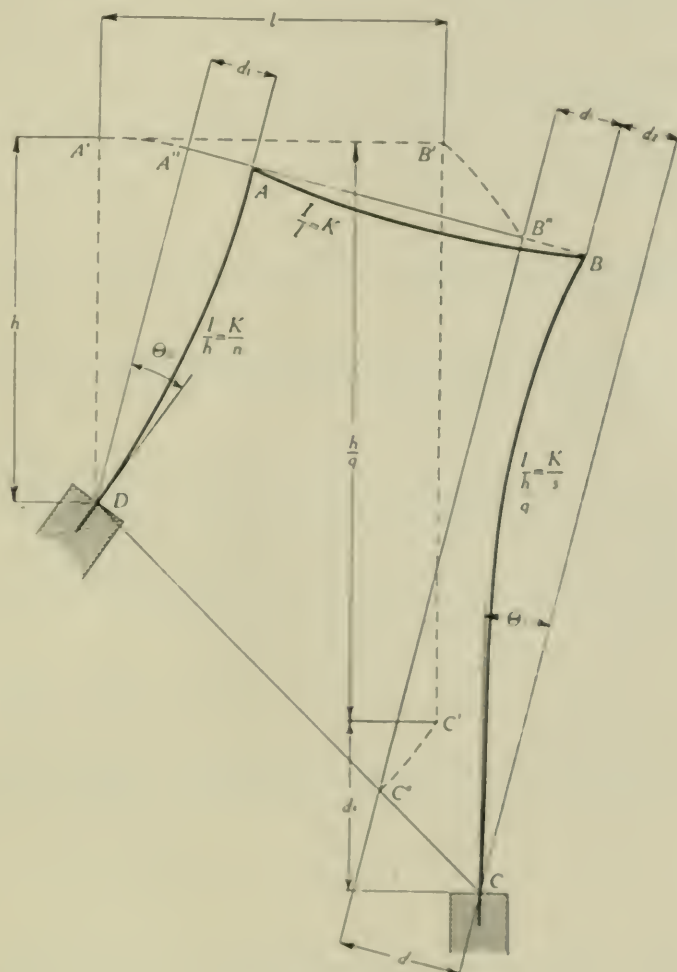


FIGURE 46

no matter what motion of C relative to D takes place, the stress in the bent depends only upon the change in the distance from C to D and upon the angles θ_D and θ_C .

The change in the distance from D to C is proportional to d . d is made up of two parts, d_1 due to the deflection of AD , and d_2 due to the deflection of BC .

in which

$$\Delta_o = 2 [ns(4+3q+4q^2) + q^2n(3ns+n+1) + s(3ns+s+1)]$$

It is to be noted that θ_C and θ_D are measured, not from the original direction of AD and BC , but from a line normal to AB .

If the tangents to the elastic curves of the legs at their bases remain vertical the moments can be expressed in terms of the vertical settlement of C relative to D . The θ 's are then measured from the original direction of AD and BC and equal zero. The displacement d is measured in a horizontal direction. Proceeding as before the moments are found to be as follows:

$$M_{AB} = \frac{2EK}{\Delta_o} \left\{ \frac{3qd}{h} [6qns + 2qn - s] - \frac{3}{(4+5q)} \frac{d_3}{l} [q^2ns(13-4q) + 2qn(2+6q+q^2) + 2s(9+q-q^2) + s^2(13-4q)] \right\} \quad (363)$$

$$M_{BC} = \frac{2EK}{\Delta_o} \left\{ \frac{3qd}{h} [6ns - qn + 2s] + \frac{3}{(4+5q)} \frac{d_3}{l} [ns(4+5q) + 2qn(1+5q+3q^2) + 2s(5+6q-2q^2) + q^2n^2(4+5q)] \right\} \quad (364)$$

$$M_{CB} = \frac{2EK}{\Delta_o} \left\{ \frac{3qd}{h} [6ns + n(4+q) + 2s + 1] - \frac{3}{(4+5q)} \frac{d_3}{l} [ns(4+5q) + n(8+16q+15q^2+6q^3) - s(3+10q-4q^2) + q^2n^2(4+5q) - 2(1-2q+q^2)] \right\} \quad (365)$$

$$M_{DA} = -\frac{2EK}{\Delta_o} \left\{ \frac{3qd}{h} [6qns + 2qn + s(1+4q) + q] - \frac{3}{(4+5q)} \frac{d_3}{l} [q^2ns(13-4q) - qn(4+2q+3q^2) + s(18+15q+20q^2-8q^3) + s^2(13-4q) + 2q(1-2q+q^2)] \right\} \quad (366)$$

Substituting $-M_{AB}$ for M_{AD} and $-M_{CD}$ for M_{CB} gives

$$-M_{AB}+M_{BC}-M_{CD}+M_{DA}=-Ph \quad . \quad . \quad . \quad . \quad . \quad . \quad (397)$$

Combining equations 389 to 397 as indicated in Table 11 gives equations 397 to 400 of Table 11.

Solving these equations simultaneously and letting Δ represent the common denominator gives

$$\Delta=22(sp n+sp+sn+np)+2(sp^2+s^2p+n^2p+p^2n+s^2+s+n^2+n)\\ +6(sn^2+s^2n+p^2+p) \text{ and}$$

$$M_{AB}=\frac{Ph}{\Delta}\left(3s^2n+5nps+2s^2p+2sp^2+6ns+6pn+5ps+3p^2 \right) \quad (401)$$

$$M_{BC}=-\frac{Ph}{\Delta}\left(3n^2s+5nps+2n^2p+2np^2+6ns+6ps+5pn+3p^2 \right)\\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (402)$$

$$M_{CD}=\frac{Ph}{\Delta}\left(3n^2s+6nps+5ns+6ps+5pn+2n+3p+2n^2 \right) \quad . \quad (403)$$

$$M_{DA}=-\frac{Ph}{\Delta}\left(3ns^2+6nps+5ns+6pn+5ps+2s+3p+2s^2 \right) \quad (404)$$

If the frame is symmetrical about the vertical center line, that is, if AB and BC , Fig. 53, have the same section, $n=s$, and equations 401 to 404 take the form

$$M_{AB}=-M_{BC}=\frac{Ph}{2}\frac{3n+p}{\beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (405)$$

$$M_{CD}=-M_{DA}=\frac{Ph}{2}\frac{3n+1}{\beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (406)$$

in which $\beta=6n+p+1$.

TABLE 11
EQUATIONS FOR RECTANGULAR FRAME WITH HORIZONTAL FORCE AT THE TOP

No. of Equa- tion	Left-hand Member of Equation				Right- hand Member of Equation	Method of Obtaining Equation
	M_{AB}	M_{BC}	M_{CD}	M_{DA}		
397	-1	+1	-1	+1	$-P_h$	Equation (397)
398	n	s	$2s+3p$	$2n+3p$	0	$(390) + (393) + 2[(389) + (394)] + 3[(395) + (396)]$
399	$-n$	$2s+1$	$2s+p$	$-n$	0	$(392) + (395) + 2[(393) + (394)] - (389) - (390)$
400	$n+1$	$s+1$	$s+p$	$n+p$	0	$(389) + (390) + (391) + (392) + (393) + (394) + (395) + (396)$

TABLE 12

EQUATIONS FOR THE MOMENTS IN THE RECTANGULAR FRAME REPRESENTED BY
FIG. 54

No. of Equation	Left-hand Member of Equation				Right-hand Member of Equation
	M_{AB}	M_{BC}	M_{CD}	M_{DA}	
411	-1	1	-1	1	$-M_D$
412	n	s	$2s+3p$	$2n+3p$	$-n(C_{AD}+2C_{DA})$
413	$-n$	$2s+1$	$2s+p$	$-n$	$n(C_{AD}+C_{DA})$
414	$n+1$	$s+1$	$s+p$	$n+p$	$-n(C_{AD}+C_{DA})$

Solving these equations simultaneously gives

$$M_{AB} = \frac{1}{\Delta} \left\{ M_D(3s^2n + 5nps + 2s^2p + 2sp^2 + 6ns + 6pn + 5ps + 3p^2) \right. \\ \left. - C_{AD}n(6sn + 2pn + 3s^2 + 17ps + 2n + 5s + 11p + 2p^2) \right. \\ \left. - C_{DA}n(3s^2 + 12ps + 6s + 10p + p^2) \right\} \quad . \quad . \quad . \quad . \quad . \quad (415)$$

$$M_{BC} = \frac{1}{\Delta} \left\{ -M_D(3sn^2 + 2n^2p + 5nps + 2np^2 + 6ns + 5pn + 6ps + 3p^2) \right. \\ \left. - C_{AD}n(3ns + 2pn + 5ps - n - 7p - 4s + 2p^2) \right. \\ \left. + C_{DA}n(3ns + 3pn - 3ps + 8p + 6s - p^2) \right\} \quad . \quad . \quad . \quad . \quad . \quad (416)$$

$$M_{CD} = \frac{1}{\Delta} \left\{ M_D(3sn^2 + 2n^2p + 6nps + 5pn + 5sn + 6ps + 2n + 3p) \right. \\ \left. + C_{AD}n(3ns + 6ps + 3n + 8p - 3s - 1) \right. \\ \left. - C_{DA}n(3ns - 4ps + 2n - 7p - pn + 5s + 2) \right\} \quad . \quad . \quad . \quad . \quad . \quad (417)$$

$$M_{DA} = \frac{1}{\Delta} \left\{ -M_D(5ns + 6nps + 3ns^2 + 2s^2 + 5ps + 6pn + 2s + 3p) \right. \\ \left. - C_{AD}n(3s^2 + 6ps + 12s + 10p + 1) \right. \\ \left. - C_{DA}n(3s^2 + 5ps + 17s + 11p + 6ns + 2pn + 2n + 2) \right\} \quad . \quad (418)$$

in which

$$\Delta = 22(spn + sp + sn + np) + 2(sp^2 + s^2p + n^2p + p^2n + s^2 + s + n^2 + n) \\ + 6(sn^2 + s^2n + p^2 + p)$$

If $n=s$, equations 415 to 418 take the form

$$M_{AB} = -\frac{1}{2} \left\{ C_{AD}n \left[\frac{n+2p}{\alpha} + \frac{3}{\beta} \right] + C_{DA}n \left[\frac{p}{\alpha} + \frac{3}{\beta} \right] - M_D \frac{3n+p}{\beta} \right\} \quad (419)$$

$$M_{BC} = -\frac{1}{2} \left\{ C_{AD}n \left[\frac{n+2p}{\alpha} - \frac{3}{\beta} \right] + C_{DA}n \left[\frac{p}{\alpha} - \frac{3}{\beta} \right] + M_D \frac{3n+p}{\beta} \right\} \quad (420)$$

$$M_{CD} = -\frac{1}{2} \left\{ C_{AD}n \left[\frac{1}{\alpha} - \frac{3}{\beta} \right] + C_{DA}n \left[\frac{n+2}{\alpha} - \frac{3}{\beta} \right] - M_D \frac{3n+1}{\beta} \right\} \quad (421)$$

$$M_{DA} = -\frac{1}{2} \left\{ C_{AD}n \left[\frac{1}{\alpha} + \frac{3}{\beta} \right] + C_{DA}n \left[\frac{n+2}{\alpha} + \frac{3}{\beta} \right] + M_D \frac{3n+1}{\beta} \right\} \quad (422)$$

in which

$$\alpha = n^2 + 2pn + 2n + 3p$$

$$\beta = 6n + p + 1$$

If $n=s$ and $p=1$ equations 419 to 422 take the form

$$M_{AB} = -\frac{1}{2} \left\{ C_{AD}n \left[\frac{n+2}{\alpha} + \frac{3}{\beta} \right] + C_{DA}n \left[\frac{1}{\alpha} + \frac{3}{\beta} \right] - \frac{M_D}{2} \right\} \quad (423)$$

$$M_{BC} = -\frac{1}{2} \left\{ C_{AD}n \left[\frac{n+2}{\alpha} - \frac{3}{\beta} \right] + C_{DA}n \left[\frac{1}{\alpha} - \frac{3}{\beta} \right] + \frac{M_D}{2} \right\} \quad (424)$$

$$M_{CD} = -\frac{1}{2} \left\{ C_{AD}n \left[\frac{1}{\alpha} - \frac{3}{\beta} \right] + C_{DA}n \left[\frac{n+2}{\alpha} - \frac{3}{\beta} \right] - \frac{M_D}{2} \right\} \quad (425)$$

$$M_{DA} = -\frac{1}{2} \left\{ C_{AD}n \left[\frac{1}{\alpha} + \frac{3}{\beta} \right] + C_{DA}n \left[\frac{n+2}{\alpha} + \frac{3}{\beta} \right] + \frac{M_D}{2} \right\} \quad (426)$$

in which

$$\alpha = (n+3)(n+1)$$

$$\beta = 2(3n+1)$$

If $n=s=p=1$, equations 423 to 426 take the form

$$M_{AB} = -\frac{1}{8} \{3C_{AD} + 2C_{DA} - 2M_D\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (427)$$

$$M_{BC} = -\frac{1}{8} \{-C_{DA} + 2M_D\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (428)$$

$$M_{CD} = \frac{1}{8} \{C_{AD} + 2M_D\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (429)$$

$$M_{DA} = -\frac{1}{8} \{2C_{AD} + 3C_{DA} + 2M_D\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (430)$$

If both vertical sides are loaded and if the frame and the loads are symmetrical about a vertical center line

$$M_{AB} = M_{BC} = -\frac{n}{\alpha} [C_{AD}(n+2p) + C_{DA}p] \quad . \quad . \quad . \quad . \quad . \quad (431)$$

$$M_{CD} = M_{DA} = -\frac{n}{\alpha} [C_{AD} + C_{DA}(n+2)] \quad . \quad . \quad . \quad . \quad . \quad (432)$$

50. *Rectangular Frame. Any System of Vertical Forces on the Top.*—Fig. 55 represents a rectangular frame subjected to any system of vertical forces on the top member AB .

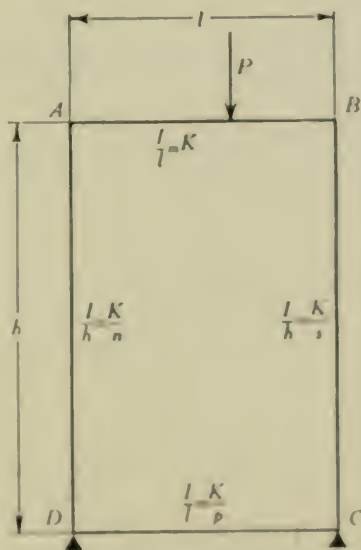


FIGURE 55

Applying the equations of Table 1 gives

$M_{AB}=2EK(2\theta_A+\theta_B)-C_{AB}$ (433)

$M_{BC}=-2EK(2\theta_B+\theta_A)-C_{BA}$ (434)

$-M_{AB}+M_{DA}+M_{BC}-M_{CD}=0$ (435)

Six other equations which are identical with equations 389, 390, and 393 to 396 of section 48 may be written. Combining these nine equations to eliminate θ and R , as indicated in the last column of Table 11, gives the equations of Table 13.

TABLE 13
EQUATIONS FOR THE MOMENTS IN THE RECTANGULAR FRAME REPRESENTED
BY FIG. 55

No. of Equation	Left-hand Member of Equation				Right-hand Member of Equation
	M_{AB}	M_{BC}	M_{CD}	M_{DA}	
435	-1	1	-1	1	0
436	n	s	$2s+3p$	$2n+3p$	0
437	$-n$	$2s+1$	$2s+p$	$-n$	$-C_{BA}$
438	$n+1$	$s+1$	$s+p$	$n+p$	$-(C_{BA}+C_{AB})$

Solving these equations simultaneously gives

$M_{AB}=-\frac{1}{\Delta}\left\{C_{BA}(10ns+s^2+12ps+6pn+3p^2)\right.$
 $\left.+C_{AB}(11ns+2s^2+2s+2n+17ps+5pn+3p^2+6p)\right\}$. (439)

$M_{BC}=-\frac{1}{\Delta}\left\{C_{BA}(11ns+2n^2+2n+2s+17pn+5ps+3p^2+6p)\right.$
 $\left.+C_{AB}(10ns+n^2+12pn+6ps+3p^2)\right\}$ (440)

$M_{CD}=\frac{1}{\Delta}\left\{C_{BA}(7ns-2n^2-2n+s-5pn+4ps-3p)\right.$
 $\left.+C_{AB}(8ns-n^2+3n-3pn+6ps+3p)\right\}$ (441)

$$M_{DA} = \frac{1}{\Delta} \left\{ C_{BA}(8ns - s^2 + 3s - 3ps + 6pn + 3p) \right. \\ \left. + C_{AB}(7ns - 2s^2 - 2s + n - 5ps + 4pn - 3p) \right\} \quad . \quad . \quad . \quad . \quad (442)$$

in which

$$\Delta = 22(sp n + sp + sn + np) + 2(sp^2 + s^2 p + n^2 p + np^2 + s^2 + s + n^2 + n) \\ + 6(sn^2 + s^2 n + p^2 + p)$$

If the load is symmetrical about the center of AB , that is, if $C_{AB} = C_{BA}$, equations 439 to 442 take the form

$$M_{AB} = -\frac{C_{AB}}{\Delta} \left[21ns + 3s^2 + 2s + 2n + 29ps + 11pn + 6p + 6p^2 \right] \quad (443)$$

$$M_{BC} = -\frac{C_{AB}}{\Delta} \left[21ns + 3n^2 + 2n + 2s + 29pn + 11ps + 6p + 6p^2 \right] \quad (444)$$

$$M_{CD} = \frac{C_{AB}}{\Delta} \left[15ns - 3n^2 + n + s - 8pn + 10ps \right] \quad . \quad . \quad . \quad . \quad (445)$$

$$M_{DA} = \frac{C_{AB}}{\Delta} \left[15ns - 3s^2 + s + n - 8ps + 10pn \right] \quad . \quad . \quad . \quad . \quad (446)$$

If $n = s$ equations 439 to 442 take the form

$$M_{AB} = -\frac{1}{2} \left\{ C_{BA} \left[\frac{2n+3p}{\alpha} - \frac{1}{\beta} \right] + C_{DA} \left[\frac{2n+3p}{\alpha} + \frac{1}{\beta} \right] \right\} \quad . \quad (447)$$

$$M_{BC} = -\frac{1}{2} \left\{ C_{BA} \left[\frac{2n+3p}{\alpha} + \frac{1}{\beta} \right] + C_{DA} \left[\frac{2n+3p}{\alpha} - \frac{1}{\beta} \right] \right\} \quad . \quad (448)$$

$$M_{CD} = \frac{1}{2} \left\{ C_{BA} \left[\frac{n}{\alpha} - \frac{1}{\beta} \right] + C_{DA} \left[\frac{n}{\alpha} + \frac{1}{\beta} \right] \right\} \quad . \quad . \quad . \quad . \quad (449)$$

$$M_{DA} = \frac{1}{2} \left\{ C_{BA} \left[\frac{n}{\alpha} + \frac{1}{\beta} \right] + C_{DA} \left[\frac{n}{\alpha} - \frac{1}{\beta} \right] \right\} \quad . \quad . \quad . \quad . \quad (450)$$

in which $\alpha = n^2 + 2pn + 2n + 3p$

$$\beta = 6n + p + 1$$

If $n=s$ and $C_{AB}=C_{BA}$

$$M_{AB}=M_{BC}=-C_{AB}\frac{2n+3p}{\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (451)$$

$$M_{CD}=M_{DA}=C_{AB}\frac{n}{\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (452)$$

If $n=s$ and $p=1$

$$M_{AB}=-\frac{1}{2}\left\{C_{BA}\left[\frac{2n+3}{\alpha}-\frac{1}{\beta}\right]+C_{AB}\left[\frac{2n+3}{\alpha}+\frac{1}{\beta}\right]\right\} \quad (453)$$

$$M_{BC}=-\frac{1}{2}\left\{C_{BA}\left[\frac{2n+3}{\alpha}+\frac{1}{\beta}\right]+C_{AB}\left[\frac{2n+3}{\alpha}-\frac{1}{\beta}\right]\right\} \quad (454)$$

$$M_{CD}=\frac{1}{2}\left\{C_{BA}\left[\frac{n}{\alpha}-\frac{1}{\beta}\right]+C_{AB}\left[\frac{n}{\alpha}+\frac{1}{\beta}\right]\right\} \quad . \quad . \quad . \quad (455)$$

$$M_{DA}=\frac{1}{2}\left\{C_{BA}\left[\frac{n}{\alpha}+\frac{1}{\beta}\right]+C_{AB}\left[\frac{n}{\alpha}-\frac{1}{\beta}\right]\right\} \quad . \quad . \quad . \quad (456)$$

in which

$$\alpha=(n+1)(n+3)$$

$$\beta=2(3n+1)$$

If $n=s$, $p=1$, and $C_{AB}=C_{BA}$

$$M_{AB}=M_{BC}=-C_{AB}\frac{2n+3}{(n+1)(n+3)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (457)$$

$$M_{CD}=M_{DA}=C_{AB}\frac{n}{(n+1)(n+3)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (458)$$

If $n=s=p=1$

$$M_{AB}=-\frac{1}{8}\left[2C_{BA}+3C_{AB}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (459)$$

$$M_{BC}=-\frac{1}{8}\left[3C_{BA}+2C_{AB}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (460)$$

$$M_{CD} = \frac{1}{8} C_{AB} (461)$$

[illegible]

If $n=s=p=1$, and $C_{AB}=C_{BA}$

[illegible]

$$M_{CD} = M_{DA} = \frac{1}{8} C_{AB} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (464)$$

Values of C_{AB} and C_{BA} to be used in equations 439 to 464 are given in Table 2.

51. *Rectangular Frame. External Moment at Upper Corner.*—

Fig. 56 represents a rectangular frame subjected to an external moment M at the upper left-hand corner.

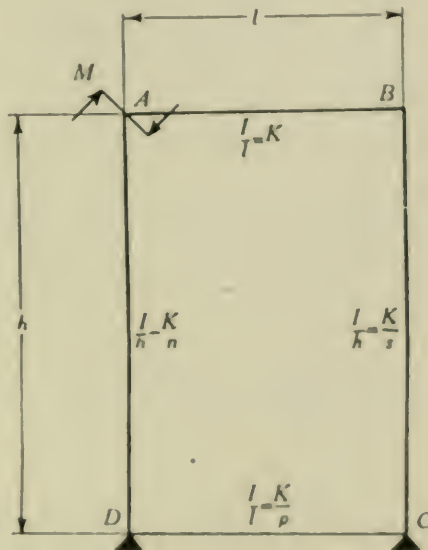


FIGURE 56

For equilibrium at A

$$M - M_{AB} - M_{AD} = 0 \text{ or}$$

$$M_{AB} = M - M_{AD}$$

From the conditions for equilibrium at D and from the relation between M_{DA} and M_{AD} it can be proved that

$$M_{DA} = \frac{M_{AD}}{2} \left[\frac{3K_{DK} + 3K_{DJ} + 2K_{DC}}{3K_{DK} + 3K_{DJ} + 2K_{DC} + 3K_{AD}} \right] \quad (520)$$

also

$$M_{IA} = \frac{M_{AI}}{2} \left[\frac{3K_{IK} + 3K_{IN} + 3K_{IL}}{3K_{IK} + 3K_{IN} + 3K_{IL} + 3K_{AI}} \right] \quad (521)$$

and

$$M_{HA} = \frac{M_{AH}}{2} \left[\frac{3K_{HN} + 3K_{HO} + 2K_{HG}}{3K_{HN} + 3K_{HO} + 2K_{HG} + 3K_{AH}} \right] \quad (522)$$

M_{DA} is made up of the three moments M_{DK} , M_{DJ} , and M_{DC} . These moments are as follows:

$$M_{DK} = -\frac{M_{AD}}{2} \left[\frac{3K_{DK}}{3K_{DK} + 3K_{DJ} + 2K_{DC} + 3K_{AD}} \right] \quad (523)$$

$$M_{DJ} = -\frac{M_{AD}}{2} \left[\frac{3K_{DJ}}{3K_{DK} + 3K_{DJ} + 2K_{DC} + 3K_{AD}} \right] \quad (524)$$

$$M_{DC} = -\frac{M_{AD}}{2} \left[\frac{2K_{DC}}{3K_{DK} + 3K_{DJ} + 2K_{DC} + 3K_{AD}} \right] \quad (525)$$

In a similar manner M_{IA} can be divided into M_{IK} , M_{IN} , and M_{IL} , and M_{HA} can be divided into M_{HN} , M_{HO} , and M_{HG} .

55. *Extremities of Members Fixed.*—If the members are fixed at O , N , L , K , and J :

From equation 506, section 52,

$$N_{AD} = \frac{4(2K_{DC} + 4K_{DJ} + 4K_{DK} + 3K_{AD})}{2K_{DC} + 4K_{DJ} + 4K_{DK} + 4K_{AD}} \quad (526)$$

$$N_{AH} = \frac{4(2K_{HG} + 4K_{HN} + 4K_{HO} + 3K_{AH})}{2K_{HG} + 4K_{HN} + 4K_{HO} + 4K_{AH}} \quad (527)$$

load are practically independent of the degree of restraint at the extremities O, N, L, K, J , etc. Therefore, the moments in the frame of Fig. 61, due to the load on AB , are given with sufficient accuracy for purposes of design by equations 513, 517, 518, 519, 520, 521, and 522, based upon the assumption that the extremities O, N, L, K, J , etc., are hinged. The equations based upon the assumption that the extremities are fixed give almost exactly the same results and could also be used.

TABLE 15

MOMENTS IN FRAME REPRESENTED BY FIG. 61

Moments are expressed in terms of $\frac{F}{l}$

Moment	Extremities of Members Hinged at O, N, L, K , and J		Extremities of Members Fixed at O, N, L, K , and J	
	All K 's Equal	K 's of Columns Equal 10 Times K 's of Girders	All K 's Equal	K 's of Columns Equal 10 Times K 's of Girders
M_{AB}	-.845	-.972	-.849	-.973
M_{AD}	+.281	+.462	+.282	+.462
M_{AH}	+.281	+.462	+.282	+.462
M_{AI}	+.283	+.048	+.285	+.049
M_{DA}	+.102	+.125	+.108	+.140
M_{HA}	+.102	+.125	+.108	+.140
M_{IA}	+.106	+.023	+.114	+.024
M_{DK}	-.038	-.011	-.043	-.012
M_{DJ}	-.038	-.107	-.043	-.122
M_{DC}	-.026	-.007	-.022	-.006
M_{HN}	-.038	-.011	-.043	-.012
M_{HO}	-.038	-.017	-.043	-.122
M_{HG}	-.026	-.007	-.022	-.006
M_{IK}	-.035	-.011	-.038	-.011
M_{IN}	-.035	-.011	-.038	-.011
M_{IL}	-.035	-.001	-.038	-.002

56. *Distribution of Loads for Maximum Moments in a Frame Composed of a Large Number of Rectangles.*—Referring to Fig. 61, a load on AB produces a moment M_{KD} having the same sign as M_{AB} . That being the case, a load on GF produces a moment M_{AB} of the same sign as the M_{AB} produced by the load on AB ; therefore if AB and GF are loaded simultaneously the moment at A in AB is greater than if either AB or GF is loaded alone. Reasoning in a similar manner, the members can be selected which, if loaded, produce a moment at A in

the member AB having the same sign as the moment at the same point due to a load on AB . If all these members are loaded simultaneously, the moment at A in the member AB is a maximum.

W_5	W_{15}	W_{25}	W_{35}	W_{45}
W_4	W_{14}	W_{24}	W_{34}	W_{44}
W_3	W_{13}	W_{23}	W_{33}	W_{43}
W_2	W_{12}	A W_{22}	B W_{32}	W_{42}
W_1	W_{11}	D W_{21}	W_{31}	W_{41}

FIGURE 62

Fig. 62 represents a frame made up of similar rectangles. All girders are equally loaded. The moments M_{AB} and M_{AD} , due to these loads, as determined by the equations of section 53 are given in Table 16. For the frame of Table 16 the K 's of all members are equal. The moments in similar frames for which the K 's of all columns are equal and the K 's of all girders are equal, but for which the K 's of the columns do not equal the K 's of the girders, have been determined. The relation between the maximum moments which it is possible to obtain in the girders, and the ratio of the K 's of the columns to the K 's of the girders, is presented graphically in Fig. 63. Similar data for the moment in the columns are given in Fig. 64.

Fig. 65 represents the loading which produces a maximum moment at A in the girder. Fig. 66 represents the loading which produces a maximum moment at A in the column.

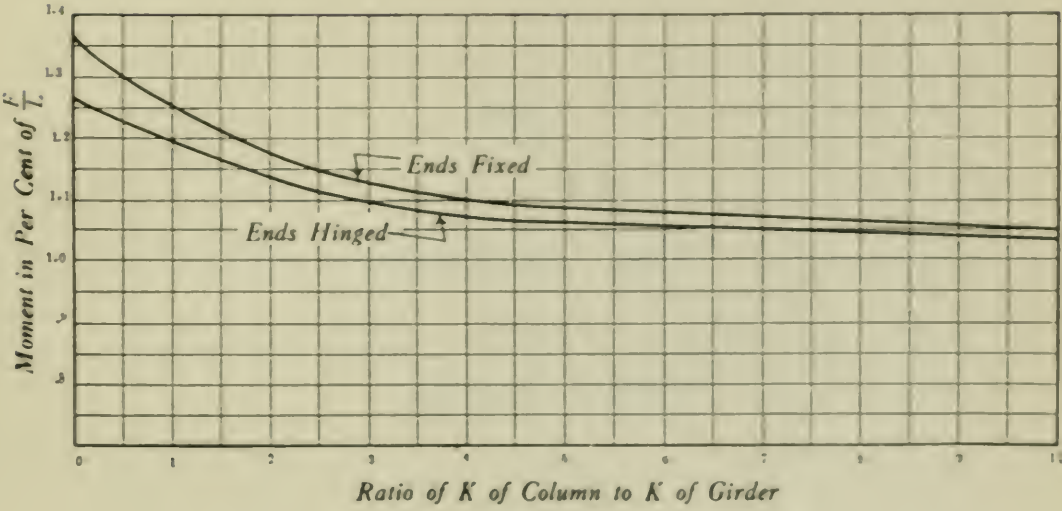


FIGURE 63

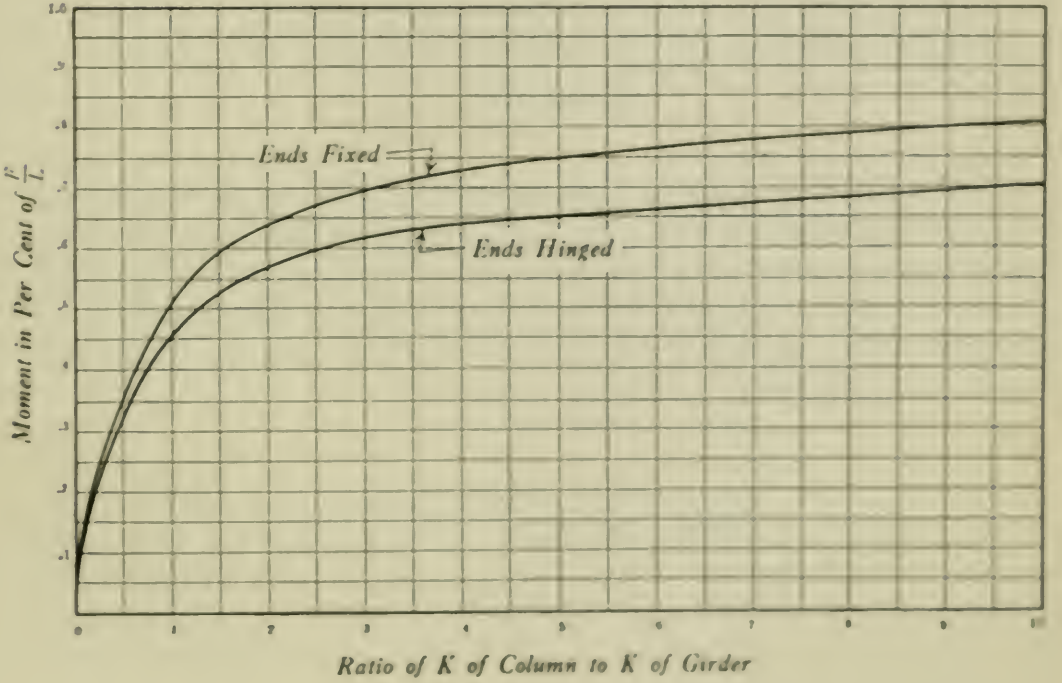


FIGURE 64

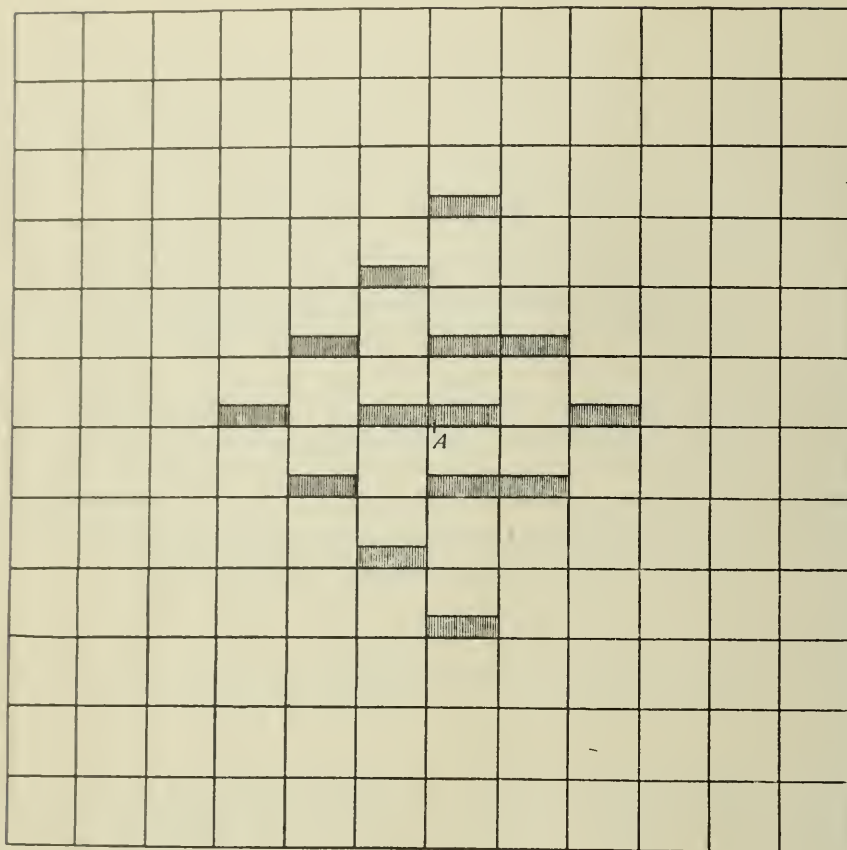


FIGURE 65

It is to be noted that for the frames of sections 52, 53, and 56 the horizontal deflection of one story of the frame relative to the other stories does not enter. If either the frame or the load is unsymmetrical there is a slight horizontal deflection. For the usual proportions of frames of engineering structures, the effect of this horizontal deflection is slightly to reduce the moments.

57. *Eccentric Load at Top of Exterior Column of a Frame. Connections of Girders to Columns Hinged.*—Fig. 67 represents a frame with eccentric loads at the tops of the exterior columns. The frame and the loading are symmetrical about the vertical center line of the frame. The connections of the girders to the columns are frictionless hinges. The columns are continuous.

The moment in the column depends upon the restraint at 1. The degree of restraint at 1 is known to be between the restraint of a column hinged at 1 and a column fixed at 1. If, therefore, the moment is deter-

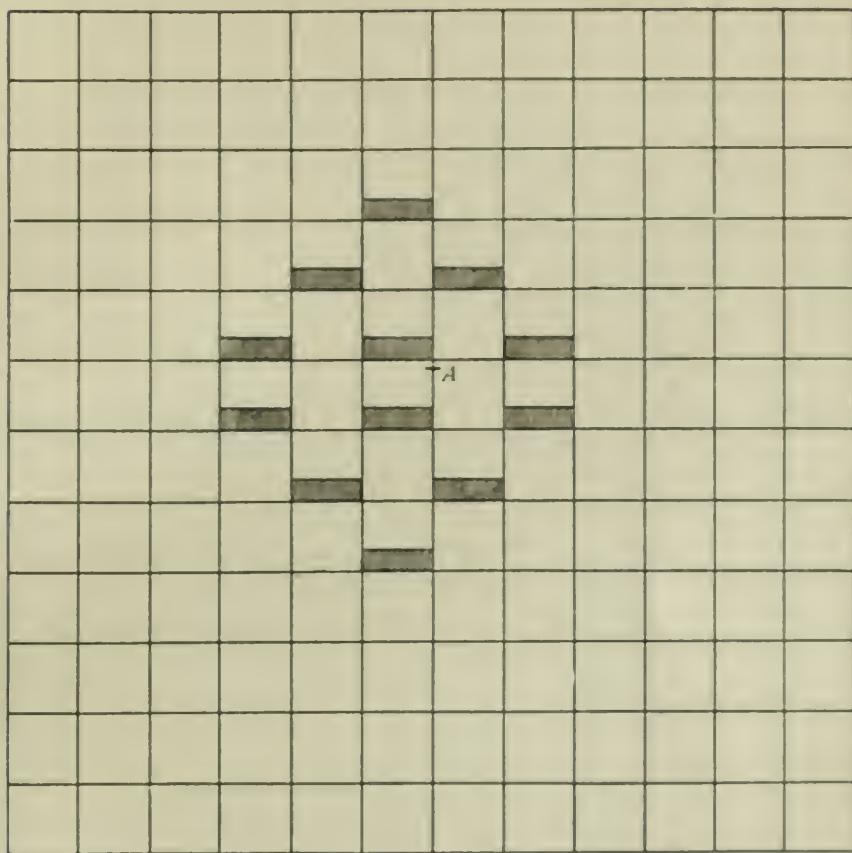


FIGURE 66

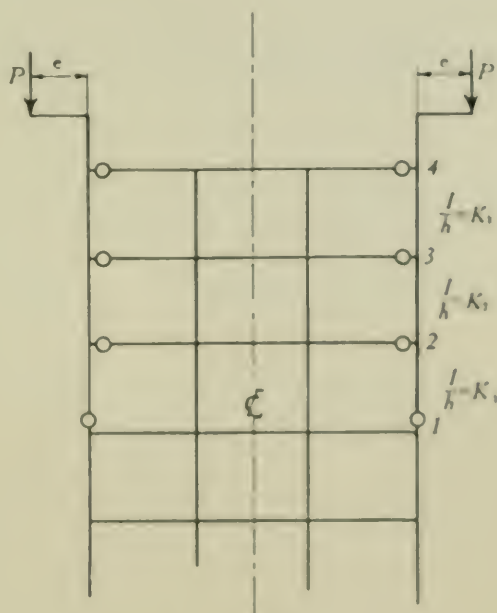


FIGURE 67

TABLE 16
MOMENTS IN FRAME REPRESENTED BY FIG. 62
K's of All Members Equal

Moment Produced by	M_{AB}		M_{AD}	
	Hinged	Fixed	Hinged	Fixed
W2			0	+ .019
W3	+ .035	+ .038	+ .035	+ .038
W11			0	+ .022
W12	+ .038	+ .043	- .102	- .108
W13	- .283	- .285	- .281	- .282
W14	+ .038	+ .043	+ .038	+ .043
W21			0	- .022
W22	- .026	- .022	+ .102	+ .108
W23	- .845	- .849	+ .281	+ .282
W24	- .026	- .022	- .038	- .043
W32	0	- .022	0	- .019
W33	+ .106	+ .114	- .035	- .038
W34	0	- .022		
W43	0	- .019		
Total	- .963	- 1.003	0	0
Maximum	- 1.180	- 1.241	$\pm .456$	$\pm .512$

W's not given in table produce only very small moments.

mined for a column hinged at 1 and for a column fixed at 1 the true moment will be located between two limits.

Consider the column to be hinged at 1

$M_{12}=0$ (532)

$M_{21}=3EK_1\theta_2$ (533)

$$M_{AB} = \frac{Pe N_1 K_1}{N_1 K_1 + N_2 K_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (559)$$

[illegible]

Also

$$M_{BA} = \frac{M_{AB}}{2} \left(\frac{3K_5 + N_2 K_2}{3K_5 + N_2 K_2 + 3K_1} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (561)$$

$$M_{BG} = -\frac{M_{AB}}{2} \left(\frac{3K_5}{3K_5 + N_2 K_2 + 3K_1} \right) \quad (562)$$

$$M_{BC} = -\frac{M_{AB}}{2} \left(\frac{N_2 K_2}{3K_1 + N_2 K_2 + 3K_3} \right) \quad \dots \quad (563)$$

$$M_{CB} = \frac{M_{BC}}{2} \left(\frac{K_3 + K_6}{K_1 + K_2 + K_3} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (564)$$

$$M_{CH} = - \frac{M_{BC}}{2} \left(\frac{K_6}{\frac{K_1}{2} + \frac{K_2}{2} + \frac{K_3}{2} + \frac{K_4}{2} + \frac{K_5}{2} + \frac{K_6}{2}} \right) \quad (565)$$

$$M_{\text{max}} = - \frac{M_{BC}}{2} \left(\frac{K_3}{K_1 + K_2 + K_3} \right) \quad (566)$$

$$M_{22} = \frac{M_{AF}}{2} \left(\frac{3K_9 + N_7 K_7}{2} \right) \quad (567)$$

$$M_{AF} (N_7 K_7) \quad (568)$$

$$M_{AF}(\quad 3K_9 \quad) \quad (560)$$

If the $\frac{I}{l}$'s of the girders are all equal and are represented by K , if the $\frac{I}{l}$'s of the columns are all equal and are represented by $\frac{K}{n}$, and if the l 's are all equal, $N_2 = N_7 = N = 4 \left(\frac{3n+6}{3n+7} \right)$, and equations 559 to 569 reduce to the form

$$M_{AB}=Pc\left[\frac{\left(\frac{3n+3+N}{3n+4+N}\right)}{\left(\frac{3n+3+N}{3n+4+N}\right)+n\left(\frac{6n+N}{7n+N}\right)}\right] \dots\dots\dots (570)$$

$$M_{AF}=Pc\left[\frac{n\left(\frac{6n+N}{7n+N}\right)}{\left(\frac{3n+3+N}{3n+4+N}\right)+n\left(\frac{6n+N}{7n+N}\right)}\right] \dots\dots\dots (571)$$

$$M_{BA}=\frac{M_{AB}}{2}\left(\frac{3n+N}{3n+3+N}\right) \dots\dots\dots (572)$$

$$M_{BG}=-\frac{M_{AB}}{2}\left(\frac{3n}{3n+3+N}\right) \dots\dots\dots (573)$$

$$M_{BC}=-\frac{M_{AB}}{2}\left(\frac{N}{3n+3+N}\right) \dots\dots\dots (574)$$

$$M_{CB}=\frac{M_{BC}}{2}\left(\frac{n+1}{n+2}\right) \dots\dots\dots (575)$$

$$M_{CH}=-\frac{M_{BC}}{2}\left(\frac{n}{n+2}\right) \dots\dots\dots (576)$$

$$M_{CD}=-\frac{M_{BC}}{2}\left(\frac{1}{n+2}\right) \dots\dots\dots (577)$$

$$M_{FA}=\frac{M_{AF}}{2}\left(\frac{3n+N}{6n+N}\right) \dots\dots\dots (578)$$

$$M_{FG}=-\frac{M_{AF}}{2}\left(\frac{N}{6n+N}\right) \dots\dots\dots (579)$$

$$M_{FJ}=-\frac{M_{AF}}{2}\left(\frac{3n}{6n+N}\right) \dots\dots\dots (580)$$

If all the K 's are equal

[illegible]

[illegible]

[illegible]

$$M_{BG} = -.078 \text{ } Pe (584)$$

[illegible]

$$M_{CB} = -.031 \text{ } Pc (586)$$

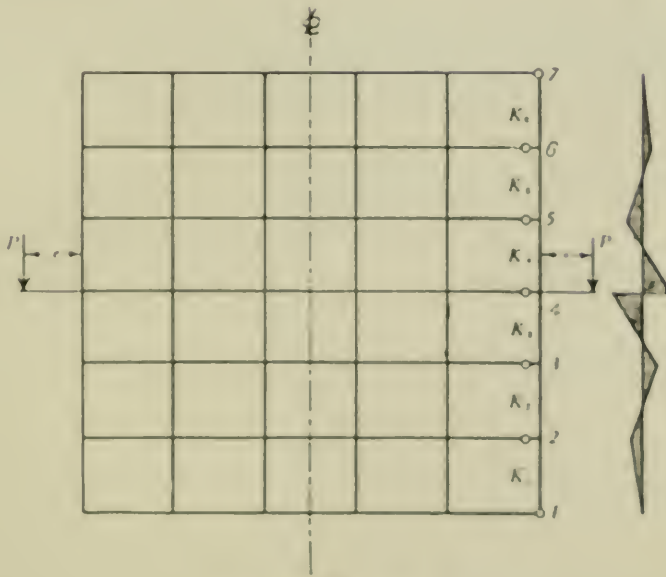
[illegible]

[illegible]

[illegible]

$$M_{FG} = -.094 Pe \quad (590)$$

[illegible]



FIGURES 69 AND 70

59. *Eccentric Load at Middle Floor Level of Exterior Column of a Frame. Connections of Girders to Columns Hinged.*—Fig. 69 represents a frame with eccentric loads at the middle floor level of the exterior columns. The frame and the loading are symmetrical about the

It is to be noted that the frame considered is symmetrical about a vertical center line and is symmetrically loaded. If there is a load on the right-hand column only, the moments in that column will be slightly smaller than the moments given by the equation, and the other columns will be subjected to a small moment. The error in the moment in the loaded column and the neglected moment in the other columns increase as the ratio of the stiffness of the loaded column to the combined stiffness of the other columns increases.

As in section 57, the error in the equations of this section due to the horizontal deflection of a frame under any vertical loading is undoubtedly negligible for purposes of design.

60. *Eccentric Load at Middle Floor Level of Exterior Column of a Frame. Connections of Girders to Columns Rigid.*—Fig. 72 represents

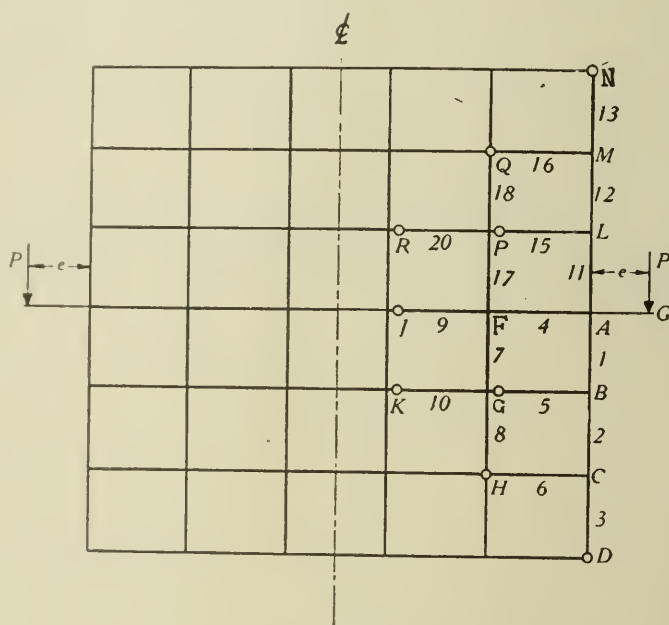


FIGURE 72

a frame with eccentric loads at the middle floor level of the exterior columns. The frame and the loading are symmetrical about the vertical center line of the frame. The connections of the girders to the columns are rigid.

From previous similar work it is known that of the moments produced by P on the right-hand side of the frame, only the moments at A , B , C , L , M , and F are large enough to be considered in the design of the frame. From previous work it is known, furthermore, that the

moments at these points are practically independent of the degree of restraint at D, H, G, K, J, R, P, Q , and N . It will therefore be considered that the girders are hinged at H, G, K, J, R, P , and Q , and that the columns are hinged at D, H, Q , and N .

Applying the general method used in section 58, it can be proved

$$\text{that letting } N_1 = 4 \left[\frac{3K_5 + N_2K_2 + 3K_1}{3K_5 + N_2K_2 + 4K_1} \right]$$

$$N_4 = 4 \left(\frac{3K_9 + N_7K_7 + N_{17}K_{17} + 3K_4}{3K_9 + N_7K_7 + N_{17}K_{17} + 4K_4} \right)$$

$$\text{and } N_{11} = 4 \left(\frac{3K_{15} + N_{12}K_{12} + 3K_{11}}{3K_{15} + N_{12}K_{12} + 4K_{11}} \right) \quad \text{gives}$$

$$M_{AB} = \frac{PeN_1K_1}{N_1K_1 + N_4K_4 + N_{11}K_{11}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (605)$$

$$M_{AF} = \frac{PeN_4K_4}{N_1K_1 + N_4K_4 + N_{11}K_{11}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (606)$$

$$M_{AL} = \frac{PeN_{11}K_{11}}{N_1K_1 + N_4K_4 + N_{11}K_{11}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (607)$$

$$M_{LA} = \frac{M_{AL}}{2} \left(\frac{3K_{15} + N_{12}K_{12}}{3K_{15} + N_{12}K_{12} + 3K_{11}} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (608)$$

$$M_{LP} = - \frac{M_{AL}}{2} \left(\frac{3K_{15}}{3K_{15} + N_{12}K_{12} + 3K_{11}} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (609)$$

$$M_{LM} = - \frac{M_{AL}}{2} \left(\frac{N_{12}K_{12}}{3K_{15} + N_{12}K_{12} + 3K_{11}} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (610)$$

$$M_{ML} = \frac{M_{LM}}{2} \left(\frac{K_{13} + K_{16}}{K_{13} + K_{16} + K_{12}} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (611)$$

$$M_{MQ} = - \frac{M_{LM}}{2} \left(\frac{K_{16}}{K_{13} + K_{16} + K_{12}} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (612)$$

$$M_{MN} = - \frac{M_{LM}}{2} \left(\frac{K_{13}}{K_{13} + K_{16} + K_{12}} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (613)$$

$$M_{BA} = \frac{M_{AB}}{2} \left(\frac{3K_5 + N_2K_2}{3K_5 + N_2K_2 + 3K_1} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (614)$$

$$M_{BG} = - \frac{M_{AB}}{2} \left(\frac{3K_5}{3K_5 + N_2K_2 + 3K_1} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (615)$$

$$M_{BC} = - \frac{M_{AB}}{2} \left(\frac{N_2K_2}{3K_5 + N_2K_2 + 3K_1} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (616)$$

$$M_{CB} = \frac{M_{BC}}{2} \left(\frac{K_3 + K_6}{K_3 + K_6 + K_2} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (617)$$

$$M_{CH} = - \frac{M_{BC}}{2} \left(\frac{K_6}{K_3 + K_6 + K_2} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (618)$$

$$M_{CD} = - \frac{M_{BC}}{2} \left(\frac{K_3}{K_3 + K_6 + K_2} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (619)$$

$$M_{FA} = \frac{1}{2} M_{AF} \left(\frac{3K_9 + N_7K_7 + N_{17}K_{17}}{3K_9 + N_7K_7 + N_{17}K_{17} + 3K_4} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (620)$$

$$M_{FG} = - \frac{1}{2} M_{AF} \left(\frac{N_7K_7}{3K_9 + N_7K_7 + N_{17}K_{17} + 3K_4} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (621)$$

$$M_{FJ} = - \frac{1}{2} M_{AF} \left(\frac{3K_9}{3K_9 + N_7K_7 + N_{17}K_{17} + 3K_4} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (622)$$

$$M_{FP} = - \frac{1}{2} M_{AF} \left(\frac{N_{17}K_{17}}{3K_9 + N_7K_7 + N_{17}K_{17} + 3K_4} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (623)$$

in which

$$N_2 = 4 \left(\frac{3K_3 + 3K_6 + 3K_2}{3K_3 + 3K_6 + 4K_2} \right)$$

$$N_7 = 4 \left(\frac{3K_8 + 3K_{10} + 3K_7}{3K_8 + 3K_{10} + 4K_7} \right)$$

$$N_{12} = 4 \left(\frac{3K_{13} + 3K_{16} + 3K_{12}}{3K_{13} + 3K_{16} + 4K_{12}} \right)$$

$$N_{17} = 4 \left(\frac{3K_{18} + 3K_{20} + 3K_{17}}{3K_{18} + 3K_{20} + 4K_{17}} \right)$$

If the $\frac{I}{l}$'s of all girders are equal and are represented by K , and if the $\frac{I}{l}$'s of all columns are equal and are represented by $\frac{K}{n}$,

$$N_2 = N_7 = N_{12} = N_{17} = N = 4 \left(\frac{3n+6}{3n+7} \right),$$

and equations 605 to 623 reduce to the form

$$M_{AB} = M_{AL} = \frac{Pe}{2} \left[\frac{\left(\frac{3n+3+N}{3n+4+N} \right)}{\left(\frac{3n+3+N}{3n+4+N} \right) + n \left(\frac{3n+N}{7n+2N} \right)} \right] \quad (624)$$

$$M_{AF} = \frac{nPe}{2} \left[\frac{\left(\frac{6n+2N}{7n+2N} \right)}{\left(\frac{3n+3+N}{3n+4+N} \right) + n \left(\frac{3n+N}{7n+2N} \right)} \right] \quad (625)$$

$$M_{BA} = M_{LA} = \frac{M_{AB}}{2} \left(\frac{3n+N}{3n+3+N} \right) \quad (626)$$

$$M_{BG} = M_{LP} = -\frac{M_{AB}}{2} \left(\frac{3n}{3n+3+N} \right) \quad (627)$$

$$M_{BC} = M_{LM} = -\frac{M_{AB}}{2} \left(\frac{N}{3n+3+N} \right) \quad (628)$$

$$M_{CB} = M_{ML} = \frac{M_{BC}}{2} \left(\frac{n+1}{n+2} \right) \quad (629)$$

$$M_{CH} = M_{MQ} = -\frac{M_{BC}}{2} \left(\frac{n}{n+2} \right) \quad (630)$$

$$M_{CD} = M_{MN} = -\frac{M_{BC}}{2} \left(\frac{1}{n+2} \right) \quad (631)$$

$$M_{FA} = \frac{M_{AF}}{2} \left(\frac{3n+2N}{6n+2N} \right) \quad (632)$$

$$M_{AL} = M_{AB} = \frac{Pe}{2} \frac{\left[\frac{4.5n+3+N\left(\frac{6n+13}{4n+8}\right)}{3n+4+N} \right]}{\left(\frac{3n+3+N}{3n+4+N}\right) + n\left(\frac{3n+N}{7n+2N}\right)} \quad \dots \quad (650)$$

M_{AF} is maximum when alternate floors are loaded. For such loading

$$M_{AF} = n \frac{Pe}{2} \frac{\left(\frac{6n+2N}{7n+2N}\right) + \left[\frac{\left(\frac{N}{2n+4}\right)}{3n+4+N} \right]}{\left(\frac{3n+3+N}{3n+4+N}\right) + n\left(\frac{3n+N}{7n+2N}\right)} \quad \dots \quad (651)$$

If the K 's are all equal and the values of Pe are all equal, the maximum moments are

$$M_{AB} = M_{AL} = 0.454 Pe \quad \dots \quad (652)$$

$$M_{AF} = 0.358 Pe \quad \dots \quad (653)$$

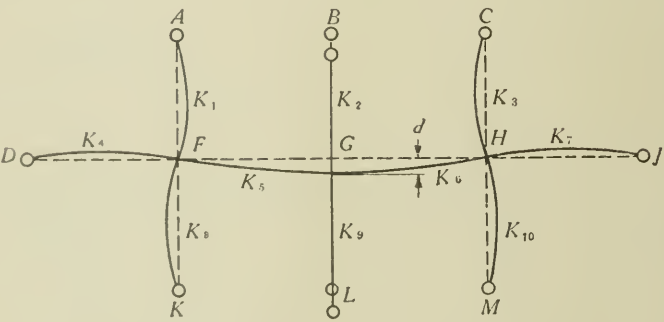


FIGURE 74

61. *Effect of Settlement of One Column of a Frame Composed of a Number of Rectangles.*—Fig. 74 represents a portion of a frame composed of a number of rectangles. The unstrained outline of the frame is represented by broken lines. The middle column BGL settles an amount represented by d . The strained outline of the frame is represented by the full lines. The points D, A, C, J, M, K, F , and H remain stationary.

For F to be in equilibrium

$$M_{FK} + M_{FD} + M_{FA} + M_{FG} = 0 \quad \dots \quad (654)$$

Substituting the values of the M 's as given by the equations of Table 1 gives

$$3E\theta_F\left(K_5+K_4+K_1\right)+2EK_5\left(2\theta_F+\theta_G-3\frac{d}{l_5}\right)=0 \quad . \quad . \quad . \quad (655)$$

or

$$\theta_F=\frac{6K_5\frac{d}{l_5}-2K_5\theta_G}{3K_4+3K_1+3K_5+4K_5} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (656)$$

$$M_{GF}=2EK_5\left(2\theta_G+\theta_F-3\frac{d}{l_5}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (657)$$

Substituting the value of θ_F from equation 656 in equation 657 gives

$$M_{GF}=EK_5\left[4\theta_G\left(\frac{3K_4+3K_1+3K_5+3K_5}{3K_4+3K_1+3K_5+4K_5}\right)-\frac{6d}{l_5}\left(\frac{3K_4+3K_1+3K_5+2K_5}{3K_4+3K_1+3K_5+4K_5}\right)\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (658)$$

Similarly for the right-hand portion of the structure

$$M_{GH}=EK_6\left[4\theta_G\left(\frac{3K_7+3K_3+3K_{10}+3K_6}{3K_7+3K_3+3K_{10}+4K_6}\right)+\frac{6d}{l_6}\left(\frac{3K_7+3K_3+3K_{10}+2K_6}{3K_7+3K_3+3K_{10}+4K_6}\right)\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (659)$$

$$M_{GB}=3EK_2\theta_G \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (660)$$

$$M_{GL}=3EK_9\theta_G \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (661)$$

For G to be in equilibrium

$$M_{GF}+M_{GH}+M_{GB}+M_{GL}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (662)$$

Substituting the value of the M 's in equation 662 gives

$$\theta_G = \frac{6d \left[\frac{K_5}{l_5} \left(\frac{3K_4 + 3K_1 + 3K_8 + 2K_5}{3K_4 + 3K_1 + 3K_8 + 4K_5} \right) - \frac{K_6}{l_6} \left(\frac{3K_7 + 3K_3 + 3K_{10} + 2K_6}{3K_7 + 3K_3 + 3K_{10} + 4K_6} \right) \right]}{4K_5 \left[\frac{3K_4 + 3K_1 + 3K_8 + 3K_5}{3K_4 + 3K_1 + 3K_8 + 4K_5} \right] + 4K_6 \left[\frac{3K_7 + 3K_3 + 3K_{10} + 3K_6}{3K_7 + 3K_3 + 3K_{10} + 4K_6} \right] + 3K_2 + 3K_9} \quad (663)$$

From equation 656

$$\theta_F = \frac{6d \frac{K_5}{l_5} - 2K_5 \theta_G}{3K_4 + 3K_1 + 3K_8 + 4K_5} \quad (664)$$

Also

$$\theta_H = - \frac{6d \frac{K_6}{l_6} + 2K_6 \theta_G}{3K_7 + 3K_3 + 3K_{10} + 4K_6} \quad (665)$$

If the portion of the frame DJ is symmetrical about G , $\theta_G = 0$, and $\theta_F = -\theta_H$.

With θ_G , θ_F , and θ_H known the moments can be determined from the equations:

$$M_{FA} = 3EK_1 \theta_F \quad (666)$$

$$M_{FD} = 3EK_4 \theta_F \quad (667)$$

$$M_{FK} = 3EK_8 \theta_F \quad (668)$$

$$M_{FG} = 2EK_5 \left(2\theta_F + \theta_G - 3 \frac{d}{l_5} \right) \quad (669)$$

$$M_{GF} = 2EK_5 \left(2\theta_G + \theta_F - 3 \frac{d}{l_5} \right) \quad (670)$$

$$M_{GB} = 3EK_2 \theta_G \quad (671)$$

$$M_{GL} = 3EK_9 \theta_G \quad (672)$$

$$M_{GH} = 2EK_6 \left(2\theta_G + \theta_H + 3 \frac{d}{l_6} \right) \quad (673)$$

TABLE 18
EQUATIONS FOR THREE-LEGGED BENT

Lengths of All Legs Different. Vertical Load on Left-hand Span. Legs Hinged at the Bases.

No. of Equation	Left-hand Member of Equation				Right-hand Member of Equation
	θ_A	θ_B	θ_F	d	
A	$3K_o+4K_1$	$2K_1$	0	$-\frac{3K_o}{h_o}$	$+\frac{C_{AB}}{E}$
B	$2K_1$	$4K_1+3K_2+4K_3$	$2K_3$	$-\frac{3K_2}{h_2}$	$-\frac{C_{BA}}{E}$
C	0	$2K_3$	$4K_3+3K_4$	$-\frac{3K_4}{h_4}$	0
D	$-\frac{3K_o}{h_o}$	$-\frac{3K_2}{h_2}$	$-\frac{3K_4}{h_4}$	$+3\left[\frac{K_o}{h_o^2}+\frac{K_2}{h_2^2}+\frac{K_4}{h_4^2}\right]$	0

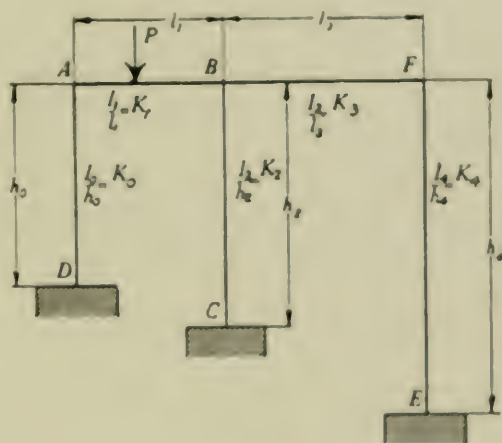


FIGURE 76

63. *Three-legged Bent. Lengths of All Legs Different. Vertical Load on Left-hand Span—Legs Fixed at the Bases.*—Fig. 76 represents a three-legged bent. The lengths of all legs are different. P represents the resultant of any system of loads on AB . The legs of the bent are fixed at D , C , and E .

contain no unknown quantities not found in the equations of section 62, and therefore they can be combined to obtain equations similar to the equations of Tables 18 and 19. The left-hand members of the equations for all bents with legs hinged at the bases will be identical with the left-hand members of the equations of Table 18, and the left-hand members of the equations for all bents with legs fixed at the bases will be identical with the left-hand members of the equations of Table 19.

The equations of Table 1 as applied to a three-legged bent having legs hinged at the bases and carrying a number of systems of loads are given in Table 20. Similar equations for a three-legged bent having legs fixed at the bases are given in Table 21. Four equations containing four unknowns, derived from the equations of Table 20 are given in Table 22, and four equations containing four unknowns based upon the equations of Table 21 are given in Table 23. The equations of Table 22 can be used to determine the stresses in a bent having legs hinged at the bases, and the equations of Table 23 can be used to determine the stresses in a bent having legs fixed at the bases. A numerical problem illustrating the use of the equation in Table 23 is presented in section 76.

TABLE 19
EQUATIONS FOR THREE-LEGGED BENT

Lengths of All Legs Different. Vertical Load on Left-hand Span. Legs Fixed at the Bases.

No. of Equa- tion	Left-hand Member of Equation				Right- hand Member of Equation
	θ_A	θ_B	θ_F	d	
A	$4K_o+4K_1$	$2K_1$	0	$-6\frac{K_o}{h_o}$	$+\frac{C_{AB}}{E}$
B	$2K_1$	$4K_1+4K_2+4K_3$	$2K_3$	$-6\frac{K_2}{h_2}$	$-\frac{C_{BA}}{E}$
C	0	$2K_3$	$4K_3+4K_4$	$-6\frac{K_4}{h_4}$	0
D	$-6\frac{K_o}{h_o}$	$-6\frac{K_2}{h_2}$	$-6\frac{K_4}{h_4}$	$+12\left[\frac{K_o}{h_o^2}+\frac{K_2}{h_2^2}+\frac{K_4}{h_4^2}\right]$	0

The θ 's and d can be determined from the equations of Table 19 by a process of elimination. Knowing the θ 's and d , the moments can be determined from equations 708 to 717.

TABLE 20
FUNDAMENTAL EQUATIONS FOR THREE-LEGGED BENTS
Lengths of All Legs Different. Any System of Loads. Legs Hinged at the Bases.

No. of Equa- tion	This part of the equation is independent of the loads	Additional term of the right-hand member of the equation, different for each system of loads, as follows:					
		Case I Ver- tical Load on Left- hand Span See Fig. 77	Case II Ver- tical Load on Right- hand Span See Fig. 78	Case III Hori- zontal Load to the Right on Left- hand Leg See Fig. 79	Case IV Hori- zontal Load to the Left on Right- hand Leg See Fig. 80	Case V Settlement and Sliding of Foundations See Fig. 81.	Case VI Exter- nal Couple at Upper Left- hand Corner See Fig. 82
1	$M_{AD} = 3EK_o \left(\theta_A - \frac{d}{h_o} \right)$	0	0	$+H_{AD}$	0	0	0
2	$M_{AB} = 2EK_1(2\theta_A + \theta_B)$	$-C_{AB}$	0	0	0	$-6EK_1 \frac{d_1}{l_1}$	0
3	$M_{BA} = 2EK_1(2\theta_B + \theta_A)$	$+C_{BA}$	0	0	0	$-6EK_1 \frac{d_1}{l_1}$	0
4	$M_{BC} = 3EK_2 \left(\theta_B - \frac{d}{h_2} \right)$	0	0	0	0	$+3EK_2 \frac{d_2}{h_2}$	0

TABLE 20—CONTINUED

5	$M_{BF} = 2EK_3(2\theta_B + \theta_F)$	0	$-C_{BF}$	0	0	$-6EK_3 \frac{(d_3 - d_1)}{l_3}$	0
6	$M_{FB} = 2EK_3(2\theta_F + \theta_B)$	0	$+C_{FB}$	0	0	$-6EK_3 \frac{(d_3 - d_1)}{l_3}$	0
7	$M_{FE} = 3EK_4 \left(\theta_F - \frac{d}{h_4} \right)$	0	0	0	0	$-H_{FE} + 3EK_4 \frac{d_4}{h_4}$	0
8	$0 = M_{AD} + M_{AB}$	0	0	0	0	0	$-M$
9	$0 = M_{BA} + M_{BF} + M_{BC}$	0	0	0	0	0	0
10	$0 = M_{FB} + M_{FE}$	0	0	0	0	0	0
11	$0 = \frac{M_{AD}}{h_2} + \frac{M_{BC}}{h_2} + \frac{M_{FB}}{h_4}$	0	0	$+\frac{M_D}{h_1}$	$+\frac{M_E}{h_4}$	0	0

M_D is the statcal moment about point D of the horizontal loads on left-hand leg; M_E is the statcal moment about point E of the horizontal loads on right-hand leg. The statcal moments are positive when the indicated rotation is clockwise.

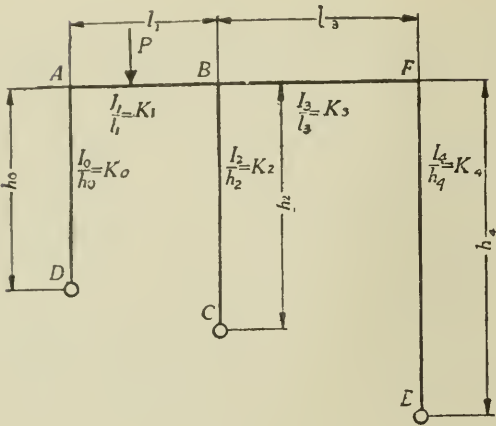


FIGURE 77

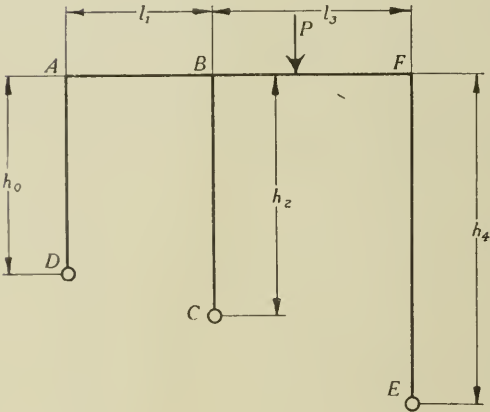


FIGURE 78

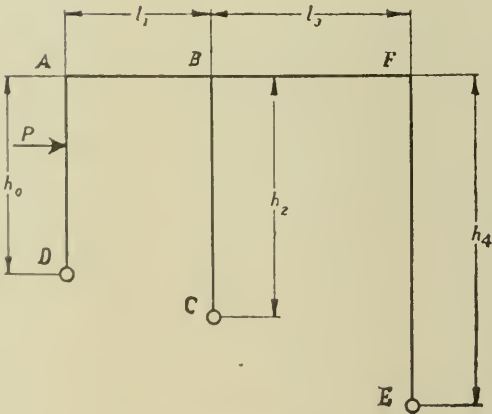


FIGURE 79

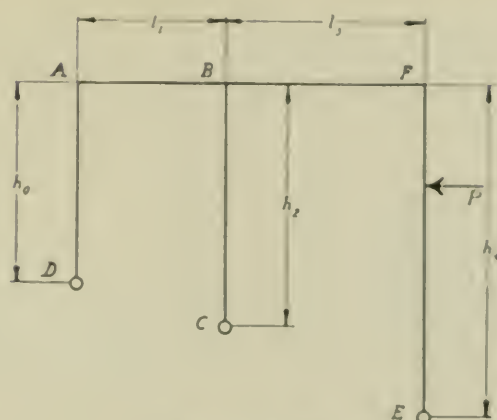


FIGURE 80

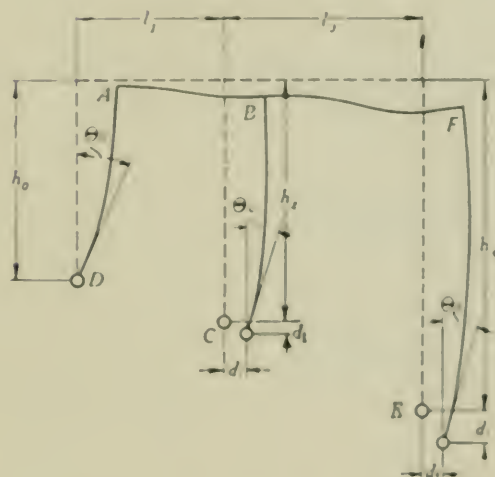


FIGURE 81

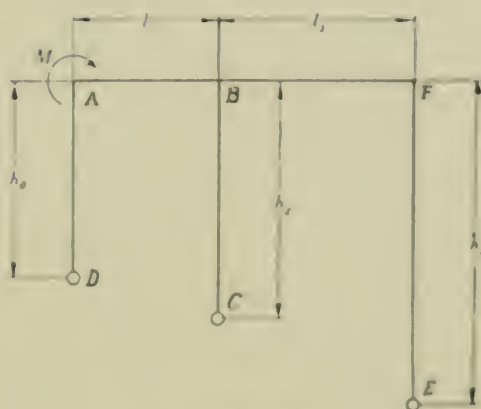


FIGURE 82

TABLE 21
FUNDAMENTAL EQUATIONS FOR THREE-LEGGED BENTS
Lengths of All Legs Different. Any System of Loads. Legs Fixed at the Bases.

No. of Equa- tion	This part of the equation is independent of the loads	Additional term of the right-hand member of the equation, different for each system of loads, as follows:					
		Case VII Vertical Load on Left- hand Span See Fig. 83	Case VIII Vertical Load on Right- hand Span See Fig. 84	Case IX Hori- zontal Load to the Right on Left- hand Leg See Fig. 85	Case X Hori- zontal Load to the Left on Right- hand Leg See Fig. 86	Case XI Settlement and Rotation of Foundations See Fig. 87	Case XII External Couple at Upper Left- hand Corner See Fig. 88
1	$M_{AD}=2EK_o\left(2\theta_A-3\frac{d}{h_o}\right)$	0	0	$+C_{AD}$	0	$+2EK_o\theta_D$	0
2	$M_{AB}=2EK_1(2\theta_A+\theta_B)$	$-C_{AB}$	0	0	0	$-6EK_1\frac{d_1}{l_1}$	0
3	$M_{BA}=2EK_1(2\theta_B+\theta_A)$	$+C_{BA}$	0	0	0	$-6EK_1\frac{d_1}{l_1}$	0
4	$M_{BC}=2EK_2\left(2\theta_B-3\frac{d}{h_2}\right)$	0	0	0	0	$+6EK_2\frac{d_2}{h_2}+2EK_2\theta_C$	0

TABLE 21—CONTINUED

5	$M_{BF} = 2EK_3(2\theta_B + \theta_F)$	0	$-C_{BF}$	0	0	$-6EK_3 \frac{d_3 - d_1}{l_3}$	0
6	$M_{FB} = 2EK_3(2\theta_F + \theta_B)$	0	$+C_{FB}$	0	0	$-6EK_3 \frac{d_3 - d_1}{l_3}$	0
7	$M_{FE} = 2EK_4 \left(2\theta_F - 3 \frac{d}{h_4} \right)$	0	0	0	$-C_{FE}$	$+6EK_4 \frac{d_4}{h_4} + 2EK_4 \theta_E$	0
8	$M_{DA} = 2EK_o \left(\theta_A - 3 \frac{d}{h_o} \right)$	0	0	$-C_{DA}$	0	$+4EK_o \theta_D$	0
9	$M_{CB} = 2EK_2 \left(\theta_B - 3 \frac{d}{h_2} \right)$	0	0	0	0	$+6EK_2 \frac{d_2}{h_2} + 4EK_2 \theta_C$	0
10	$M_{EF} = 2EK_4 \left(\theta_F - 3 \frac{d}{h_4} \right)$	0	0	0	$+C_{EF}$	$+6EK_4 \frac{d_4}{h_4} + 4EK_4 \theta_E$	0
11	$0 = M_{AD} + M_{AB}$	0	0	0	0	0	$-M$
12	$0 = M_{BA} + M_{BF} + M_{BC}$	0	0	0	0	0	0
13	$0 = M_{FB} + M_{FE}$	0	0	0	0	0	0
14	$0 = \frac{M_{AD} + M_{DA}}{h_o} + \frac{M_{BC} + M_{CB}}{h_2} + \frac{M_{FE} + M_{EF}}{h_4}$	0	0	$+\frac{M_D}{h_o}$	$+\frac{M_E}{h_4}$	0	0

For definition of M_D and M_E see Table 20.

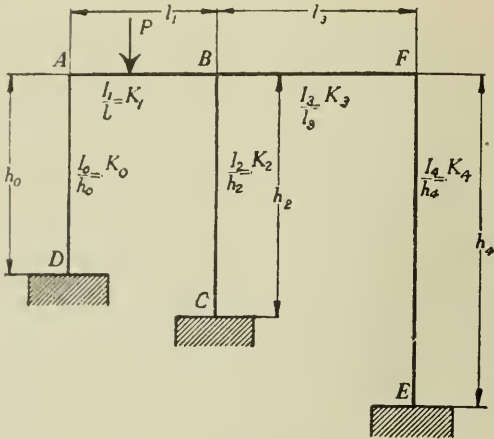


FIGURE 83

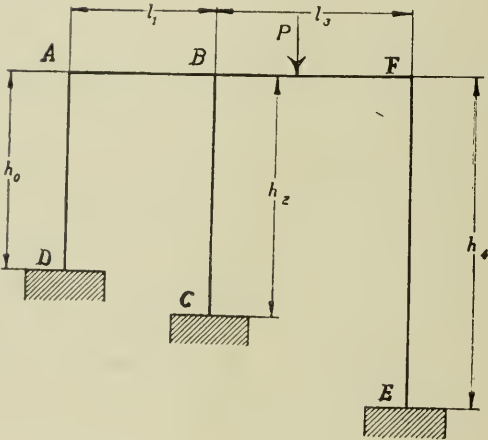


FIGURE 84

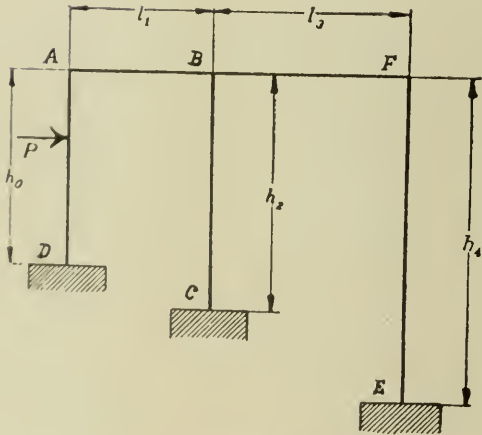


FIGURE 85

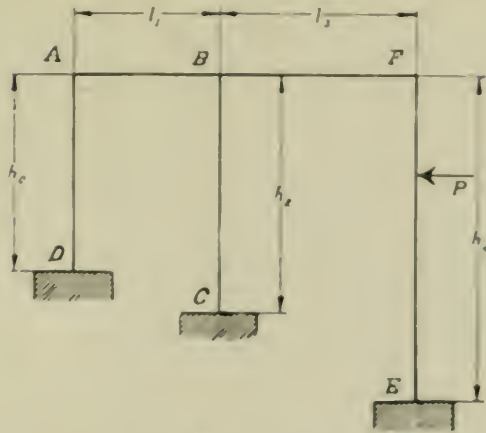


FIGURE 86

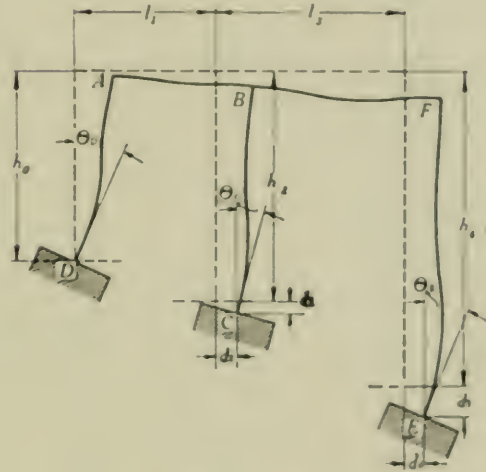


FIGURE 87

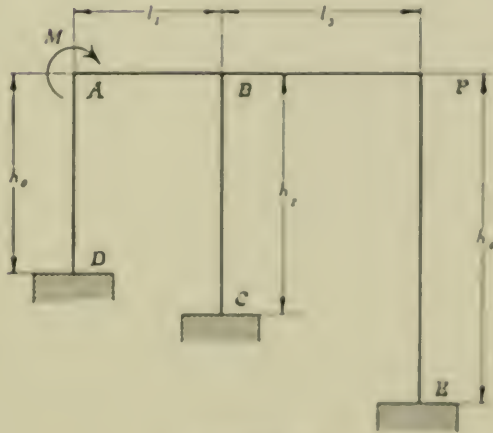


FIGURE 88

TABLE 22
EQUATIONS FOR THREE-LEGGED BENT
Lengths of All Legs Different. Any System of Loads. Legs Hinged at the Bases.

No. of Equa- tion	Left-hand Member of Equation				Right-hand Member of Equation					
	θ_A	θ_B	θ_F	d	Case I Vertical Load on Left- hand Span See Fig. 77	Case II Vertical Load on Right- hand Span See Fig. 78	Case III Hori- zontal Load to the Right on Left- hand Leg See Fig. 79	Case IV Hori- zontal Load to the Left on Right- hand Leg See Fig. 80	Case V Settlement and Sliding of Foundations See Fig. 81	Case VI External Couple at Upper Left- hand Corner See Fig. 82
A	$3K_o+4K_1$	$2K_1$	0	$-3\frac{K_o}{h_o}$	$\frac{C_{AB}}{E}$	0	$-\frac{H_{AD}}{E}$	0	$6K_1\frac{d_1}{l_1}$	$\frac{M}{E}$
B	$2K_1$	$4K_1+3K_2+4K_3$	$2K_3$	$-3\frac{K_2}{h_2}$	$-\frac{C_{BA}}{E}$	$\frac{C_{BF}}{E}$	0	0	$6K_1\frac{d_1}{l_1}+6K_3\frac{d_3-d_1}{l_3}-3K_2\frac{d_2}{h_2}$	0
C	0	$2K_3$	$4K_3+3K_4$	$-3\frac{K_4}{h_4}$	0	$-\frac{C_{FB}}{E}$	0	$\frac{H_{FE}}{E}$	$6K_3\frac{(d_3-d_1)}{l_3}-3K_4\frac{d_4}{h_4}$	0
D	$\frac{K_o}{h_o}$	$\frac{K_2}{h_2}$	$\frac{K_4}{h_4}$	$-\left[\frac{K_o}{h_o^2}+\frac{K_2}{h_2^2}+\frac{K_4}{h_4^2}\right]$	0	0	$\frac{1}{-3Eh_o}\left[M_D+H_{AD}\right]$	$\frac{1}{3Eh_4}\left[M_E-H_{FE}\right]$	$-K_2\frac{d_2}{h_2^2}-K_4\frac{d_4}{h_4^2}$	0

For definition of M_D and M_E see Table 20.

TABLE 23

EQUATIONS FOR THREE-LEGGED BENT

Lengths of All Legs Different. Any System of Loads. Legs Fixed at the Bases.

No. of Equation	Left-hand Member of Equation			Right-hand Member of Equation						
	θ_A	θ_B	θ_F	d	Case VII Vertical Load on Left-hand Span See Fig. 83	Case VIII Vertical Load on Right-hand Span See Fig. 84	Case IX Horizontal Load to the Right on Left-hand Leg See Fig. 85	Case X Horizontal Load to the Left on Right-hand Leg See Fig. 86	Case XI Settlement and Rotation of Foundations See Fig. 87	Case XII External Couple at Upper Left-hand Corner See Fig. 88
A	$4K_o+4K_1$	$2K_1$	0	$-6\frac{K_o}{h_o}$	$+\frac{C_{AB}}{E}$	0	$-\frac{C_{AD}}{E}$	0	$6K_1\frac{d_1}{l_1}-2K_o\theta_D$	$\frac{E}{M}+$
B	$2K_1$	$4K_1+4K_2+4K_3$	$2K_3$	$-6\frac{K_2}{h_2}$	$-\frac{C_{BA}}{E}$	$+\frac{C_{BF}}{E}$	0	0	$6K_1\frac{d_1}{l_1}-6K_2\frac{d_2}{h_2}+6K_3\frac{d_3-d_1}{l_3}-2K_2\theta_C$	0
C	0	$2K_3$	$4K_3+4K_4$	$-6\frac{K_4}{h_4}$	0	$-\frac{C_{FB}}{E}$	0	$+\frac{C_{FE}}{E}$	$6K_3\frac{d_3-d_1}{l_3}-6K_4\frac{d_4}{h_4}-2K_4\theta_E$	0
D	$\frac{K_o}{h_o}$	$\frac{K_2}{h_2}$	$\frac{K_4}{h_4}$	$-2\left[\frac{K_o}{h_o^2}+\frac{K_2}{h_2^2}+\frac{K_4}{h_4^2}\right]$	0	0	$-\frac{1}{6Eh_o}\left[M_D+C_{AD}-C_{DA}\right]$	$-\frac{1}{6Eh_4}\left[M_F+C_{FE}-C_{FE}\right]$	$-2K_2\frac{d_2}{h_2^2}-2K_4\frac{d_4}{h_4^2}-\frac{K_o\theta_D}{h_o}-\frac{K_2\theta_C}{h_2}-\frac{K_4\theta_E}{h_4}$	0

The definition of M and M_E see Table 20

65. *Three-legged Bent. Lengths and Sections of All Legs Equal. Lengths and Sections of Top Members Equal. Any System of Loads. Let $\frac{I}{l}$ of a Top Member be n times $\frac{I}{h}$ of a Leg. Legs Hinged at the Bases.*—

Fig. 89 represents a three-legged bent. The lengths of the legs are equal, and the lengths of the top members are equal. The $\frac{I}{l}$ of a top member is designated as K , and the $\frac{I}{h}$ of a leg is designated as $\frac{K}{n}$. The legs of the bent are hinged at D , C , and E .

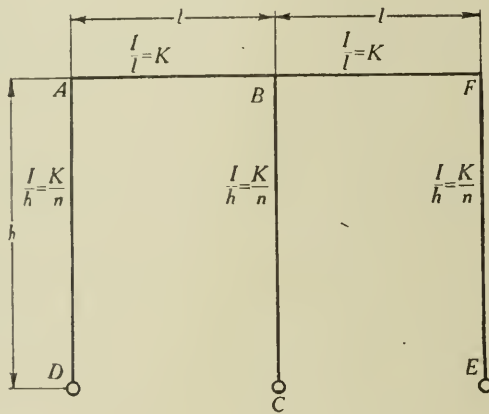


FIGURE 89

The equations of Table 1 as applied to the three-legged bent represented in Fig. 89 are given in Table 24 for a number of systems of loads. Four equations containing four unknowns, derived from the equations

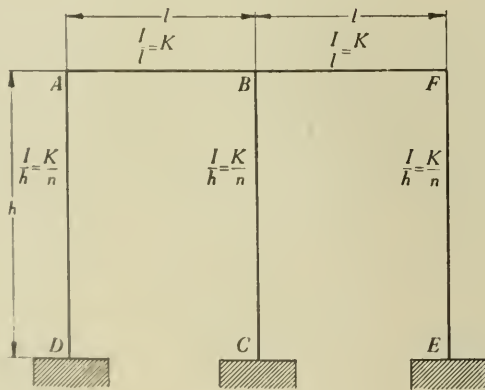


FIGURE 90

of Table 24, are given in Table 25, and the moments at the ends of the members as found from the equations of Tables 24 and 25 are given in Table 26.

66. *Three-legged Bent. Lengths and Sections of All Legs Equal. Lengths and Sections of Top Members Equal. Any System of Loads—Legs Fixed at the Bases.*—Fig. 90 represents a three-legged bent. The lengths of the legs are equal, and the lengths of the top members are equal. The $\frac{I}{l}$ of a top member is designated as K , and the $\frac{I}{h}$ is designated as $\frac{K}{n}$. The legs of the bent are fixed at D , C , and E .

The equations of Table 1 as applied to the three-legged bent represented in Fig. 90 are given in Table 27 for a number of systems of loads. Four equations containing four unknowns, derived from the equations of Table 27, are given in Table 28, and the moments at the ends of the members as found from the equations of Tables 27 and 28 are given in Table 29.

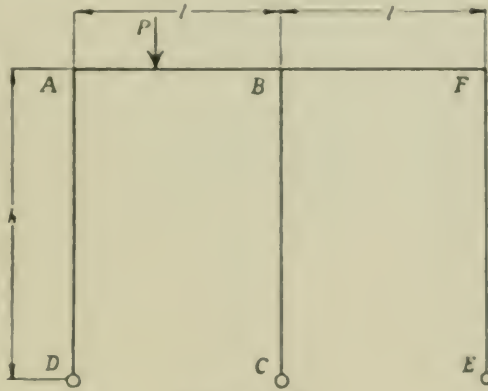


FIGURE 91

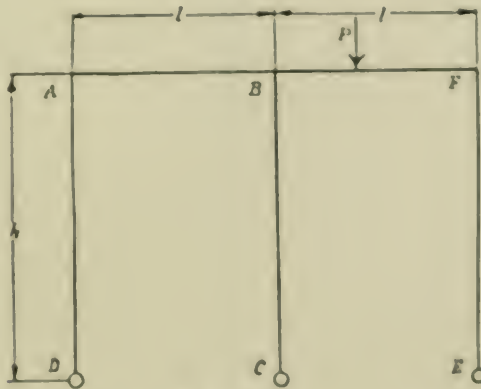


FIGURE 92

TABLE 24
FUNDAMENTAL EQUATIONS FOR THREE-LEGGED BENT

Lengths and Sections of Top Members Equal. $\left(\frac{l}{l} = K\right)$

Lengths and Sections of All Legs Equal. $\left(\frac{l}{h} = \frac{K}{n}\right)$

Any System of Loads. Legs Hinged at the Bases.

No. of Equa- tion	This part of the equation is independent of the loads	Additional terms of the right-hand member of the equation, different for each system of loads, as follows:					
		Case I Vertical Load on Left- hand Span See Fig. 91	Case II Vertical Load on Right- hand Span See Fig. 92	Case III Hori- zontal Load to the Right on Left- hand Leg See Fig. 93	Case IV Hori- zontal Load to the Left on Right- hand Leg See Fig. 94	Case V Settlement and Sliding of Foundations See Fig. 95	Case VI External Couple at Upper Left- hand Corner See Fig. 96
1	$M_{AD} = 3E \frac{K}{n} \left(\theta_A - R \right)$	0	0	$+H_{AD}$	0	0	0
2	$M_{AB} = 2EK(2\theta_A + \theta_B)$	$-C_{AB}$	0	0	0	$-6EK \frac{d_1}{l}$	0
3	$M_{BA} = 2EK(2\theta_B + \theta_A)$	$+C_{BA}$	0	0	0	$-6EK \frac{d_1}{l}$	0

TABLE 24—CONTINUED

4	$M_{BC} = 3E \frac{K}{n} (\theta_B - R)$	0	0	0	0	$+3E \frac{K}{n} \frac{d_2}{h}$	0
5	$M_{BF} = 2EK(2\theta_B + \theta_F)$	0	$-C_{BF}$	0	0	$-6EK \frac{d_3 - d_1}{l}$	0
6	$M_{FB} = 2EK(2\theta_F + \theta_B)$	0	$+C_{FB}$	0	0	$-6EK \frac{d_3 - d_1}{l}$	0
7	$M_{FE} = 3E \frac{K}{n} (\theta_F - R)$	0	0	0	$-H_{FE}$	$+3E \frac{K}{n} \frac{d_4}{h}$	0
8	$0 = M_{AD} + M_{AB}$	0	0	0	0	0	$-M$
9	$0 = M_{BA} + M_{BF} + M_{BC}$	0	0	0	0	0	0
10	$0 = M_{FB} + M_{FE}$	0	0	0	0	0	0
11	$0 = M_{AD} + M_{BC} + M_{FE}$	0	0	$+M_D$	$+M_E$	0	0

M_D is the statical moment about point D of the horizontal loads on left-hand leg.

M_E is the statical moment about point E of the horizontal loads on right-hand leg.

The statical moment is positive when the indicated rotation is clockwise.

TABLE 25
EQUATIONS FOR THREE-LEGGED BENT
Lengths and Sections of Top Member Equal. $\left(\frac{l}{l} = K\right)$ Lengths and Sections of All Legs Equal. $\left(\frac{l}{h} = \frac{K}{n}\right)$
Any System of Loads Legs Hinged at the Bases.

No. of Equa- tion	Left-hand Member of Equation			Right-hand Member of Equation						
	θ_A	θ_B	θ_F	$3R$	Case I Vertical Load on Left- hand Span See Fig. 91	Case II Vertical Load on Right- hand Span See Fig. 92	Case III Hori- zontal Load to the Right on Left- hand Leg See Fig. 93	Case IV Hori- zontal Load to the Left on Right- hand Leg See Fig. 94	Case V Settlement and Sliding of Foundations See Fig. 95	Case VI External Couple at Upper Left- hand Corner See Fig. 96
A	$4n+3$	$2n$	0	-1	$\frac{n.C_{AB}}{EK}$	0	$-\frac{n.H_{AD}}{EK}$	0	$6n \frac{d_1}{l}$	$Mn \over EK$
B	$2n$	$8n+3$	$2n$	-1	$-\frac{n.C_{BA}}{EK}$	$\frac{n.C_{BF}}{EK}$	0	0	$6n \cdot \frac{d_3}{l} - \frac{3d_2}{h}$	0
C	0	$2n$	$4n+3$	-1	0	$-\frac{n.C_{FB}}{EK}$	0	$\frac{n.H_{FE}}{EK}$	$\frac{6n(d_3-d_1)}{l} - \frac{3d_4}{h}$	0
D	1	1	1	-1	0	0	$\left[\begin{matrix} -\frac{n}{3EK} \\ M_D \end{matrix} \right] + H_{AD}$	$\left[\begin{matrix} -\frac{n}{3EK} \\ M_E \end{matrix} \right] - H_{FE}$	$-\frac{d_2+d_4}{h}$	0

For definition of M_D and M_E see Table 24.

TABLE 26

MOMENTS DUE TO VARIOUS SYSTEMS OF LOADS ON A THREE-LEGGED BENT

Lengths and Sections of Top Members Equal. $\left(\frac{l}{t} = K\right)$ Lengths and Sections of All Legs Equal. $\left(\frac{l}{h} = \frac{K}{n}\right)$

Legs Hinged at the Bases.

No. of Equation	Moment	Case I Vertical Load on Left-hand Span See Fig. 91	Case II Vertical Load on Right-hand Span See Fig. 92	Case III Horizontal Load to the Right on Left-hand Leg See Fig. 93	Case IV Horizontal Load to the Left on Right-hand Leg See Fig. 94	Case V Settlement and Sliding of Foundation See Fig. 95	Case VI External Couple at Upper Left-hand Corner See Fig. 96
1	$M_{AD} =$	$+\frac{C_{AB}(10n+9)+C_{BA}(4n+3)}{4(n+1)(4n+3)}$	$-\frac{C_{BF}(4n+3)-C_{FB}(2n+3)}{4(n+1)(4n+3)}$	$+\frac{2nH_{AD}(16n+15)-M_D(4n+3)^2}{12(n+1)(4n+3)}$	$+\frac{2nH_{FE}(8n+9)-M_E(4n+3)^2}{12(n+1)(4n+3)}$	$+\frac{EK}{2hl} \frac{18h(n+1)(2d_1-d_3)-l[d_2(8n+6)+d_4(8n+9)]}{(n+1)(4n+3)}$	$+\frac{M(10n+9)}{4(n+1)(4n+3)}$
2	$M_{AB} =$	$-\frac{C_{AB}(10n+9)+C_{BA}(4n+3)}{4(n+1)(4n+3)}$	$+\frac{C_{BF}(4n+3)-C_{FB}(2n+3)}{4(n+1)(4n+3)}$	$-\frac{2nH_{AD}(16n+15)-M_D(4n+3)^2}{12(n+1)(4n+3)}$	$-\frac{2nH_{FE}(8n+9)-M_E(4n+3)^2}{12(n+1)(4n+3)}$	$-\frac{EK}{2hl} \frac{18h(n+1)(2d_1-d_3)-l[d_2(8n+6)+d_4(8n+9)]}{(n+1)(4n+3)}$	$+\frac{M(16n^2+18n+3)}{4(n+1)(4n+3)}$
3	$M_{BA} =$	$+\frac{(2n+3)[C_{AB}(2n+1)+C_{BA}(4n+3)]}{4(n+1)(4n+3)}$	$+\frac{C_{BF}(4n+3)(2n+1)+C_{FB}(4n^2-3)}{4(n+1)(4n+3)}$	$-\frac{(2n+3)[2nH_{AD}-M_D(4n+3)]}{12(n+1)(4n+3)}$	$-\frac{2nH_{FE}(10n+9)-M_E(4n+3)(2n+3)}{12(n+1)(4n+3)}$	$-\frac{EK}{2hl} \frac{6h(n+1)(2n+3)(2d_1-d_3)+l[d_2(8n+6)-d_4(10n+9)]}{(n+1)(4n+3)}$	$+\frac{M(2n+1)(2n+3)}{4(n+1)(4n+3)}$
4	$M_{BC} =$	$-\frac{C_{AB}+C_{BA}}{2(n+1)}$	$+\frac{C_{BF}+C_{FB}}{2(n+1)}$	$-\frac{2nH_{AD}+M_D(2n+3)}{6(n+1)}$	$+\frac{2nH_{FE}-M_E(2n+3)}{6(n+1)}$	$+\frac{EK}{h} \frac{2d_2-d_4}{n+1}$	$-\frac{M}{2(n+1)}$
5	$M_{BF} =$	$-\frac{C_{AB}(4n^2-3)+C_{BA}(4n+3)(2n+1)}{4(n+1)(4n+3)}$	$-\frac{(2n+3)[C_{BF}(4n+3)+C_{FB}(2n+1)]}{4(n+1)(4n+3)}$	$+\frac{2nH_{AD}(10n+9)+M_D(4n+3)(2n+3)}{12(n+1)(4n+3)}$	$+\frac{(2n+3)[2nH_{FE}+M_E(4n+3)]}{12(n+1)(4n+3)}$	$+\frac{EK}{2hl} \frac{6h(n+1)(2n+3)(2d_1-d_3)-l[d_2(8n+6)+d_4(2n+3)]}{(n+1)(4n+3)}$	$-\frac{M(4n^2-3)}{4(n+1)(4n+3)}$
6	$M_{FB} =$	$+\frac{C_{AB}(2n+3)-C_{BA}(4n+3)}{4(n+1)(4n+3)}$	$+\frac{C_{BF}(4n+3)+C_{FB}(10n+9)}{4(n+1)(4n+3)}$	$+\frac{2nH_{AD}(8n+9)+M_D(4n+3)^2}{12(n+1)(4n+3)}$	$+\frac{2nH_{FE}(16n+15)+M_E(4n+3)^2}{12(n+1)(4n+3)}$	$+\frac{EK}{2hl} \frac{18h(n+1)(2d_1-d_3)+l[d_2(8n+6)-d_4(16n+15)]}{(n+1)(4n+3)}$	$+\frac{M(2n+3)}{4(n+1)(4n+3)}$
7	$M_{FE} =$	$-\frac{C_{AB}(2n+3)-C_{BA}(4n+3)}{4(n+1)(4n+3)}$	$-\frac{C_{BF}(4n+3)+C_{FB}(10n+9)}{4(n+1)(4n+3)}$	$-\frac{2nH_{AD}(8n+9)+M_D(4n+3)^2}{12(n+1)(4n+3)}$	$-\frac{2nH_{FE}(16n+15)+M_E(4n+3)^2}{12(n+1)(4n+3)}$	$-\frac{EK}{2hl} \frac{18h(n+1)(2d_1-d_3)+l[d_2(8n+6)-d_4(16n+15)]}{(n+1)(4n+3)}$	$-\frac{M(2n+3)}{4(n+1)(4n+3)}$

For definition of M_D and M_E see Table 24.

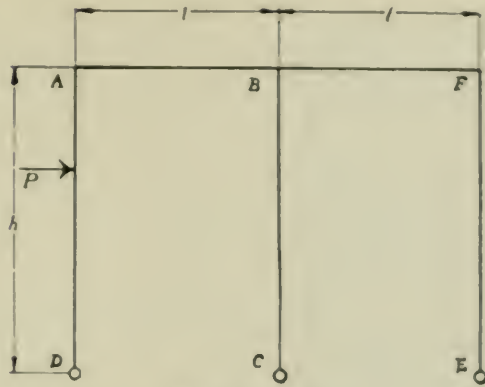


FIGURE 93

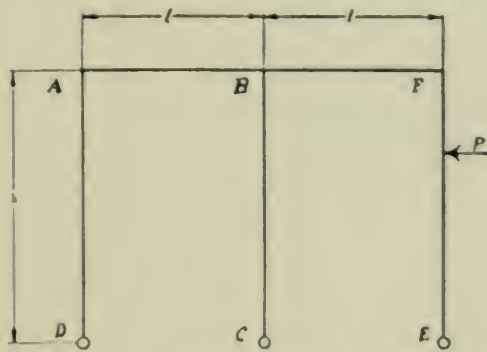


FIGURE 94

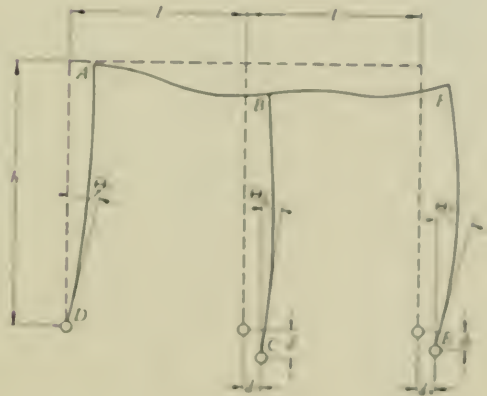


FIGURE 95

TABLE 27

FUNDAMENTAL EQUATIONS FOR THREE-LEGGED BENT

Lengths and Sections of Top Members Equal. $\left(\frac{l}{l} = K\right)$ Lengths and Sections of All Legs Equal. $\left(\frac{l}{h} = \frac{K}{n}\right)$

Any System of Loads. Legs Fixed at the Bases.

No. of Equa- tion	This part of the equation is independent of the loads	Additional term of the right-hand member of the equation, different for each system of loads, as follows:				
		Case VII Vertical Load on Left- hand Span See Fig. 97	Case VIII Vertical Load on Right- hand Span See Fig. 98	Case IX Horizontal Load to the Right on Left- hand Leg See Fig. 99	Case X Horizontal Load to the Left on Right- hand Leg See Fig. 100	Case XII External Couple at Upper Left-hand Corner See Fig. 101
1	$M_{AD} = 2E \frac{K}{n} (2\theta_A - 3R)$	0	0	$+C_{AD}$	0	0
2	$M_{AB} = 2EK (2\theta_A + \theta_B)$	$-C_{AB}$	0	0	0	0
3	$M_{BA} = 2EK (2\theta_B + \theta_A)$	$+C_{BA}$	0	0	0	0
4	$M_{BC} = 2E \frac{K}{n} (2\theta_B - 3R)$	0	0	0	0	0
5	$M_{BF} = 2EK (2\theta_B + \theta_F)$	0	$-C_{BF}$	0	0	0

TABLE 27—CONTINUED

6	$M_{FH} = 2EK(2\theta_F + \theta_H)$	0	$+C_{FB}$	0	0	0
7	$M_{FE} = 2E \frac{K}{n} (2\theta_F - 3R)$	0	0	0	$-C_{FE}$	0
8	$M_{DA} = 2E \frac{K}{n} (\theta_A - 3R)$	0	0	$-C_{DA}$	0	0
9	$M_{CH} = 2E \frac{K}{n} (\theta_H - 3R)$	0	0	0	0	0
10	$M_{HF} = 2E \frac{K}{n} (\theta_F - 3R)$	0	0	0	$+C_{HF}$	0
11	$0 = M_{AD} + M_{AH}$	0	0	0	0	$-M$
12	$0 = M_{BA} + M_{BF} + M_{BC}$	0	0	0	0	0
13	$0 = M_{FB} + M_{FH}$	0	0	0	0	0
14	$0 = M_{AD} + M_{DA} + M_{BC} + M_{CB} + M_{FB} + M_{FF}$	0	0	$+M_D$	$+M_b$	0

M_H is the external moment about point D of the horizontal loads on left-hand leg.
 M_F is the external moment about point E of the horizontal loads on right-hand leg.
The external moment is positive when the indicated rotation is clockwise.

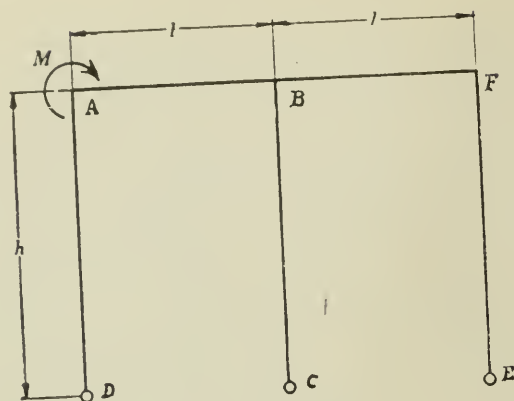


FIGURE 96

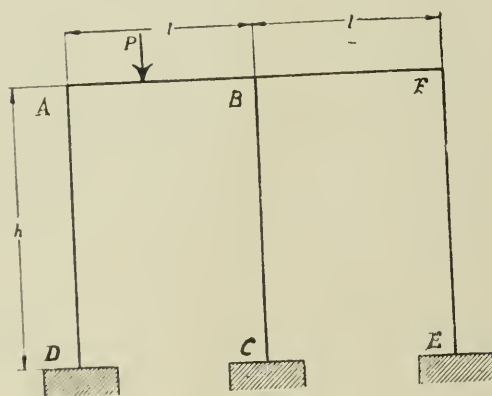


FIGURE 97

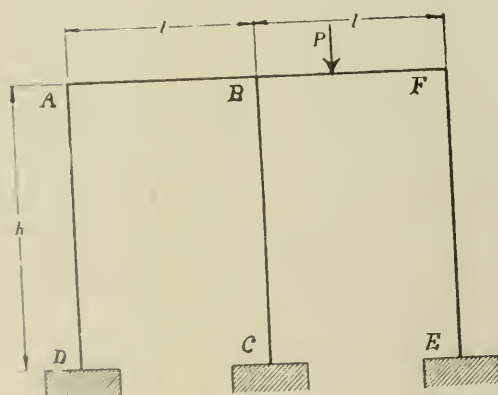


FIGURE 98

TABLE 28
EQUATIONS FOR THREE-LEGGED BENT

Lengths and Sections of Top Members Equal. $\left(\frac{l}{l} = K\right)$
Lengths and Sections of All Legs Equal. $\left(\frac{l}{h} = \frac{K}{n}\right)$

Any System of Loads. Legs Fixed at the Bases.

No. of Equa- tion	Left-hand Member of Equation			Right-hand Member of Equation					
	θ_A	θ_n	θ_F	$3R$	Case VII Vertical Load on Left- hand Span See Fig. 97	Case VIII Vertical Load on Right- hand Span See Fig. 98	Case IX Horizontal Load to the Right on Left-hand Leg See Fig. 99	Case X Horizontal Load to the Left on Right-hand Leg See Fig. 100	Case XII External Couple at Upper Left-hand Corner See Fig. 101
A	$2n+2$	n	0	-1	$+\frac{nC_{AB}}{2EK}$	0	$-\frac{nC_{AD}}{2EK}$	0	$+\frac{Mn}{2EK}$
B	n	$4n+2$	n	-1	$-\frac{nC_{BA}}{2EK}$	$\frac{nC_{BF}}{2EK}$	0	0	0
C	0	n	$2n+2$	-1	0	$-\frac{nC_{FH}}{2EK}$	0	$+\frac{nC_{FE}}{2EK}$	0
D	1	1	1	-2	0	0	$-\frac{n(M_D+C_{AD}-C_{DA})}{6EK}$	$-\frac{n(M_F+C_{BF}-C_{FE})}{6EK}$	0

For definition of M_D and M_F see Table 27.

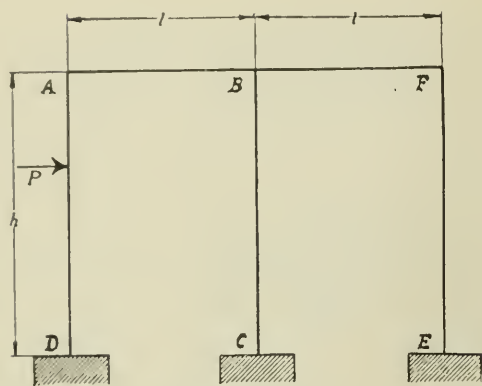


FIGURE 99

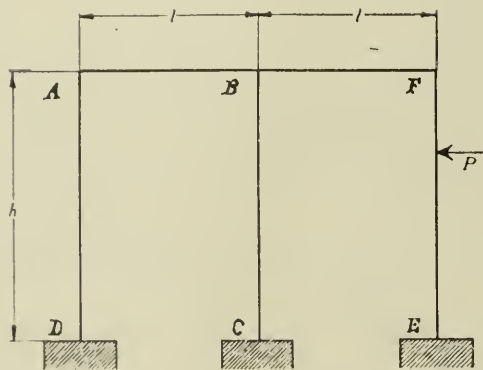


FIGURE 100

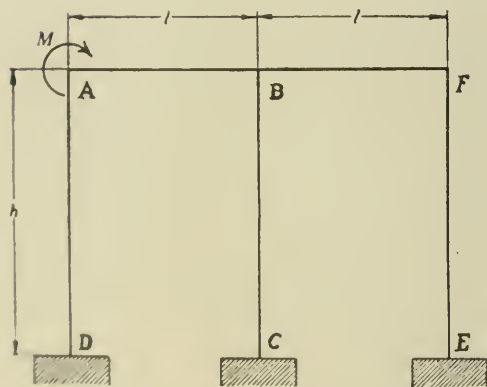


FIGURE 101

TABLE 29
MOMENTS DUE TO VARIOUS SYSTEMS OF LOADS ON A THREE-LEGGED BENT

Lengths and Sections of Top Members Equal $\left(\frac{l}{l} = K\right)$

Lengths and Sections of All Legs Equal $\left(\frac{l}{h} = \frac{K}{n}\right)$
Legs Fixed at the Bases.

No. of Equation	Moment	Case VII Vertical Load on Left-hand Span See Fig. 97	Case VIII Vertical Load on Right-hand Span See Fig. 98	Case IX Horizontal Load to the Right on Left-hand Leg See Fig. 99	Case X Horizontal Load to the Left on Right-hand Leg See Fig. 100	Case XII External Couple at Upper Left-hand Corner See Fig. 101
1	$M_{AD} =$	$+\frac{C_{AB}(11n^2+15n+2)+4nC_{BA}(n+1)}{2(n+1)(6n^2+9n+1)}$	$-\frac{4nC_{BF}(n+1)-nC_{FB}(n+3)}{2(n+1)(6n^2+9n+1)}$	$+\frac{nC_{AD}(10n^2+15n+3)-2n(M_D-C_{DA})(n+1)^2}{2(n+1)(6n^2+9n+1)}$	$+\frac{nC_{FE}(2n^2+3n-1)-2n(M_E+C_{EF})(n+1)^2}{2(n+1)(6n^2+9n+1)}$	$+\frac{M(11n^2+15n+2)}{2(n+1)(6n^2+9n+1)}$
2	$M_{AB} =$	$-\frac{C_{AB}(11n^2+15n+2)+4nC_{BA}(n+1)}{2(n+1)(6n^2+9n+1)}$	$+\frac{4nC_{BF}(n+1)-nC_{FB}(n+3)}{2(n+1)(6n^2+9n+1)}$	$-\frac{nC_{AD}(10n^2+15n+3)-2n(M_D-C_{DA})(n+1)^2}{2(n+1)(6n^2+9n+1)}$	$-\frac{nC_{FE}(2n^2+3n-1)-2n(M_E+C_{EF})(n+1)^2}{2(n+1)(6n^2+9n+1)}$	$+\frac{Mn(3n+1)(4n+5)}{2(n+1)(6n^2+9n+1)}$
3	$M_{BA} =$	$+\frac{nC_{AB}(3n^2+8n+4)+C_{BA}(n+1)(6n^2+13n+2)}{2(n+1)(6n^2+9n+1)}$	$+\frac{nC_{BF}(n+1)(6n+5)+nC_{FB}(3n^2+n-3)}{2(n+1)(6n^2+9n+1)}$	$+\frac{n(n+2)[C_{AD}(2n+1)-(M_D-C_{DA})(n+1)]}{2(n+1)(6n^2+9n+1)}$	$-\frac{nC_{FE}(4n^2+4n-1)-n(M_E+C_{EF})(n+1)(n+2)}{2(n+1)(6n^2+9n+1)}$	$+\frac{Mn(n+2)(3n+2)}{2(n+1)(6n^2+9n+1)}$
4	$M_{BC} =$	$-\frac{7nC_{AB}+2C_{BA}(4n+1)}{2(6n^2+9n+1)}$	$+\frac{2C_{BF}(4n+1)+7nC_{FB}}{2(6n^2+9n+1)}$	$-\frac{nC_{AD}(2n-3)+2n(M_D-C_{DA})(n+2)}{2(6n^2+9n+1)}$	$+\frac{nC_{FE}(2n-3)-2n(M_E+C_{EF})(n+2)}{2(6n^2+9n+1)}$	$-\frac{7Mn}{2(6n^2+9n+1)}$
5	$M_{BF} =$	$-\frac{nC_{AB}(3n^2+n-3)+nC_{BA}(n+1)(6n+5)}{2(n+1)(6n^2+9n+1)}$	$-\frac{C_{BF}(n+1)(6n^2+13n+2)+nC_{FB}(3n^2+8n+4)}{2(n+1)(6n^2+9n+1)}$	$+\frac{nC_{AD}(4n^2+4n-1)+n(M_D-C_{DA})(n+1)(n+2)}{2(n+1)(6n^2+9n+1)}$	$+\frac{n(n+2)[C_{FE}(2n+1)+(M_E+C_{EF})(n+1)]}{2(n+1)(6n^2+9n+1)}$	$-\frac{Mn(3n^2+n-3)}{2(n+1)(6n^2+9n+1)}$
6	$M_{FB} =$	$+\frac{nC_{AB}(n+3)-4nC_{BA}(n+1)}{2(n+1)(6n^2+9n+1)}$	$+\frac{4nC_{BF}(n+1)+C_{FB}(11n^2+15n+2)}{2(n+1)(6n^2+9n+1)}$	$+\frac{nC_{AD}(2n^2+3n-1)+2n(M_D-C_{DA})(n+1)^2}{2(n+1)(6n^2+9n+1)}$	$+\frac{nC_{FE}(10n^2+15n+3)+2n(M_E+C_{EF})(n+1)^2}{2(n+1)(6n^2+9n+1)}$	$+\frac{Mn(n+3)}{2(n+1)(6n^2+9n+1)}$
7	$M_{FE} =$	$-\frac{nC_{AB}(n+3)-4nC_{BA}(n+1)}{2(n+1)(6n^2+9n+1)}$	$-\frac{4nC_{BF}(n+1)+C_{FB}(11n^2+15n+2)}{2(n+1)(6n^2+9n+1)}$	$-\frac{nC_{AD}(2n^2+3n-1)+2n(M_D-C_{DA})(n+1)^2}{2(n+1)(6n^2+9n+1)}$	$-\frac{nC_{FE}(10n^2+15n+3)+2n(M_E+C_{EF})(n+1)^2}{2(n+1)(6n^2+9n+1)}$	$-\frac{Mn(n+3)}{2(n+1)(6n^2+9n+1)}$
8	$M_{DA} =$	$+\frac{nC_{AB}(4n+5)+C_{BA}(n+1)(2n+1)}{2(n+1)(6n^2+9n+1)}$	$-\frac{C_{BF}(n+1)(2n+1)-C_{FB}(2n^2+4n+1)}{2(n+1)(6n^2+9n+1)}$	$-\frac{C_{AD}(6n^2+27n^2+26n+2)}{6(n+1)(6n^2+9n+1)} - \frac{M_D(6n^2+9n+2)+C_{DA}(30n^2+45n+4)}{6(6n^2+9n+1)}$	$+\frac{C_{FE}(6n^2+9n^2-n-1)-(M_E+C_{EF})(n+1)(6n^2+9n+2)}{6(n+1)(6n^2+9n+1)}$	$+\frac{Mn(4n+5)}{2(n+1)(6n^2+9n+1)}$
9	$M_{CB} =$	$-\frac{C_{AB}(5n+1)+4nC_{BA}}{2(6n^2+9n+1)}$	$+\frac{4nC_{BF}+C_{FB}(5n+1)}{2(6n^2+9n+1)}$	$-\frac{C_{AD}(6n^2-3n-1)+2(M_D-C_{DA})(3n^2+6n+1)}{6(6n^2+9n+1)}$	$+\frac{C_{FE}(6n^2-3n-1)-2(M_E+C_{EF})(3n^2+6n+1)}{6(6n^2+9n+1)}$	$-\frac{M(5n+1)}{2(6n^2+9n+1)}$
10	$M_{EF} =$	$-\frac{C_{AB}(2n^2+4n+1)-C_{BA}(n+1)(2n+1)}{2(n+1)(6n^2+9n+1)}$	$-\frac{C_{BF}(n+1)(2n+1)+nC_{FB}(4n+5)}{2(n+1)(6n^2+9n+1)}$	$-\frac{C_{AD}(6n^2+9n^2-n-1)+(M_D-C_{DA})(n+1)(6n^2+9n+2)}{6(n+1)(6n^2+9n+1)}$	$+\frac{C_{FE}(6n^2+27n^2+26n+2)}{6(n+1)(6n^2+9n+1)} - \frac{M_E(6n^2+9n+2)-C_{EF}(30n^2+45n+4)}{6(6n^2+9n+1)}$	$-\frac{M(2n^2+4n+1)}{2(n+1)(6n^2+9n+1)}$

For definitions of M and M_E see Table 27.

XII. EFFECT OF ERROR IN ASSUMPTIONS

67. *Error Due to Assumption that Axial Deformation is Zero.*—

In determining the stresses in frames it has been assumed that the members of the frame do not change in length, but since the members are subjected to axial stress there must be a corresponding axial deformation. It remains to determine the effect of the axial deformation upon the stresses in the frames.

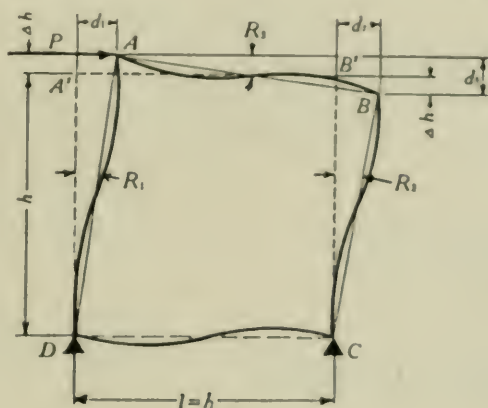


FIGURE 102

Fig. 102 represents a square frame with all sides identical in section. The frame is subjected to a single horizontal force P at the top, and is in equilibrium under the action of P and the reactions at D and C . Since there are no loads except at the ends of the members, H and C of Table 2 are zero for all members.

Case 1. Assume horizontal reactions at C and D equal. From equations 407 and 408 of section 48

$$M_{AB} = \frac{Ph}{4}$$

$$M_{BC} = -\frac{Ph}{4}$$

$$M_{CD} = \frac{Ph}{4}$$

$$M_{DA} = -\frac{Ph}{4}$$

$$\text{Shear in } AD = \frac{M_{AD} + M_{DA}}{h} = \frac{1}{2} P$$

Likewise

$$\text{Shear in } BC = \frac{1}{2} P$$

$$\text{Shear in } AB = \frac{1}{2} P$$

$$\text{Shear in } DC = \frac{1}{2} P$$

$$\text{Stress in } AB = \frac{1}{2} P \text{ compression}$$

$$\text{Stress in } CD = 0$$

$$\text{Stress in } AD = \frac{1}{2} P \text{ tension}$$

$$\text{Stress in } BC = \frac{1}{2} P \text{ compression}$$

Since AB , AD , and BC have the same sections, the same lengths, and are subjected to axial stresses of the same magnitudes, Δh , representing the change in length due to axial stress, is the same for all members.

Referring to Fig. 102

$$R_1 - R_2 = \frac{\Delta h}{h} = \frac{P}{2AE}$$

$$R_3 = \frac{2\Delta h}{l} = \frac{P}{AE}$$

Applying the equations of Table 1 gives

$$M_{AB} = 2EK(2\theta_A + \theta_B - 3R_3) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (719)$$

$$M_{AB} = -2EK(2\theta_A + \theta_D - 3R_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (720)$$

$$M_{BC} = 2EK(2\theta_B + \theta_C - 3R_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (721)$$

$$M_{BC} = -2EK(2\theta_B + \theta_A - 3R_3) \quad (722)$$

$$M_{CD} = 2EK(2\theta_C + \theta_D) \quad (723)$$

$$M_{CD} = -2EK(2\theta_C + \theta_B - 3R_2) \quad (724)$$

$$M_{DA} = 2EK(2\theta_D + \theta_A - 3R_1) \quad (725)$$

$$M_{DA} = -2EK(2\theta_D + \theta_C) \quad (726)$$

For AD and BC to be in equilibrium

$$-M_{AB} + M_{BC} - M_{CD} + M_{DA} + Pl = 0 \quad (727)$$

Combining equations 719 to 727, substituting values for R_1 , R_2 , and R_3 , and solving for the moments gives

$$M_{AB} = \frac{Ph}{4} \left[1 - \frac{3K}{2Ah} \right] \quad (728)$$

$$M_{BC} = -\frac{Ph}{4} \left[1 - \frac{9K}{2Ah} \right] \quad (729)$$

$$M_{CD} = \frac{Ph}{4} \left[1 + \frac{3K}{2Ah} \right] \quad (730)$$

$$M_{DA} = -\frac{Ph}{4} \left[1 + \frac{9K}{2Ah} \right] \quad (731)$$

In these equations $\frac{9K}{2Ah}$ represents the largest error due to neglecting the axial deformation in the members, and the ratio of $\frac{9K}{2Ah}$ to 1 represents the relative error.

Case 2. Assume horizontal reaction at C only. If the horizontal reaction is all taken at C

$$\text{Stress in } AB = \frac{1}{2}P \text{ compression}$$

$$\text{Stress in } CD = \frac{1}{2}P \text{ compression}$$

Comparing Case 1 with Case 2, it is apparent that the error due to neglecting the axial deformations is smaller if the horizontal reaction is all at one corner than if half of it comes at each of the lower corners.

The maximum relative error, for Case 1, is $\frac{9K}{2Ah}$. Substituting for K its value $\frac{I}{l}$, the expression $\frac{9K}{2Ah}$ becomes $\frac{9I}{2Ah^2}$. That is, the error due to neglecting the axial deformation in a square frame with all sides identical, subjected to a single horizontal force at the top, varies directly with the moments of inertia of the section of the members, inversely with the area of the members, and inversely with the square of the length.

If for Case 1 the frame, instead of being square as shown in Fig. 102, is a rectangle twice as wide as it is high, and if the sections of all members are identical, the moments in the frame due to a single horizontal force P at the top are

$$M_{AB} = \frac{Ph}{4} \left(1 + \frac{9}{4} \frac{K}{Ah} \right) \quad (736)$$

$$M_{BC} = -\frac{Ph}{4} \left(1 - \frac{15}{4} \frac{K}{Ah} \right) \quad (737)$$

$$M_{CD} = \frac{Ph}{4} \left(1 - \frac{9}{4} \frac{K}{Ah} \right) \quad (738)$$

$$M_{DA} = -\frac{Ph}{4} \left(1 + \frac{15}{4} \frac{K}{Ah} \right) \quad (739)$$

in which h and K are for the vertical members.

The maximum relative error in this case is $\frac{15}{4} \frac{K}{Ah}$ divided by 1.

It is to be noted that $\frac{15}{4}$, the coefficient of $\frac{K}{Ah}$ in equation 739, is less than $\frac{9}{2}$, the coefficient of $\frac{K}{Ah}$, in equation 731. Hence for two frames, one a rectangle twice as wide as it is high with sections of all members identical, and the other a square having all sides identical with the vertical sides of the rectangle, and both frames having a

the error is well under 2 per cent. For concrete frames the ratio $\frac{I}{A}$ is much less than for steel frames, and the error due to neglecting the axial deformations is correspondingly less for concrete frames than for steel frames.

TABLE 30
ERROR IN STRESSES IN RECTANGULAR FRAMES DUE TO NEGLECTING AXIAL DEFORMATION

Frame is subjected to a single horizontal force at the top. The horizontal reaction at the bottom is equally divided between the two lower corners. The sections of all members of a frame are identical.

Section	Square Frame		Frame Having Width Twice Height			Frame Having Height Twice Width		
	Height and Width feet	Error per cent	Height feet	Width feet	Error per cent	Height feet	Width feet	Error per cent
27"—I—83 lb.	20	.92	20	40	.77	40	20	.33
	15	1.64	15	30	1.37	30	15	.58
	10	3.70	10	20	3.08	20	10	1.31
24"—I—80 lb.	20	.70	20	40	.58	40	20	.25
	15	1.24	15	30	1.03	30	15	.43
	10	2.80	10	20	2.33	20	10	.99
20"—I—65 lb.	20	.48	20	40	.40	40	20	.17
	15	.85	15	30	.71	30	15	.30
	10	1.92	10	20	1.60	20	10	.68
15"—I—42 lb.	20	.28	20	40	.23	40	20	.10
	15	.49	15	30	.41	30	15	.17
	10	1.10	10	20	.92	20	10	.39
12"—I—31.5 lb.	20	.18	20	40	.15	40	20	.06
	15	.32	15	30	.27	30	15	.11
	10	.73	10	20	.61	20	10	.26

68. *Error Due to Assumption that Shearing Deformation is Zero.*—In the derivation of the fundamental propositions upon which the analyses are based, deflection due to shear was not considered. That being the case, R in the equations of Table 1 depends upon the bending deflection only. In the analyses of the rectangular frames the R 's of the two vertical members are taken equal. This is true of the bending

deflections only when the shearing deformations in the two vertical members are equal. (Axial deformation is here neglected.) Likewise, in considering that the deflection due to bending for the top of the frame is zero, the shear deflection in the top of the frame is neglected.

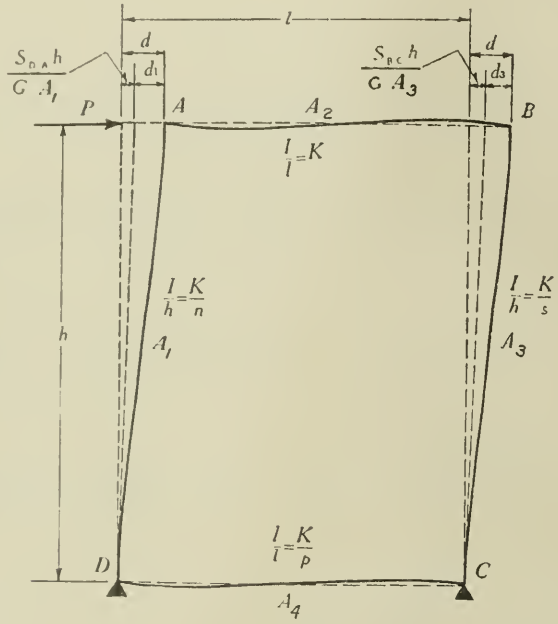


FIGURE 103

Fig. 103 represents a rectangular frame having a horizontal force applied at the top. The members of the frame are subjected to shear as follows:

$$\text{Shear in } DA, S_{DA} = \frac{M_{DA} - M_{AB}}{h} = -\frac{Ph}{h} \frac{\Delta_{DA} + \Delta_{AB}}{\Delta} \quad (744)$$

$$\text{Shear in } AB, S_{AB} = \frac{M_{AB} - M_{BC}}{l} = \frac{Ph}{l} \frac{\Delta_{AB} + \Delta_{BC}}{\Delta} \quad (745)$$

$$\text{Shear in } BC, S_{BC} = \frac{M_{BC} - M_{CD}}{h} = \frac{Ph}{h} \frac{\Delta_{BC} + \Delta_{CD}}{\Delta} \quad (746)$$

$$\text{Shear in } CD, S_{CD} = \frac{M_{CD} - M_{DA}}{l} = \frac{Ph}{l} \frac{\Delta_{CD} + \Delta_{DA}}{\Delta} \quad (747)$$

In these equations Δ_{AB} , Δ_{BC} , Δ_{CD} , and Δ_{DA} represent the quantities in the parentheses of equations 401, 402, 403, and 404, of section 48, respectively, and Δ represents the common denominator of the same equations.

The points D and C are considered to remain stationary, and axial strain is neglected.

The total horizontal deflection of A equals the total horizontal deflection of B . These deflections are represented in Fig. 103 by d .

The shear deflection of DA is $\frac{S_{DA}h}{GA_1}$ and the shear deflection of BC is $\frac{S_{BCh}}{GA_3}$. The bending deflection of DA , d_1 , is therefore given by the equation

$$d_1 = d - \frac{S_{DA}h}{GA_1} \quad (748)$$

and the bending deflection of BC , d_3 , is given by the equation

$$d_3 = d - \frac{S_{BCh}}{GA_3} \quad (749)$$

The point C does not move vertically relatively to D , but there is a shear S_{CD} which produces a shear deflection in CD of $-\frac{S_{CD}l}{GA_4}$; therefore there must be an equal and opposite moment deflection of $\frac{S_{CD}l}{GA_4}$. Similarly, there is a bending deflection in AB of $\frac{S_{AB}l}{GA_2}$.

From the preceding equations

$$R_1 = \frac{d_1}{h} = \frac{d}{h} - \frac{S_{DA}}{GA_1} \quad (750)$$

$$R_2 = \frac{d_2}{l} = \frac{S_{AB}}{GA_2} \quad (751)$$

$$R_3 = \frac{d_3}{h} = \frac{d}{h} - \frac{S_{BC}}{GA_3} \quad (752)$$

$$R_4 = \frac{d_4}{l} = \frac{S_{CD}}{GA_4} \quad (753)$$

$$M_{CD} = \begin{bmatrix} M_{CD} \text{ when shearing strain is} \\ \text{neglected, a positive quantity} \\ \text{(see equation 403, section 48).} \end{bmatrix} - \left[\frac{6PK}{\Delta^2} \frac{E}{G} \right] [B_{CD}D + O_{CD}Q] \quad (765)$$

$$M_{DA} = \begin{bmatrix} M_{DA} \text{ when shearing strain is} \\ \text{neglected, a negative quantity} \\ \text{(see equation 404, section 48).} \end{bmatrix} - \left[\frac{6PK}{\Delta^2} \frac{E}{G} \right] [B_{DA}D - O_{DA}Q] \quad (766)$$

in which

$$B_{AB} = 6ns + 5ps + 2pn + p^2 + 2n + p - s$$

$$B_{BC} = 6ns + 5pn + 2ps + p^2 - n + 2s + p$$

$$B_{CD} = 6ns + 5n + 2ps + p - pn + 2s + 1$$

$$B_{DA} = 6ns + 5s - ps + p + 2pn + 2n + 1$$

$$O_{AB} = \frac{h}{l} (ns - pn + s^2 + 5ps + 2n + 2s + 6p)$$

$$O_{BC} = \frac{h}{l} (ns + 5pn - ps + n^2 + 2n + 6p + 2s)$$

$$O_{CD} = \frac{h}{l} (ns + 2ps + 5n + 6p + n^2 + 2pn - s)$$

$$O_{DA} = \frac{h}{l} (s^2 + 2ps + 5s + 6p + ns + 2pn - n)$$

$$D = \frac{11ns + 11nps + 6ns^2 + 2s^2 + 10ps + 12pn + 2s + 3p + 2s^2p + 2sp^2 + 3p^2}{A_1}$$

$$= \frac{6sn^2 + 2n^2 + 11nps + 10pn + 11sn + 12ps + 2n + 3p + 2n^2p + 2np^2 + 3p^2}{A_2}$$

$$Q = \frac{10ns + 12nps + 3ns^2 + 2s^2 + 11ps + 11pn + 2s + 6p + 3sn^2 + 2n^2 + 2n}{A_4}$$

$$= \frac{12ns + 10nps + 3ns^2 + 11ps + 11pn + 2s^2p + 2sp^2 + 6p^2 + 3sn^2 + 2n^2p + 2np^2}{A_2}$$

$$\Delta = 22(pns + ps + ns + np) + 2(p^2s + ps^2 + n^2p + p^2n + s^2 + n^2 + n) + 6(n^2s + ns^2 + p^2 + p)$$

If the members AB and CD are identical, equations 763, 764, 765, and 766 reduce to the form

$M_{AB} = M_{AB}$ when shearing deformation is neglected (positive quantity) $+k$ (767)

$M_{BC} = M_{BC}$ when shearing deformation is neglected (negative quantity) $+k$ (768)

$M_{CD} = M_{CD}$ when shearing deformation is neglected (positive quantity) $-k$ (769)

$M_{DA} = M_{DA}$ when shearing deformation is neglected (negative quantity) $-k$ (770)

in which

$$k = 6 \frac{E}{G} \frac{PK}{(n+s+6)^2} \left(\frac{s+3}{A_1} - \frac{n+3}{A_3} \right)$$

If the members AB and CD are identical, and if DA and BC are also identical but not necessarily identical with AB and CD , k of equations 767 to 770 is zero, showing that for a rectangular frame with opposite sides identical the shearing deformation does not affect the moment in the frame.

The error due to shearing deformation in a number of frames is given in Table 31.

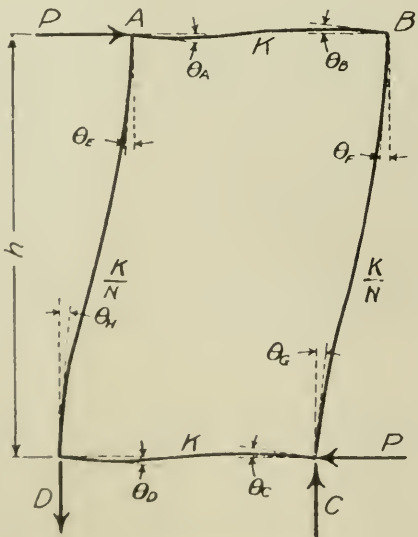


FIGURE 104

TABLE 31

ERROR IN STRESSES IN RECTANGULAR FRAMES DUE TO NEGLECTING SHEARING DEFORMATIONS

All frames are subjected to a single horizontal force at the top.

	Description of Frame						Error Due to Shearing Strain Per cent			
	Height in feet	Width in feet	Member	Area of Section in. ²	I in. ⁴	K in. ³	M_{AB}	M_{BC}	M_{CD}	M_{DA}
No Two Members Alike	30	50	AB	20	2500	4.16	.07	.03	.09	.02
			BC	20	300	.83				
			CD	40	5000	8.33				
			DA	100	6000	16.67				
	24	40	AB	20	2500	5.21	.115	.05	.14	.03
			BC	20	300	1.04				
			CD	40	5000	10.41				
			DA	100	6000	20.81				
	15	25	AB	20	2500	8.33	.30	.14	.35	.07
			BC	20	300	1.67				
			CD	40	5000	16.67				
			DA	100	6000	33.33				
AB and CD identical	30	50	AB and CD	20	2500	4.16	.032	.078	.078	.032
			BC	20	300	0.83				
			DA	100	6000	16.67				
	24	40	AB and CD	20	2500	5.21	.050	.122	.122	.050
			BC	20	300	1.04				
			DA	100	6000	20.81				
	15	25	AB and CD	20	2500	8.33	.127	.313	.313	.127
			BC	20	300	1.67				
			DA	100	6000	33.33				
	50	30	AB and CD	20	2500	6.95	.016	.072	.072	.016
			BC	20	300	0.50				
			DA	100	6000	10.00				
	40	24	AB and CD	20	2500	8.69	.025	.113	.113	.025
			BC	20	300	0.62				
			DA	100	6000	12.50				
	25	15	AB and CD	20	2500	13.90	.063	.290	.290	.063
			BC	20	300	1.00				
			DA	100	6000	20.00				

69. *Effect of Slip in the Connections upon the Moments in a Rectangular Frame Having a Single Horizontal Force at the Top.*—Fig. 104 represents a rectangular frame having a single horizontal force applied at the top. The sections of opposite sides of the frame are identical. The connections at the corners of the frame slip.

The changes in the slopes at the ends of the member AB are represented by θ_A at A and θ_B at B ; likewise for the member CD the change in slope at C is represented by θ_C and at D by θ_D ; for the member AD the change in slope at A is represented by θ_E and at D by θ_H ; and for the member BC the change in slope at B is represented by θ_F and at C by θ_G . That is: slip at A equals $\theta_E - \theta_A$; slip at B equals $\theta_F - \theta_B$; slip at C equals $\theta_G - \theta_C$; and slip at D equals $\theta_H - \theta_D$.

Represent $\frac{I}{l}$ for AB and CD by K , and $\frac{I}{l}$ for AD and BC by $\frac{K}{n}$. Applying equation A, Table 1, and equating the sum of the moments at each of the points A , B , C , and D to zero and also equating the sums of the moments at the ends of the two members AD and BC to $-Ph$ gives

$$2EK(2\theta_A + \theta_B) + \frac{2EK}{n}(2\theta_E + \theta_H - 3R) = 0 \quad . \quad . \quad . \quad . \quad . \quad (771)$$

$$2EK(2\theta_B + \theta_A) + \frac{2EK}{n}(2\theta_F + \theta_G - 3R) = 0 \quad . \quad . \quad . \quad . \quad . \quad (772)$$

$$2EK(2\theta_C + \theta_D) + \frac{2EK}{n}(2\theta_G + \theta_F - 3R) = 0 \quad . \quad . \quad . \quad . \quad . \quad (773)$$

$$2EK(2\theta_D + \theta_C) + \frac{2EK}{n}(2\theta_H + \theta_E - 3R) = 0 \quad . \quad . \quad . \quad . \quad . \quad (774)$$

$$\frac{2EK}{n} (2\theta_E + \theta_H - 3R + 2\theta_H + \theta_E - 3R + 2\theta_F + \theta_G - 3R + 2\theta_G + \theta_F - 3R) + Ph = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (775)$$

Letting A represent the quantity, slip at A divided by R , likewise letting B , C , and D represent the corresponding quantities at the points B , C , and D , respectively, gives

$$A = \frac{\theta_E}{R} - \frac{\theta_A}{R}$$

$$B = \frac{\theta_F}{R} - \frac{\theta_B}{R}$$

$$C = \frac{\theta_G}{R} - \frac{\theta_C}{R}$$

$$D = \frac{\theta_H}{R} - \frac{\theta_D}{R}$$

Substituting the values of θ_E , θ_F , θ_G , and θ_H from these equations in the preceding equations gives

$$\frac{2\theta_A}{nR}(1+n) + \frac{\theta_B}{R} + \frac{\theta_D}{nR} = \frac{1}{n}(3-2A-D) \quad (776)$$

$$\frac{2\theta_B}{nR}(1+n) + \frac{\theta_A}{R} + \frac{\theta_C}{nR} = \frac{1}{n}(3-2B-C) \quad (777)$$

$$\frac{2\theta_C}{nR}(1+n) + \frac{\theta_D}{R} + \frac{\theta_B}{nR} = \frac{1}{n}(3-2C-B) \quad (778)$$

$$\frac{2\theta_D}{nR}(1+n) + \frac{\theta_C}{R} + \frac{\theta_A}{nR} = \frac{1}{n}(3-2D-A) \quad (779)$$

$$2EKR = \frac{\frac{1}{3}Phn}{-\left[\frac{\theta_A}{R} + \frac{\theta_B}{R} + \frac{\theta_C}{R} + \frac{\theta_D}{R}\right] + 4 - (A+B+C+D)} \quad (780)$$

These five equations contain four unknown angles and one unknown deflection. Solving these equations and substituting the values for the θ 's and R in the expressions for the moments gives

$$M_{AD} = \frac{1}{3} \frac{Ph}{4 - (A+B+C+D)} \left[\frac{(6A+3B-9) + n(16A+5B+4C+5D-30) + n^2(6A+3D-9)}{(n+3)(3n+1)} \right] \quad (781)$$

$$M_{DA} = \frac{1}{3} \frac{Ph}{4 - (A + B + C + D)} \left[\frac{(6D + 3C - 9) + n(16D + 5C + 4B + 5A - 30) + n^2(6D + 3A - 9)}{(n + 3)(3n + 1)} \right] \dots \dots \dots (782)$$

$$M_{BC} = \frac{1}{3} \frac{Ph}{4 - (A + B + C + D)} \left[\frac{(6B + 3A - 9) + n(16B + 5A + 4D + 5C - 30) + n^2(6B + 3C - 9)}{(n + 3)(3n + 1)} \right] \dots \dots \dots (783)$$

$$M_{CB} = \frac{1}{3} \frac{Ph}{4 - (A + B + C + D)} \left[\frac{(6C + 3D - 9) + n(16C + 5D + 4A + 5B - 30) + n^2(6C + 3B - 9)}{(n + 3)(3n + 1)} \right] \dots \dots \dots (784)$$

The values of $\frac{\theta_A}{R}$, $\frac{\theta_B}{R}$, $\frac{\theta_C}{R}$, and $\frac{\theta_D}{R}$ determined from equations 776

to 779 substituted in equation 780 gives

$$R = \frac{1}{4} \left[(RA + RB + RC + RD) + \frac{1}{6} \frac{Ph(n + 1)}{EK} \right] \dots \dots \dots (785)$$

in which RA , RB , RC , and RD represent the slips in the connections at A , B , C , and D respectively. If the slips are measured, R may be computed from equation 786. Knowing R and the slips, the values of A , B , C , and D may be computed. Substituting the values of A , B , C , and D in equations 781 to 784, the moments in a frame having connections which are not rigid may be determined.

If there is no slip in the connections the moment at each corner of the frame of Fig. 104 is $-\frac{1}{4}Ph$. The differences between $-\frac{1}{4}Ph$ and the moments given by equations 781, 782, 783, and 784 represent the effect of the slip in the connections.

$$M_{AD} = 3,030,000 \text{ in. lb.}$$

$$M_{DA} = 3,240,000 \text{ in. lb.}$$

$$M_{BC} = 3,050,000 \text{ in. lb.}$$

$$M_{CB} = 3,300,000 \text{ in. lb.}$$

If there is no slip in the connections, each of these moments is 3,150,000 in. lb. The errors in the moments due to neglecting the slip in the connections are as follows:

$$M_{AD}, \text{ error} = -3.8 \text{ per cent}$$

$$M_{DA}, \text{ error} = +3.0 \text{ per cent}$$

$$M_{BC}, \text{ error} = -3.0 \text{ per cent}$$

$$M_{CB}, \text{ error} = +4.6 \text{ per cent}$$

For a given slip, the error due to slip is greater for a frame having short stiff members than it is for a frame having long flexible members.

XIII. NUMERICAL PROBLEMS

The following numerical problems illustrate the use of the equations which have been derived.

70. *Girder Restrained at the Ends. Supports on Different Levels. Any System of Vertical Loads.*—Fig. 106 represents a 12 in.-31.5 lb. I-beam embedded in masonry at both ends. Because of upheaval by frost, or other causes, the beam which was originally horizontal now has one end higher than the other and the tangents to the elastic curve of the beam at the ends are inclined to the horizontal. The beam carries a concentrated load and a uniform load as shown. It is required to find the bending moments in the beam.

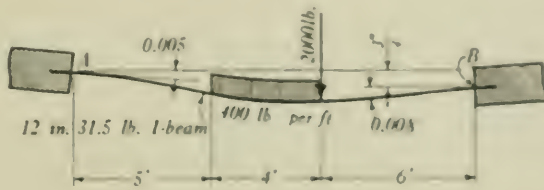


FIGURE 106

Equations 25 and 26, section 13, are applicable. E for steel is 30,000,000 lb. per sq. in.

$$d_B = .75 \text{ inch}$$

$$\theta_A = +.005$$

$$I = 215.8 \text{ in.}^4$$

$$\theta_B = -.008$$

$$l = 180 \text{ in.}$$

$$R = \frac{.75}{180} = .00416$$

$$K = \frac{215.8}{180} = 1.2 \text{ in.}^3$$

From Table 2, C_{AB} for a single concentrated load is

$$\frac{Pab^2}{l^2} \text{ in which}$$

$$a = 108 \text{ in.}$$

$$b = 72 \text{ in.}$$

$$P = 2000 \text{ lb.}$$

$$\frac{Pab^2}{l^2} = 34560 \text{ in. lb.}$$

Also from Table 2, C_{AB} for a uniform load over a portion of the span is $\frac{w}{12l^2} \left[d^3(4l-3d) - b^3(4l-3b) \right]$ in which.

$$d = 120 \text{ in.}$$

$$b = 72 \text{ in.}$$

$$w = 400 \text{ lb. per ft.} = \frac{400}{12} \text{ lb. per in.}$$

$$\frac{w}{12l^2} \left[d^3(4l-3d) - b^3(4l-3b) \right] = 37,200 \text{ in. lb.}$$

$$\text{Total } C_{AB} = 37,200 + 34,560 = 71,760 \text{ in. lb.}$$

Substituting the values of the constants in equation 25, section 13, gives

$$M_{AB} = -826,300 \text{ in. lb.}$$

From Table 2, C_{BA} for the concentrated load is 52,000 in. lb., and C_{BA} for the uniform load is 32,700 in. lb.

$$\text{Total } C_{BA} = 52,000 + 32,700 = 84,700 \text{ in. lb.}$$

Substituting the values of the constants in equation 26, section 13, gives

$$M_{BA} = -1,606,300 \text{ in. lb.}$$

It is to be noted in the solution of this problem, in solving for both M_{AB} and M_{BA} , that θ_A and R are both positive whereas θ_B is negative. The minus sign, moreover, is used before C_{AB} and the plus sign is used before C_{BA} . The signs are in accordance with the conventional method of fixing signs given at the bottom of Table 1.

If the supports had been on the same level and if the tangents to the elastic curves at the ends of the beams had been horizontal, the moments would have been given by the equations of section 11. Using the same values of C_{AB} and C_{BA} as before

$$M_{AB} = -C_{AB} = -71,760 \text{ in. lb.}$$

$$M_{BA} = +C_{BA} = +84,700 \text{ in. lb.}$$

71. *Girder Continuous over Four Supports. Supports on Different Levels. Any System of Vertical Loads.*—Fig. 107 represents a 20 in.-80 lb. I-beam supported on four supports and having hinged ends. The supports are all on different levels. It is required to find the moments in the girder.

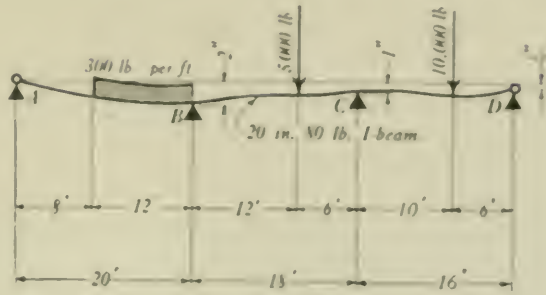


FIGURE 107

Equations (a), Table 6, are applicable.

$$I_o = I_1 = I_2 = 1466 \text{ in.}^4$$

$$E = 30,000,000 \text{ lb. per sq. in.}$$

$$l_o = 240 \text{ in.}$$

$$d_A = 0$$

$$l_1 = 216 \text{ in.}$$

$$d_B = 2 \text{ in.}$$

$$l_2 = 192 \text{ in.}$$

$$d_C = 1 \text{ in.}$$

$$K_o = \frac{1466}{240} = 6.10 \text{ in.}^3$$

$$d_D = .5 \text{ in.}$$

$$K_1 = \frac{1466}{216} = 6.79 \text{ in.}^3$$

From Table 2

$$K_2 = \frac{1466}{192} = 7.64 \text{ in.}^3$$

$$H_{BA} = \frac{3600 \times 144}{8 \times 240 \times 240} (2 \times 240 - 144)^2 = 127,000 \text{ in. lb.}$$

$$n_o = \frac{6.79}{6.10} = 1.11$$

$$H_{BC} = \frac{5000 \times 144 \times 72 (216 + 72)}{2 \times 216 \times 216} = 160,000 \text{ in. lb.}$$

$$n_1 = \frac{7.64}{6.79} = 1.125$$

$$H_{CH} = \frac{5000 \times 144 \times 72 (216 + 144)}{2 \times 216 \times 216} = 200,000 \text{ in. lb.}$$

$$H_{CD} = \frac{10000 \times 120 \times 72 (192 + 72)}{2 \times 192 \times 192} = 310,000 \text{ in. lb.}$$

Substituting these values in equations (a) of Table 6 gives

$$M_{BC} = 4,071,500 \text{ in. lb.}$$

$$M_{CD} = -1,987,500 \text{ in. lb.}$$

If the supports had all been on the same level, the moments would have been

$$M_{BC} = -88,500 \text{ in. lb.}$$

$$M_{CD} = -227,500 \text{ in. lb.}$$

72. *Girder Continuous over Five Supports. Supports on Different Levels. Any System of Vertical Loads. Ends of Girder Restrained.*—Fig. 108 represents a girder continuous over five supports. All the supports are on different levels and the girder is restrained at the ends. The slopes of the elastic curve at the ends are known. It is required to determine the moments in the girder in terms of I , the moment of inertia of the girder.

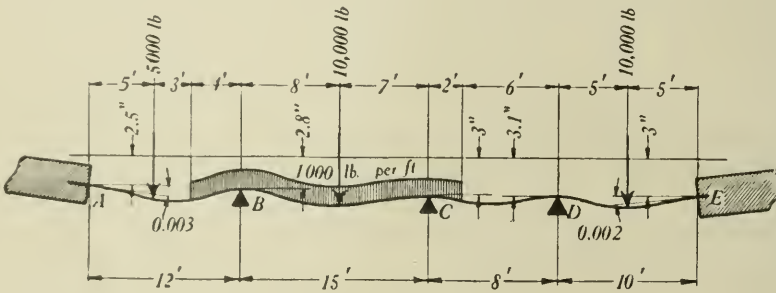


FIGURE 108

The solution of the problem involves writing the equations of three moments and solving these equations for the moments.

Cases (a), (d), and (e), Table 4, and the equations of Table 2 are applicable.

$$K_o = \frac{I}{12 \times 12} = \frac{I}{144} \text{ in.}^3 \quad n_1 = \frac{K_1}{K_o} = \frac{4}{5} = .800 \quad l_o = 12 \times 12 = 144 \text{ in.}$$

$$K_1 = \frac{I}{12 \times 15} = \frac{I}{180} \text{ in.}^3 \quad n_2 = \frac{K_2}{K_1} = \frac{15}{8} = 1.875 \quad l_1 = 12 \times 15 = 180 \text{ in.}$$

$$K_2 = \frac{I}{12 \times 8} = \frac{I}{96} \text{ in.}^3 \quad n_3 = \frac{K_3}{K_2} = \frac{4}{5} = .800 \quad l_2 = 12 \times 8 = 96 \text{ in.}$$

$$K_3 = \frac{I}{12 \times 10} = \frac{I}{120} \text{ in.}^3 \quad l_3 = 12 \times 10 = 120 \text{ in.}$$

$$d_A = 2.5 \text{ in.} \quad d_B - d_A = 2.8 - 2.5 = +0.3 \text{ in.} \quad \theta_A = +.003$$

$$d_B = 2.8 \text{ in.} \quad d_C - d_B = 3.0 - 2.8 = +0.2 \text{ in.} \quad \theta_E = -.002$$

$$d_C = 3.0 \text{ in.} \quad d_D - d_C = 3.1 - 3.0 = +0.1 \text{ in.} \quad E = 30,000,000 \text{ lb. per sq. in.}$$

$$d_D = 3.1 \text{ in.} \quad d_E - d_D = 3.0 - 3.1 = -0.1 \text{ in.}$$

$$d_E = 3.0 \text{ in.}$$

$$\begin{aligned} H_{AB} &= \frac{5000 \times 5 \times 7 \times 19 \times 12}{2 \times 144} + \frac{12000(16) \times (2 \times 144 - 16)}{8 \times 144} \\ &= -183,900 \text{ in. lb.} \end{aligned}$$

$$\begin{aligned} H_{BA} &= \frac{5000 \times 5 \times 7 \times 17 \times 12}{2 \times 144} + \frac{12000}{8 \times 144} \left((144 - 64) \right) \left(2 \times 144 - 144 - 64 \right) \\ &= 190,700 \text{ in. lb.} \end{aligned}$$

$$H_{BC} = \frac{10000 \times 8 \times 7 \times 22 \times 12}{2 \times 225} + \frac{12000}{8} \times 225 = 665,500 \text{ in. lb.}$$

$$H_{CB} = \frac{10000 \times 8 \times 7 \times 23 \times 12}{2 \times 225} + \frac{12000}{8} \times 225 = 681,500 \text{ in. lb.}$$

$$H_{CD} = \frac{12000}{8 \times 64} \times (64 - 36) \left(2 \times 64 - 36 - 64 \right) = 18,400 \text{ in. lb.}$$

$$H_{DC} = \frac{12000}{8 \times 64} \left(4 \right) \left(2 \times 64 - 4 \right) = 11,600 \text{ in. lb.}$$

$$H_{DE} = \frac{10000 \times 5 \times 5(15) \times 12}{2 \times 100} = 225,000 \text{ in. lb.}$$

$$H_{ED} = \frac{10000 \times 5 \times 5(15) \times 12}{200} = 225,000 \text{ in. lb.}$$

Substituting the values of these quantities in the equations of Table 4 gives

From equation (d)

$$6.4 M_{BC} + 2.0 M_{CB} = 1030 I - 2,978,000 \quad (786)$$

From equation (a)

$$1.875 M_{BC} + 5.75 M_{CD} + M_{DE} = 130 I - 2,593,000 \quad . \quad . \quad . \quad (787)$$

From equation (e)

$$1.6 M_{CD} + 6.2 M_{DE} = 3900 I - 487,100 \quad . \quad . \quad . \quad . \quad . \quad . \quad (788)$$

From equations 786, 787, and 788

$$M_{DE} = (672 I + 9,500) \text{ in. lb.}$$

$$M_{CD} = -(166 I + 341,000) \text{ in. lb.}$$

$$M_{BC} = (213 I - 358,000) \text{ in. lb.}$$

Substituting the values of M_{BC} and M_{CD} in equation (a) of Table 4 gives

$$M_{AB} = (467 I + 18,750) \text{ in. lb.}$$

Also from equation (a) of Table 4

$$M_{ED} = -(530 I - 193,400) \text{ in. lb.}$$

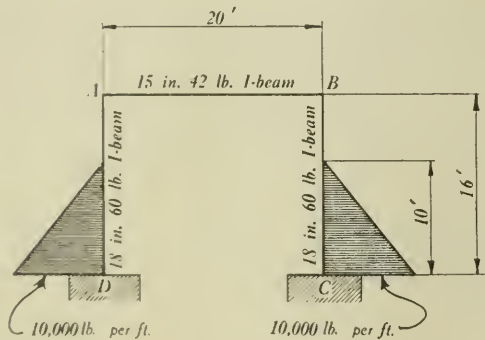


FIGURE 109

73. *Two-legged Rectangular Bent with Legs Fixed at the Bases. Bent and Loading Symmetrical about a Vertical Center Line.*—Fig. 109 represents a two-legged rectangular bent with legs fixed at the bases. Both legs are subjected to hydrostatic pressure. The bent and loading are symmetrical about a vertical center line. It is required to determine the moments in the frame.

Equations 126 and 127, section 25, are applicable.

$$n = \frac{K \text{ for } AB}{K \text{ for } AD} = \frac{\frac{441.8}{240}}{\frac{841.8}{192}} = .42$$

From Table 2

$$C_{AD} = \frac{Wa^2}{30l^2}(5l-3a) \text{ in which}$$

$$C_{DA} = \frac{Wa}{30l^2}(3a^2-10al+10l^2)$$

$$W = \frac{10,000 \times 10}{2} = 50,000 \text{ lb}$$

$$a = 120 \text{ inches}$$

$$l = 192 \text{ inches}$$

$$C_{AD} = 391,000 \text{ in. lb.}$$

$$C_{DA} = 984,400 \text{ in. lb.}$$

$$M_{AB} = M_{BC} = -\frac{0.42}{0.42+2} \times 391,000 = -67,900 \text{ in. lb.}$$

$$M_{CB} = -M_{DA} = \frac{1}{0.42+2} \times 391,000 + 984,400 = 1,145,900 \text{ in. lb.}$$

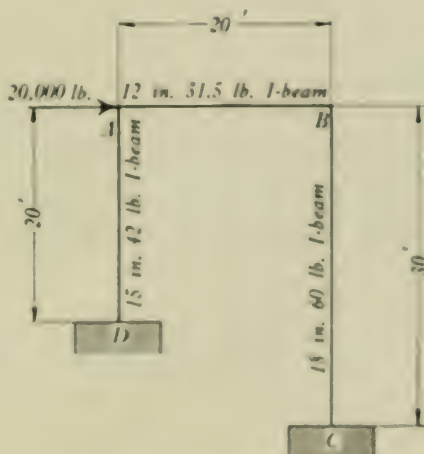


FIGURE 110

74. *Two-legged Rectangular Bent. One Leg Longer than the Other. Concentrated Horizontal Load at the Top. Legs Fixed at the Bases.—*

Fig. 110 represents a rectangular bent having one leg longer than the other. There is a horizontal force of 20,000 lb. applied at the top. The bases of the legs are fixed. It is required to determine the moments in the bent.

Equations 252, 253, 254, and 255, section 33, are applicable.

$$P = 20,000 \text{ lb.}$$

$$l = 240 \text{ inches}$$

$$h = 240 \text{ inches}$$

$$q = \frac{240}{360} = 0.667$$

$$K \text{ for } AB = \frac{215.8}{240} = 0.90 \text{ in.}^3$$

$$K \text{ for } AD = \frac{441.8}{240} = 1.84 \text{ in.}^3$$

$$K \text{ for } BC = \frac{841.8}{360} = 2.34 \text{ in.}^3$$

$$n = \frac{K \text{ for } AB}{K \text{ for } AD} = 0.49$$

$$s = \frac{K \text{ for } AB}{K \text{ for } BC} = 0.385$$

$$\begin{aligned} \Delta_o = & 2(3 \times .49 \times .385 \times .385 + 4 \times .49 \times .385 + .385 \times .385 + .385 + 3 \\ & \times .49 \times .385 \times .667 + 3 \times .49 \times .49 \times .385 \times .667 \times .667 + .49 \\ & \times .49 \times .667 \times .667 + .49 \times .667 \times .667 + 4 \times .49 \times .385 \times .667 \\ & \times .667) = 5.334 \end{aligned}$$

Substituting the values of the quantities in equations 252, 253, 254, and 255, of section 33, gives

$$M_{AB} = 1,100,000 \text{ in. lb.}$$

$$M_{BA} = 958,000 \text{ in. lb.}$$

$$M_{DA} = -1,950,000 \text{ in. lb.}$$

$$M_{CB} = -1,664,000 \text{ in. lb.}$$

75. *Two-legged Rectangular Bent. Settlement of Foundation.*—Fig. 111 represents a rectangular portal. Due to upheaval by frost or other causes the foundation originally at C' moves 2 inches to the right, settles 3 inches, and turns in a negative direction an angle of .01 radians. It is required to determine the moments in the portal.

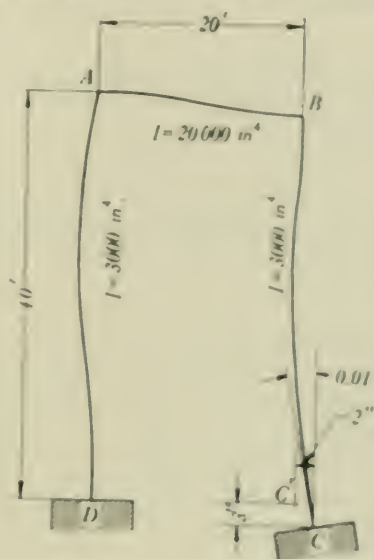


FIGURE 111

Equations 204, 205, 206, and 207, section 31, are applicable.

$$l = 240 \text{ inches}$$

$$h = 480 \text{ inches}$$

$$K = \frac{20,000}{240} = 83.4 \text{ in.}^3$$

$$n = \frac{K \text{ for } AB}{K \text{ for } AD} = \frac{83.4}{6.25} = 13.3$$

$$\theta_C = -.01 - \frac{3}{242} = -.0224$$

$$\theta_D = 0$$

$$d = 2 \text{ inches}$$

$$E = 30,000,000 \text{ lb. per sq. in.}$$

$$M_{AB} = 2,502,000,000 [0.000816 - 0.001461 - 0.000831] \\ = -3,700,000 \text{ in. lb.}$$

$$M_{BC} = 2,502,000,000 [0.000816 - 0.001461 + 0.000831] \\ = +470,000 \text{ in. lb.}$$

$$M_{CB} = 2,502,000,000 [0.000877 - 0.003255 - 0.000831] \\ = -8,040,000 \text{ in. lb.}$$

$$M_{DA} = -2,502,000,000 [0.000877 - 0.003255 + 0.000831] \\ = +3,875,000 \text{ in. lb.}$$

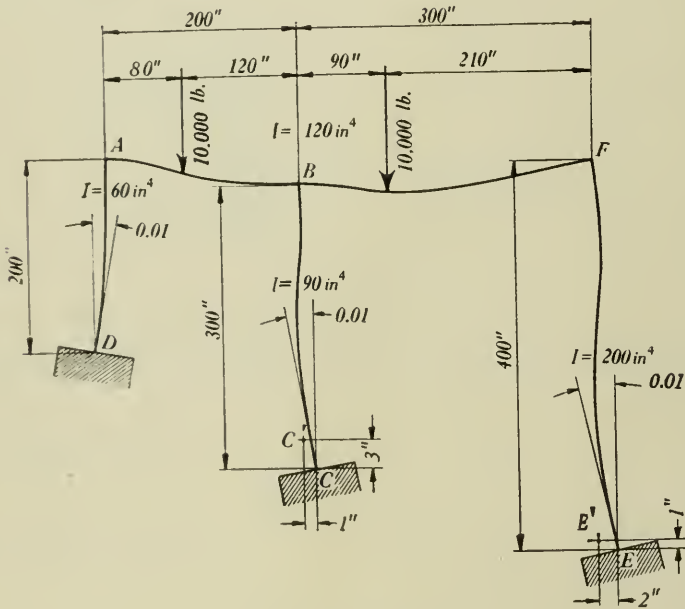


FIGURE 112

76. *Three-legged Bent. Lengths of All Legs Different. Loads on Top and Settlement of Foundations.*—Fig. 112 represents a three-legged bent having vertical loads on top. The legs are restrained at the bases. Due to upheaval by frost or other causes the foundation at D has rotated in a positive direction through an angle of 0.01 radians. Likewise the foundations at C and E have rotated in a negative direction through an angle 0.01. The foundation, originally at C' , has moved, moreover, to the right 1 inch and has settled 3 inches. Likewise the foundation, originally at E' , has moved to the right 2 inches and has settled 1 inch. It is required to determine the moments in the frame.

The equations of Table 21 are applicable. Since there are no horizontal loads on the legs nor external couples at the top, Cases IX, X, and XII will not enter.

The total right-hand members of the equations of Table 21 will therefore be the algebraic sum of the right-hand members due to Case VII, Case VIII, and Case XI.

$h_o = 200 \text{ in.}$	$I_o = 60 \text{ in.}^4$
$h_2 = 300 \text{ in.}$	$I_1 = 120 \text{ in.}^4$
$h_4 = 400 \text{ in.}$	$I_2 = 90 \text{ in.}^4$
$l_1 = 200 \text{ in.}$	$I_3 = 120 \text{ in.}^4$
$l_3 = 300 \text{ in.}$	$I_4 = 200 \text{ in.}^4$
$K_o = 0.30 \text{ in.}^5$	$C_{AB} = 2,880,000 \text{ in. lb.}$
$K_1 = 0.60 \text{ in.}^3$	$C_{BA} = 1,920,000 \text{ in. lb.}$
$K_2 = 0.30 \text{ in.}^3$	$C_{BF} = 4,410,000 \text{ in. lb.}$
$K_3 = 0.40 \text{ in.}^3$	$C_{FB} = 1,890,000 \text{ in. lb.}$
$K_4 = 0.50 \text{ in.}^3$	$E = 30,000,000 \text{ lb. per sq. in.}$
$\frac{6d_1K_1}{l_1} = \frac{6 \times 3 \times 0.6}{200} = 0.054 \text{ in.}^3$	$K_o\theta_D = 0.003 \text{ in.}^3$
$\frac{6d_2K_2}{h_2} = \frac{6 \times 1 \times 0.3}{300} = 0.006 \text{ in.}^3$	$K_2\theta_C = -0.003 \text{ in.}^3$
$\frac{6(d_3-d_1)K_3}{l_3} = \frac{6 \times (-2) \times 0.4}{300}$ $= -0.016 \text{ in.}^3$	$K_4\theta_E = -0.005 \text{ in.}^3$
$\frac{6d_4K_4}{h_4} = \frac{6 \times 2 \times 0.5}{400} = 0.015 \text{ in.}^3$	

Substituting the values of these quantities in the equations of Table 23 gives equations 1, 2, 3, and 4 of Table 32. It is to be noted in the equations of Table 32 that the quantities under the headings Case VII and Case VIII are coefficients of $\frac{100}{E}$. Solving equations 1, 2, 3, and 4 by the method of elimination, as given in Table 32, gives

$$d = \frac{1}{E} \left[49,373,300 - 19,722,100 \right] + 1.19314$$

$\theta_F = \frac{1}{E} \left[238,155 - 797,034 \right] - 0.0047062$

$\theta_B = \frac{1}{E} \left[-608,835 + 1,039,263 \right] + 0.0061134$

$\theta_A = \frac{1}{E} \left[1,126,370 - 395,727 \right] + 0.014278$

Substituting these values of the θ 's and d in the equations of Table 21 gives the moments in the frame. The moments are itemized and presented in Table 33.

TABLE 32
EQUATIONS FOR THE THREE-LEGGED BENT OF FIG. 112

No. of Equation	Left-hand Member of Equation				Right-hand Member of Equation		
	θ_A	θ_B	θ_F	d	Case VII Coefficients of $\frac{100}{E}$	Case VIII Coefficients of $\frac{100}{E}$	Case XI
1	3.6	1.2	0	-.0090	+28800	0	.04800
2	1.2	5.2	.80	-.0060	-19200	+44100	.03800
3	0	.8	3.60	-.0075	0	-18900	-.02100
4 $\times 1000$	1.5	1.0	1.25	-.02792	0	0	-.01167
1	+1.0	+ .3333	0	-.002500	+ 8000	0	+.013333
2	+1.0	+4.3333	+ .6667	-.005000	-16000	+36750	+.031667
3	0						
4	+1.0	+ .6667	+ .8333	-.018611	0	0	-.007777
(2-1) = a	0	+4.0000	+ .6667	-.002500	-24000	+36750	+.018333
(2-4) = b	0	+3.6667	- .1667	+.013611	-16000	+36750	+.039444
a		+1.0	+.166667	-.000625	-6000	+ 9187.5	+.004583
b		+1.0	-.045454	+.003712	-4363.63	+10022.7	+.010757
3		+1.0	+4.500000	-.009375	0	-23625.0	-.026250
(3-a) = c		0	+4.333333	-.008750	+6000	-32812.5	-.030833
(3-b) = d		0	+4.545454	-.013087	+4363.63	-33647.7	-.037007
c			+1.0	-.0020192	+1384.61	-7572.11	-.0071154
d			+1.0	-.0028792	+ 960.00	-7402.50	-.0081415
(c-d) = e			0	+.0008600	+424.61	-169.61	+.0010261
e				+1.0	+493733	-197221	+1.19314

TABLE 33
MOMENTS IN THREE-LEGGED BENT OF FIG. 112
Moments are expressed in inch-pounds.

Moment	Due to Load on <i>AB</i>	Due to Load on <i>BF</i>	Due to Settlement of Foundations	Total
<i>M_{DA}</i>	+ 231500	− 59900	+294900	+ 466500
<i>M_{AB}</i>	− 907300	+ 297400	−371800	− 981700
<i>M_{BA}</i>	+ 1810400	+ 2019400	−665800	+ 3164000
<i>M_{BC}</i>	− 1026800	+ 1365400	+ 5300	+ 343900
<i>M_{CB}</i>	− 661500	+ 741900	−284700	− 204300
<i>M_{BP}</i>	− 783600	− 3384800	+660500	− 3507900
<i>M_{FB}</i>	− 406000	+ 1446200	+400800	+ 1741000
<i>M_{EP}</i>	− 132100	− 649100	−559600	− 1340800

XIV. CONCLUSIONS

Some outstanding features of the use of the slope-deflection equations, as brought out by the analysis in Part II, may well be emphasized.

Two general methods of using the equations have been illustrated. In one case, after the equations have been written for each member of a frame, by equating the sum of the moments at each joint to zero and employing one equation of statics, a number of equations is obtained which contain values of θ and R as the only unknowns. From these equations can be found values of θ and R , which, when substituted in the original slope-deflection equations, give values of the various moments. This method applies especially well to a frame in which a large number of members meet at each joint. Such a problem is generally best solved in numerical terms. Examples of this method are found in sections 23, 61, 64, and 65.

The procedure in the other case is more direct. The slope-deflection equations for each member may be combined to eliminate values of θ and R , leaving equations involving the unknown moments, the properties of the members, and the given loading of the frame. These equations may be solved directly for the moments. Examples of this method are found in sections 24, 35, and 49.

Special attention is called to the form of the equations, which are independent of the magnitude and location of the individual loads, except as the magnitude and location of such loads influence the numerical values of the quantities C and H of the equations. The quantities C and H are determinate and their numerical values may be readily found for any known system of loads.

It is well to note also that the treatment of continuous girders for which the supports are not on the same level is comparatively simple. This part of the work is of considerable value inasmuch as it permits the determination of the effect of the settlement of supports. The treatment of the settlement of foundations for two-legged bents is of equal importance.

Advantages of the slope-deflection equations which are worthy of appreciation are:

- (1) The general form of the fundamental equation is easily memorized, and the equations may be written for all members of a structure with little effort. The value of the quantities C or H

for loaded members may be calculated by reference to Tables 2 and 3. It is frequently possible to simplify the equations through noting where values of θ and P must be equal to zero from the conditions of the problem.

(2) No integrations need be performed except possibly to find values of C or H , and there is little danger of the omission of the effect of a single indeterminate quantity, as there is in methods involving the work of internal forces or moments.

(3) The physical conception of a problem is easier than in the case where differentiation or integration is performed. When the slopes and deflections are determined, it is easy to visualize the approximate shape of the elastic curve of a member, whereas an expression involving the work of an indeterminate force or moment may have little physical meaning. Neither does the method of cutting a member and equating expressions for the linear and angular movement of the adjoining ends give so clear an idea of the actual deformation. To one unfamiliar with such a method, the determination of the sign of the movement of the ends of the member cut is also more or less difficult.

(4) It is shown in sections 67, 68, and 69 that the effect of axial and shearing deformations and of slip of joints may be calculated by the use of the slope-deflection equations. This makes possible a complete treatment of any problem, though it is shown that it is seldom necessary to make use of such refinements in an analysis.

(5) The use of the quantity K for $\frac{I}{l}$ of a member, and also of quantities n , s , and p , as ratios of K 's for different members, is of great help in writing equations in a workable form. The restraint factor N is also useful in simplifying both analyses and final equations.

(6) Although the fact has not been brought out in this bulletin, these equations may be applied to many structures not composed of rectangular units. The determination of secondary stresses in bridge trusses is an example of such use which has been in print for some time. With trapezoidal and triangular frames, care must be taken in the use of the term R for adjoining members.

(7) While the method is readily applicable to all the problems solved in this bulletin, its advantage over other methods is seen when applied to structures which are statically indeterminate to a high degree and in which a number of members meets at each joint.

The use of statically indeterminate structures in recent years has grown rapidly and many new types of structures have been evolved. With the use of riveted connections in steel frames and the development of monolithic reinforced concrete structures of all sorts, it often happens that statically indeterminate stresses cannot be avoided. On the other hand, structures are frequently made of an indeterminate type for the purpose of securing economy of material. Rational methods of design will do much to inspire confidence in the reliability and economy of such structures, thus insuring their more widespread use.

It is felt that the treatment of statically indeterminate structures given in this bulletin will be helpful in giving information regarding such structures. The method has been explained in sufficient detail to enable the designing engineer to use it in the solution of his particular problems. It is believed that the fundamental principles can be quickly coördinated with the ordinary principles of mechanics so that the more complex problems and even the simpler ones may be studied from a new viewpoint.

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DECEMBER, 1918

THE ORIFICE AS A MEANS OF
MEASURING FLOW OF WATER
THROUGH A PIPE

BY

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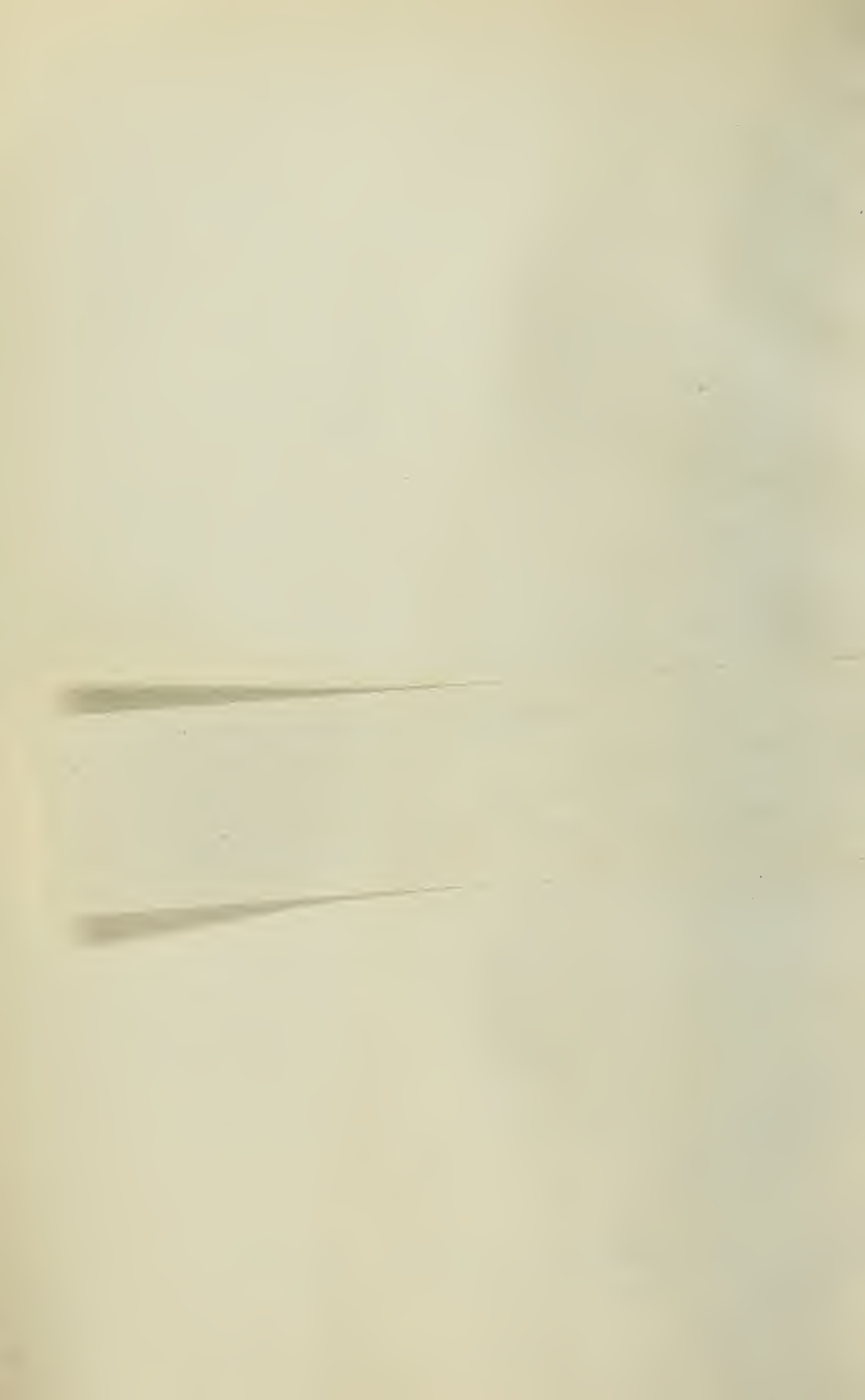
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CONTENTS

I. INTRODUCTION

	PAGE
1. Scope of Bulletin	7
2. Other Experiments	8
3. Acknowledgment	8
4. Notation	9

II. ANALYTICAL RELATION BETWEEN VELOCITY, DROP IN PRESSURE HEAD, AND LOST HEAD

5. Relation between Drop in Pressure Head and Velocity of Flow	11
6. Expression for Lost Head	12

III. TESTS AND RESULTS

7. Scope of Experiments	14
8. Apparatus	16
9. Water Supply and Measurement	16
10. Pressure Changes near Orifice	19
11. Deductions on Behavior of the Jet.	21
12. Relation between Drop in Pressure Head and Discharge.	22
13. Simplified Equations for Velocity and Discharge.	31
14. Application to Other Sizes of Pipes.	34
15. Empirical Relation between Velocity and Drop in Pressure Head	35
16. Empirical Formula for Lost Head.	35
17. Choice of Orifice	36
18. Computations for Discharge	39
19. Errors and Precautions	40

IV. EFFECT OF DEVIATION FROM STANDARD CONDITIONS

20. General Remarks	43
21. Throttling	43

CONTENTS (CONTINUED)

	PAGE
22. Position and Number of Gage Connections	44
23. Eccentricity	45
24. Bevel-edged Orifices	46

V. SUMMARY

25. General Applicability of Pipe Orifice Method. . .	50
26. Conclusions.	50

LIST OF FIGURES

NO	PAGE
1. Behavior of Jet	11
2. Apparatus for Observing Pressure Changes	17
3. Pressure Variations near Orifice	20
4. Pipe Connections near Orifice	23
5. Differential Gages	24
6. Experimental Discharge Curves— 4-inch pipe.	28
7. Experimental Discharge Curves— 6-inch pipe.	29
8. Experimental Discharge Curves—12-inch pipe.	30
9. Coefficients of Discharge	32
10. Diameter Factor	34
11. Relation between Lost Head and Drop in Pressure Head	37
12. Relation between Maximum Drop in Pressure and Size of Orifice.	37
13. Emergency Differential Gage	41
14. Coefficients of Discharge for Bevel-edged Orifices	47

LIST OF TABLES

NO	PAGE
1. Diameters of Orifices and Pipes	14
2. Position of Sections at Pressure Connections	19
3. Sample Data	25—27
4. Coefficient of Discharge	30
5. Velocity Modulus	33
6. Approximate Values of Drop in Pressure and Lost Head.	38
7. Coefficients of Discharge for Bevel-edged Orifices	46

THE ORIFICE AS A MEANS OF MEASURING FLOW OF WATER THROUGH A PIPE

I. INTRODUCTION

1. *Scope of Bulletin.*—In this bulletin are given the results of tests made to determine the practicability of measuring the flow of water by means of the thin-plate circular orifice inserted in a pipe, to determine the experimental coefficients for calculating the velocity of the flow in the pipe and the discharge, and to determine the conditions most favorable to the use of such an orifice as a flow measuring device. An orifice thus inserted causes an abrupt change in the conditions of flow as the stream approaches and passes through the orifice and this change in the conditions of flow is accompanied by a considerable change in pressure head which may be readily measured and which varies with the velocity of flow in the pipe. The orifice inserted in a pipe line is looked upon more especially as a temporary or field device for measuring the flow of water through a pipe, being inexpensive, simple to construct, light in weight, and easy to install. There may be opportunities for its use in permanent installations where a continuous record is not necessary, loss in head is not an important factor, and the expense of any one of the common flow meters is not justified. In flanged pipe systems the thin-plate orifice may be inserted at a joint with little or no disturbance of existing piping; and in long pipe lines the loss in head caused by the orifice will be inconsiderable as compared with other losses. It would seem that this device might well be utilized, for example, in testing the efficiency of certain forms of pumps, in measuring the performance of individual wells of water works systems, in determining the water consumption for individual purposes in mills and factories, in measuring the discharge through city mains, and in distributing water for irrigation purposes. In planning and conducting the tests the conditions under which the orifice would ordinarily be found useful have been borne in mind, and no attempt has been made to refine apparatus or methods of observing beyond what might be expected in practice.

Tests were made to determine: (1) the positions of two cross-sections of the pipe, one section upstream from the orifice, the other downstream from the orifice, at which pressure head may be measured and the drop in pressure from one section to the other may be most favorably determined, (2) the relation between this drop in pressure head and the rate of discharge through the pipe, (3) the lost head occasioned by the orifice, (4) the effect of small deviations from what may be termed standard conditions, and (5) the proper size of orifice for given conditions. In the presentation of the results of these tests special attention has been given to the probable sources, magnitudes, and effects of the accidental and constant errors incidental to the observations. Attempt has been made through the use of tables and curves to render the results easily adaptable to all sizes of pipe from 4 inches to 20 inches in diameter and for all sizes of orifice up to five-sixths of the diameter of the pipe. In the belief that others than experienced hydraulicians may find use for the results, some attention has been given to matters connected with the construction, installation, and use of a form of apparatus adapted to normal practice.

2. *Other Experiments.*—The method of measuring water by inserting an orifice in the pipe line is not new. Partially closed valves have been calibrated as orifices for such use and experiments of a preliminary nature have been conducted recently with thin-plate orifices. In "Experiments on Water Flow through Pipe Orifices,"* Horace Judd reports the progress of somewhat similar experiments conducted at Ohio State University, the orifices being in plates of Monel metal 1/32 inch thick inserted in a 5-inch pipe.

In "Diaphragm Method of Measuring the Velocity of Fluid Flow in Pipes,"† Holbrook Gaskell, Jr., gives an account of a brief series of tests on 6-inch and 8-inch pipe.

3. *Acknowledgment.*—The tests were made in the Laboratory of Applied Mechanics of the University of Illinois in 1914-15 as graduate work in the Department of Theoretical and Applied Mechanics and in 1916-17 as an investigation of the University of Illinois Engineering Experiment Station. The investigations were under the general supervision of ARTHUR N. TALBOT, Professor of Municipal and Sanitary Engineering and in Charge of the Department of Theoretical and

*Jour. of the Am. Soc. of M. E., Sept. 1916.

†Minutes of Proc. of the Inst. of C. E., 1914.

Applied Mechanics. To Professor Talbot credit is due for valuable suggestions made during the preparation of this bulletin. To PROFESSORS M. L. ENGER and FRED B SELLY, of the Department of Theoretical and Applied Mechanics, credit is due for many helpful suggestions made during the progress of the work.

The tests treated in this bulletin are the outgrowth of brief experiments by C. S. MULVANEY, of the class of 1914, conducted under the direction of V. R. FLEMING, Assistant Professor of Applied Mechanics. The results of the experiments of Mr. Mulvaney were presented as an undergraduate thesis.

4. *Notation.*—Throughout the bulletin the following notation will be used:

V = mean velocity (ft. per sec.) in pipe at a section of normal uniform flow.

v = velocity (ft. per sec.) of the jet issuing from the orifice at the section of greatest contraction.

A = cross-sectional area of the pipe (sq. ft.).

Q = rate of discharge (cu. ft. per sec.).

D = diameter of the pipe (in.).

d = diameter of the orifice (in.).

h_a = lost head (ft. of water) caused by the orifice, including the loss due both to contraction and to sudden expansion of the jet.

h_b = drop in pressure head (ft. of water); or change in pressure head as registered by a differential gage one column of which is connected to the pipe at a section of normal uniform flow upstream from the orifice and the other column of which is connected to the pipe in the same plane as the section of greatest contraction of the jet downstream from the orifice.

C = coefficient of discharge.

C_c = coefficient of contraction.

C_v = coefficient of velocity.

g = acceleration of gravity (ft. per sec.²).

C_p = factor depending on diameter of pipe.

K = velocity modulus; or velocity corresponding to a drop in pressure head (h_b) of 1 ft.

*the main
stage* ~~C_r~~ = ratio factor, depending on $\frac{D}{d}$.

F = factor showing the dependence of h_a on diameter of pipe.

J = coefficient expressing the relation between h_a and h_b .

C' = coefficient of discharge for bevel-edged orifices.

II. ANALYTICAL RELATION BETWEEN VELOCITY, DROP IN PRESSURE HEAD, AND LOST HEAD

5. *Relation between Drop in Pressure Head and Velocity of Flow.*—In the use of the pipe orifice the velocity of flow in the pipe (or the rate of discharge) is calculated from the drop in the pressure head between a section upstream from the orifice and a section downstream from the orifice. It is desirable, therefore, to develop a simple rational expression for the velocity in terms of the drop in the pressure head. Furthermore, since the loss of head caused by the orifice in the pipe may be a determining factor in the selection of the proper orifice it is desirable that a rational expression be found for the lost head. Assumptions have been made concerning the behavior of the jet which have some experimental justification as will be discussed later.

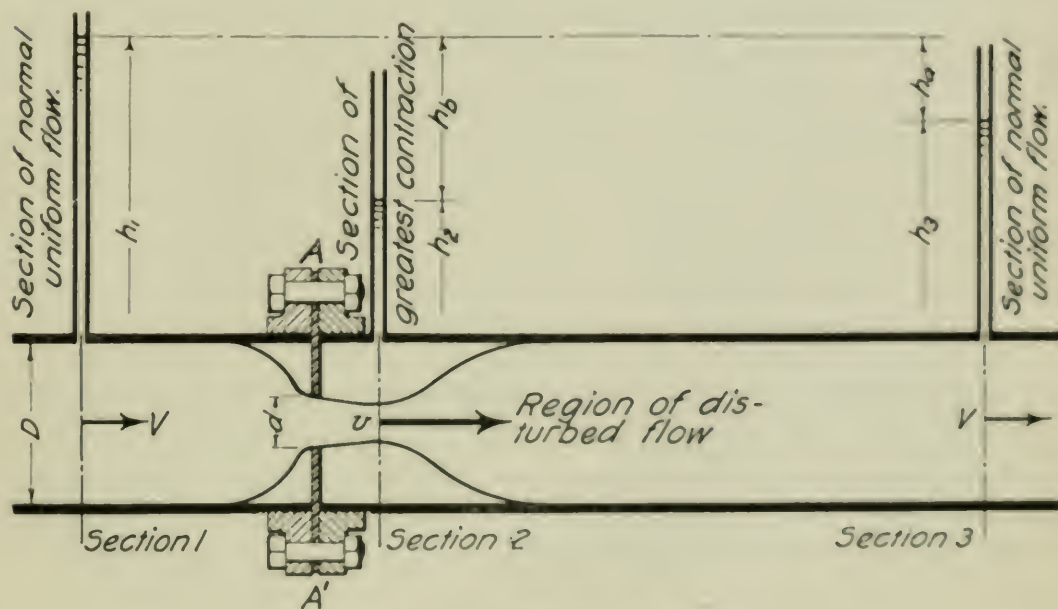


FIG. 1. BEHAVIOR OF JET

Fig. 1 represents a longitudinal section of a pipe with the water passing through an orifice at AA' . Let section 1 be a section of normal uniform flow on the upstream side of the orifice. As the stream approaches the orifice from this section it converges and, if it may be

assumed to behave not unlike the jet issuing from the standard orifice in the open air, it continues to converge after leaving the orifice until the greatest contraction takes place at, let us say, section 2. The jet then expands until normal flow takes place at section 3.

Let h_1 , h_2 , and h_3 respectively be the pressure heads registered by piezometer tubes inserted at sections 1, 2, and 3, and let it be assumed that the pressure head h_2 is the same as it would be if the piezometer tube at section 2 were projecting through the wall of the pipe to the periphery of the jet at the contracted section.

Neglecting pipe friction, the drop in pressure head between the section of beginning of convergence and the section of greatest contraction (Fig. 1) is equal to the change in velocity head between the two sections plus the loss in head due to the contraction of the jet. Then by Bernoulli's Law

$$h_b = h_1 - h_2 = \frac{v^2}{2gC_v^2} - \frac{V^2}{2g}$$

Since $\frac{v}{V} = \frac{D^2}{C_c d^2}$ and $C = C_c C_v$

$$h_b = \left[\left(\frac{D}{d} \right)^4 \frac{1}{C^2} - 1 \right] \frac{V^2}{2g} \quad \dots \dots \dots (1)$$

$$V = \sqrt{\frac{2g h_b}{\left(\frac{D}{d} \right)^4 \frac{1}{C^2} - 1}} \quad \dots \dots \dots (2)$$

It is evident that this drop in pressure head, h_b , is the maximum drop in head that may be found near the orifice and is somewhat greater than the sum of all losses occasioned by the orifice, since beyond the contracted section as the velocity decreases from v to V a portion of the velocity head is transformed into pressure head.

6. *Expression for Lost Head.*—Between two sections of uniform flow, one section on each side of the orifice, such as section 1 and section 3 (Fig. 1), a change in pressure head is accompanied by three losses—the loss due to contraction, the loss due to sudden expansion, and the loss due to pipe friction. If the latter factor is eliminated from consideration, then by methods similar to those used in the

derivation of the expression in the preceding article it may be shown that

$$h_a = h_1 - h_3 = \frac{V^2}{2g} \left[\left(\frac{D}{d} \right)^4 \left(\frac{1}{C^2} - \frac{1}{C_c^2} \right) + \left(\left(\frac{D}{d} \right)^2 \frac{1}{C} - 1 \right)^2 \right] \quad (3)$$

This is the loss of head caused by the orifice. It is evidently always less than the drop in pressure head of equation (1) but approaches that quantity as the ratio $\frac{D}{d}$ increases.

In order properly to apply these equations, the sections previously referred to, particularly the section of beginning of convergence and the section of greatest contraction, must be studied experimentally. The method of modifying the rational equations to conform with the experimental results will be discussed later.

III. TESTS AND RESULTS

7. *Scope of Experiments.*—The principal tests were made on 4-inch, 6-inch and 12-inch pipe with eight sizes of orifices for each size of pipe as recorded in Table 1. It will be noted that the diameters

TABLE 1
DIAMETERS OF ORIFICES AND PIPES

4-inch Pipe $D=4.06$ in.		6-inch Pipe $D=6.12$ in.		12-inch Pipe $D=12.15$ in.	
d	$\frac{D}{d}$	d	$\frac{D}{d}$	d	$\frac{D}{d}$
0.497	8.17	0.75	8.16	1.50	8.10
0.667	6.09	1.00	6.12	2.00	6.08
1.00	4.06	1.49	4.10	3.00	4.05
1.34	3.04	2.00	3.06	4.00	3.04
1.67	2.43	2.50	2.44	5.00	2.43
2.00	2.03	3.01	2.03	6.00	2.03
2.68	1.52	4.02	1.52	8.00	1.52
3.33	1.22	5.01	1.22	10.00	1.22

of the orifices range from one-eighth to five-sixths the diameter of the pipe, and that the eight ratios of diameter of pipe to diameter of orifice for the 4-inch series are approximately the same as for the 6-inch and 12-inch series.

The mean velocity in the pipe ranged from 0.01 ft. per sec. with the smallest orifice of each series to a maximum velocity of about 23 ft. per sec. with the largest orifice of the 4-inch series, 14 ft. per sec. with the 6-inch series, and $3\frac{1}{4}$ ft. per sec. with the 12-inch series.

Experiments were conducted to investigate phases of the problem as follows:

(1) Pressure Variations and Behavior of Stream

By observations of pressure variations along the pipe the position of the following sections was found: (a) the section at which the flow in the pipe changes from normal uniform flow and the stream begins to converge towards the orifice; (b) the section at which the jet issuing from the orifice becomes fully expanded and normal uniform flow is resumed; and (c) the section of greatest contraction of

the jet issuing from the orifice. The change in pressure head was taken to indicate the change in area of stream flow, the section of greatest contraction being located where the pressure head is the least. The terms normal uniform flow, beginning of convergence, section of greatest contraction, and section where jet becomes fully expanded and normal uniform flow is resumed are used for convenience to describe the general phenomena at the several sections; they are not to be taken to define strictly the conditions of flow. The general behavior of the stream in the vicinity of the orifice was also studied.

(2) Relation between Drop in Pressure and Discharge

The observations of pressure variations along the pipe indicated that the two most favorable sections for gage connections, the sections giving the steadiest pressure and the most reliable drop in pressure head were a section at or slightly upstream from the section of beginning of convergence and one at the section of greatest contraction. For these sections a greater change in pressure head was found than for any other two sections in the vicinity of the orifice.

For measured rates of discharge the drop in pressure head as registered by a differential gage connected at approximately the section of beginning of convergence and at the section of greatest contraction was observed. From these tests coefficients of discharge will be derived.

(3) Lost Head

The lost head caused by the orifice, if pipe friction is disregarded, is represented by the change in pressure head between the section of beginning of convergence upstream from the orifice and the section of resumed normal uniform flow down stream from the orifice, or is the difference between the pressure heads at two sections of normal uniform flow, one section above and one below the orifice. Observations were made to determine the relation between lost head and discharge.

(4) Effect of Varying the Conditions

Tests were made to determine: (a) the effect of a small lateral displacement of the orifice, (b) the effect of throttling the gage on the precision of observations, (c) whether or not there is a measure

able difference between the drop in pressure head with gage connections made at the top of each section and the drop in pressure head with the gage connections made at the bottom of each section, (d) the effect of number of connections at each section on the precision of observations, and (e) the effect of beveling the upstream edge of the orifice.

The drop in pressure was measured between a section at approximately the beginning of convergence and the section of greatest contraction. The tests were made only on 4-inch and 6-inch pipe.

8. *Apparatus.*—The pipe upon which tests were made was commercial steel pipe which had been in service some years. In general there was a small but measurable difference between the diameters of the adjoining sections of pipe between which the orifice plate was inserted, and, due to roughnesses and other variations, the error of measuring the diameter was perhaps 0.02 inch. The pipe was horizontal and straight for a sufficient distance on each side of the orifice to make the effect from bends negligible.

All orifices were circular in shape and were cut in 3/16-inch steel plates. In general the edges of the orifices were square. To find the effect of a deviation from this standard, experiments were also made with bevel-edged orifices, a bevel of 45 degrees being made on the upstream side of the orifice in such a way as to leave a thickness of metal of 1/32 inch at the throat.

Drop in pressure head was measured by the usual form of U-tube differential gage, the water gage being used for small differences, and the mercury gage for large differences.

The gage connections to the pipe at the sections under consideration were 1/4-inch pipe nipples. Care was taken that no burr was left at the inner edge of the tapped holes and that the nipples did not protrude beyond the inner surface of the pipe. The pipe nipples were connected with the gages by ordinary rubber tubing.

9. *Water Supply and Measurement.*—The supply of water was obtained from the 60-foot standpipe of the Hydraulic Laboratory. For most of the tests the water level in the standpipe was within a few feet of the top and a nearly constant head was maintained by a 2200-gallon duplex pump, the pump being automatically regulated. For most work the head remained nearly constant during a given run,

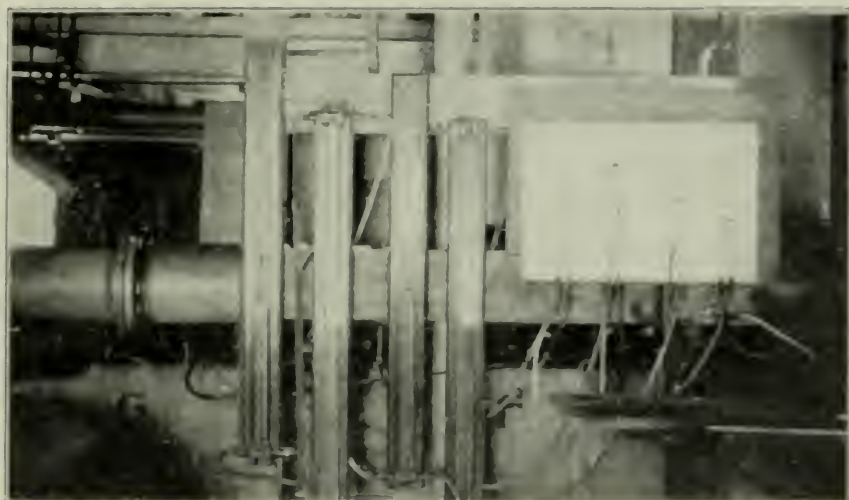


FIG. 2. APPARATUS FOR OBSERVING PRESSURE CHANGES

but for large discharges surges occurred in the standpipe resulting in momentary fluctuations in pressure in the pipe.

When less than about $1/3$ cu. ft. per sec., the discharge was measured by weight; when greater, by displacement.

Time of discharge was observed with a calibrated stop watch.

10. *Pressure Changes near Orifice.*—For these tests gage connections to the pipe were inserted at the sections indicated in Table 2.

TABLE 2

POSITION OF SECTIONS AT PRESSURE CONNECTIONS

Diameter of pipe and distances from the orifice to the various sections are given in inches.

Diam. of Pipe	Sec. A to Orif.	Sec. B to Orif.	Orif. to Sec. C	Orif. to Sec. D	Orif. to Sec. E	Orif. to Sec. F	Orif. to Sec. G	Orif. to Sec. H	Orif. to Sec. I	Sec. A to Sec. I
4.06	$3\frac{1}{4}$	2	$\frac{1}{2}$	2	$3\frac{1}{2}$	$9\frac{1}{2}$	$15\frac{1}{2}$	$25\frac{1}{2}$	$49\frac{1}{2}$	53
6.12	$4\frac{1}{2}$	$2\frac{1}{2}$	$\frac{1}{2}$	$2\frac{1}{4}$		$8\frac{1}{4}$	20	38	74	77
12.15	5			$4\frac{1}{4}$		16	40	76	127	132

section A being upstream from the orifice and the change in pressure head between adjacent sections was observed at from 20 to 40 rates of flow for each orifice. The maximum changes in pressure observed ranged from 0.01 ft. to 50 ft.

Fig. 2 shows the apparatus for observing changes in pressure head in the 12-inch pipe, the orifice plate being between the flanges near the left of the picture and the direction of flow being from left to right. At the sections adjacent to the orifice (section A and section D, Table 2), gage connections were inserted both in the top and in the bottom of the pipe, and the two nipples at each of these sections were brought to a union. Two connections were also used at section I. At other sections a single nipple was inserted in the bottom of the pipe.

The results of the observations are shown graphically in Fig. 3. For ease in making comparison, changes in pressure head along the pipe are shown in terms of the lost head caused by the orifice (ratio of change in pressure head to lost head), an allowance for pipe friction having been made.

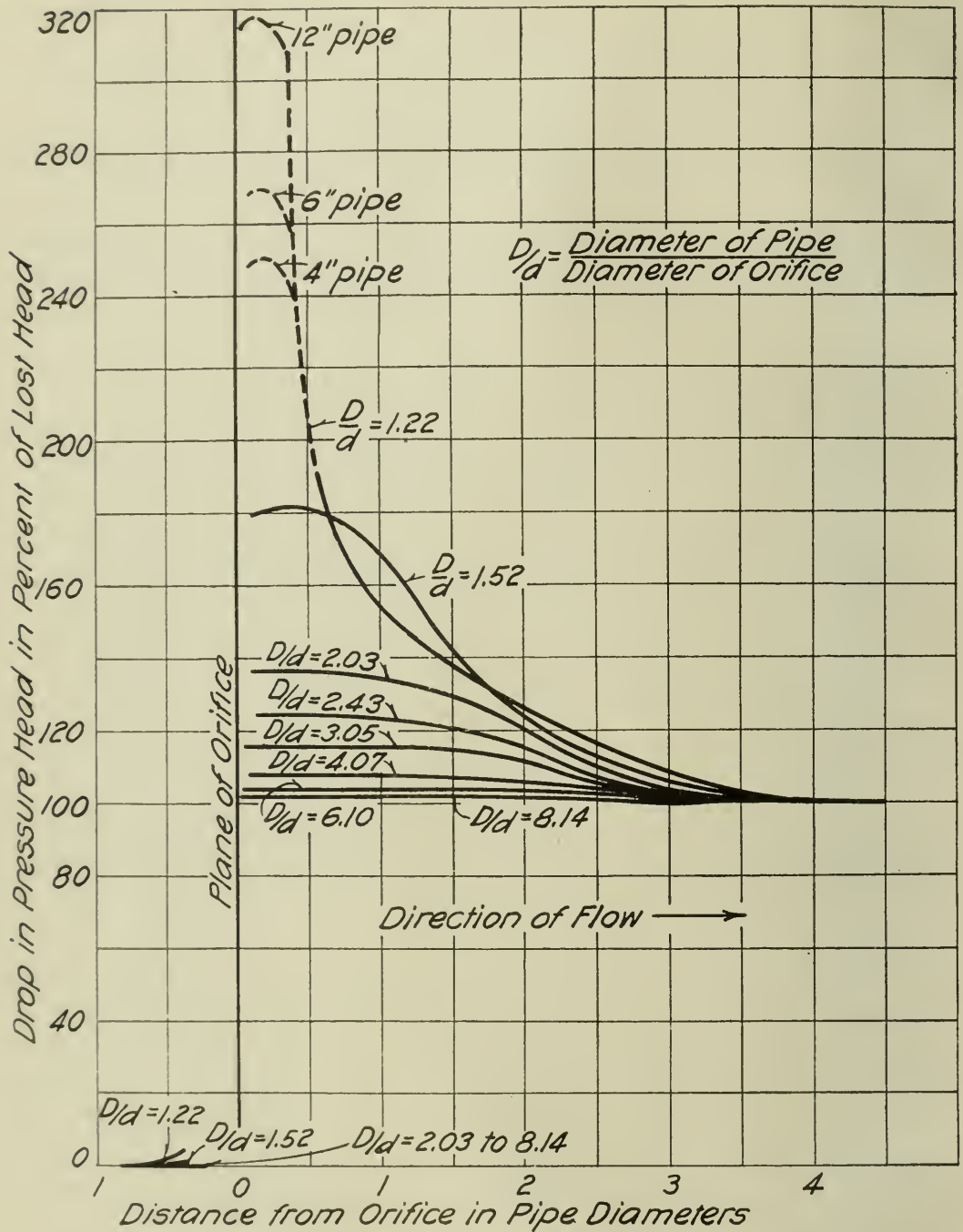


FIG. 3. PRESSURE VARIATIONS NEAR ORIFICE

For each $\frac{D}{d}$ the ordinate of the curve at any section was obtained by averaging the ratios for all rates of flow, there being nothing to indicate a consistent variation between these ratios at low rates of flow

and at high rates of flow. That there are not three curves shown for each $\frac{D}{d}$ merely indicates that there is close agreement between the general behavior of the jet for one size of pipe and the general behavior for other sizes, and does not indicate that the size of pipe has no influence on the relation between velocity and change in pressure head. The dotted portion of the curve for $\frac{D}{d}=1.22$ is regarded as uncertain.

11. *Deductions on Behavior of the Jet.*—A study of the observations of the changes in pressure head (which are too numerous to be shown here) and of the curves in Fig. 3 warrants the following deductions:

(1) The location of the section of beginning of convergence as the stream approaches the orifice is approximately 0.8 of the diameter of the pipe upstream from the plane of the orifice, for $\frac{D}{d}=1.2$, and it gradually approaches the orifice as $\frac{D}{d}$ increases.

(2) The section of greatest contraction is at a fairly constant distance of 0.4 of the pipe diameter downstream from the plane of the orifice for values of $\frac{D}{d}$ of 1.5 or greater. As $\frac{D}{d}$ becomes less than 1.5, the section of greatest contraction gradually approaches the orifice.

(3) The distance from the orifice to the section at which the jet has fully expanded and normal uniform flow is resumed varies between 3 and 4 pipe diameters, increasing as $\frac{D}{d}$ decreases until $\frac{D}{d}$ becomes 1.5, and thereafter probably decreasing.

(4) Downstream from the contracted section there is a region of greatly disturbed pressure. As the diameter of the orifice approaches the diameter of the pipe, this region approaches the section of greatest contraction and the pressure fluctuations as shown by gage readings become more violent.

(5) The proportional accidental error of observing drop in pressure head between the section of beginning of convergence

upstream from the orifice and sections downstream from the orifice is in general least when the downstream section is at the point of greatest contraction. The error increases as the distance from this section is increased until the region of maximum pressure fluctuation is reached, and decreases thereafter as the downstream section approaches the section of resumed normal uniform flow.

(6) Except for $\frac{D}{d}=1.2$ (the largest orifice used with each size of pipe), there is no indication of a variation between the general behavior of the flow passing through the orifice used with one size of pipe and that passing the corresponding orifices used with other sizes of pipe. That is, for a particular value of $\frac{D}{d}$, except $\frac{D}{d}=1.2$, there is no indication that the curve showing the pressure variation for the 4-inch pipe should not coincide with that for the 6-inch pipe and for the 12-inch pipe. For $\frac{D}{d}=1.2$ there is evidence of considerable variation, though whether or not this is to any considerable extent due to size of pipe is problematical.

The preceding deductions indicate that, for the purpose of flow measurement, gage readings of drop in pressure should be taken with gage connections at the section of beginning of convergence or upstream therefrom and at the section of greatest contraction. The distances to these sections, as far as effect on measurement of drop in pressure head is concerned, may be considered as $0.8 D$ upstream from the orifice and $0.4 D$ downstream from the orifice. In the tests to determine the relation between drop in pressure and discharge these distances were used.

12. *Relation between Drop in Pressure Head and Discharge.*—Since the use of the pipe orifice as a flow measuring device depends upon the proper relations between the drop in pressure head and the velocity (or rate of discharge) a wide range of tests was made to determine their relation after the preliminary experiments had shown the proper arrangement of the apparatus as already discussed.

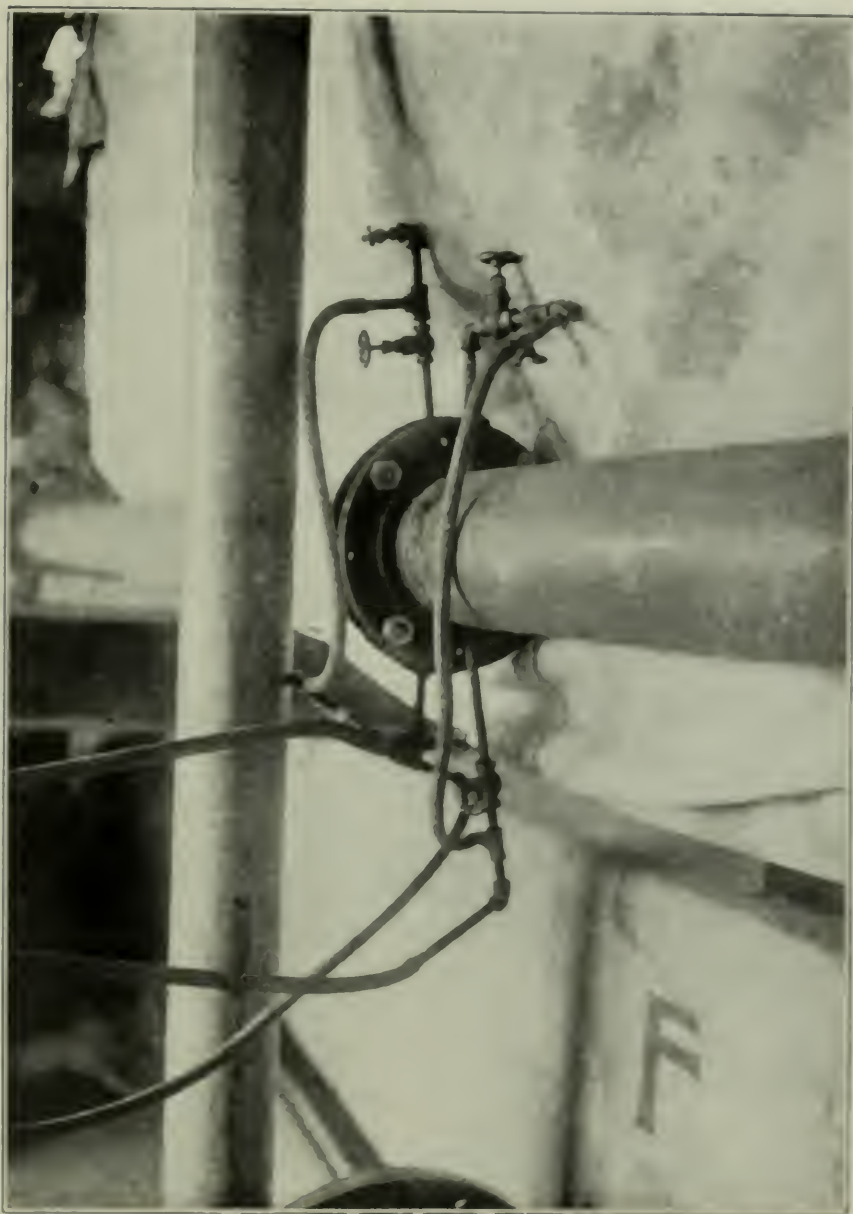


FIG. 4. PIPE CONNECTIONS NEAR ORIFICE

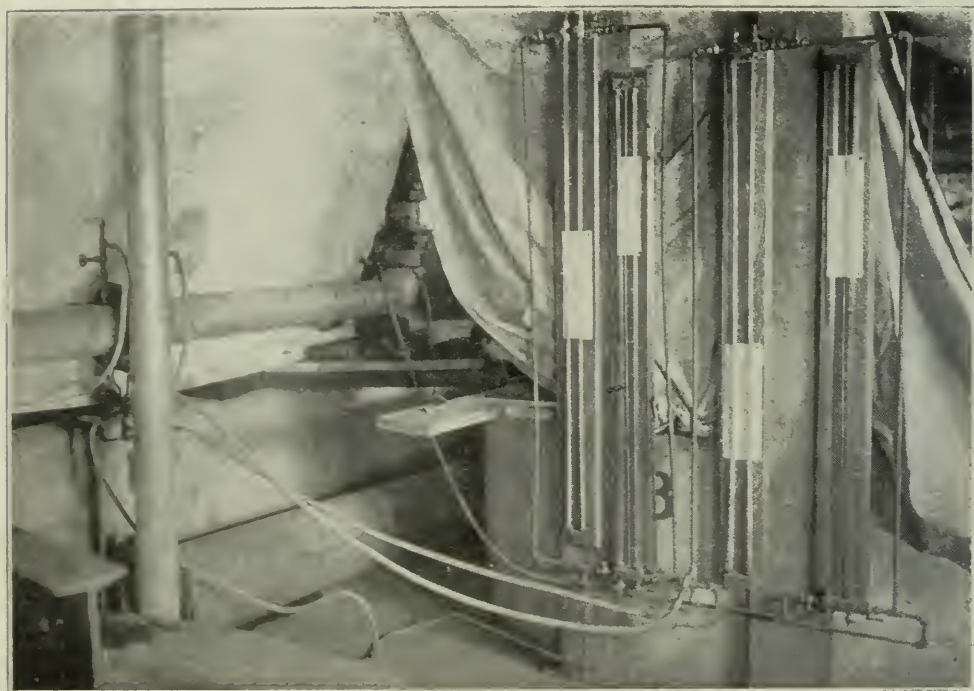


FIG. 5. DIFFERENTIAL GAGES

For various rates of flow the discharge was measured during an observed time interval and the drop in pressure head was read from the differential gage. The velocities in the pipe ranged from 0.01 ft. per sec. with the smallest orifice of each series to 24 ft. per sec. with the largest orifice of the 4-in. series; the drop in pressure ranged from 0.01 ft. to 56 ft. On the average 30 observations for as many rates of flow were made for each size of orifice.

Fig. 4 shows the gage connections with the pipe at the two sections adjacent to the orifice, the distances to these sections being 0.8

TABLE 3
SAMPLE DATA
Orifices in 4 in. pipe

Head in ft. of water. Time in sec. Discharge in cu. ft. per sec.

Obs. No.	Net Wt.	Cor. Time	Lost Head	Drop in Pressure Head	Discharge
Orifice 1.67 in. diameter.					
183	345	240	0.073	0.088	0.0230
184	394	254½	0.086	0.104	0.0247
185	414	240	0.105	0.127	0.0276
186	332	180	0.123	0.151	0.0295
187	447	221½	0.144	0.174	0.0323
188	534	240	0.172	0.213	0.0356
189	1076	420	0.229	0.283	0.0410
190	525	180	0.293	0.360	0.0467
191	583	179½	0.363	0.454	0.0519
192	660	180	0.468	0.585	0.0587
193	751	182½	0.591	0.738	0.0658
194	839	180	0.760	0.950	0.0746
195	938	180	0.95	1.18	0.0834
196	977	165	1.22	1.52	0.0948
197	912	132½	1.68	2.05	0.110
198	960	121½	2.21	2.76	0.126
199	1515	160	3.17	3.95	0.152
200	1739	180	3.26	4.00	0.155
201	1884	179½	3.86	4.76	0.168
202	1857	164	4.45	5.48	0.181
203	1854	149½	5.36	6.68	0.199
204	1852	135	6.56	8.07	0.219
205	1825	120	7.98	9.97	0.244
206	1821	106	10.1	12.7	0.276
207	3256	174½	12.9	15.9	0.298
208	3373	155	16.7	20.6	0.348
209	3385	140	20.3	25.4	0.387
210	3129	120	23.7	29.5	0.417
Orifice 2.00 in. diameter					
211	252	360	0.008	0.009	0.0112
212	301	360	0.011	0.013	0.0134
213	223	360	0.013	0.015	0.0144
214	284	300	0.013	0.018	0.0152
215	519	484	0.017	0.022	0.0172
216	246	200	0.020	0.026	0.0197
217	247	200	0.021	0.030	0.0197
218	683	490	0.030	0.039	0.0223
219	818	560	0.034	0.044	0.0234
220	578	360	0.040	0.052	0.0257
221	576	330	0.046	0.060	0.0279

TABLE 3 (Continued)

SAMPLE DATA

Orifices in 12-in. pipe

Head in ft. of water. Time in sec. Discharge in cu. ft. per sec.

Obs. No.	Net Wt.	Cor. Time	Lost Head	Drop in Pressure Head	Discharge
Orifice 1.50 in. diameter					
613a	224	600	0.011	0.011	0.00598
614a	210	500	0.013	0.013	0.00672
615a	198	400	0.017	0.018	0.00793
616a	205	360	0.021	0.022	0.00911
617	209	339	0.028	0.029	0.00987
618	216	300	0.036	0.036	0.0115
619	202	261	0.042	0.044	0.0124
620	203	241	0.050	0.052	0.0135
621	208	215	0.067	0.067	0.0155
622	258	240	0.079	0.080	0.0172
623	376	332	0.090	0.092	0.0181
624	342	240	0.112	0.113	0.0228
625	300	241	0.135	0.135	0.0199
626	372	240	0.176	0.176	0.0248
627	347	199	0.216	0.216	0.0279
628	392	199	0.274	0.277	0.0315
629	448	200	0.350	0.353	0.0358
630	497	200	0.428	0.437	0.0398
631	517	191	0.516	0.523	0.0433
632	585	191	0.663	0.675	0.0490
633	596	180	0.773	0.790	0.0530
634	650	180	0.915	0.940	0.0578
635	730	184	1.12	1.14	0.0635
636	830	179	1.50	1.53	0.0742
637	1040	202	1.86	1.91	0.0825
638	1512	260	2.39	2.44	0.0930
639	1542	240	2.97	3.04	0.103
640	1725	240	3.66	3.72	0.115
641	1644	200	4.79	4.89	0.132
642	1679	180	6.18	6.31	0.149
643	1902	180	7.88	8.04	0.169
644	1787	155	9.56	9.70	0.184
645	3286	259	11.5	11.7	0.203
646	3218	215	15.9	16.2	0.239
647	1800	103½	21.2	21.6	0.279
648	3254	166½	27.2	27.8	0.312
649	3110	138	36.2	36.9	0.361
650	3398	145	41.4	42.1	0.375
Orifice 2.00 in. diameter					
651	224	360	0.009	0.010	0.00997
652	241	300	0.012	0.013	0.0129
653	206	240	0.016	0.018	0.0137
654	223	270	0.017	0.017	0.0132
655	258	240	0.026	0.028	0.0172
656	246	188½	0.037	0.040	0.0209
657	287	180	0.056	0.060	0.0255

D upstream and $0.4 D$ downstream from the orifice and there being two pressure openings at diametrically opposite points at each section. Fig. 5 shows the differential gages with connecting hose. The two gages on the left, one for mercury, the other for water, are connected to the sections adjacent to the orifice and register the drop in pressure head; the two gages on the right register the lost head due to the orifice plus

TABLE 3 (Concluded)

SAMPLE DATA
Orifices in 12-in. pipe

Head in ft. of water. Time in sec. Discharge in cu. ft. per sec.

Obs. No.	Cubic Feet			Cor. Time	Lost Head	Drop in Pressure Head	Cu. Ft Sec
	Initial	Final	Net				
Orifice 6.00 in. diameter							
783	48.2	167.3	119.1	293½	0.136	0.179	0.407
784	54.7	170.3	115.6	233½	0.197	0.265	0.497
785	57.7	168.3	110.6	271½	0.136	0.183	0.407
786	54.5	189.4	134.9	228½	0.276	0.370	0.589
787	53.3	200.9	147.6	217	0.371	0.497	0.680
788	57.2	209.6	152.4	194	0.490	0.66	0.786
789	45.1	193.7	148.6	168½	0.61	0.82	0.882
790	44.5	178.7	134.2	134	0.80	1.06	1.00
791	37.0	177.0	140.0	122½	1.04	1.40	1.14
792	13.6	190.9	177.3	133½	1.40	1.89	1.31
793	14.1	187.9	173.8	113½	1.84	2.47	1.53
794	1.4	204.1	205.5	119	2.38	3.18	1.72
795	15.5	184.3	168.8	78	3.76	5.07	2.16
796	4.4	209.9	205.5	85½	4.50	6.23	2.39
797	6.8	226.6	219.8	84½	5.27	7.12	2.60
798	11.0	204.5	193.5	72	6.46	8.94	2.66
799	2.3	236.6	234.3	79½	6.89	9.32	2.95
Orifice 8.00 in. diameter.							
Observed discharge given in pounds							
800			1443	122	0.005	0.010	0.189
801			1771	122½	0.008	0.0155	0.232
802			3245	235	0.007	0.014	0.221
803			3187	200	0.010	0.018	0.255
804			3197	176½	0.013	0.024	0.260
805			3148	162	0.015	0.0295	0.313
Discharge in Cubic Feet							
806	55.6	165.4	109.8	302½	0.023	0.040	0.303
807	57.8	169.1	111.3	264½	0.029	0.050	0.421
808	60.3	169.8	109.5	241½	0.035	0.063	0.454
809	11.3	167.4	156.1	293½	0.047	0.085	0.532
810	50.5	167.2	116.7	187½	0.070	0.110	0.622
811	48.2	170.6	122.4	164½	0.092	0.148	0.745
812	45.2	176.3	131.1	156½	0.125	0.200	0.838
813	52.7	182.1	129.4	136½	0.154	0.250	0.948
814	42.8	205.8	163.0	152½	0.206	0.325	1.07
815	51.1	202.4	141.3	125½	0.27	0.435	1.21
816	41.3	215.7	174.4	129½	0.30	0.51	1.35
817	36.3	225.2	188.9	121½	0.41	0.70	1.56
818	35.4	223.1	187.7	102	0.53	0.97	1.84
819	39.4	225.3	185.9	97	0.60	1.19	1.92
820	27.4	221.2	193.8	91	0.76	1.38	2.11
821	25.9	236.7	210.8	88	0.95	1.70	2.49
822	7.1	230.5	223.4	91½	1.07	1.78	2.44

the loss due to pipe friction within the length between gage connections. The lost head will be discussed in section 16.

A sample of the data thus obtained is shown in Table 3. The values of lost head there recorded have been corrected for pipe friction.

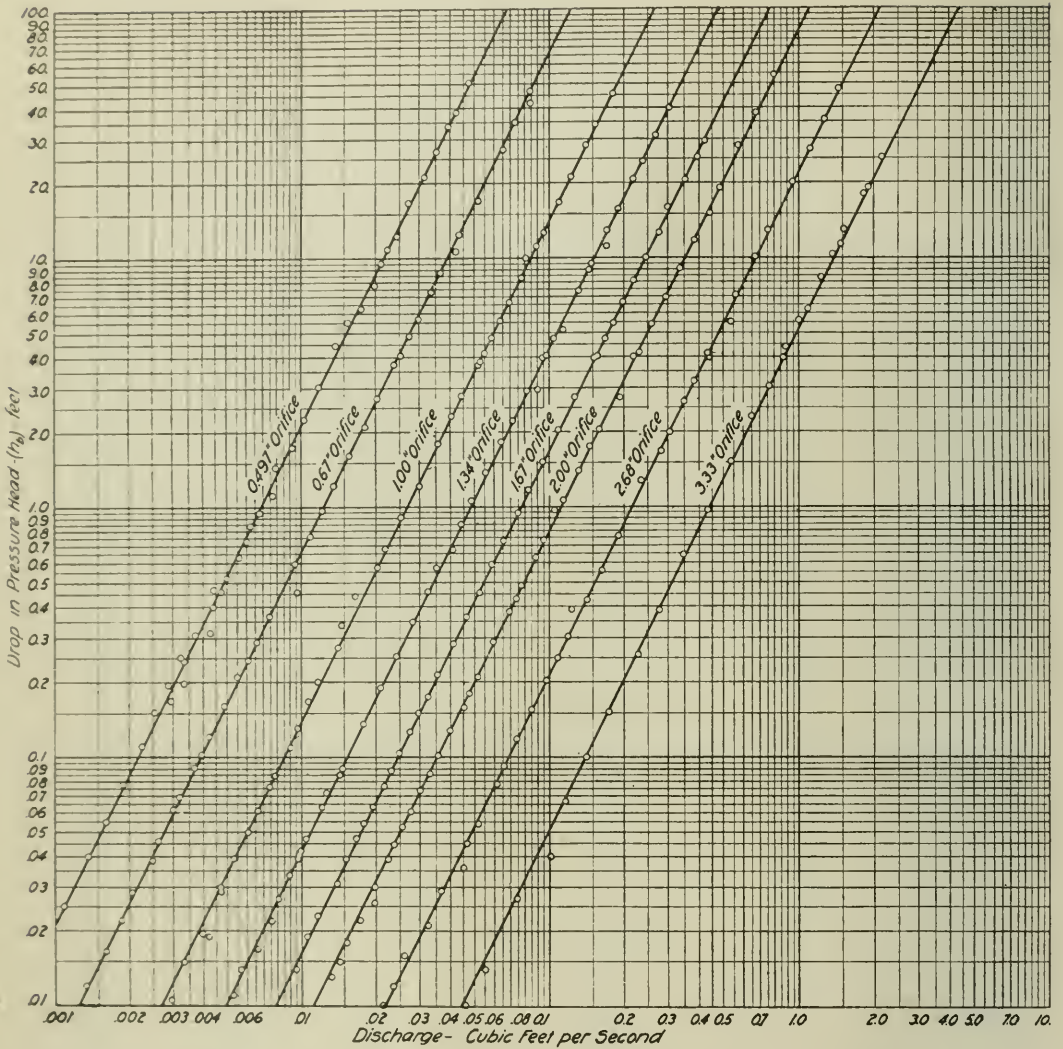


FIG. 6. EXPERIMENTAL DISCHARGE CURVES—4-INCH PIPE

The data of the tests are plotted logarithmically in Figs. 6, 7 and 8, the observed drop in pressure head in feet being plotted against measured discharge in cubic feet per second. The lines which have been drawn through the mean of the plotted points are seen to be straight and very nearly parallel. It will also be noted that the curves for one series bear the same general relation to one another as do the curves for the other two series.

The mean of the measured slopes of the 24 curves varies only slightly from 2.00. A study of the curves indicates that the variation of the slopes of the individual curves from the mean value is partly

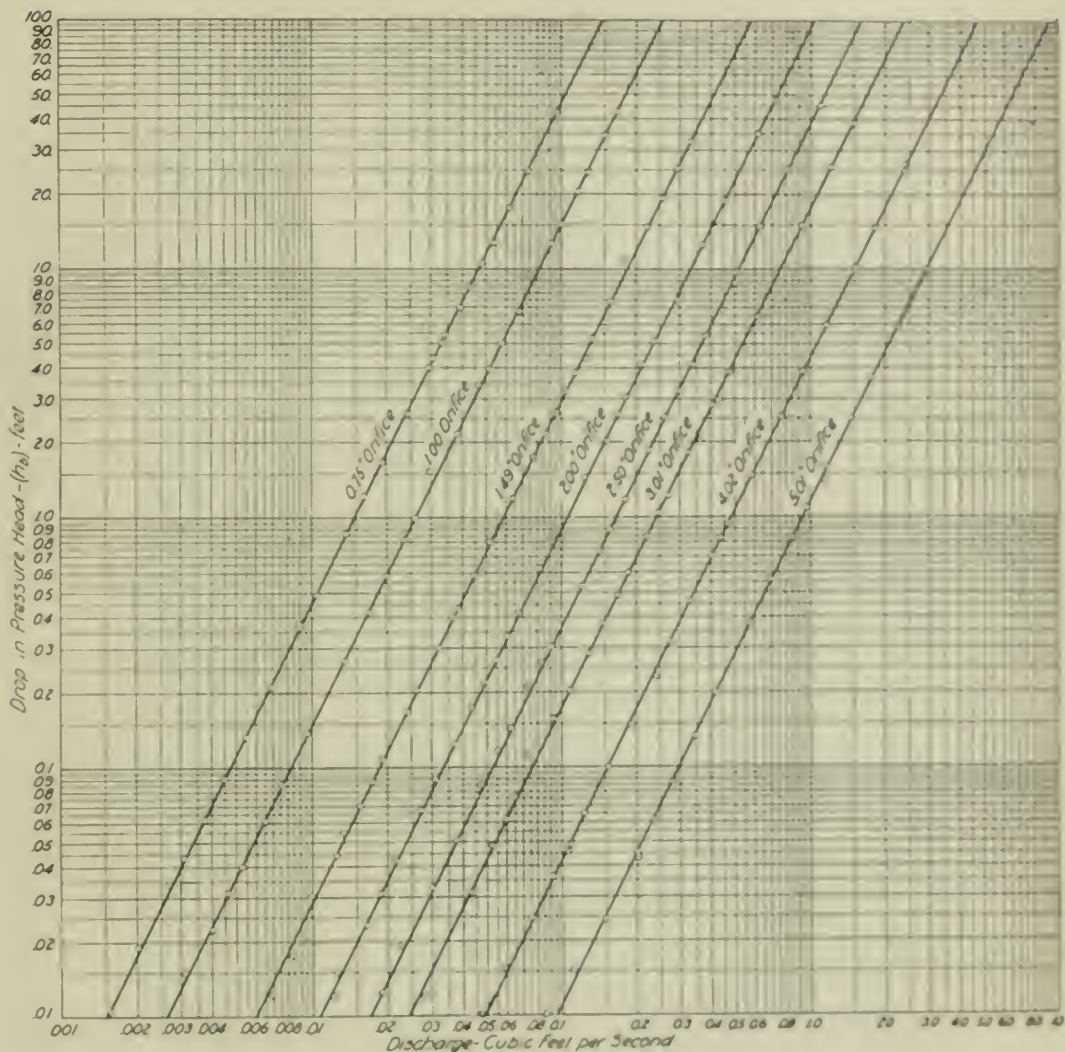


FIG. 7. EXPERIMENTAL DISCHARGE CURVES—6-INCH PIPE

accidental. Since the curves are plotted to logarithmic scale and have a slope of 2, h_b may be said to vary as V^2 , which is in accord with the theoretical expression, equation (1) p. 12. To make the theoretical equation exactly fit the experimental curves it must be assumed that for a particular size of orifice and pipe the coefficient of discharge decreases slightly as the drop in pressure head increases, as may be seen in Table 4.

The values of the coefficients of discharge in Table 4 were obtained by substituting in equation (1) values of h_b and $V = \frac{Q}{A}$ taken from the experimental curves. It will be seen that for a particular value of $\frac{D}{d}$

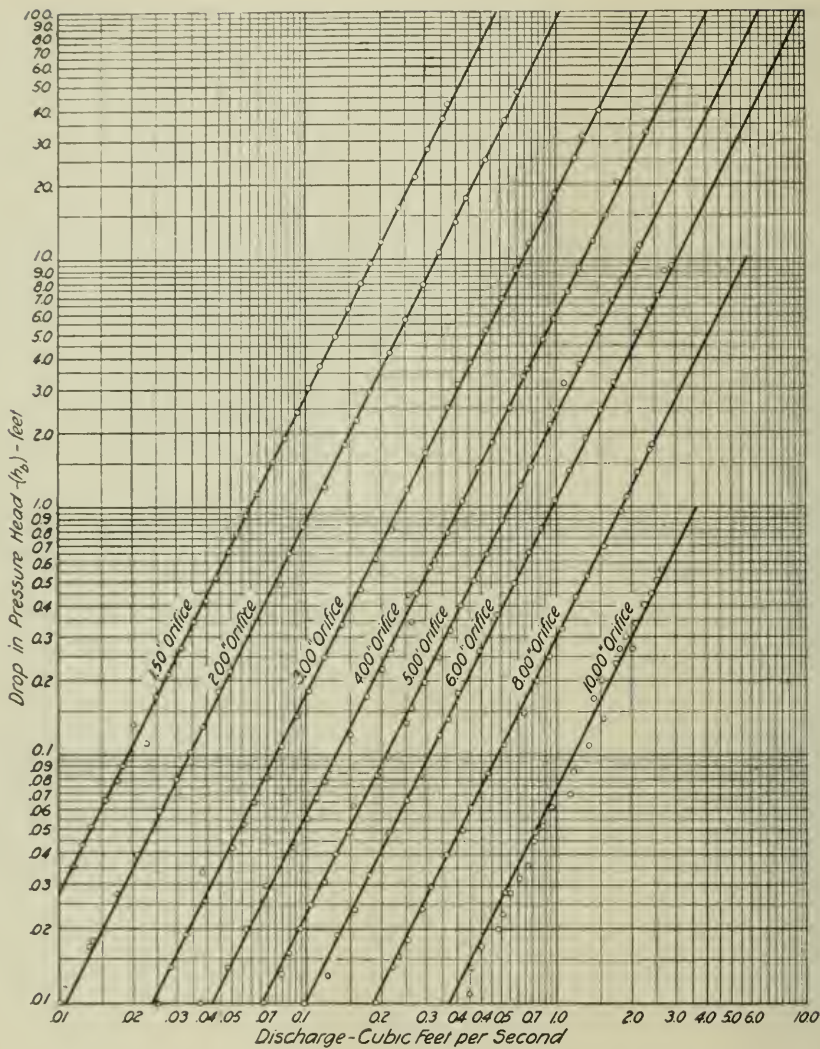


FIG. 8. EXPERIMENTAL DISCHARGE CURVES—12-INCH PIPE

TABLE 4
COEFFICIENT OF DISCHARGE—C

$\frac{D}{d}$	4-in. Pipe			6-in. Pipe			12-in. Pipe		
	$h_b=0.1$	$h_b=1.00$	$h_b=50$	$h_b=0.1$	$h_b=1.00$	$h_b=50$	$h_b=0.1$	$h_b=1.00$	$h_b=10$
1.22	0.771	0.760		0.753	0.746		0.742		
1.52	0.670	0.663	0.651	0.659	0.651		0.638	0.631	
2.03	0.633	0.628	0.617	0.620	0.616	0.608	0.615	0.609	0.603
2.43	0.622	0.619	0.609	0.609	0.606	0.600	0.608	0.602	0.596
3.05	0.617	0.614	0.606	0.604	0.602	0.597	0.605	0.600	0.594
4.07	0.618	0.615	0.608	0.606	0.604	0.598	0.605	0.600	0.595
6.10	0.626	0.624	0.618	0.613	0.611	0.601	0.610	0.606	0.602
8.14	0.638	0.636	0.630	0.619	0.616	0.604	0.610	0.606	0.602

the coefficient of discharge not only decreases slightly as the drop in pressure head increases but also decreases as the diameter of the pipe increases.

For a better understanding of the general behavior of the coefficient of discharge, and also that the coefficient of discharge may be determined readily for other values of $\frac{D}{d}$ than those of the experiments, the data of Table 4 for the 4-inch pipe are shown graphically by the two upper curves of Fig. 9, values of the coefficients being plotted against $\frac{D}{d}$. The two curves—the full line for $h_b=0.1$ ft. and the dash line for $h_b=50$ ft.—are intended to show the extremes of the coefficients which may possibly be used. For $\frac{D}{d}$ greater than 3.0 the coefficient of discharge increases nearly as a straight line, a peculiarity which may perhaps be explained through the fact that as $\frac{D}{d}$ increases the distance from the wall of the pipe to the periphery of the jet at the section of greatest contraction also increases and there is consequently less likelihood of the pressure at the wall of the pipe being the same as that which exists at the periphery of the jet at the section of greatest contraction. As $\frac{D}{d}$ decreases from 3.00 the coefficient of discharge increases, and the rate at which the coefficient changes for small values of $\frac{D}{d}$ still further emphasizes the impracticability of attempting to make precise measurements of discharge when the diameter of the orifice is greater than two-thirds the diameter of the pipe.

The C curves for the 4-inch pipe are typical of those for the 6-inch and 12-inch for the lower values of h_b and $\frac{D}{d}$, but as h_b becomes larger the rate of increase of C diminishes for the higher values of $\frac{D}{d}$. The two lower curves of Fig. 9 illustrate this change in rate of increase of C .

13. *Simplified Equations for Velocity and Discharge.*—The theoretical equation for the determination of velocity in the pipe in terms

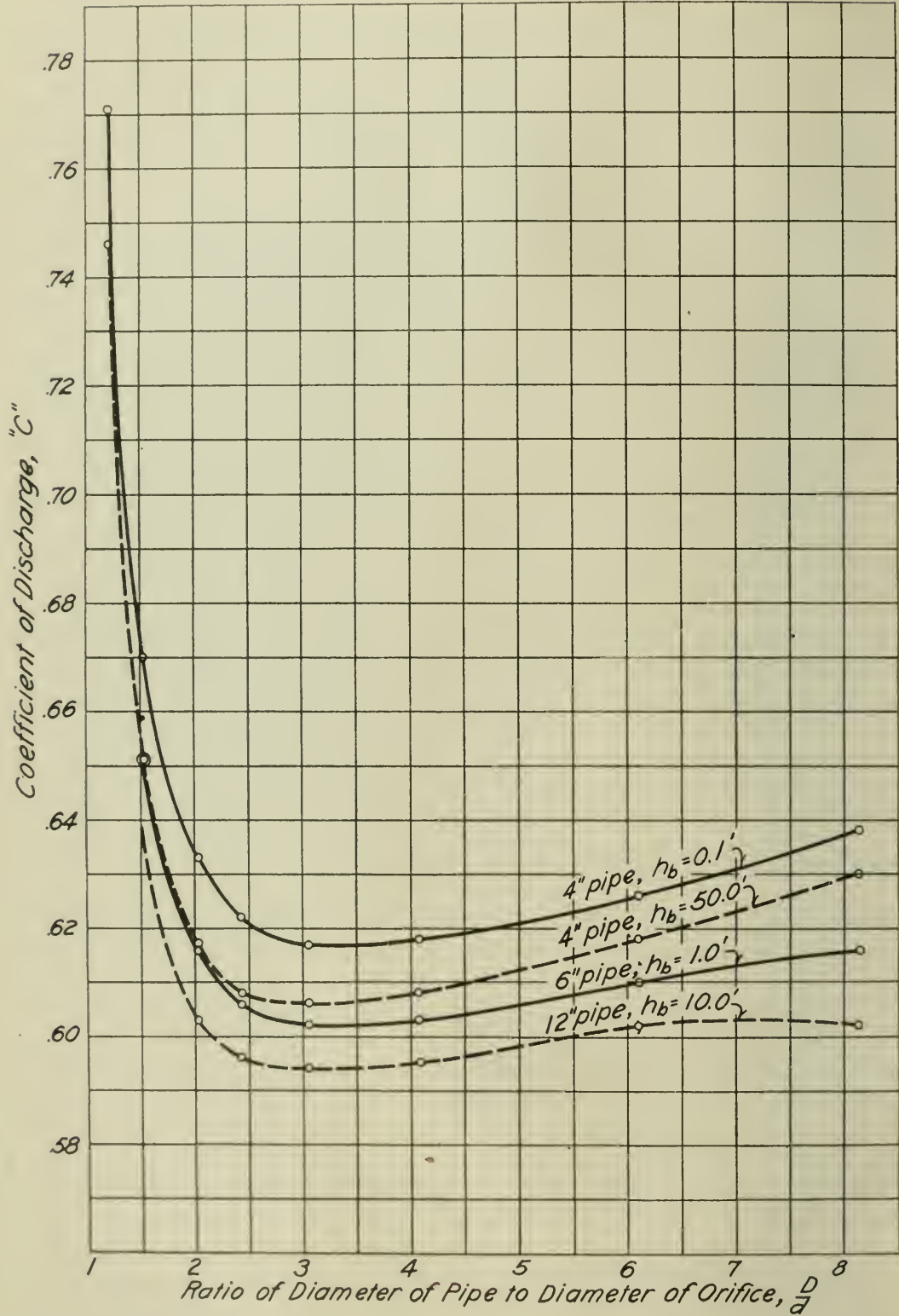


FIG. 9. COEFFICIENTS OF DISCHARGE

of the drop of pressure head (equation (2), p. 12) may be put in the form

$$V = K \sqrt{h_b} \quad (4)$$

in which $K = \sqrt{\frac{2g}{\left(\frac{D}{d}\right)^4 - C^2}}$

The expression for the rate of discharge then is

$$Q = AK \sqrt{h_b} \quad (5)$$

Within the range of the drop in pressure likely to be used in practice, K may with small error be regarded as a constant for a particular size of pipe and orifice. To simplify the work of computing velocities, values of K for various sizes of pipes are given in Table 5. For convenience, the term K will be called the *velocity modulus*. Some of the values of the velocity modulus have been carried to one more sig-

TABLE 5
VELOCITY MODULUS— K
Values of K in Equation $V = K \sqrt{h_b}$

$\frac{D}{d}$	Velocity Modulus					
	4-in. Pipe	5-in. Pipe	6-in. Pipe	8-in. Pipe	10-in. Pipe	12-in. Pipe
2.00	1.275	1.250	1.240	1.240	1.240	1.240
2.10	1.150	1.123	1.111	1.110	1.110	1.110
2.20	1.030	1.015	1.010	1.010	1.010	1.010
2.30	0.944	0.930	0.925	0.925	0.925	0.925
2.40	0.865	0.850	0.845	0.845	0.845	0.845
2.50	0.798	0.785	0.779	0.779	0.779	0.779
2.60	0.736	0.725	0.720	0.720	0.720	0.720
2.70	0.684	0.672	0.667	0.667	0.667	0.667
2.80	0.635	0.625	0.620	0.620	0.620	0.620
2.90	0.592	0.583	0.578	0.577	0.576	0.576
3.00	0.553	0.545	0.540	0.540	0.539	0.539
3.25	0.473	0.465	0.462	0.461	0.461	0.461
3.50	0.405	0.399	0.395	0.395	0.395	0.395
3.75	0.351	0.345	0.342	0.342	0.342	0.342
4.00	0.310	0.305	0.303	0.302	0.302	0.302
4.50	0.244	0.240	0.238	0.237	0.237	0.237
5.00	0.197	0.194	0.192	0.192	0.192	0.192
5.50	0.165	0.161	0.160	0.160	0.160	0.160
6.00	0.138	0.136	0.135	0.135	0.135	0.135

nificant figure than the data may warrant in order to permit more nearly accurate interpolation. In interpolating for other values of $\frac{D}{d}$ or other sizes of pipe than those shown it may be assumed that the velocity modulus varies as a straight line.

The computations for the velocity modulus were based on the coefficients of discharge for a drop in pressure head of 1.0 ft. as shown in Table 4 and by the curves of Figs. 9 and 10. (Fig. 10 is explained in the following section.) Since K varies as C and since C decreases as h_b increases, the values of the velocity modulus are for a large drop in pressure head slightly too large and for a small drop in pressure head slightly too small; but within the limits $h_b=0.1$ ft. to $h_b=10$ ft., which seem to be about the limits which would be found practicable under ordinary conditions of flow, the error introduced by use of Table 5 is negligible.

14. *Application to Other Sizes of Pipes.*—In order to use equation (2) or (4) for sizes of pipe other than those used in the experiments herein recorded it is important to determine the effect of the diameter of the pipe on the coefficient of discharge. The proper coefficient of discharge is obtained by applying a *diameter factor*, C_p , which is the quantity by which the coefficient of discharge for a 6-inch pipe must be multiplied to produce the coefficient of discharge for a given size of pipe. The values of the diameter factor for various diameters of pipe are shown in Fig. 10. Since the curve is based on only

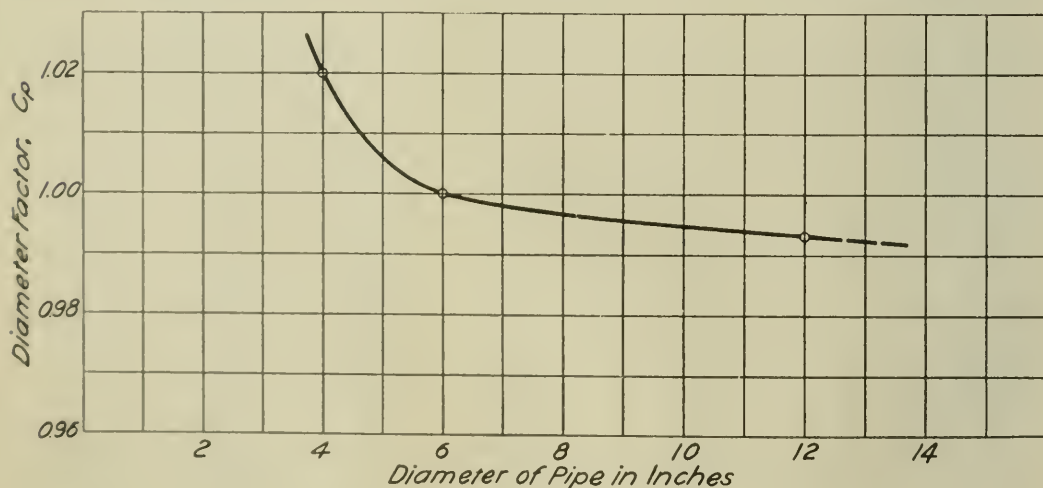


FIG. 10. DIAMETER FACTOR

three plotted points it is likely to be somewhat in error. Its use should be confined to values of $\frac{D}{d}$ between 2 and 6 and between 1 ft. and 10 ft. for h_b if accurate results are desired.

15. *Empirical Relation between Velocity and Drop in Pressure Head.*—From the discharge curves of Figs. 6, 7, and 8 the following empirical equation has been derived for the velocity in terms of the drop in pressure head,

$$V = 4.85 \frac{\sqrt{h_b}}{\left(\frac{D}{d}\right)^2 - \left(\frac{d}{D}\right)^3} \quad \dots \dots \dots (6)$$

This equation has an advantage over the rational expression, equation (2), in that the variable coefficient of discharge is eliminated. This empirical equation will give approximately the same results as equation (2) for those values of $\frac{D}{d}$ which are recommended for use in engineering practice, namely, for values of $\frac{D}{d}$ between 2 and 6.

16. *Empirical Formula for Lost Head.*—Since the amount of lost head caused by the orifice may be an important factor in the selection of the orifice to be used, or even in accepting the pipe orifice method of measuring the discharge, it is important to know the relation between the drop in pressure head and the lost head. In Fig. 5 the gages on the right registered the lost head due to the orifice plus the loss due to pipe friction within the length between gage connections.

The results of the experiments to determine this relation indicate that the rational expression, equation (3) p. 13, holds good when the ratio $\frac{D}{d}$ is large but gives results which are increasingly too large as $\frac{D}{d}$ decreases, until for $\frac{D}{d} = 1.2$ the lost head as computed by equation (3) is about 3 per cent less than the lost head as determined by experiment. In arriving at this conclusion, the coefficients of discharge as determined from the experiments on drop in pressure head and

as shown in Table 4 were used in the rational expression and the coefficients of contraction employed were derived assuming a coefficient of velocity of 0.98.

Equation (3) is too complicated an expression to be used readily in computing. A simpler expression for the lost head and the discharge is given by the empirical equation

$$h_a = 0.0366 F \left[\left(\frac{D}{d} \right)^2 - \frac{d}{D} \right]^{2.02} V^2 \quad . \quad . \quad . \quad (7)$$

in which F is a factor depending on the size of pipe and having the values 0.98 for 4-inch pipe, 1.02 for 6-inch pipe and 1.04 for 12-inch pipe. This expression gives lost heads correct within about 2 per cent except when $\frac{D}{d}$ is less than 1.5. For $\frac{D}{d} = 1.2$ the expression gives results about 3 per cent too small.

A convenient expression for determining the lost head, having given the drop in pressure head, is given by the equation

$$h_a = 0.84 h_b \frac{\left[\left(\frac{D}{d} \right)^2 - \frac{d}{D} \right]^{2.02}}{\left[\left(\frac{D}{d} \right)^2 - \left(\frac{d}{D} \right)^3 \right]^{1.99}} = J h_b \quad . \quad . \quad . \quad (8)$$

The values of J in the foregoing equation for given values of h_b are shown by the curve in Fig. 11.

17. *Choice of Orifice.*—The choice of the size of orifice to be used under given conditions may depend upon four factors, the rate of flow in the pipe, the lost head that may be allowed, the desired precision of the discharge measurement, and the maximum drop in pressure head which the differential gage will register. Table 6 which shows approximate values of the drop in pressure head and the lost head for several velocities and values of $\frac{D}{d}$ is intended to be of assistance in estimating the size of orifice best adapted to particular requirements. In Fig. 12 the drop in pressure head for various ratios of $\frac{D}{d}$ is given in terms of the lost head.

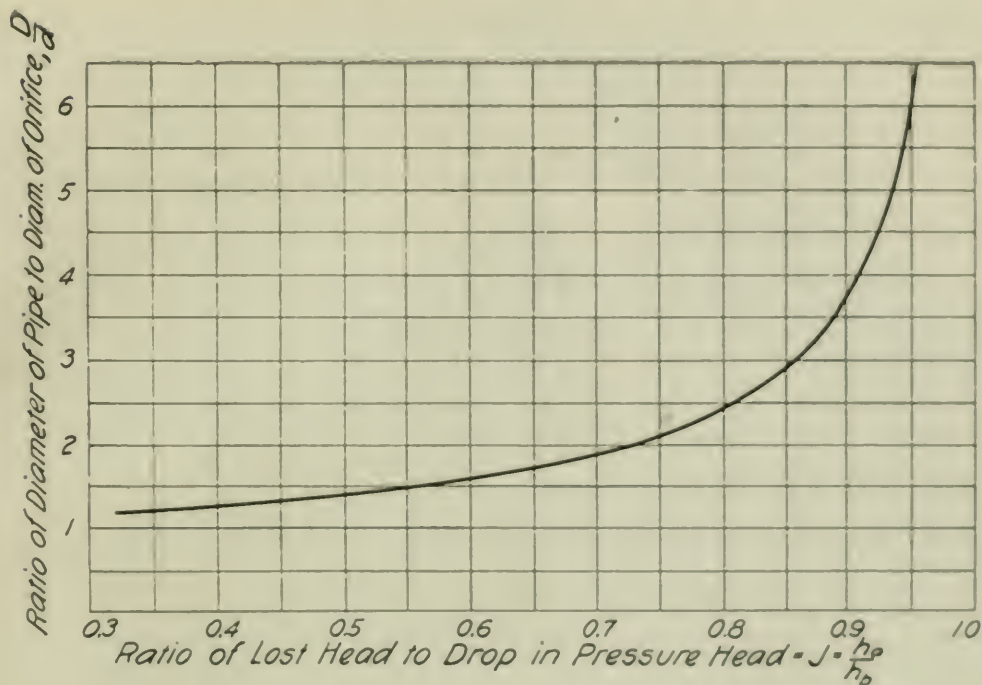


FIG. 11. RELATION BETWEEN LOST HEAD AND DROP IN PRESSURE HEAD

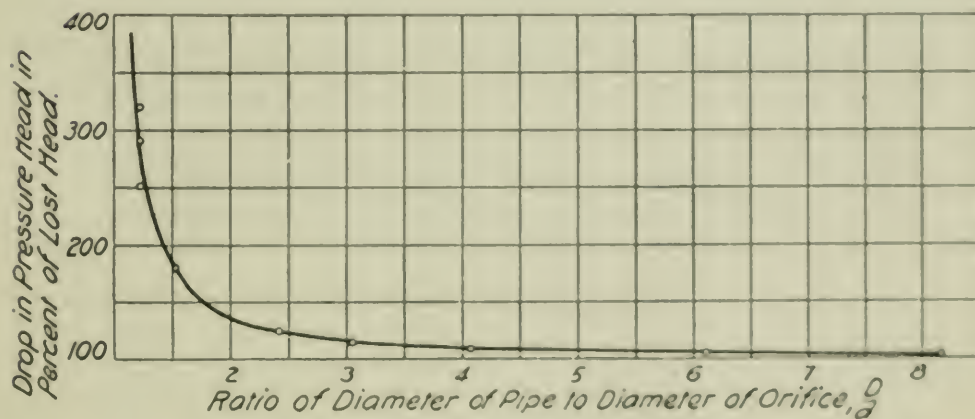


FIG. 12. RELATION BETWEEN MAXIMUM DROP IN PRESSURE AND SIZE OF ORIFICE

The relative error in the observation of the drop in pressure head varies inversely as $\frac{D}{d}$ but for $\frac{D}{d}$ equal to 2.0 or greater the error may be reduced to a negligible quantity. There are other factors which make it advisable to use a diameter of orifice not greater than one-half that of the pipe when conditions will allow. If the magnitude of

TABLE 6

APPROXIMATE VALUES OF DROP IN PRESSURE AND LOST HEAD

h_b = drop in pressure head in feet; h_a = lost head in feet

V	$\frac{D}{d} = 1.2$		$\frac{D}{d} = 1.5$		$\frac{D}{d} = 2.0$		$\frac{D}{d} = 2.5$		$\frac{D}{d} = 3.0$		$\frac{D}{d} = 4.0$	
	h_b	h_a	h_b	h_a	h_b	h_a	h_b	h_a	h_b	h_a	h_b	h_a
0.2											0.43	0.39
0.5					0.16	0.12	0.41	0.33	0.86	0.73	2.7	2.5
1			0.17	0.09	0.64	0.46	1.6	1.3	3.4	2.9	10.8	9.8
2			0.66	0.36	2.6	1.9	6.5	5.3	14	12	43	39
3	0.37	0.12	1.5	0.8	5.8	4.2	15	12	31	26		
4	0.65	0.21	2.6	1.4	10	7	26	21	55	47		
5	1.0	0.3	4.1	2.3	16	12	41	33				
6	1.5	0.5	5.9	3.2	23	17	59	48				
10	4.1	1.3	17	9	64	46						

the lost head is not an important factor, it is best that the drop in pressure head be greater than 1.0 ft. so that the relative error of measurement will be small, but the indications are that gage heights as small as 0.2 ft. may be measured with good results, provided the observer is experienced. Reliable observations may be taken with an orifice having a diameter two-thirds that of the pipe, but the probable error of reading the gage is likely to be several times what it is for the smaller orifices, even though the gage be carefully throttled. An orifice having a diameter much greater than two-thirds that of the pipe should not be used except for approximate discharge measurements.

As an illustrative example let it be required to measure the rate of discharge through an 8-inch pipe the velocity in which varies during the day through a large range, say from about 1 to 6 ft. per sec. Should the pipe-orifice method be used and, if so, what size of orifice should be installed? From Table 6 it will be noted that for $\frac{D}{d}=2$ the drop in pressure head is 0.64 ft. for a velocity of 1 ft. per sec. in the pipe, with a lost head of 0.46 ft. Likewise the drop in pressure head is 23 ft. for a velocity of 6 ft. per sec. with a lost head of 17 ft. Assuming that a lost head of 10 ft. is the greatest that should be allowed and that both a water gage and a mercury gage would be used for a range in drop of pressure head of 23 ft., it is seen that an

orifice larger than 4 in. should be selected. One with $\frac{D}{d} = 1.5$ gives a range in drop in pressure head from 0.17 to 5.9 ft. corresponding to a range in lost head of 0.09 to 3.2 ft. An orifice with $\frac{D}{d} = 1.5$ would give somewhat less nearly accurate results than one with $\frac{D}{d} = 2$ (as already discussed) but it would answer the purpose well, although perhaps it would be advisable to use a slightly larger orifice, one with $\frac{D}{d} = 1.6$ or $\frac{D}{d} = 1.75$.

18. *Computations for Discharge.*—For most precise computations use equation (2), p. 12, with coefficient of discharge taken from Table 4, p. 30. For sizes of pipe other than those for which coefficients are shown use the diameter factor as explained in section 14, p. 34. With $\frac{D}{d}$ from 1.5 to 6 and with care in making the observations, the error introduced may be kept lower than 2 per cent. With wider ranges of $\frac{D}{d}$ or with very high or low values in drop in pressure head this percentage will be increased, because of difficulties in reading the gage properly.

For less precise computations use equation (4) or equation (5), p. 33, with the proper velocity modulus taken from Table 5, p. 33. With pipe from 4 to 12 inches in diameter and with $\frac{D}{d}$ from 1.5 to 6, the maximum error introduced will be about 3 per cent. With wider ranges of $\frac{D}{d}$ or with very high or low values of drop in pressure head this percentage will be correspondingly increased.

The use of equation (6), p. 35, eliminates variable coefficients. For sizes of pipe from 4 to 12 inches, with $\frac{D}{d}$ from 2 to 4 the maximum error will not be greater than 3 per cent. With $\frac{D}{d}$ less than 2 or greater than 4 the error introduced may be 5 per cent or more.

Diagrams similar to Figs. 6, 7, and 8 may be constructed, from which the discharge for the more common sizes of pipe, with the proper

range of $\frac{D}{d}$, may be obtained directly. The calculation, however, from equations (4) and (5) and Tables 6 and 7 is very simple. See note

To determine the lost head, having given $\frac{D}{d}$ and the drop in pressure head, use equation (8), p. 36.

19. *Errors and Precautions.*—In the use of a flow measuring device it is important to know the source of the errors likely to be met and to know the precautions and limitations which if observed will help to reduce the errors.

It is important that the edges of the orifices be sharp and square. The orifice plates in the present tests were 3/16 inch thick, but there is no reason for believing that the coefficients derived from the experimental data would not apply equally well to orifices in plates of lesser thickness.

The orifice should be placed in a region where there is approximately uniform flow. This will necessitate the pipe upstream from the orifice being straight and free from abrupt changes in cross-sectional area for 10 diameters or more.

On account of the possibility of small openings becoming clogged it seems inadvisable to use gage connections having a diameter less than $\frac{1}{4}$ inch, particularly for permanent installation.

It is important that all burr be removed and also that the nipple does not protrude beyond the inner surface of the pipe; for small projections, by altering flow conditions, are likely to produce a systematic error of considerable magnitude in the gage readings.

For $\frac{D}{d}$ equal to 2.0 or greater a single nipple at each section will be sufficient. Although in the tests from which the coefficients of discharge were derived the sections at which nipples were placed were $0.8 D$ upstream from the orifice and $0.4 D$ downstream from the orifice, these distances may be altered somewhat without appreciably changing pressure conditions. (See Fig. 3, p. 20).

For $\frac{D}{d}$ less than 2.0 two opposite nipples at each section produce more reliable gage readings than do the single nipples, the pair at each section being joined together as illustrated in Fig. 4, p. 23. The indications are that the distance from the orifice to the section

downstream from the orifice may not be appreciably altered from $0.4 D$ and that the distance from the orifice to the section upstream from the orifice should not be less than $0.8 D$. On account of pipe friction this latter distance should not greatly exceed $0.8 D$, particularly for high velocities.

A differential gage of the U-tube type, similar to those shown in Fig. 5, p. 24, answers all the requirements, is simple to construct, and may be used with mercury or water. The gage board should be graduated to 0.01 ft. and the graduations should extend under the gage tubes. For flushing the gage and the connecting tubes, and for regulating the height of the air columns when water is the differential, there should be a pet cock at the end of each gage tube. Stop cocks for throttling the gage should be placed near the ends of the gage tubes as shown in Fig. 5.

A gage that may be quickly constructed in an emergency and the essential parts of which are easily obtainable and readily portable is shown in Fig. 13. The glass tubes need not be more than a foot

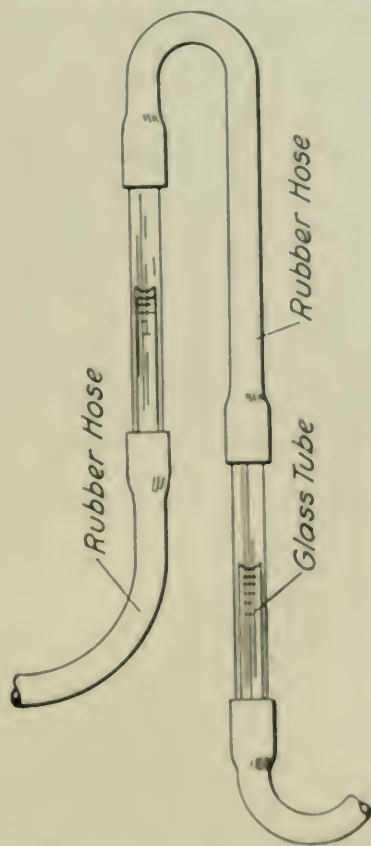


FIG. 13. EMERGENCY DIFFERENTIAL GAGE

long. The piece of rubber tubing connecting the glass tubes makes it possible to adjust the relative height of the tubes to suit the drop in pressure head. The gage may be fixed to an ordinary leveling rod or may be fixed to a plain board and the gage height measured with the ordinary pocket rule. This form of gage is not suitable for high pressures on account of the difficulty of making tight joints.

If the pressure openings are in the upper side of the pipe, it is important that provision be made for removing air, which is likely to become pocketed on either side of the orifice and in the gage connections, without forcing it through the hose or pipe connecting the pressure openings with the gage. If there are no pressure openings in the upper side of the pipe, pet cocks should be inserted. When opposite openings are inserted at each section provision should be made for ascertaining whether or not the tubes are free from obstruction and full of water. Fig. 4, p. 23, illustrates an arrangement of valves and pet cocks which makes this possible.

Preliminary to taking observations, with water flowing in the pipe, all cocks should be opened and the pipe and gage connections should be flushed. It is important that there should be no air in the pipe in the vicinity of the orifice, nor in the gage connections, for the presence of air is likely to change radically the gage reading.

If the foregoing precautions are observed under favorable conditions, the rate of discharge as measured by a pipe orifice should be accurate within 2 per cent. This means compares favorably with many other methods of measuring water. For unfavorable conditions the accuracy of the method will be as great as with most other methods under similarly unfavorable conditions.

IV. EFFECT OF DEVIATION FROM STANDARD CONDITIONS

20. *General Remarks.*—With the possibilities of utilizing the relation between drop in pressure and discharge for determining the rate of discharge through a pipe line in mind, it is important that something should be known concerning the effect of slight deviations from the standard conditions under which the tests were made, either in the arrangement or design of the apparatus or in the methods of handling the apparatus. To this end series of tests were made: (1) with gage connections fully opened and then with gage throttled, (2) with a single gage connection at each section, first with the nipple inserted in the upper side of the pipe and then in the under side, (3) with $\frac{1}{4}$ -inch eccentricity between center of orifice and center of pipe, first with a single nipple at each section, and then with the standard connection of two opposite nipples at each section, (4) with 45 degree bevel-edged orifices, the bevel facing upstream.

21. *Throttling.*—The purpose of throttling is to reduce the effect of momentary pressure fluctuations, thereby causing, for a constant discharge, the fluid in the columns of the differential gage to remain in a nearly stationary position. The experiments indicate that for values of $\frac{D}{d}$ of 2 or greater, when there are no systematic surges along the pipe line (such as might be caused by pump action), there is no appreciable reduction in the magnitude of the accidental errors of observing brought about by throttling; but for $\frac{D}{d}$ less than 2, when the region of greatly disturbed flow (to which reference has previously been made) is at or near the section of greatest contraction, the accidental errors may be reduced to 20 or 25 per cent of what they are with gage connections fully open. The results of the tests also make it clear that systematic errors are likely to enter into the gage readings unless the throttling is done very carefully. If the gage is throttled too much and a change in the rate of flow takes place, it is likely to be several minutes before the columns become fully adjusted to the change in pressure. If the throttling is done

quickly, a gage reading taken immediately after is not likely to represent the mean difference in pressure head.

Briefly summed up, the experience of the observers warrants the following suggestions:

(1) Throttle only when necessary, which in general will be for values of $\frac{D}{d}$ less than 2, unless there are systematic surges along the pipe line.

(2) Throttling too much is often worse than not throttling at all, particularly if the rate of discharge is variable. Accurate observations may be taken when there is a considerable fluctuation in the heights of the gage columns.

(3) When throttling, best results are to be obtained if the throttle cocks are opened prior to each observation and simultaneously and slowly closed the desired amount. It is important that there should be no leaks in the gage connections. A leakage of a few drops per minute from the throttle cock is likely to produce a large error in gage reading.

(4) When using the water gage, observations may be most nearly accurately taken when the gage is so throttled that the two water columns fluctuate the same amount and in unison. In general this arrangement will require that the gage column connection for the section of greatest contraction be throttled more than that for the section of beginning of convergence.

22. *Position and Number of Gage Connections.*—The tests to determine the effect of the position and number of gage connections warrant the following statements:

(1) If the orifice is concentric with the pipe, a change in the circumferential position of the pipe nipples inserted at the section of beginning of convergence and the section of greatest contraction will cause no variation in gage readings. If only one nipple is to be inserted at each section, there is an advantage in placing it in the under side of the pipe if air is present in the discharge, and in the upper side of the pipe if sediment is being carried.

(2) For $\frac{D}{d}$ equal to 2 or more there is no material advantage

in having more than one pressure opening at each section. For $\frac{D}{d}$ less than 2 the momentary fluctuations in pressure, as registered by the gage, may be appreciably reduced by having two opposite openings at each section, these being connected as shown in Fig. 4, p. 23. It seems probable that additional openings would still further reduce the gage fluctuations.

23. *Eccentricity.*—The tests to determine the effect of eccentricity were made on only the 4-inch pipe. The eccentricity was $\frac{1}{4}$ inch and the displacement of the orifice was vertical. The apparatus was so arranged that the gage could be made to register the difference in pressure obtained for the two sections by connecting to the differential gage (a) the openings in the top of the pipe, (b) the openings in the bottom of the pipe, and (c) all four openings.

The results indicate that regardless of the size of orifice an eccentricity of as much as $D/16$ will produce no effect on gage readings, provided the gage is connected to two opposite pressure openings at each section. But for $\frac{D}{d}$ less than 2 the gage readings will be changed considerably when the gage is connected to a single pressure opening at each section. For $\frac{D}{d}=1.5$ this change is about 4 per cent, and for $\frac{D}{d}=1.2$, about 18 per cent. It seems probable that a corresponding eccentricity with other sizes of pipe would produce a similar effect. Considering the difficulties of ascertaining whether or not the orifice is properly centered under the ordinary conditions of practice, the necessity for gage connections on opposite sides of the pipe at each section, when $\frac{D}{d}$ is less than 2, will be readily appreciated.

The effect of small longitudinal displacements of the gage connections is well illustrated by the curves of Fig. 3, p. 20. It is evident that for $\frac{D}{d}$ less than 2, in order that no measurable change in the gage readings be produced, no considerable change in the longitudinal position of the pressure openings for the section of beginning of convergence may be made, nor may the pressure openings for the section

of greatest contraction be placed nearer the orifice than four-tenths the diameter of the pipe.

24. *Bevel-edged Orifices.*—The method of conducting tests with bevel-edged orifices was identical with that used in determining the relation between drop in pressure and discharge for thin square-edged orifices. Tests were made on 4-inch and 6-inch pipe with five sizes of orifice for each size of pipe. For each of the two series of tests the ratios of diameter of pipe to diameter of orifice were 1.22, 1.52, 2.03, 3.04, and 6.08.

The orifices were cut in 3/16-inch plates with the upstream edge on a 45-degree bevel. The thickness of metal at the small diameter of each orifice was 1/32 inch, and the large diameter was 5/16 inch greater than the small diameter.

In instituting the tests the thought was that the effect of the beveled edge might go to produce a coefficient of discharge less affected by small changes in $\frac{D}{d}$ as the diameter of the orifice approaches that of the pipe, and thus make it practicable, with high velocities, to use a larger sized orifice than the experiments with thin-plate orifices indicate may be used with precision.

TABLE 7
COEFFICIENTS OF DISCHARGE FOR BEVEL-EDGED ORIFICES— C'

$\frac{D}{d}$	Pipe Diam.	Coef. of Discharge					Pipe Diam.	Coef. of Discharge				
		$h_b = 0.01$	$h_b = 0.1$	$h_b = 1.0$	$h_b = 10$	$h_b = 50$		$h_b = 0.01$	$h_b = 0.1$	$h_b = 1.0$	$h_b = 10$	$h_b = 50$
1.22	4-in.	0.870	0.852	0.832	0.811	0.800	6-in.	0.820	0.813	0.795	0.775	0.764
1.52		0.802	0.784	0.765	0.747	0.738		0.762	0.744	0.727	0.709	0.701
2.03		0.773	0.759	0.745	0.731	0.724		0.735	0.722	0.709	0.696	0.690
3.04		0.753	0.741	0.730	0.718	0.713		0.735	0.724	0.713	0.702	0.697
6.08		0.756	0.750	0.745	0.739	0.737		0.746	0.740	0.735	0.729	0.727

The coefficients of discharge, deduced from discharge curves similar to those of Figs. 6 and 7, pp. 28 and 29, are shown in Table 7. A comparison of these coefficients with corresponding values in Table 4, p. 30, will show that the effect of beveling has been materially to increase the coefficients of discharge and also to produce a coefficient of discharge that, as the drop in pressure head increases and as the size of pipe increases, decreases much more rapidly. In Fig. 14 the coeffi-

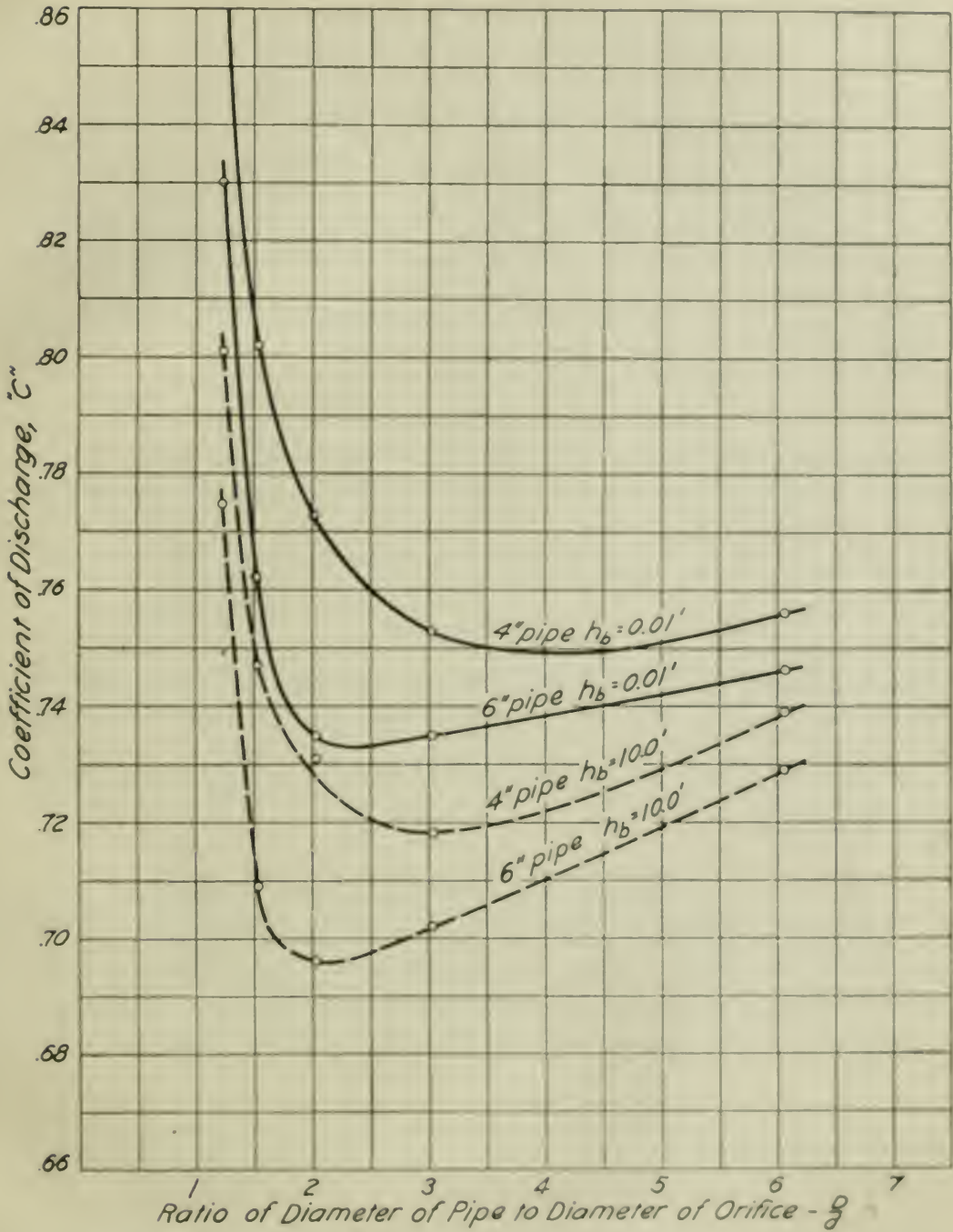


FIG. 14. COEFFICIENTS OF DISCHARGE FOR BEVEL-EDGED ORIFICES

coefficients of Table 7 for $h_b = 0.01$ ft. and $h_b = 10$ ft. are plotted against values of $\frac{D}{d}$. A comparison of the curves drawn through these points with corresponding curves for square-edged orifices, Fig. 9, p. 32, will

still further emphasize the extreme variability of the coefficients of discharge under consideration. As an offset to these undesirable characteristics, for a given change in the value of $\frac{D}{d}$ when that ratio is less than 2, there is considerably less variation in the coefficient of discharge for the bevel-edged orifice than for the square-edged orifice. This statement is especially true for values of $\frac{D}{d}$ less than 1.5, where the rate of change for the bevel-edged orifice is about two-thirds that for the square-edged orifice.

It is worth noting that the coefficient of discharge for each orifice approaches that for the corresponding square-edged orifice as the drop in pressure head increases, and that the rate of this increase becomes greater as $\frac{D}{d}$ decreases and becomes greater as the diameter of the pipe increases. For example, for the 4-inch pipe and $\frac{D}{d}=2.03$ the variation between the two coefficients of discharge is 17 per cent when $h_v=0.1$ ft. and 15 per cent when $h_v=50$ ft.; for the 6-inch and $\frac{D}{d}=2.03$ the variation is 14 per cent when $h_v=0.1$ ft. and 12 per cent when $h_v=50$ ft.

A study of the observations indicates that the accidental errors of reading the gage are about the same for the bevel-edged orifice as for the square-edged orifice. The systematic errors, however, or those which will not be eliminated by increasing the number of observations, are those to which most attention must be given; and among the sources of systematic error when $\frac{D}{d}$ is small, that source most likely to be productive of the greatest error in the computed discharge under the conditions of ordinary practice seems to be in the determination of the ratio $\frac{D}{d}$. Since for small values of $\frac{D}{d}$ the coefficient of discharge for bevel-edged orifices varies less for a given change in $\frac{D}{d}$ than does the coefficient for square-edged orifices, it seems reasonable to believe that for values of $\frac{D}{d}$ less than 1.5 there may be a slight advantage in using

the bevel-edged orifice, although the present experiments are not sufficiently comprehensive to confirm fully this belief.

The curves in Figs. 9 and 14 are of value as an indication of the importance of having the edges of the square-edged orifice sharp and square if the coefficients of discharge for square-edged orifices are expected to hold good. In permanent installations this will make it important that the orifice plate be of some material which does not rust or corrode easily.

V. SUMMARY

25. *General Applicability of Pipe Orifice Method.*—The foregoing discussion has shown that the thin-plate orifice inserted in a pipe may be used with confidence for measuring the discharge of water through pipes. Like nearly all methods of measuring water it is subject to some limitations although it helps to fill a growing need which has been partly filled by the pitometer and by the injection of chemicals. The pipe orifice is in effect a portable Venturi meter, the disadvantage of the pipe orifice being the relatively large lost head caused by the obstruction of the orifice plate; however since the pipe orifice method is probably best adapted to temporary use the lost head may in general be unimportant. In a long pipe line also the lost head caused by the orifice would be relatively small. Cases in which the pipe orifice should be of particular value have already been suggested in the introduction.

Although all the deductions and conclusions given in this summary apply to the measurement of water, attention should be called to the fact that the pipe orifice is adapted to measuring the discharge of air, gas, and steam through pipes.

26. *Conclusions.*—The following points are important as a guide to the proper use of the pipe orifice method of measuring the discharge of water through a pipe:

(1) The two sections of the pipe between which change in pressure head may be most reliably determined are the section at which normal flow is discontinued and the stream begins to converge as it approaches the orifice and the section of greatest contraction of the jet after it leaves the orifice. Regardless of the size of pipe, for all sizes of orifice which it is feasible to use, the distance from the plane of the orifice to the section of beginning of convergence may be taken as eight-tenths the pipe diameter, and the distance to the section of greatest contraction as four-tenths the pipe diameter (section 11, p. 21).

(2) The drop in pressure head between these two sections is greater than that to be found for any other two sections near the orifice.

(3) Having given the measured difference between the pressure head at the section of beginning of convergence and the pressure head at the section of greatest contraction the discharge may be determined through the use of equation (2), p. 12, or through the use of equation (6), p. 35.

(4) The coefficient of discharge to be used in equation (2) is a variable quantity (Table 4, p. 30). It decreases as the size of pipe increases; it decreases slightly as the drop in pressure head increases; it has a minimum value for orifices having a diameter of one-third that of the pipe and increases as the diameter of the orifice becomes greater or becomes less than one-third the diameter of the pipe.

(5) The lost head caused by any given orifice in the pipe in terms of the velocity in the pipe may be determined by equation (3), p. 13.

(6) The lost head is always less than the drop in pressure head but approaches it in value as the ratio of the diameter of the pipe to the diameter of the orifice increases (section 6, p. 13).

(7) Due to the fluctuations of the liquid in the gage tubes the systematic error of reading the gage increases as the ratio of the diameter of the pipe to that of the orifice decreases, but when that ratio $\frac{D}{d}$ is 2 or greater the error may under normal conditions of flow be reduced to a negligible quantity by a proper manipulation of apparatus. As $\frac{D}{d}$ becomes less than 2 the accidental error of reading the gage increases very rapidly (section 19, p. 40) and also small errors in the measurement of the diameter of the pipe or the diameter of the orifice are likely to be the constant sources of an error of increasing magnitude in the computed discharge (section 14, p. 34). The indications are that, for favorable conditions of flow and with care in installing the apparatus and in observing, discharge may be determined generally within 2 per cent when the diameter of the orifice is not in excess of two-thirds that of the pipe, but this size of orifice seems to be about the maximum that can be used except for approximate determinations of discharge. When the magnitude of the lost head is not the controlling factor in the choice of size of orifice, best results

are likely to be obtained if the diameter of the orifice is not greater than one-half that of the pipe.

(8) For orifices having a diameter greater than one-half that of the pipe the use of two opposite pressure openings at each section is important because of the probability of the orifice being somewhat eccentric with the pipe, unless greater care is taken in placing the orifice than will usually be found practicable (sections 22, p. 44, and 23, p. 45). Systematic errors of observing may be greatly reduced by proper throttling (section 21, p. 43).

(9) The coefficient of discharge for bevel-edged orifices is a much more variable quantity and is materially greater than the coefficient of discharge for thin square-edged orifices. The use of the bevel-edged orifice seems not to be practicable, except for approximate measurements when the orifice diameter is greater than two-thirds of the pipe diameter (section 24, p. 46).

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PASSENGER TRAIN RESISTANCE

BY

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CONTENTS

	PAGE
I. INTRODUCTION	7
1. Preliminary Statement	7
2. Acknowledgments	8
II. SUMMARY	10
3. The Trains	10
4. The Track	10
5. The Results	10
III. THE TRAINS, THE TRACK AND THE TEST CAR	12
6. The Trains Tested	12
7. The Track	14
8. The Test Car	15
IV. TEST CONDITIONS, METHODS AND CALCULATIONS.	16
9. Test Conditions	16
10. Test Methods	16
11. Methods Employed in Calculating the Results.	18
12. The Derivation of the Resistance Curves.	20
V. THE RESULTS OF THE TESTS	21
13. Results of the Individual Tests.	21
14. Results of All the Tests	22
15. The Results Expressed as Resistance-Speed Curves.	29
16. The Results Expressed in Tabular Form	31
17. Final Results	33
18. Comparison with Other Experiments	34
APPENDIX. TYPICAL TABLES OF RESULTS OF THE INDIVIDUAL TESTS	36

LIST OF FIGURES

NO.	PAGE
1. Diagrams Showing the Relation between Train Resistance and Speed for Six Trains.	21
2. Diagrams Showing the Relation between Train Resistance and Speed for Four Trains	22
3. Diagrams Showing the Relation between Train Resistance and Speed for Six Trains	23
4. Diagrams Showing the Relation between Train Resistance and Speed for Six Trains	24
5. Diagrams Showing the Relation between Train Resistance and Speed for Six Trains.	25
6. Curves Showing the Relation between Train Resistance and Speed for the Twenty-eight Trains Tested	26
7. Diagrams Showing the Relation between Train Resistance and Average Car Weight for Seven Different Speeds	27
8. Curves Showing the Average Relation between Resistance and Average Car Weight for Seven Different Speeds.	30
9. Curves Showing the Relation between Train Resistance and Speed for Average Car Weight between Thirty and Seventy-five Tons. . .	31
10. Comparative Curves Showing the Relation between Train Resistance and Speed for Train Composed of Eight 40-ton Cars.	34
11. Comparative Curves Showing the Relation between Train Resistance and Speed for Train Composed of Ten 60-ton Cars.	35

LIST OF TABLES

NO.	PAGE
1. Train Data and Weather Conditions	17
2. Values of Resistance at Various Speeds Derived from the Curves for the Individual Tests.	28
3. Values of Resistance at Various Speeds and for Various Average Car Weights	32
4. The Results of Test No. 7	37
5. The Results of Test No. 24.	38
6. The Results of Test No. 19.	39

PASSENGER TRAIN RESISTANCE

I. INTRODUCTION

1. *Preliminary Statement.*—The Department of Railway Engineering of the University of Illinois has for some years carried on experiments to determine the tractive resistance of steam railway trains and electric railway motor cars.* The results relating to steam railways thus far published have applied only to freight trains. This bulletin presents for the first time the results of the experiments with passenger trains. These experiments were begun in 1908 on light local trains and were resumed during the spring and summer of 1916 on heavy equipment in through passenger service.

The results here presented define the resistance of the trains tested throughout the ordinary range of speed of passenger trains. The fact is also made clear that in passenger as in freight trains the resistance expressed in pounds per ton varies with the average weight of the cars composing the train, being less for trains made up of heavy cars than it is for trains of light cars. As regards freight train resistance, this influence of car weight has been recognized for some years, and this recognition found application in freight service in the making of tonnage ratings even before complete experimental data were at hand. Today tonnage ratings almost universally embody distinctions based upon this influence of car weight.

It has always seemed likely that the specific resistance of passenger trains was similarly affected by the weight of the cars in the train. Notwithstanding this probability, however, the influence of car weight has frequently been ignored in dealing with problems in passenger train resistance. It is embodied in only a few of the formulas currently used for that purpose, and even in some of these its influence is recognized only to the extent to which it affects the air

*See Bulletin 43, entitled "Freight Train Resistance: Its Relation to Car Weight," 1910, Bulletin 59, entitled "The Effects of Cold Weather upon Train Resistance and Tonnage Rating," 1912, Bulletin 74, entitled "The Tractive Resistance of a 28-ton Electric Car," 1914, and Bulletin 92, entitled "The Tractive Resistance on Curves of a 28-ton Electric Car," 1916, of the Engineering Experiment Station of the University of Illinois.

resistance component of net resistance, its effect on journal and rolling resistance being neglected. This situation has arisen primarily because of the lack of experimental data applying to passenger cars of different weights, and because the problems requiring the prediction of passenger train resistance have not the economic importance of similar problems arising in connection with freight trains. The error involved by ignoring car weight is not, however, so great in the former case as in the latter; for while passenger cars ordinarily vary in gross weight only from about thirty-five to sixty-five tons, the corresponding range in freight service is from about eighteen to ninety-eight tons. Despite this difference in the necessity of accurately determining resistance in the two instances, the prediction of passenger train resistance is nevertheless important, and data such as are here presented have been needed.

The results here set forth define the resistance of passenger trains made up of cars ranging in average weight from thirty-three to seventy-one tons, and, it is believed, present for the first time full experimental data showing the influence of car weight on the resistance of entire steam railroad passenger trains throughout the range of car weight now prevalent in American practice.* Throughout this bulletin the terms "resistance," "net resistance," and "specific resistance" mean the number of pounds of tractive force required for each ton of train weight in order to keep the train moving at uniform speed on straight level track. Ordinarily these terms apply to resistance in still air. Here, however, they include the effects on resistance of such winds as prevailed during the tests, which, as later appears, were from various directions and of velocities as high as thirty-one miles per hour. The bulletin deals exclusively with the resistance of the cars alone—locomotive and tender resistance were not measured and are not discussed.

The results of the last eighteen tests included in this report were separately summarized and discussed in a paper on "Passenger Train Resistance" published in February, 1917 in Bulletin 194 of the American Railway Engineering Association.

2. *Acknowledgments.*—Through the courtesy of W. L. PARK, Vice President, and R. W. BELL, General Superintendent of Motive

*Data concerning the resistance of two trains composed of cars of two different weights (46.2 tons and 62.7 tons) are given in Bulletin No. 26 of the Pennsylvania Railroad Company Test Department.

Power of the Illinois Central Railroad, the tests were made on the lines of that company between Champaign and Centralia, Illinois, and by means of a dynamometer car which since 1900 has been jointly owned by the Railroad Company and the University. The interest and the coöperation of E. H. BAKER, Trainmaster at Champaign, under whose immediate direction the test car was handled, are also gratefully acknowledged.

II. SUMMARY

At the expense of some duplication of statement, the following summary is presented at this point in order to provide a general view of the work.

3. *The Trains.*—The tests were undertaken to measure the resistance of passenger trains at all speeds up to seventy miles per hour and for average car weights varying throughout the entire current range in weight. The chief characteristics of the twenty-eight trains tested were as follows:

	Minimum	Maximum
Total weight (excluding locomotive and tender),		
tons	138	727
Number of cars in the train	4	12
Average gross weight per car, tons	33.6	71.1

Nine of these trains were in “local” service, the others in “through” service. Of the 240 cars composing these twenty-eight trains, 178 had six-wheel trucks, and 62 had four-wheel trucks.

4. *The Track.*—The experiments were made upon well constructed and well maintained main line track, nearly all of which is laid with either 85-pound or 90-pound rail. Except through station grounds where screenings or cinders are used for ballast, the track is ballasted with broken stone.

5. *The Results.*—The average values of the resistance of the trains tested are defined by the curves of Fig. 9. From these curves has been prepared Table 3, which shows probable average values of resistance for passenger trains composed of cars varying in weight from thirty to seventy-five tons, and at speeds ranging from five to seventy-five miles per hour. Individual trains will occasionally be met with whose general average resistance will exceed the values previously defined by from 8 to 10 per cent.

The tests were made in fair and moderate or warm weather during which the maximum wind velocity encountered at any time was, with one exception, twenty-five miles per hour. The results are probably safely applicable to passenger trains running under like conditions of track and weather. They should not, however, be applied without modification to trains running under widely different conditions.

III. THE TRAINS, THE TRACK AND THE TEST CAR

6. *The Trains Tested.*—The twenty-eight passenger trains included in the report were tested in two series. Series I comprises ten tests which were made in 1908-1909 in conjunction with freight resistance tests then in progress. At that time the test car was operated from Champaign to Mattoon, Illinois, in freight service and it was usually returned in local passenger train No. 24 from Mattoon to Champaign. The test car was operated upon one occasion at this period in through train No. 5, and Series I includes, therefore, nine runs with train 24 and one with train 5. The average weight of the cars in train 24 varied during the nine tests then made only from 33.6 to 40.7 tons, and the results based on this limited range in weight did not warrant generalizations. The opportunity to continue the tests with heavy trains did not present itself until May of 1916 when the work was resumed and continued during the summer with trains No. 5 and No. 2. Eleven tests were then made with train No. 5, "Fast Mail," running from Champaign to Centralia, and seven tests with train No. 2, "Through Mail," running from Centralia to Champaign. These eighteen tests constitute Series 2.

Considering both series together the range in train weight was from 138 to 727 tons, in average car weight from 33.6 to 71.1 tons, and the number of cars per train varied from 4 to 12. For each of the three classes of trains the variations in train weight, car weight, number of cars, and truck construction were as follows:

Train No.	Number of Trains Tested	Gross Weight of Train, Tons		Average Weight per Car, Tons		Number of Cars in the Train		Number of Cars per Train Having 4-wheel Trucks	
		Min.	Max.	Min.	Max.	Min.	Max.	Min.	Max.
24	9	138.0	234.2	33.60	40.70	4	6	3	4
5	12	370.6	615.5	46.32	55.90	10	12	2	3
2	7	549.0	727.0	63.49	71.10	8	11	0	2

Similar data and other details concerning the make-up of each of the trains are presented in Table 1.

Train No. 24 is scheduled to cover the 45 miles from Mattoon to Champaign in 1 hour and 22 minutes, a schedule speed of 32.9 miles per hour. A typical running time for this train is 1 hour and 10 minutes, equivalent to a running speed of 38.6 miles per hour. Train No. 5 is allowed by the schedule 3 hours and 35 minutes to cover the 124.5 miles from Champaign to Centralia; thus its schedule speed is 34.8 miles per hour. Its running time is about 2 hours and 51 minutes, which corresponds to a running speed of 43.7 miles per hour. Train No. 2 is scheduled between Centralia and Champaign at 3 hours and 5 minutes, a schedule speed of 40.4 miles per hour. For this train a typical running time, 2 hours and 37 minutes, is equivalent to a running speed of 47.5 miles per hour.

In eight of the nine tests of train 24 the resistance of the dynamometer car itself is included in the records and its weight is included in the train weight. The weight of the test car, 29 tons, is however not greatly different from the average weight of the cars in this train and its inclusion has lowered only slightly the average car weight. The test car weight is likewise included in five of the tests with trains 2 and 5, and in these instances its relatively light weight has lowered appreciably the normal average car weight. In the remaining fourteen tests the dynamometer car was coupled with its measuring draw-bar toward the rear of the train, and in these cases its own resistance is excluded from the test car records. Its weight is consequently excluded from the train weights and has not affected the normal average car weights.

The net weights of all cars were obtained from the records of the Illinois Central Railroad and the Pullman Company. To determine the car loads passengers were counted and allowed for at 140 pounds each; the weights of baggage and mail were estimated by personal inspection and count of the contents of each car; the weight of express was determined by reference to the bills, supplemented by inspection and estimate. The error entailed by these methods certainly does not exceed fifteen per cent of the weight of the load, and since among all the trains the maximum load amounted to only 6.6 per cent of the gross train weight, the maximum error in gross train weight itself caused by inaccuracies in the processes described is less than one per cent.

Of the three classes of trains tested, local train No. 24 had the lowest average car weight and, naturally, the highest proportion of

four-wheel truck cars. Of the 45 cars composing these nine local trains, 28 (or 62 per cent) had four-wheel trucks. In the twelve trains No. 5 there were included altogether 129 cars, of which 27 (or 21 per cent) were equipped with four-wheel trucks. Train No. 2 had the heaviest cars and the smallest proportion of four-wheel truck cars. The seven trains of this class included 66 cars, of which only 7 (or 10.6 per cent) had four-wheel trucks. All other cars in the three classes of trains had six-wheel trucks.

Since car weight affects the specific resistance, not only through its influence on air resistance, but through its influence on journal and rolling resistance as well, there is apparently an inconsistency in method in grouping—as is done in this report—trains including both four-wheel and six-wheel truck cars, and especially in thus grouping trains which have in their make-up different proportions of the two kinds of trucks. This apparent inconsistency, considering the purpose of the investigation, is not so objectionable as may appear. It was not possible under the conditions under which the tests were made to control the make-up of the trains. The tests had to be made in regular service and the trains had to be accepted with their usual make-up. This limitation has not defeated the main purpose of the tests, for they were undertaken, not to distinguish the resistance of four-wheel and six-wheel truck cars, but to measure the resistance of ordinary passenger trains of widely different average car weight; and wide variation in car weight carries with it, in American practice, a variation in truck construction similar to that encountered in the trains here discussed. Any train of 35 to 40 tons average car weight is sure to include in its make-up four-wheel truck cars,—and in about the proportion which prevailed in these tests. Even the heaviest through trains are likely occasionally to include a car or two with four-wheel trucks, as was the case with Train No. 2.

7. *The Track*.—The track, formerly part of one of the oldest single-track lines in the State, was converted about twenty years ago into a double-track road. The roadbed is in good condition and the drainage in general is excellent. The track was especially surveyed for the purposes of these tests.

Except on a few short stretches through station grounds, where cinders and screenings are used for ballast, both tracks are ballasted with broken limestone. The cross-ties, 6 inches by 8 inches by 8 feet,

are spaced about 20 inches between centers and are of treated red oak, treated soft wood or untreated white oak. Seventy miles of the southbound track are laid with 90-pound rail of American Railway Association section, while the remaining 54.5 miles are laid with 85-pound rail of American Society of Civil Engineers section; 98.6 miles of the northbound track are laid with 85-pound rail of American Society of Civil Engineers section, while the remaining 25.9 miles are laid with 90-pound rail of American Railway Association section. All rails are laid with broken joints supported on two ties. During eight months of the year there is employed in maintaining this portion of the road a force of men averaging one man per mile of track. During the remaining four months this force is reduced to one man for each two miles.

8. *The Test Car.*—The dynamometer car used in these experiments has been described in detail in Bulletin 43 of the Engineering Experiment Station of the University of Illinois and in the *Railway Age Gazette* for February 19, 1909. The recording apparatus within the car produces continuous graphical records of the gross resistance of the train, speed, time, brake cylinder pressure, wind direction, wind velocity, and location of mile posts and other reference points. A reproduction of one of the test car records is given in Bulletin 43 previously mentioned. When supplemented by an accurate profile and train data, these records permit the calculation of train resistance at any point or section on the road. The methods by which these calculations were made and the precautions observed to ensure accuracy are explained later.

IV. TEST CONDITIONS, METHODS AND CALCULATIONS

9. *Test Conditions.*—In Table 1 are set forth for each test the weather conditions, the average velocity and the direction of the wind, and the air temperatures prevailing at the beginning and at the end of the test. With only two exceptions the tests were made on fair days and in moderate or warm weather. The average air temperature varied from 40 degrees F. during one test to 93 degrees F. in another. The average wind velocity was 20 m. p. h. or less during all but three tests. During two tests the average wind velocity was 25 m. p. h. and on only one it was 31 m. p. h.

While the wind velocity was recorded for each test, the uncertainties regarding its influence have made a definite evaluation of its share of the total resistance impracticable. It has seemed necessary, therefore, to regard the wind velocity as one of the incidental variables which is likely to be met with in train operation and which should be included in the final resistance values. It has, accordingly, been included and the terms “resistance” and “specific resistance” as here used include the resistance due to such winds as prevailed during the tests.

10. *Test Methods.*—As has already been stated the tests were run in regular service, no attempt being made to modify schedules or to control speed. During each test the data cited in the description of the test car were produced and from them resistance was calculated for as great a variety of speeds as the records presented. These resistance values were then plotted in a diagram such as any one of the curves in Fig. 1, which shows for the train in question the relation between resistance and speed. The production of such a diagram was the immediate purpose of each test, and it was the expectation that when the resistance-speed curves of the individual tests were brought together, analysis of the curves would reveal the relations existing between train resistance and car weight.

The pressure in the brake cylinder was recorded merely to make it possible to distinguish the periods when the brakes were applied, it being obviously necessary to avoid such portions of the record when

PASSENGER TRAIN RESISTANCE

TABLE I
TRAIN DATA AND WEATHER CONDITIONS

Test Number	Date of Test	Weather	Air Temperature, Degrees F.		Approximate Average Wind Velocity, Miles per Hour	Range of Direction of the Wind with Respect to the Track		Train Length, Feet	Weights		Total Number of Cars in the Train	No. of Cars Having 2 Axles per Truck	No. of Cars Having 3 Axles per Truck	Kind of Cars						
			At Beginning of Test	At End of Test		From	To		Gross Train Weight, Tons	Average Gross Weight per Car, Tons				Test	Baggage and Express	Mail	Couch	Pullman	Diner	Special
1	10-31-08	Fair	60	60	11	+20° R	-81° R	300	202.9	40.58	5	3	3	1	1	1	1	1	1	1
2	11-7-08	"	60	55	13	+45° L	-88° L	300	201.5	40.70	5	3	3	1	1	1	1	1	1	1
3	11-21-08	"	62	60	8	+8° R	+39° R	300	185.1	37.02	5	3	3	1	1	1	1	1	1	1
4	1-23-09	"	66	70	20	+16° R	+39° R	345	234.2	39.03	6	3	3	1	1	1	1	1	1	1
5	1-28-09	Rainy	40	41	18	+28° R	+55° R	300	189.4	37.90	5	3	3	1	1	1	1	1	1	1
6	2-2-09	Fair	45	45	11	+27° R	+33° L	300	185.5	37.10	5	3	3	1	1	1	1	1	1	1
7	3-12-09	Cloudy	41	41	11	+34° L	+60° R	300	185.6	37.12	5	3	3	1	1	1	1	1	1	1
8	3-27-09	"	40	45	13	+36° L	+60° L	330	168.0	33.60	5	4	4	1	1	1	1	1	1	1
9	4-17-09	"	74	71	31	+36° R	+59° R	255	138.0	34.50	4	3	3	1	1	1	1	1	1	1
10	7-8-08	"	50	60	14	+10° L	+22° L	480	370.6	46.32	8	3	3	6	1	3	1	1	1	1
11	5-25-16	"	82	86	"	"	"	700	604.5	54.06	11	2	2	9	6	2	2	1	1	1
12	5-25-16	"	86	90	"	"	"	700	570.0	51.80	11	2	2	9	6	2	2	1	1	1
13	6-9-16	"	72	72	20	-30° R	+55° R	700	615.5	55.90	11	2	2	9	4	4	4	2	1	1
14	6-9-16	"	72	71	15	-10° R	+25° R	700	551.3	55.13	11	2	2	10	1	4	4	2	1	1
15	6-9-16	"	72	69	10	-15° R	+10° R	640	727.0	66.10	11	2	2	1	1	1	1	1	1	1
16	6-22-16	"	75	77	"	"	"	660	548.5	49.04	11	2	2	9	5	2	2	1	1	1
17	6-22-16	"	77	80	"	"	"	660	562.0	51.18	11	3	3	8	6	2	2	1	1	1
18	6-22-16	"	83	82	20	-70° L	+25° R	520	615.2	65.29	9	1	8	1	1	1	1	3	3	1
19	6-22-16	"	82	81	20	-70° R	+15° R	580	634.9	63.49	10	2	2	8	1	1	1	3	3	1
20	7-6-16	"	88	90	25	+0°	+20° L	650	601.6	50.13	12	3	3	9	1	2	2	3	1	1
21	7-6-16	"	90	92	25	+0°	+10° L	580	535.2	48.65	11	3	3	8	1	1	1	1	1	1
22	7-6-16	"	92	91	20	+15° L	+65° R	690	618.0	68.66	9	1	1	7	1	1	1	3	3	1
23	7-6-16	"	91	90	20	+0°	+35° R	630	549.0	68.63	8	1	1	7	1	1	1	3	3	1
24	7-13-16	"	86	89	20	+15° L	+40° R	615	570.1	51.83	11	2	2	9	6	2	2	1	1	1
25	7-13-16	"	89	93	15	+5° R	+40° R	630	540.0	49.10	11	2	2	9	5	2	2	1	1	1
26	7-13-16	"	93	92	15	+30° L	+35° R	765	708.3	70.83	10	1	1	9	1	1	1	3	4	1
27	7-13-16	"	92	90	15	-60° R	+30° L	720	639.9	71.10	9	2	9	1	1	1	1	2	4	1
28	8-2-16	"	72	72	20	+25° L	+45° L	675	606.2	55.10	11	2	9	1	1	1	1	2	1	1

1 Direction is designated by the angle made with the track. A wind which has any component of its velocity helping the train forward is marked +; winds with opposing velocity components are marked -. Winds from the right side of the track are designated as R; from the left side as L; thus +40° R means a wind blowing from the rear and from the right hand side, whose direction makes an angle of 40 degrees with the track.

choosing sections for calculation. The wind velocity and direction relative to the test car were obtained by means of an anemometer and windvane mounted on its roof. When compounded with the known speed and direction of motion of the car, these data permit the determination of the actual wind direction and wind velocity with respect to the track. The location of the test car upon the road was defined by marking upon the chart the position of mile posts and stations at the moment they were passed by the car. This record makes it possible to correlate any portion of the chart with the road profile. In addition to the weight of the train there were recorded its length, the car names and numbers, and facts concerning the truck construction.

11. *Methods Employed in Calculating the Results.*—To produce for each train the resistance-speed curve referred to in the preceding section involves calculating the train resistance at various positions of the train upon the track, and the first step towards this end is the inspection of the test car record in order to select suitable points or sections at which the resistance may be calculated. The considerations of most importance in this selection are that the points represent finally as great a speed range as possible and that the speeds be approximately evenly distributed within this range. Points and sections were selected only where the entire train was running and continued to run upon straight track; resistance due to track curvature is therefore entirely eliminated. The data essential to the process of calculation are the drawbar pull of the engine, the train speed and its acceleration, the tonnage, and the profile. The pull and the speed, as previously stated, are determined from continuous curves drawn on the test car chart. Two processes have been used, designated here as method 1 and method 2. By method 1, the momentary values of pull, speed, acceleration, and grade were determined for a particular position of the train upon the road; by method 2 the average values of these quantities were determined for the period during which the test car was passing over a definite section of the track.

Under method 1, the point on the road having been chosen, the pull and the speed were found by direct readings from the chart. This pull divided by the tonnage gives the gross train resistance at this speed, and this gross resistance was then corrected for both acceleration and grade resistances. The acceleration was determined by graphical methods from the speed curve, and the grade was found by cor-

relating the position of the train with the profile. The points were all selected so that at the moment under consideration the entire train was on a nearly uniform grade. Method 1 results in momentary values of train resistance at the points considered.

By method 2 the average value of train resistance was determined for the period during which the test car at the head of the train was passing a selected section of the track. This track section corresponds to a certain length on the test car record. The section was selected so that the speed of the car when entering was nearly equal to its speed at exit and moreover so that no considerable variations in speed occurred during transit over the section. These portions of the chart having been chosen, the average pull was next found by determining the average ordinate of the curve of drawbar pull, and the average speed was found by means of the section length and the time record. Gross resistance in pounds per ton was next derived by dividing this value of pull by the tonnage, and this gross resistance was accordingly corrected for the resistances due to acceleration and grade, as in method 1. In this case the average acceleration is found by consideration of the speeds at entrance to and exit from the section. In order to correct for grade, the elevation of the center of gravity of the train was determined for that position of the train at which the test car entered the section, and again for the position at which the car left the section. The difference between these elevations establishes the effective average grade, which either helps or opposes the locomotive while the train passes the section. These elevations of the center of gravity of the train may not be determined with sufficient accuracy unless the train at the moment is on a practically uniform grade. The section limits were therefore so chosen. Method 2 results in a value of *average* train resistance for the *average* speed at which the train passes the section under consideration. It would be rigidly correct if train resistance varied uniformly with speed, in other words, if the curve showing the relation of resistance to speed were a straight line. This, of course, is not the relation, and the process accordingly gives results which are slightly in error. As previously stated, however, the section was chosen so that the difference between the speeds at entrance to and exit from the section was small, and for the speed range represented by this difference, the curve of train resistance deviates but little from a straight line. Such error as does result from the process is, therefore, very small, and is of no moment whatever when compared with

variations, due to natural causes, which occur in the resistance itself.

The two methods are fundamentally alike. Although the first is the less laborious, it requires the determination of acceleration at a point on the speed curve, which is occasionally difficult to make accurately. Method 2 is, accordingly, generally preferable, and it is also to be preferred because it deals with average values and therefore tends to eliminate from the results the incidental momentary variations which frequently occur in train resistance. Method 2 has consequently been used during these tests whenever it was possible to do so, but it has been necessary occasionally to resort to the use of method 1, especially for the determination of resistance at low speed, because in passenger service low speeds are rarely maintained for a long enough period to permit the use of the second method. Of all the resistance values included in this report only about 16 per cent were derived by method 1. These values are represented in the resistance speed diagrams by the circles, whereas values derived by method 2 are represented by the black dots.

In general the methods used during these tests were like those set forth in Bulletin 43 of the Engineering Experiment Station of the University of Illinois and described in detail in sections 11 to 15 and in Appendix 4 of that bulletin. The precautions taken to ensure accuracy which are there described, particularly in the determination of the acceleration corrections, were strictly observed in these experiments.

12. *The Derivation of the Resistance Curves.*—The calculations result, for each test, in a series of values of net train resistance at a variety of speeds. These values of resistance were plotted with respect to speed, and gave such a diagram as in Fig. 1. The curve, such as is shown there, was next drawn to express, for the test in question, the relation existing between resistance and speed. In order to draw this curve, the plotted points were assumed to be arranged in a number of groups, and for each group the averages of the values of speed and of resistance were determined. By these averages a new point or “center of gravity” of the group was then plotted. The curve was drawn by confining attention to the few points thus determined. The groups of points were arbitrarily selected so that the resulting “centers of gravity” would be distributed nearly equidistantly throughout the speed range. All curves presented in the report, except those shown in Fig. 9, were drawn by this process.

V. THE RESULTS OF THE TESTS

13. *Results of the Individual Tests.*—The immediate result of each test is a curve which shows for the train under consideration the relation existing between train resistance and speed. Fig. 1 includes such a curve for the train of test 1, in which the average weight of the cars was 40.58 tons. Similar curves for each of the twenty-eight trains tested are presented in Figs. 1 to 5 inclusive.

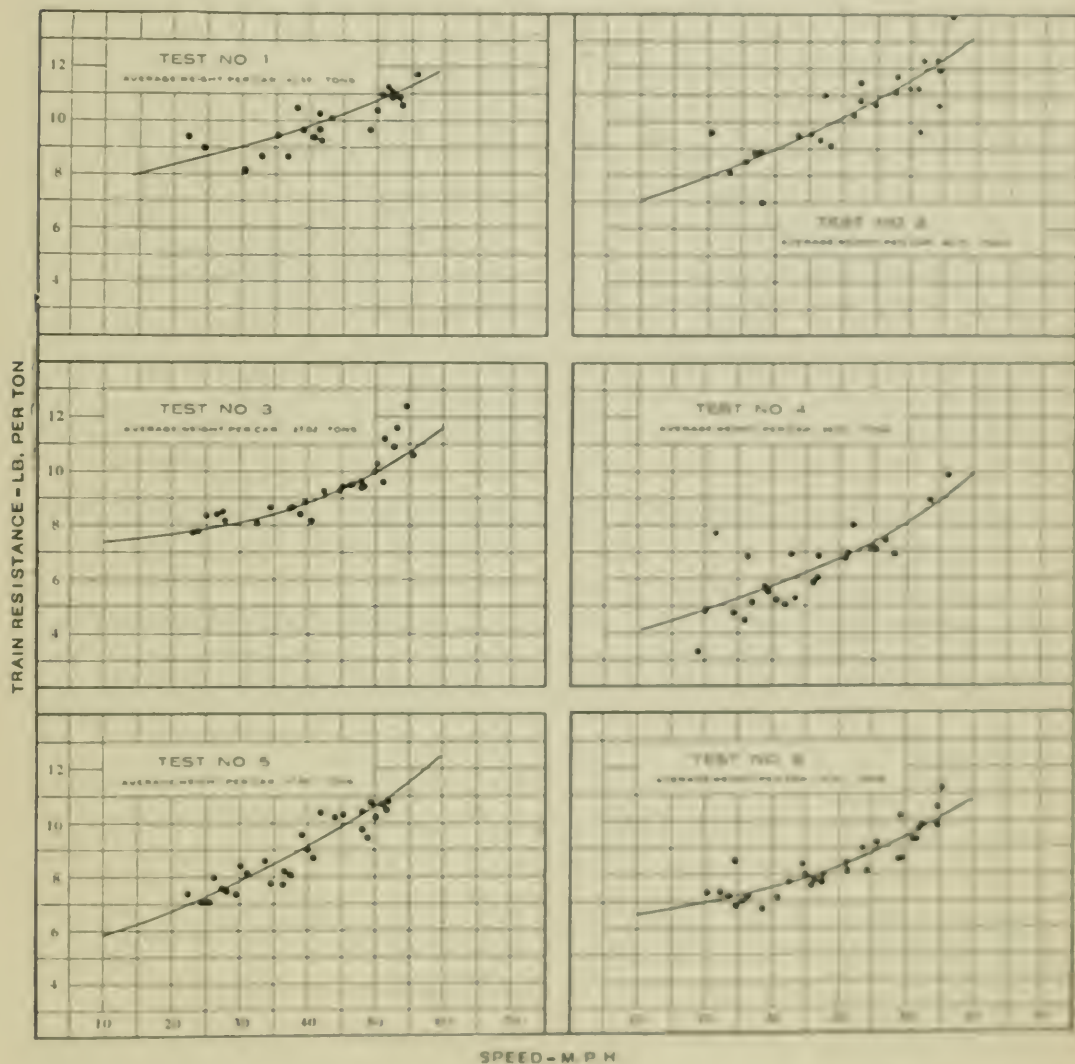


FIG. 1. DIAGRAMS SHOWING THE RELATION BETWEEN TRAIN RESISTANCE AND SPEED FOR SIX TRAINS

There is considerable variation among the points on the individual plots of Figs. 1 to 5, as well as considerable variation of the points from the mean defined by the curves. It is possible that part of this variation may be due to accumulated errors in instruments or in the

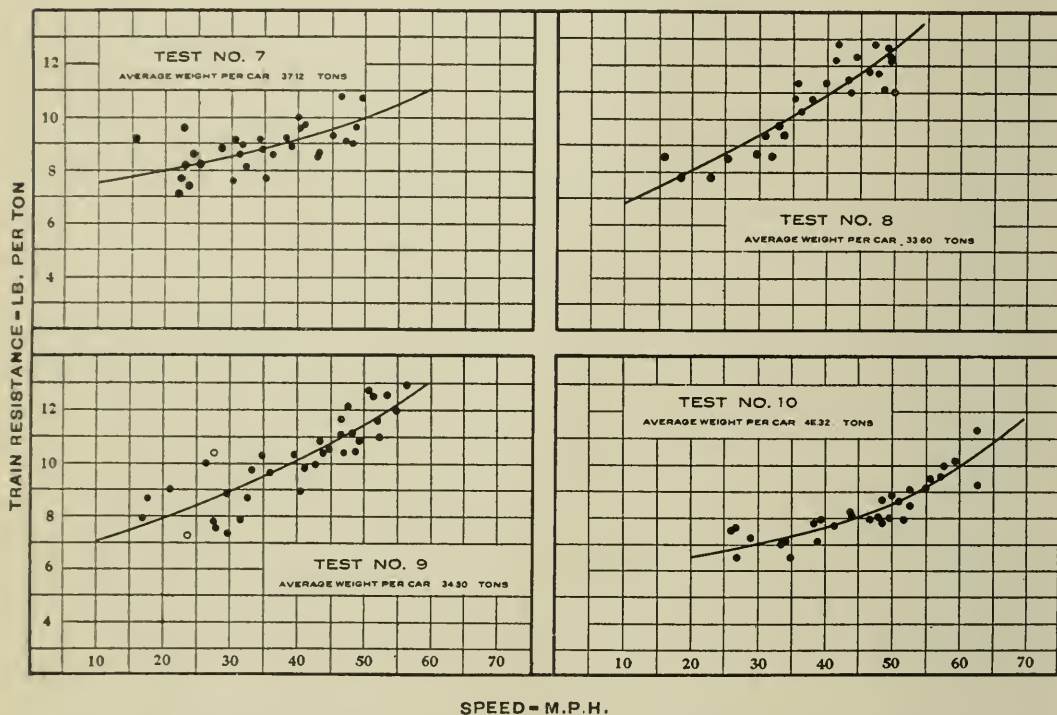


FIG. 2. DIAGRAMS SHOWING THE RELATION BETWEEN TRAIN RESISTANCE AND SPEED FOR FOUR TRAINS

calculations, but since every precaution was taken to avoid such errors, they are undoubtedly of little consequence. The data, particularly for certain tests during which wind conditions changed considerably, indicate that the usual variations in the direction and velocity of the wind during any test are sufficient to account for the differences in train resistance shown in these figures. Variations in such other components of train resistance as journal friction and oscillatory resistances are also undoubtedly accountable in a large degree for the differences discussed. The data do not afford a means of evaluating the influence of such components of resistance.

14. *Results of All the Tests.*—The twenty-eight curves from Figs. 1 to 5 have been brought together in Fig. 6 which embodies, therefore, the immediate results of all the tests. These curves make

it clear that there was a maximum variation of about 75 per cent in the resistance of different trains. Difference in wind velocity fails to account for this variation, except in a few selected cases, but there

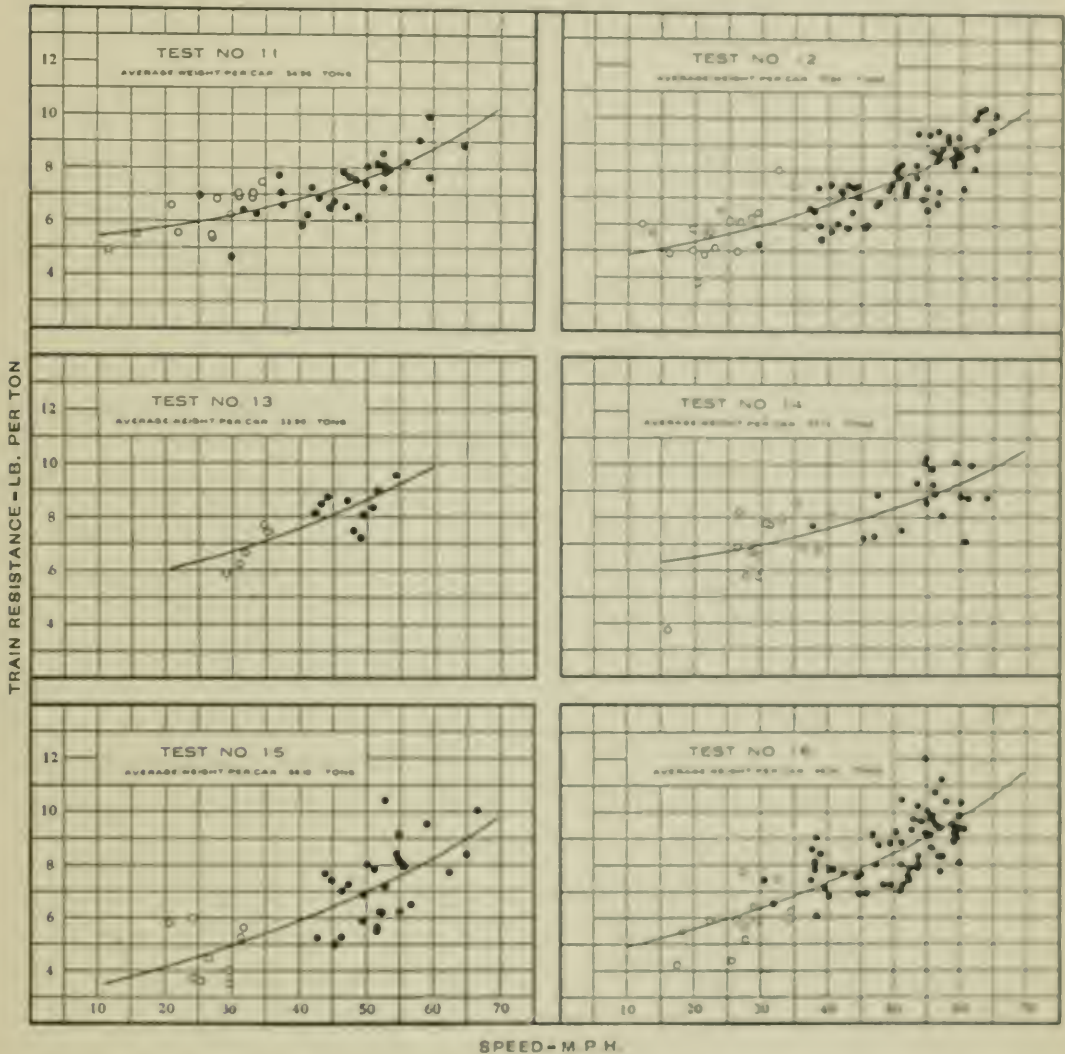


FIG. 3. DIAGRAMS SHOWING THE RELATION BETWEEN TRAIN RESISTANCE AND SPEED FOR SIX TRAINS

was every reason to believe that as in the freight train resistance tests reported in Bulletin 43 of the Engineering Experiment Station the divergence of these curves might be explained by the variation in the average weight of the cars going to make up the different trains. Comparison with the tabular data shows that the four upper curves are derived from tests of trains in which the average car weight was 36.5 tons while the four lower curves are from tests of trains in which the

average car weight was 69.8 tons. These facts serve as a rough indication of the part played by car weight in effecting changes in train resistance. As has been previously stated, this conclusion was anticipated when the work was begun and tests were made therefore with trains which cover the ordinary range of passenger car weight.

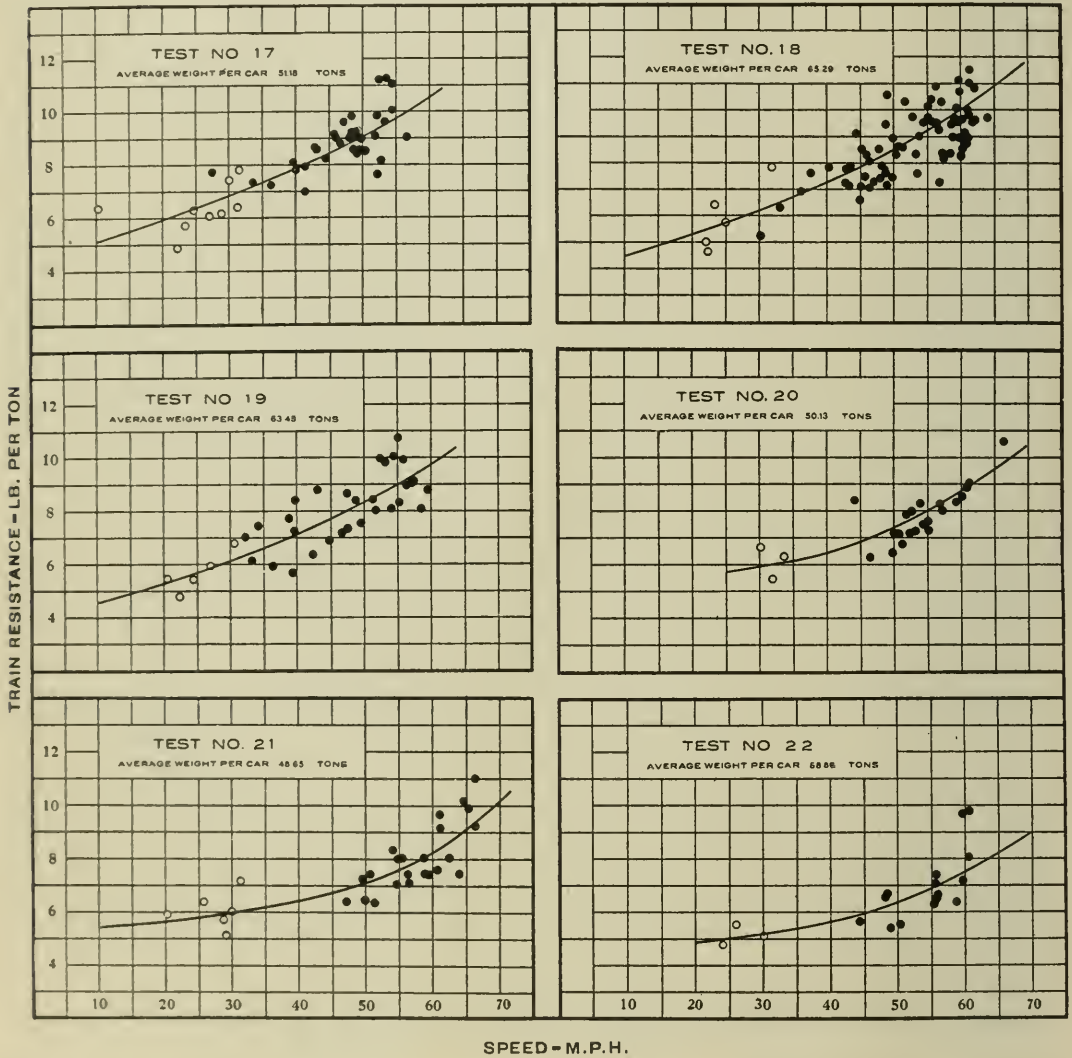


FIG. 4. DIAGRAMS SHOWING THE RELATION BETWEEN TRAIN RESISTANCE AND SPEED FOR SIX TRAINS

If in Fig. 6 at the point corresponding to 20 miles per hour a perpendicular is erected, it will cut the curves in twenty-eight points, each of which pertains to a particular train and defines for that train the average value of resistance at a speed of 20 miles per hour. If each of these resistance values is plotted with respect to the average

car weight of the train to which it pertains, the diagram for 20 miles per hour shown in Fig. 7 is obtained. Since in this diagram the values of resistance all relate to a common speed, the influence of speed on resistance is eliminated, and the figure shows the relation which at 20 miles per hour exists between resistance and average car weight. Fig.

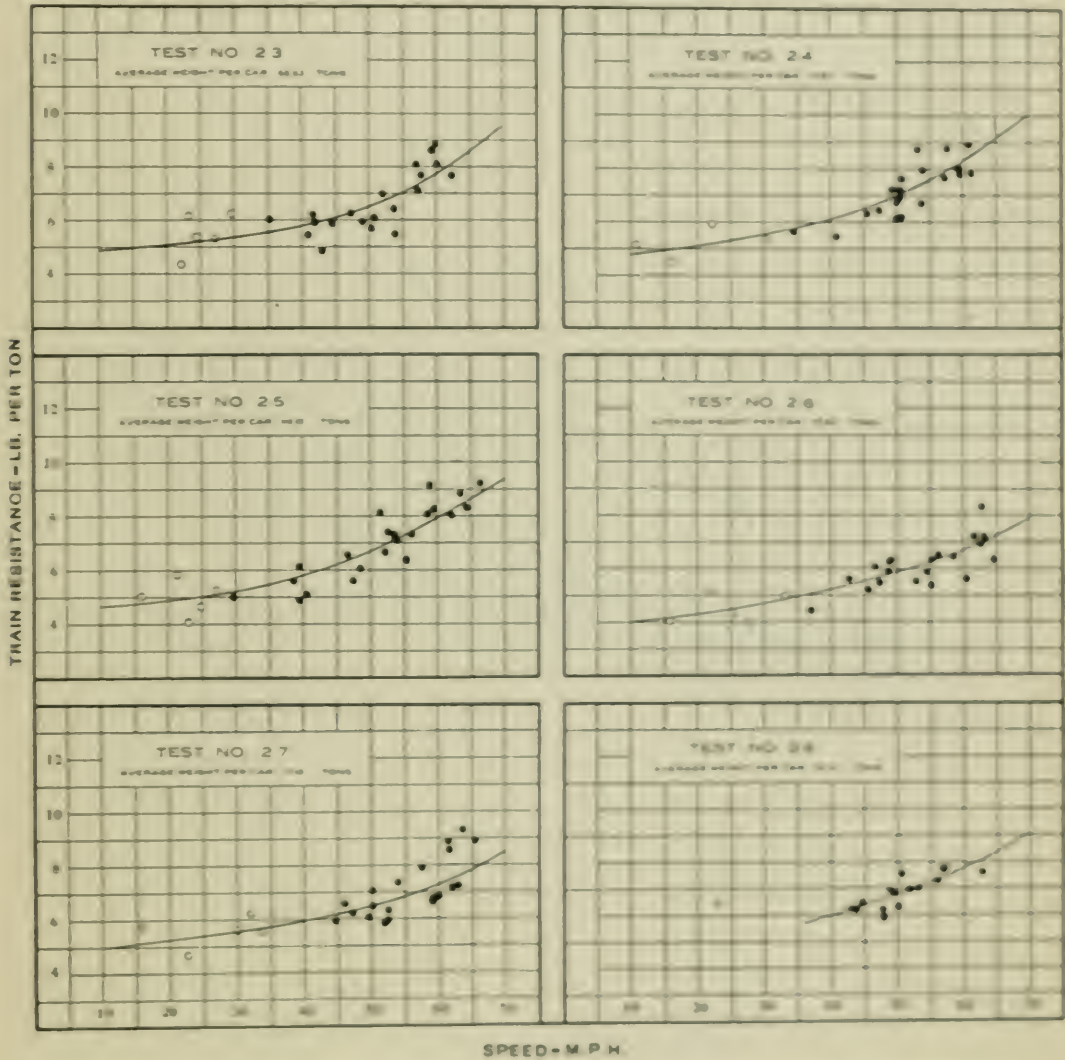


FIG. 5. DIAGRAMS SHOWING THE RELATION BETWEEN TRAIN RESISTANCE AND SPEED FOR SIX TRAINS

7 includes seven diagrams obtained by this process for speeds of 10, 20, 30, 40, 50, 60 and 70 miles per hour. The coördinates of the points plotted in Fig. 7 are given in Table 2. In it the tests are arranged in the order of the average car weights.

Both Fig. 7 and Table 2 show that the points on the individual plots fall into three groups. The group having the lowest average car weight was obtained from tests made on local train number 24 which has been described on page 12 and in Table 1. It has been noted that the number of cars in these trains having four-wheel trucks varied from 50 per cent to 80 per cent. The intermediate group of points was derived from tests with train number 5 in which the num-

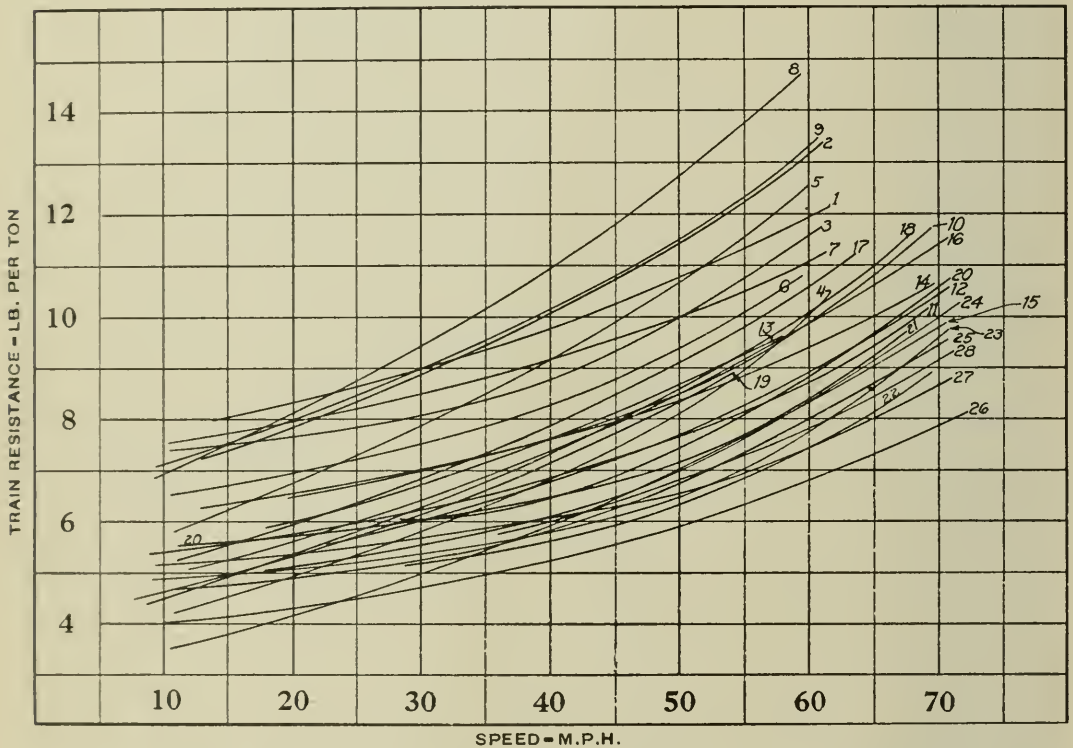


FIG. 6. CURVES SHOWING THE RELATION BETWEEN TRAIN RESISTANCE AND SPEED FOR THE TWENTY-EIGHT TRAINS TESTED

ber of cars having four-wheel trucks varied from 18 per cent to 27 per cent. The group of points pertaining to trains having a high average car weight applies to tests made on train number 2 in which the number of cars having four-wheel trucks varied from 0 per cent to 20 per cent.

In considering the variation among the points on the individual plots of Fig. 7 it should be borne in mind that each point represents the average resistance of a particular train at a given speed. With the exception of test number 4 which results in abnormally low resistance values the average variation of all the points from the mean

curves is 8 per cent. There are three trains which at some speed gave resistances greater than 20 per cent above or below the mean curve. The majority, however, vary from the curve by 10 per cent or less.

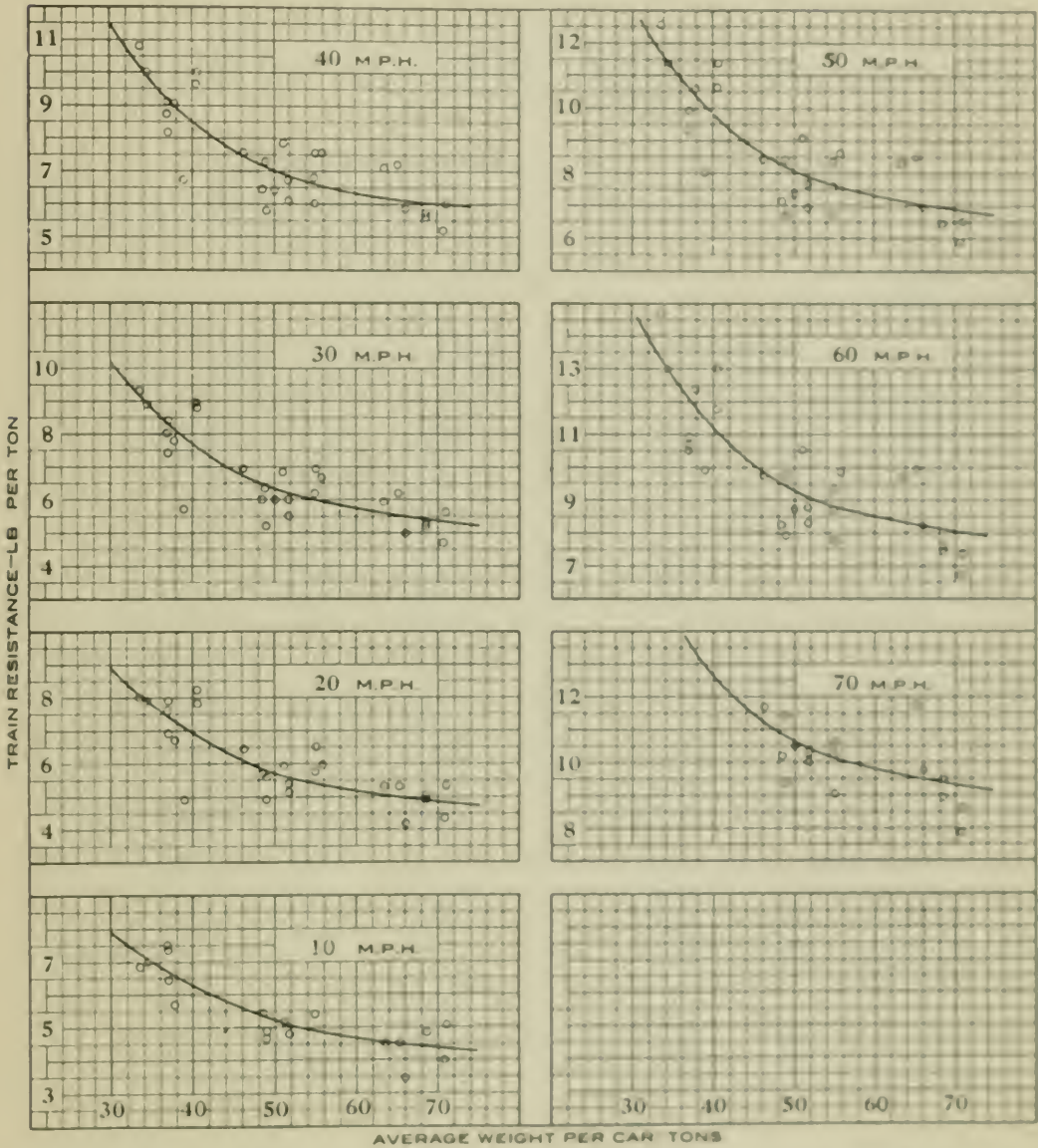


FIG. 7. DIAGRAMS SHOWING THE RELATION BETWEEN TRAIN RESISTANCE AND AVERAGE CAR WEIGHT FOR SEVEN DIFFERENT SPEEDS

In the following table are given the average deviations above and below the mean for all the points in Fig. 7.

No satisfactory explanation for these differences is obtainable from the data. They are probably due to varying external conditions

such as wind and temperature as well as to variations in train condition and make up. Variations similar to those recorded in the foregoing table may be expected with any train.

TABLE 2

VALUES OF RESISTANCE AT VARIOUS SPEEDS DERIVED FROM THE CURVES FOR THE INDIVIDUAL TESTS

This table provides the coördinates of the points plotted in Fig. 7

Test No.	Average Weight per Car Tons	Train Resistance—Pounds per Ton						
		10 M. P. H.	20 M. P. H.	30 M. P. H.	40 ^{mi} M. P. H.	50 M. P. H.	60 ^{mi} M. P. H.	70 M. P. H.
8	33.60	6.83	8.00	9.30	10.80	12.56	14.72	
9	34.50	7.01	7.90	8.90	10.00	11.39	13.00	
3	37.02	7.35	7.61	8.03	8.73	9.91	10.50	
6	37.10	6.45	6.90	7.43	8.18	9.30	10.67	
7	37.12	7.47	7.90	8.42	9.10	9.91	10.92	
5	37.90	5.70	6.70	7.80	9.05	10.58	12.38	
4	39.03		4.90	5.72	6.73	8.00	9.90	
1	40.58		8.23	8.89	9.65	10.60	11.78	
2	40.70		7.81	8.80	10.00	11.35	13.00	
10	46.32		6.42	6.93	7.57	8.42	9.76	11.68
21	48.65	5.44	5.70	6.00	6.45	7.14	8.23	10.20
16	49.04	4.90	5.60	6.35	7.30	8.40	9.73	11.40
25	49.10	4.67	4.89	5.21	5.80	6.70	7.91	9.38
20	50.13			6.00	6.40	7.35	8.70	10.50
17	51.18	5.18	5.92	6.82	7.85	9.08	10.52	
12	51.80	4.85	5.38	6.00	6.72	7.65	8.78	10.30
24	51.83	4.80	5.12	5.51	6.10	6.93	8.30	10.00
11	54.96	5.40	5.75	6.20	6.81	7.60	8.70	10.25
28	55.10				6.00	6.76	7.76	9.05
14	55.13		6.50	6.95	7.55	8.33	9.31	10.60
13	55.90		5.96	6.68	7.56	8.60	9.85	
19	63.49	4.52	5.30	5.94	7.10	8.30	9.70	
1	65.29	4.51	5.30	6.20	7.22	8.48	9.94	11.75
15	66.10	3.46	4.17	4.98	5.87	6.94	8.22	9.77
23	68.63	4.84	5.02	5.30	5.74	6.45	7.62	9.50
22	68.66		4.90	5.20	5.64	6.38	7.45	8.94
26	70.83	4.00	4.32	4.70	5.20	5.88	6.71	7.87
27	71.10	5.08	5.32	5.62	6.00	6.50	7.32	8.60

Although there is considerable variation among the points, they indicate clearly a decrease in the resistance as the car weight increases. The rate of this decrease is shown by the curves drawn to represent the points on the individual plots of Fig. 7. These curves have been drawn by the same method as the curves of Figs. 1 to 5 inclusive.

The seven curves from the individual plots of Fig. 7 have been brought together in Fig. 8, which shows, therefore, the average rela-

tion between resistance and average car weight for each of seven different speeds. Fig. 8 shows in graphical form the results of the whole research.

AVERAGE DEVIATION OF ALL POINTS IN FIG. 7 FROM THE MEAN AS SHOWN BY THE CURVES THERE DRAWN EXPRESSED AS PERCENTAGES OF THE CURVE ORDINATES

Speed.....	Miles per Hour						
	10	20	30	40	50	60	70
Points above the Curve.....	8.7	6.5	7.1	9.0	9.0	10.2	6.7
Points below the Curve.....	6.6	8.3	7.3	8.5	8.6	7.1	6.6

15. *The Results Expressed as Resistance-Speed Curves.*—Fig. 8, however, presents the relations in unusual form. Ordinarily train resistance is expressed as a curve or equation which defines the relation between resistance and speed instead of the relation between resistance and car weight as in this figure. Obviously a single curve will not suffice to express the results of these experiments in the usual form, since the influence of car weight cannot thereby be made evident. A number of curves will be required for this purpose, each of which will apply only to a particular average car weight. Fig. 9 presents such a group of resistance speed curves which have been derived directly from the lines of Fig. 8, and Fig. 9 shows, therefore, in different form, only such information as is obtainable from Fig. 8.

The relation between the two figures will be made clear by explaining the derivation of the curve in Fig. 9 applying to an average car weight of 40 tons. In Fig. 8 the ordinate corresponding to an average car weight of 40 tons cuts the seven lines there drawn at seven points, at which the mean resistance values are 6.3, 7.0, 7.7, 8.6, 9.8, 11.2, and 12.7 pounds per ton, corresponding to speeds of 10, 20, 30, 40, 50, 60 and 70 miles per hour respectively. These values are the coördinates of seven points on a resistance-speed curve applying to a car weight of 40 tons. These seven points have been plotted in Fig. 9 and define there the curve for 40 tons. The curves for 30, 35, 45, 50, 60, and 75 tons average weight per car have been derived in like manner. The curves of Fig. 9 reproduce quite exactly the facts presented in Fig. 8 and present the final results of the experiments. Three additional curves corresponding to 55, 65, and 70 tons per car which were included in the original diagram have been omitted from Fig. 9 to avoid confusion.

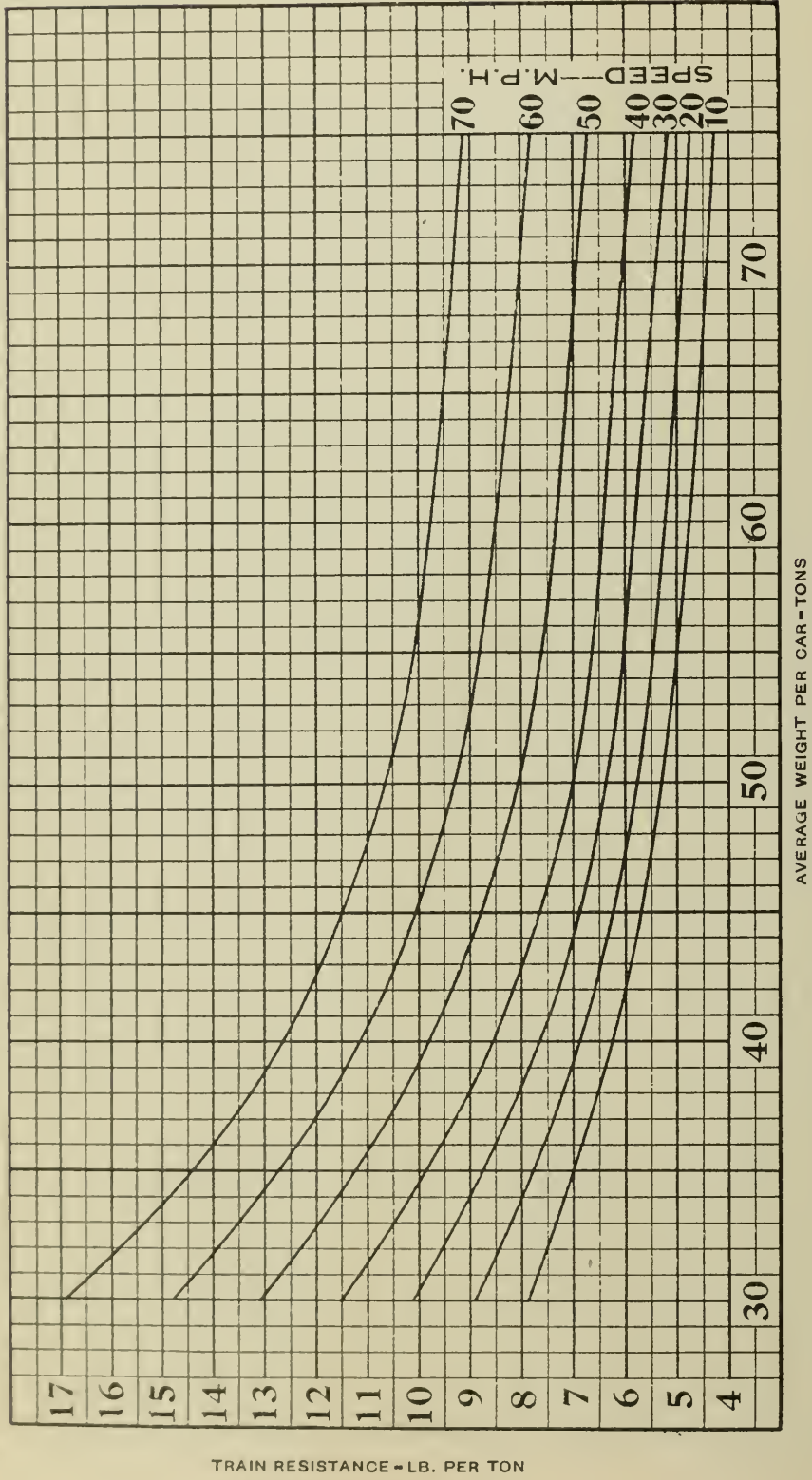


FIG. 8. CURVES SHOWING THE AVERAGE RELATION BETWEEN RESISTANCE AND AVERAGE CAR WEIGHT FOR SEVEN DIFFERENT SPEEDS

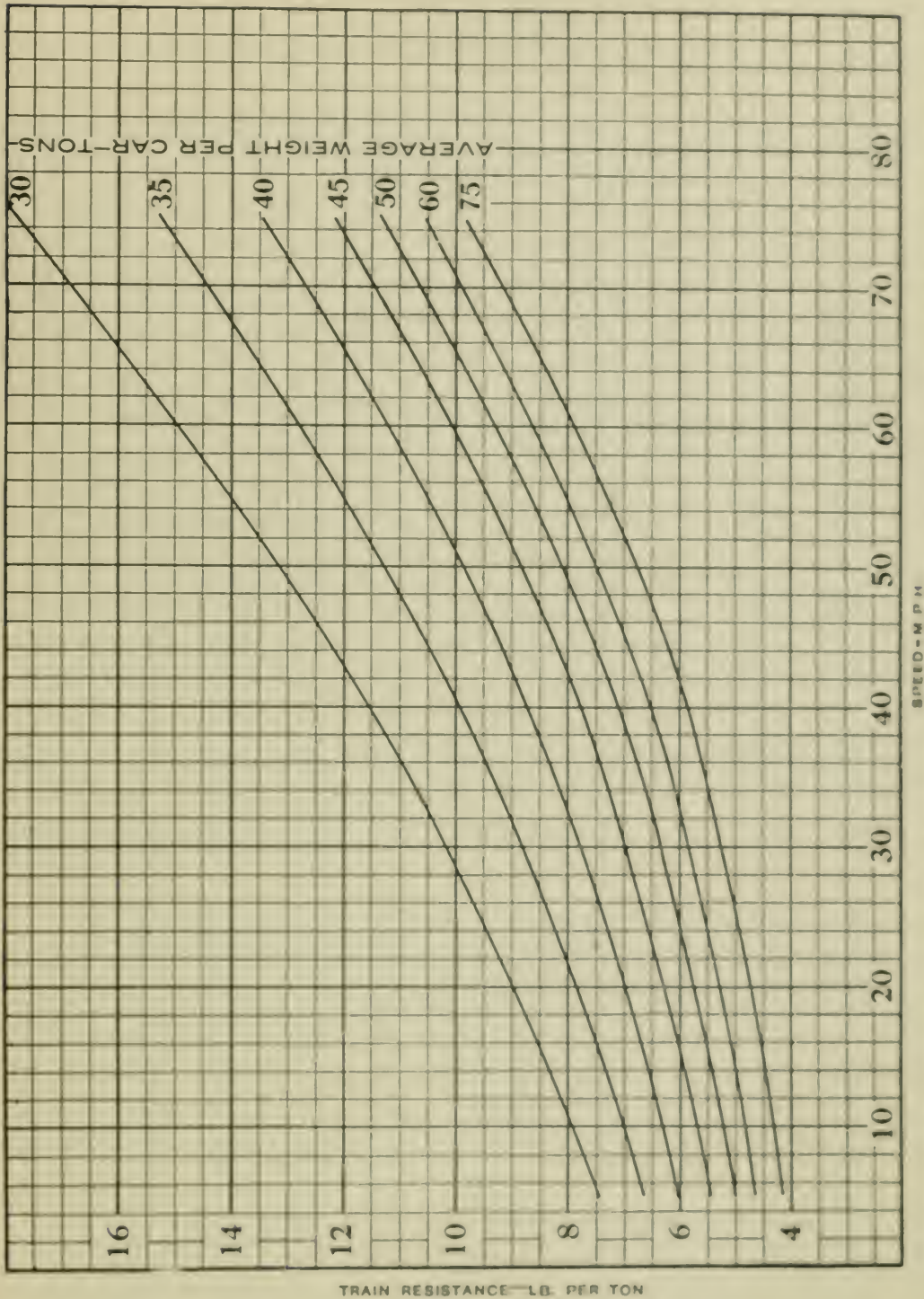


FIG. 9. CURVES SHOWING THE RELATION BETWEEN TRAIN RESISTANCE AND SPEED FOR AVERAGE CAR WEIGHT BETWEEN THIRTY AND SEVENTY FIVE TONS

16. *The Results Expressed in Tabular Form.*—For convenience the values of resistance defined by the curves of Figs. 8 and 9 have been tabulated in Table 3 which gives the resistance values at speeds

TABLE 3

VALUES OF RESISTANCE AT VARIOUS SPEEDS AND FOR VARIOUS AVERAGE
CAR WEIGHTS

The values are derived directly from the curves of Fig. 9 and present the final results of the tests

Speed Miles per Hour	Train Resistance—Pounds per Ton										Speed Miles per Hour
	Column Headings Indicate the Average Weights per Car—Tons										
	30	35	40	45	50	55	60	65	70	75	
5	7.4	6.6	6.0	5.4	5.0	4.8	4.6	4.4	4.3	4.1	5
6	7.5	6.7	6.0	5.5	5.0	4.8	4.6	4.4	4.3	4.2	6
7	7.6	6.8	6.1	5.5	5.1	4.8	4.7	4.5	4.3	4.2	7
8	7.7	6.8	6.2	5.6	5.1	4.9	4.7	4.5	4.4	4.2	8
9	7.8	6.9	6.2	5.6	5.2	4.9	4.8	4.6	4.4	4.3	9
10	7.9	7.0	6.3	5.7	5.2	5.0	4.8	4.6	4.4	4.3	10
11	8.0	7.1	6.4	5.8	5.2	5.0	4.8	4.7	4.5	4.3	11
12	8.1	7.1	6.4	5.8	5.3	5.0	4.9	4.7	4.5	4.4	12
13	8.2	7.2	6.5	5.9	5.3	5.1	4.9	4.7	4.5	4.4	13
14	8.3	7.3	6.5	5.9	5.4	5.1	5.0	4.8	4.6	4.4	14
15	8.4	7.4	6.6	6.0	5.4	5.2	5.0	4.8	4.6	4.5	15
16	8.5	7.5	6.7	6.0	5.5	5.2	5.0	4.9	4.7	4.5	16
17	8.6	7.6	6.7	6.1	5.5	5.3	5.1	4.9	4.7	4.5	17
18	8.7	7.7	6.8	6.1	5.6	5.3	5.1	5.0	4.8	4.6	18
19	8.8	7.7	6.9	6.2	5.7	5.4	5.2	5.0	4.8	4.6	19
20	9.0	7.8	7.0	6.3	5.7	5.4	5.2	5.0	4.9	4.7	20
21	9.1	7.9	7.0	6.3	5.8	5.5	5.3	5.1	4.9	4.7	21
22	9.2	8.0	7.1	6.4	5.8	5.5	5.3	5.1	5.0	4.8	22
23	9.3	8.1	7.2	6.4	5.9	5.6	5.4	5.2	5.0	4.8	23
24	9.4	8.2	7.2	6.5	5.9	5.6	5.4	5.2	5.0	4.9	24
25	9.5	8.3	7.3	6.6	6.0	5.7	5.5	5.3	5.1	4.9	25
26	9.6	8.4	7.4	6.6	6.1	5.7	5.6	5.4	5.1	5.0	26
27	9.8	8.5	7.5	6.7	6.1	5.8	5.6	5.4	5.2	5.0	27
28	9.9	8.6	7.5	6.8	6.2	5.9	5.7	5.5	5.2	5.1	28
29	10.0	8.7	7.6	6.8	6.2	5.9	5.7	5.5	5.3	5.1	29
30	10.1	8.8	7.7	6.9	6.3	6.0	5.8	5.6	5.4	5.2	30
31	10.3	8.9	7.8	7.0	6.4	6.0	5.8	5.6	5.4	5.2	31
32	10.4	9.0	7.9	7.1	6.4	6.1	5.9	5.7	5.5	5.3	32
33	10.5	9.1	8.0	7.1	6.5	6.2	6.0	5.8	5.5	5.4	33
34	10.7	9.2	8.0	7.2	6.6	6.2	6.0	5.8	5.6	5.4	34
35	10.8	9.3	8.1	7.3	6.7	6.3	6.1	5.9	5.7	5.5	35
36	10.9	9.4	8.2	7.4	6.7	6.4	6.2	6.0	5.7	5.5	36
37	11.1	9.5	8.3	7.4	6.8	6.5	6.2	6.0	5.8	5.6	37
38	11.2	9.6	8.4	7.5	6.9	6.5	6.3	6.1	5.9	5.7	38
39	11.4	9.8	8.5	7.6	7.0	6.6	6.4	6.2	5.9	5.7	39
40	11.5	9.9	8.6	7.7	7.0	6.7	6.4	6.2	6.0	5.8	40
41	11.7	10.0	8.7	7.8	7.1	6.8	6.5	6.3	6.1	5.9	41
42	11.8	10.1	8.8	7.9	7.2	6.9	6.6	6.4	6.2	6.0	42
43	12.0	10.3	9.0	8.0	7.3	6.9	6.7	6.5	6.3	6.0	43
44	12.2	10.4	9.1	8.1	7.4	7.0	6.8	6.6	6.4	6.1	44
45	12.3	10.5	9.2	8.2	7.5	7.1	6.9	6.7	6.4	6.2	45
46	12.5	10.7	9.3	8.3	7.6	7.2	7.0	6.8	6.5	6.3	46
47	12.6	10.8	9.4	8.4	7.7	7.3	7.1	6.9	6.6	6.4	47
48	12.8	11.0	9.6	8.5	7.8	7.4	7.2	7.0	6.7	6.5	48
49	13.0	11.1	9.7	8.6	7.9	7.5	7.3	7.1	6.8	6.6	49
50	13.1	11.2	9.8	8.8	8.0	7.6	7.4	7.2	7.0	6.7	50
51	13.3	11.4	9.9	8.9	8.1	7.7	7.5	7.3	7.1	6.8	51
52	13.5	11.5	10.1	9.0	8.2	7.8	7.6	7.4	7.2	6.9	52
53	13.7	11.7	10.2	9.1	8.4	7.9	7.7	7.5	7.3	7.0	53
54	13.8	11.8	10.3	9.2	8.5	8.1	7.8	7.6	7.4	7.1	54
55	14.0	12.0	10.5	9.4	8.6	8.2	8.0	7.7	7.5	7.2	55

TABLE 3—(CONTINUED)

Speed Miles per Hour	Train Resistance—Pounds per Ton										Speed Miles per Hour
	Column Headings Indicate the Average Weights per Car—Tons										
	30	35	40	45	50	55	60	65	70	75	
56	14.2	12.1	10.6	9.5	8.7	8.3	8.1	7.8	7.6	7.3	56
57	14.4	12.3	10.8	9.6	8.9	8.4	8.2	8.0	7.7	7.5	57
58	14.6	12.5	10.9	9.8	9.0	8.6	8.3	8.1	7.8	7.6	58
59	14.8	12.6	11.1	9.9	9.1	8.7	8.4	8.2	8.0	7.7	59
60	15.0	12.8	11.2	10.0	9.2	8.8	8.6	8.3	8.1	7.8	60
61	15.1	12.9	11.3	10.2	9.4	8.9	8.7	8.4	8.2	7.9	61
62	15.3	13.1	11.5	10.3	9.5	9.1	8.8	8.6	8.3	8.1	62
63	15.5	13.2	11.7	10.5	9.7	9.2	9.0	8.7	8.5	8.2	63
64	15.7	13.4	11.8	10.6	9.8	9.3	9.1	8.8	8.6	8.3	64
65	15.9	13.6	11.9	10.7	9.9	9.5	9.2	8.9	8.7	8.4	65
66	16.1	13.8	12.1	10.9	10.0	9.6	9.3	9.1	8.8	8.6	66
67	16.3	13.9	12.2	11.0	10.2	9.7	9.4	9.2	9.0	8.7	67
68	16.5	14.1	12.4	11.1	10.3	9.9	9.6	9.3	9.1	8.8	68
69	16.7	14.3	12.5	11.3	10.5	10.0	9.7	9.5	9.2	9.0	69
70	16.9	14.4	12.7	11.4	10.6	10.1	9.9	9.6	9.4	9.1	70
71	17.1	14.6	12.8	11.6	10.7	10.3	10.0	9.7	9.5	9.2	71
72	17.3	14.8	13.0	11.7	10.9	10.4	10.1	9.9	9.6	9.4	72
73	17.5	15.0	13.1	11.9	11.0	10.6	10.3	10.0	9.8	9.5	73
74	17.7	15.1	13.3	12.0	11.2	10.7	10.4	10.1	9.9	9.7	74
75	17.9	15.3	13.5	12.1	11.3	10.8	10.5	10.3	10.0	9.8	75

between 5 and 75 miles per hour and for car weights ranging from 30 to 75 tons. Table 3 also includes the coördinates of the resistance curves corresponding to 55, 65 and 70 tons per car and omitted from Fig. 10.

17. *Final Results.*—The final results of the research are presented in the two forms just stated, namely, Fig. 9 and Table 3, from which the resistance of ordinary passenger trains may be predicted with reasonable accuracy. It should be borne in mind, however, that these results are applicable to trains running at uniform speed, on level tangent track of good construction, during weather when the temperature is not lower than 40 degrees F. and when the wind velocity does not exceed 25 m. p. h.*

It is also significant that a considerable number of the tests developed resistances averaging about 8 per cent in excess of the mean resistance which would be predicted by the use of Figs. 7, 8 or 9. It should be understood that this allowance is intended to cover probable variations in the resistance of different trains under normal op-

*Twenty-five of the tests here reported were made when the wind velocity did not exceed 20 m. p. h. During two tests it did not exceed 25 m. p. h. and during only one test did it exceed 30 m. p. h.

erating conditions. This allowance in no way takes the place of that additional reserve which must be allowed to cover unusual variations in resistance due to low temperatures, or high winds or of that reserve in tractive effort of the locomotive necessitated by operating conditions which reduce the efficiency of the locomotive itself.

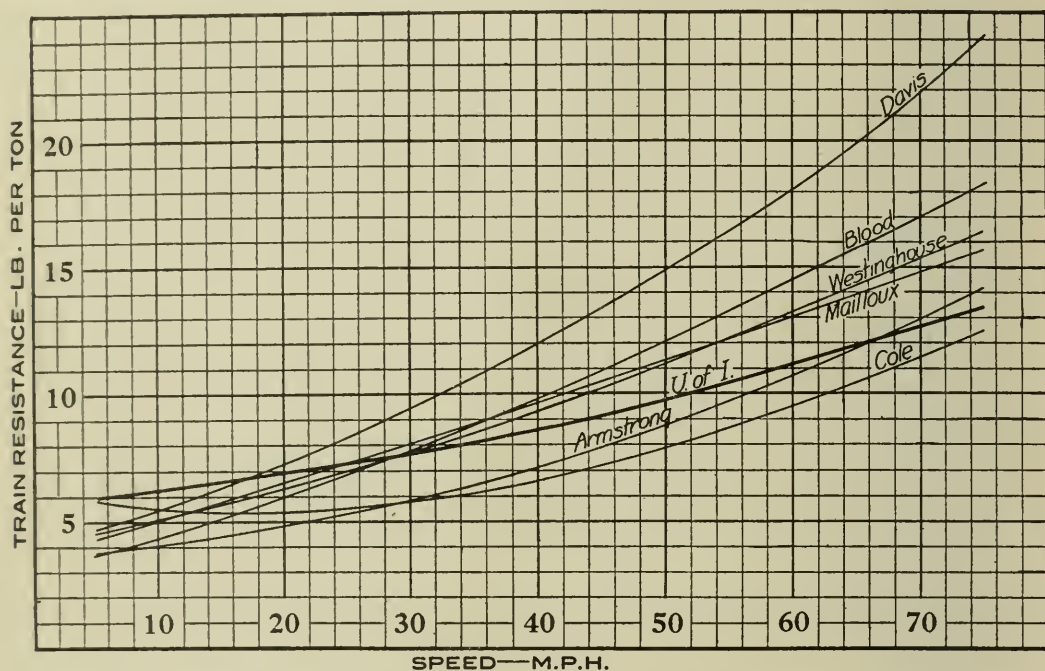


FIG. 10. COMPARATIVE CURVES SHOWING THE RELATION BETWEEN TRAIN RESISTANCE AND SPEED FOR TRAIN COMPOSED OF EIGHT 40-TON CARS

18. *Comparison with Other Experiments.*—For purposes of comparison with the results of other investigations the following formulas are given, together with Figs. 10 and 11. The curves in Fig. 10 have been calculated for a train composed of eight 40-ton cars while those of Fig. 11 have been calculated for a train of ten 60-ton cars.

$$\text{Armstrong*} \quad R = \frac{50}{\sqrt{W}} + 0.035S + \frac{0.002 a S^2}{W} \left[1 + \frac{n-1}{10} \right]$$

$$\text{Blood†} \quad R = 3 + 0.12S + \left[0.0014 + \frac{0.40}{W} \right] S^{1.8}$$

*"Standard Handbook for Electrical Engineers," Section 13, p. 947, 1910.

†Proc. Amer. Soc. Mech. Eng., Vol. 24, p. 945, 1903.

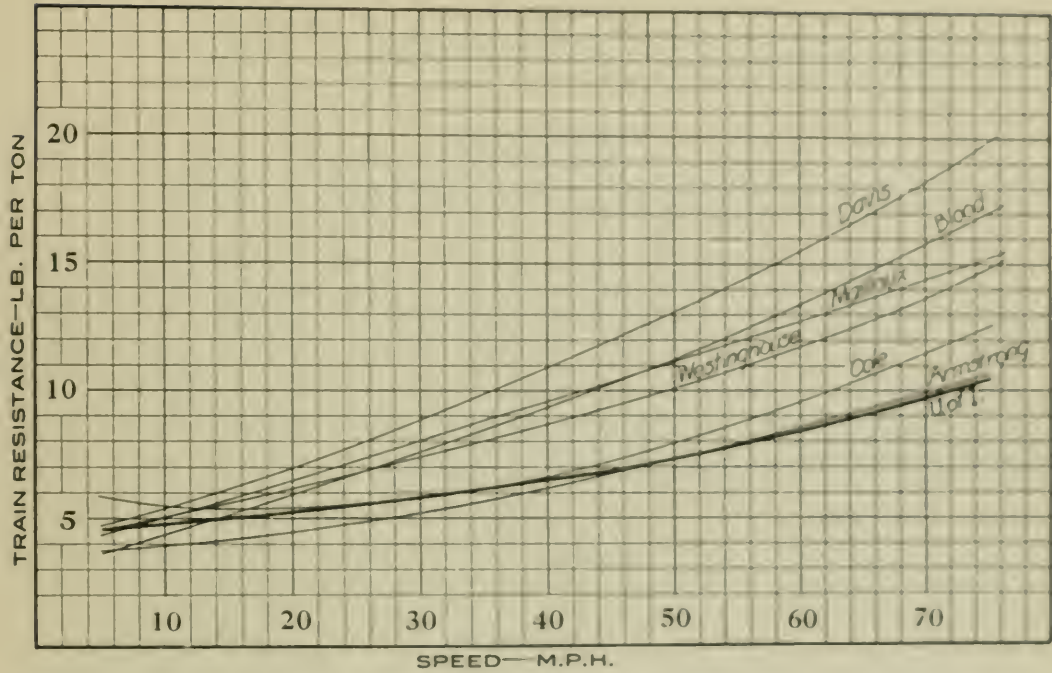


FIG. 11. COMPARATIVE CURVES SHOWING THE RELATION BETWEEN TRAIN RESISTANCE AND SPEED FOR TRAIN COMPOSED OF TEN 60-TON CARS

Cole*
$$R = 5.4 + 0.002 [S - 15]^2 + \frac{100}{[S + 2]^3}$$

Davis†
$$R = 4 + 0.13 S + \frac{0.33 S^2}{W} \left[1 + \frac{n - 1}{10} \right]$$

Mailloux‡
$$R = 3.5 + 0.15 S + \left[\frac{(0.02 N + 0.25)}{N W} \right] S^2$$

Westinghouse¶
$$R = 4 + \frac{S}{10} + \frac{10 S^2}{36 W}$$

Notation: R = train resistance, pounds per ton.
 S = speed, miles per hour.
 W = weight of the train, tons.
 N = number of cars in the train.
 A = cross sectional area.

*Railway Age Gazette, Vol. 47 n. s., pp. 361, 411, 451, 503, 546, 583, 1903.

†Street Railway Journal, Vol. 24, p. 1003, 1904.

‡Proc. Amer. Inst. Elec. Eng., Vol. 23, p. 781, 1904.

¶Westinghouse Company.

APPENDIX

TYPICAL TABLES OF RESULTS OF THE INDIVIDUAL TESTS

In this appendix three tables are given, each of which shows the principal results of the calculations for one test. The tests have been chosen so that each one is fairly representative of all the tests made with a particular class of train. Table 4, for example, is typical of the results obtained with train No. 24, Table 5 representative of the tests with train No. 5, and Table 6 representative of the tests with train No. 2. The final values of net resistance on tangent, level track, at uniform speed are given in column 12, and the corresponding values of speed in column 11. These two columns give the coördinates of the points plotted on the individual diagrams of Figs. 1 to 5, inclusive.

It has been customary in bulletins of the Engineering Experiment Station of this nature to present complete tables of data for all the tests. This practice has been maintained largely for the purpose of showing the range of conditions such as wind velocity and direction, acceleration, grade and length of sections. It has also served to show the division of the gross train resistance into such items as acceleration, grade, and net resistance. In view of the fact that these data have been given so fully in previous publications and since the relation between the various items is not materially different in this case, it is thought that the presentation of typical tables of data is sufficient.

TABLE 4
THE RESULTS OF TEST NO. 7

Method of Calculation	Item No.	Length of Section Feet	Total Drawbar Pull Pounds	Acceleration Miles per Hour per Second	Speeds		Grade + Up — Down Feet per Mile	Wind		Speed m. p. h.	Net Train Resistance Pounds per Ton
					At Entrance to Section m. p. h.	At Exit from Section m. p. h.		Approximate Direction	Approximate Velocity m. p. h.		
1	2	3	4	5	6	7	8	9	10	11	12
Section	1	1380	7060		00.00	24.05	— 1.15	+60° R	16	15.7	9.13
"	2	2700	4650		24.05	35.45	— 4.30	+60° R	16	30.7	9.10
"	3	3564	3510		35.45	43.90	— 9.90	+60° R	16	40.6	9.52
"	4	4112	2975		43.90	48.00	— 2.80	+60° R	16	46.7	10.68
"	5	4744	2675		48.00	51.55	— 3.90	+60° R	16	49.8	10.66
"	6	1008	6520		18.50	25.50	+11.00	+50° R	12	22.9	9.56
"	7	2800	4925		25.50	35.90	+ 4.30	+50° R	12	31.8	8.92
"	8	3532	3925		35.90	42.90	+ 0.70	+50° R	12	40.2	9.93
"	9	4160	2900		42.90	48.32	— 4.70	+50° R	12	47.3	9.07
"	10	4260	2275		48.32	45.00	+10.00	+50° R	12	48.4	8.97
"	11	2144	4650		17.60	29.25	— 3.40	+60° R	12	24.4	8.53
"	12	3024	3580		29.25	37.70	— 7.70	+60° R	12	34.4	9.12
"	13	3628	2825		37.70	43.25	— 8.30	+60° R	12	41.2	9.68
"	14	2036	4790		15.75	28.05	— 2.30	+35° R	10	23.2	8.12
"	15	2840	3540		28.05	34.45	+ 3.00	+35° R	10	32.3	8.07
"	16	3204	2860		34.45	37.40	+ 5.90	+35° R	10	36.4	8.53
"	17	2624	4960		21.80	33.25	+ 1.30	+40° R	10	28.7	8.78
"	18	3348	3850		33.25	40.92	— 0.60	+40° R	10	38.3	9.17
"	19	2540	5475		17.65	30.25	+ 6.60	+25° R	9	25.5	8.15
"	20	3652	4165		30.25	37.20	+ 7.60	+27° R	9	34.7	8.71
"	21	1948	4830		14.40	27.25	— 0.80	+60° R	10	22.1	7.07
"	22	2760	3540		27.25	33.60	+ 1.90	+60° R	10	31.4	8.56
"	23	3196	3050		33.60	37.40	+ 7.60	+60° R	10	35.3	7.63
"	24	2448	2710		37.40	40.10	+ 4.00	+60° R	10	39.2	8.83
"	25	3796	2325		41.95	43.25	+ 5.30	+60° R	10	43.1	8.47
"	26	1984	2920		15.85	26.55	+ 9.00	+45° R	8	22.6	7.04
"	27	2970	3925		26.55	32.95	+ 9.50	+45° R	8	30.4	7.55
"	28	3820	2425		40.95	44.40	— 2.50	+50° R	10	43.4	8.61
"	29	3004	2250		44.40	46.25	— 0.90	+50° R	10	45.5	9.25
"	30	4292	1950		46.25	49.00	— 8.00	+50° R	10	48.9	9.89
"	31	2964	4830		15.85	28.40	— 0.00	+25° R	7	23.7	7.17

TABLE 5
THE RESULTS OF TEST No. 24

Method of Calculation	Item No.	Length of Section Feet	Total Drawbar Pull Pounds	Acceleration Miles per Hour per Second	Speeds		Grade + Up — Down Feet per Mile	Wind		Speed m. p. h.	Net Train Resistance Pounds per Ton
					At Entrance to Section m. p. h.	At Exit from Section m. p. h.		Approximate Direction	Approximate Velocity m. p. h.		
1	2	3	4	5	6	7	8	9	10	11	12
Point	3		10630	+1528			— 1.32	+13° L	3	16.20	4.50
"	4		18030	+2745			+ 0.79	+40° R	6	11.00	5.15
"	5		13750	+2185			+ 7.04	+40° R	6	22.30	5.91
"	6		8600	+1330			— 7.74	+40° R	6	30.40	5.36
Section	1	4484	5400		50.45	51.90	+ 2.51	+ 0°	8	50.90	6.20
"	2	5200	6150		49.00	53.40	— 6.10	+10° R	18	50.60	6.96
"	3	2968	4840		57.50	58.65	— 9.18	+10° R	24	57.80	8.79
"	4	4812	4000		60.20	60.80	— 5.01	+ 5° R	24	59.60	7.86
"	5	1788	500		61.70	61.70	—14.60	+ 5° R	23	61.30	6.42
"	6	3676	5700		49.90	51.30	+ 1.48	+10° R	22	50.10	6.74
"	7	3676	5600		51.15	51.00	+10.46	+10° R	22	50.10	6.15
"	8	3964	6480		54.75	54.75	+ 7.37	+10° R	20	54.00	8.56
"	9	4016	7300		45.35	47.10	+ 9.67	+20° R	18	45.60	6.33
"	10	4120	6890		50.80	52.80	+ 2.27	+20° R	20	51.00	7.67
"	38	1560	5000		52.40	52.80	— 0.74	+ 0°	8	53.20	9.68
"	39	2084	7150		46.40	48.85	— 4.33	+10° R	18	47.40	6.47
"	40	1976	5250		55.10	55.30	— 0.83	+15° R	22	53.90	6.73
"	41	2180	2400		60.50	60.50	—10.12	+ 5° R	24	59.40	8.04
"	42	1788	2100		60.80	61.40	—21.84	+ 5° R	23	61.00	8.94
"	43	2412	600		40.45	42.40	+ 0.57	+10° R	17	41.10	5.48
"	44	2028	8400		38.65	41.10	+ 6.04	+15° R	31	34.50	5.67
"	45	2972	6740		51.30	50.80	+15.47	+20° R	20	50.70	7.23
"	46	1816	7040		50.80	50.60	+14.85	+20° R	20	49.50	7.27
"	47	2944	4970		57.70	58.15	+ 2.74	+20° R	20	57.40	7.68

TABLE 6
THE RESULTS OF TEST NO. 19

Method of Calculation	Item No.	Length of Section Feet	Total Drawbar Pull Pounds	Acceleration Miles per Hour per Second	Speeds		Grade + Up - Down Feet per Mile	Wind		Speed m. p. h.	Net Train Resistance Pounds per Ton
					At Entrance to Section m. p. h.	At Exit from Section m. p. h.		Approximate Direction	Approximate Velocity m. p. h.		
1	2	3	4	5	6	7	8	9	10	11	12
Point	1	15800	15800	+ 1610			+ 12.66	+ 15° R	13	22.40	4.78
	2	15180	15180	+ 1920			0.00	+ 65° R	12	24.40	5.40
	3	12150	12150	+ 1626			- 7.92	+ 45° R	5	30.60	6.78
	4	12850	12850	+ 1842			- 7.92	+ 45° R	5	27.00	5.94
Z-Drop	5	16330	16330	+ 2258			- 2.64	+ 85° R	29	20.50	5.16
	6	6310	6310	0.00	55.60	55.60	0.00	+ 25° R	24	55.90	9.92
	7	10510	10510	+ 13.08	37.85	37.85	+ 13.08	+ 25° R	19	39.40	5.68
	8	13250	13250	+ 12.05	32.30	32.30	+ 12.05	+ 20° R	14	33.20	6.12
Z-Drop	9	8600	8600	- 5.28	49.00	49.00	- 5.28	+ 25° R	24	49.65	7.53
	10	9850	9850	+ 5.36	40.00	41.95	+ 5.36	+ 30° R	19	42.49	6.35
	11	9900	9900	0.00	35.40	40.00	+ 5.18	+ 30° R	19	39.60	7.25
	12	10380	10380	0.00	33.00	35.40	- 5.38	+ 30° R	19	36.40	5.90
Z-Drop	13	7700	7700	+ 9.90	49.80	49.80	+ 9.90	+ 30° R	24	51.90	8.01
	14	7880	7880	+ 5.51	49.00	49.00	+ 5.51	+ 25° R	20	51.45	8.43
	15	8180	8180	+ 8.54	45.00	46.60	+ 8.54	+ 30° R	22	47.72	7.35
	16	8880	8880	+ 19.80	45.00	44.40	+ 19.80	+ 20° R	21	46.85	7.18
Z-Drop	17	9000	9000	0.00	40.40	42.00	0.00	+ 25° R	21	42.93	7.78
	18	12540	12540	0.00	30.10	30.60	0.00	+ 15° R	18	34.20	7.43
	19	4250	4250	0.00	57.70	58.40	0.00	+ 30° R	28	59.60	8.78
	20	6020	6020	0.00	56.90	57.70	0.00	+ 30° R	25	58.60	8.07
Z-Drop	21	6040	6040	0.00	55.30	55.70	0.00	+ 40° R	20	57.30	9.06
	22	7100	7100	0.00	53.80	55.00	0.00	+ 40° R	20	56.40	8.94
	23	7800	7800	- 10.00	49.70	50.40	- 10.00	+ 70° R	19	53.40	9.87
	24	8720	8720	- 10.01	44.90	45.40	- 10.01	+ 70° R	19	48.00	8.42
Z-Drop	25	9000	9000	- 19.92	40.60	41.60	- 19.92	+ 65° R	12	44.90	6.50
	26	10020	10020	0.00	34.80	38.30	0.00	+ 65° R	12	38.75	7.74
	27	6000	6000	- 7.92	54.10	54.10	- 7.92	+ 65° R	17	55.30	8.30
	28	6040	6040	- 7.92	51.70	53.00	- 7.92	+ 65° R	17	54.50	10.02
Z-Drop	29	6860	6860	+ 16.00	51.00	51.00	+ 16.00	+ 40° R	21	54.15	8.00
	30	6900	6900	0.00	50.20	53.20	0.00		21	55.10	10.75
	31	7180	7180	0.00	50.00	50.20	0.00	+ 60° R	21	52.10	9.09
	32	8150	8150	0.00	45.00	46.00	0.00	+ 60° R	20	47.50	8.68
Z-Drop	33	8000	8000	- 4.93	41.40	44.40	- 4.93	+ 70° R	15	44.40	9.02
	34	1748	1748	- 10.02	36.40	36.80	- 10.02	+ 70° R	16	39.76	8.88
	35	1900	1900	- 8.03	37.10	38.90	- 8.03	+ 65° R	12	32.30	7.01
	36	2162	2162								

LIST OF PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION

- Bulletin No. 1.* Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*
- Circular No. 1.* High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None available.*
- Bulletin No. 2.* Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*
- Circular No. 2.* Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*
- Circular No. 3.* Fuel Tests with Illinois Coal (Compiled from tests made by the Technological Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904-1907), by L. P. Breckenridge and Paul Diserens. 1908. *Thirty cents.*
- Bulletin No. 3.* The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*
- Bulletin No. 4.* Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906. *Forty-five cents.*
- Bulletin No. 5.* Resistance of Tubes to Collapse, by Albert P. Carman and M. L. Carr. 1906. *None available.*
- Bulletin No. 6.* Holding Power of Railroad Spikes, by Roy I. Webber. 1906. *None available.*
- Bulletin No. 7.* Fuel Tests with Illinois Coals, by L. P. Breckenridge, S. W. Parr, and Henry B. Dirks. 1906. *None available.*
- Bulletin No. 8.* Tests of Concrete: I, Shear; II, Bond, by Arthur N. Talbot. 1906. *None available.*
- Bulletin No. 9.* An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Revised Edition 1912. *Fifty cents.*
- Bulletin No. 10.* Tests of Concrete and Reinforced Concrete Columns, Series of 1906, by Arthur N. Talbot. 1907. *None available.*
- Bulletin No. 11.* The Effect of Scale on the Transmission of Heat through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *None available.*
- Bulletin No. 12.* Tests of Reinforced Concrete T-Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*
- Bulletin No. 13.* An Extension of the Dewey Decimal System of Classification Applied to Architecture and Building, by N. Clifford Ricker. 1907. *None available.*
- Bulletin No. 14.* Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*
- Bulletin No. 15.* How to Burn Illinois Coal without Smoke, by L. P. Breckenridge. 1908. *None available.*
- Bulletin No. 16.* A Study of Roof Trusses, by N. Clifford Ricker. 1908. *None available.*
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